GATE 2026 MA Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :65

General Instructions

Please read the following instructions carefully:

1. This question paper is divided into three sections:
   - **General Aptitude (GA):** 10 questions (5 questions × 1 mark + 5 questions × 2 marks) for a total of 15 marks.
   - **Environmental Science and Engineering + Engineering Mathematics:**
     - **Part A (Mandatory):** 36 questions (1 questions × 1 mark + 19 questions × 2 marks) for a total of 55 marks.
     - **Part B (Section 1):** Candidates can choose either Part B1 (Surveying and Mapping) or Part B2 (Section 2). Each part contains 16 questions (8 questions × 1 mark + 11 questions × 2 marks) for a total of 30 marks.
2. The total number of questions is **65**, carrying a maximum of **100 marks**.
3. The duration of the exam is **3 hours**.
4. Marking scheme:
   - For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.
   - For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.
   - No negative marking for numerical answer type (NAT) questions.
   - No marks will be awarded for unanswered questions.
5. Ensure you attempt questions only from the optional section (Part B1 or Part B2) you have selected.
6. Follow the instructions provided during the exam for submitting your answers.

1. Let A be a square matrix. If $A^2 = A$, then the matrix A is called:

(A) Nilpotent
(B) Idempotent
(C) Involutory
(D) Singular

Correct Answer: (B) Idempotent

Solution:

Step 1: Understanding the given condition.
The condition given in the question is $A^2 = A$. This means that when the matrix A is multiplied by itself, the result is the same matrix A.

Step 2: Definition of an idempotent matrix.
A square matrix A is called an **idempotent matrix** if it satisfies the condition

$$A^2 = A$$

This definition directly matches the given condition in the question.

Step 3: Analysis of the given options.
(A) Nilpotent: A nilpotent matrix satisfies $A^k = 0$ for some positive integer k, which is not given here.
(B) Idempotent: Correct — an idempotent matrix satisfies $A^2 = A$.
(C) Involutory: An involutory matrix satisfies $A^2 = I$, where I is the identity matrix.
(D) Singular: A singular matrix is one whose determinant is zero, which is unrelated to the given condition.

Step 4: Conclusion.
Since the matrix satisfies $A^2 = A$, it is correctly classified as an **idempotent matrix**.

> **Quick Tip**
>
> Remember these key matrix properties: $A^2 = A$ (Idempotent), $A^2 = I$ (Involutory), $A^k = 0$ (Nilpotent).

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2. The limit $\lim_{x \to 0} \frac{\sin x}{x}$ is equal to:

(A) 0
(B) 1
(C) ∞
(D) Does not exist

Correct Answer: (B) 1

Solution:

Step 1: Understanding the meaning of the limit.
The expression

$$\lim_{x \to 0} \frac{\sin x}{x}$$

asks for the value that the ratio $\frac{\sin x}{x}$ approaches as x gets very close to zero from both the positive and negative sides.

Step 2: Behavior of numerator and denominator near zero.
As $x \to 0$,

$$\sin x \to 0 \quad \text{and} \quad x \to 0$$

So the expression is of the indeterminate form $\frac{0}{0}$, which means we must evaluate the limit carefully rather than substituting directly.

Step 3: Using a fundamental trigonometric identity.
One of the most important standard limits in calculus is:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

This result is obtained using geometric arguments involving the unit circle or by comparing areas of sectors and triangles.

Step 4: Conceptual explanation.
For very small values of x (measured in radians), the value of $\sin x$ becomes almost equal to x. Hence, the ratio $\frac{\sin x}{x}$ becomes closer and closer to 1 as x approaches zero.

Step 5: Analysis of the given options.
(A) 0**:** Incorrect, because $\sin x$ decreases at the same rate as x, not faster.
(B) 1**:** Correct — the ratio approaches 1 as $x \to 0$.
(C) ∞**:** Incorrect, the expression remains finite near zero.
(D) Does not exist: Incorrect, since the left-hand and right-hand limits are equal.

Step 6: Final conclusion.
Since the ratio $\frac{\sin x}{x}$ approaches 1 from both sides as $x \to 0$, the value of the limit is **1**.

> Quick Tip
>
> This limit is the foundation of derivatives of trigonometric functions, especially $\frac{d}{dx}(\sin x) = \cos x$.

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3. A continuous function on a closed and bounded interval is always:

(A) Differentiable
(B) Monotonic

(C) Bounded and attains its bounds
(D) Periodic

Correct Answer: (C) Bounded and attains its bounds

Solution:

Step 1: Understanding the given condition.
The function is stated to be **continuous** on a **closed and bounded interval**. Let the interval be $[a, b]$, where a and b are real numbers, $a < b$, and both endpoints are included.

Step 2: Applying the Extreme Value Theorem.
According to the **Extreme Value Theorem**, if a function is continuous on a closed and bounded interval $[a, b]$, then:
- The function is bounded on $[a, b]$, and
- The function attains both its maximum and minimum values at least once in $[a, b]$.

Step 3: Analysis of the given options.
(A) Differentiable: Incorrect — continuity does not guarantee differentiability. A function can be continuous but not differentiable.
(B) Monotonic: Incorrect — a continuous function may increase and decrease within the interval.
(C) Bounded and attains its bounds: Correct — this follows directly from the Extreme Value Theorem.
(D) Periodic: Incorrect — periodicity is unrelated to continuity on a closed interval.

Step 4: Conclusion.
Since every continuous function on a closed and bounded interval is bounded and achieves both its maximum and minimum values, the correct answer is **(C)**.

> Quick Tip
>
> Always associate "continuous + closed and bounded interval" with the Extreme Value Theorem.

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4. The general solution of $\frac{dy}{dx} = y$ is:

(A) $y = x + C$
(B) $y = Ce^x$
(C) $y = Cx$

(D) $y = e^{Cx}$

Correct Answer: (B) $y = Ce^x$

Solution:

Step 1: Identifying the type of differential equation.
The given equation

$$\frac{dy}{dx} = y$$

is a first-order differential equation in which variables can be separated. Hence, it is a **separable differential equation**.

Step 2: Separating the variables.
Rewriting the equation, we get:

$$\frac{1}{y}\,dy = dx$$

This separates all y terms on one side and all x terms on the other side.

Step 3: Integrating both sides.
Integrating both sides,

$$\int \frac{1}{y}\,dy = \int dx$$

which gives

$$\ln|y| = x + C$$

where C is the constant of integration.

Step 4: Removing the logarithm.
Taking exponential on both sides,

$$|y| = e^{x+C}$$

This can be written as

$$y = Ce^x$$

where C is an arbitrary constant (positive or negative).

Step 5: Analysis of the given options.
(A) $y = x + C$**:** Incorrect — this satisfies $\frac{dy}{dx} = 1$, not y.
(B) $y = Ce^x$**:** Correct — differentiating gives $\frac{dy}{dx} = Ce^x = y$.
(C) $y = Cx$**:** Incorrect — derivative is constant C.
(D) $y = e^{Cx}$**:** Incorrect — this does not represent the general solution form.

Step 6: Conclusion.
The general solution of the differential equation $\dfrac{dy}{dx} = y$ is

$$\boxed{y = Ce^x}$$

Quick Tip

For equations of the form $\frac{dy}{dx} = ky$, the general solution is always $y = Ce^{kx}$.

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5. If a function is analytic in a domain, then it is necessarily:

(A) Continuous only
(B) Differentiable only once
(C) Infinitely differentiable
(D) Bounded

Correct Answer: (C) Infinitely differentiable

Solution:

Step 1: Understanding the meaning of an analytic function.
A complex function is said to be **analytic** in a domain if it is complex differentiable at every point of that domain. Complex differentiability is a much stronger condition than real differentiability.

Step 2: Important property of analytic functions.
One of the fundamental results of complex analysis states that if a function is analytic in a domain, then it possesses derivatives of all orders in that domain. That is, the function is infinitely differentiable.

Step 3: Theoretical justification.
Analytic functions satisfy the Cauchy–Riemann equations and can be represented by a power series in a neighborhood of every point in the domain. This power series representation guarantees the existence of derivatives of all orders.

Step 4: Analysis of the given options.
(A) Continuous only: Incorrect — analytic functions are not just continuous, they have much stronger smoothness properties.
(B) Differentiable only once: Incorrect — analyticity implies differentiability of all orders.
(C) Infinitely differentiable: Correct — this is a direct consequence of analyticity.
(D) Bounded: Incorrect — an analytic function need not be bounded in its domain.

Step 5: Conclusion.
Since every analytic function has derivatives of all orders in its domain, it is necessarily **infinitely differentiable**.

> **Quick Tip**
>
> In complex analysis, remember: **Analytic** $\Rightarrow$ **infinitely differentiable** $\Rightarrow$ **power series expansion**.

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6. If two events A and B are independent, then:

(A) $P(A \cap B) = P(A) + P(B)$
(B) $P(A \cap B) = P(A)P(B)$
(C) $P(A|B) = P(A) + P(B)$
(D) $P(A \cup B) = P(A)P(B)$

Correct Answer: (B) $P(A)P(B)$

Solution:

Step 1: Meaning of independent events.
Two events A and B are called independent if the occurrence of one event does not influence the occurrence of the other. In simple terms, knowing whether A has occurred gives no information about B, and vice versa.

Step 2: Conditional probability viewpoint.
By definition, if A and B are independent, then

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

This means the probability of A remains the same even after B has occurred.

Step 3: Deriving the required formula.
We know the general formula of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Since $P(A|B) = P(A)$ for independent events, we get:

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

Multiplying both sides by $P(B)$,

$$P(A \cap B) = P(A)P(B)$$

Step 4: Checking the given options.
(A) Incorrect — addition rule applies to mutually exclusive events, not independent ones.
(B) Correct — this is the defining condition of independent events.
(C) Incorrect — for independent events, $P(A|B) = P(A)$, not a sum.
(D) Incorrect — probability of union follows a different formula.

Step 5: Final conclusion.
If events A and B are independent, then the probability of their intersection is

$$\boxed{P(A \cap B) = P(A)P(B)}$$

Quick Tip

Independence in probability always leads to **multiplication of probabilities**, never addition.

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7. Which method is commonly used to find roots of nonlinear equations?

(A) Euler's method
(B) Runge–Kutta method
(C) Newton–Raphson method
(D) Gauss elimination method

Correct Answer: (C) Newton–Raphson method

Solution:

Step 1: Understanding nonlinear equations.
A nonlinear equation is an equation of the form

$$f(x) = 0$$

where $f(x)$ is a nonlinear function. Such equations usually cannot be solved exactly using algebraic methods.

Step 2: Need for numerical methods.
Since analytical solutions are often not possible, numerical methods are used to approximate the roots of nonlinear equations with increasing accuracy.

Step 3: Basic idea of Newton–Raphson method.
The Newton–Raphson method is based on the idea of approximating a nonlinear function by its tangent line at a chosen point. The point where the tangent cuts the x-axis gives a better approximation of the root.

Step 4: Mathematical formula.
If x_n is the current approximation of the root, the next approximation is given by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

This process is repeated until the successive values converge to the actual root.

Step 5: Why Newton–Raphson is preferred.
The Newton–Raphson method converges very fast when the initial guess is close to the true root. This makes it one of the most efficient and commonly used root-finding techniques.

Step 6: Analysis of options.
(A) Euler's method: Used for solving differential equations.
(B) Runge–Kutta method: Also used for differential equations.
(C) Newton–Raphson method: Correct — used for finding roots of nonlinear equations.
(D) Gauss elimination method: Used for solving systems of linear equations.

Step 7: Final conclusion.
The most commonly used numerical technique for finding roots of nonlinear equations is the

$$\boxed{\text{Newton–Raphson method}}$$

Quick Tip

Newton–Raphson method has **quadratic convergence**, which makes it faster than many other numerical methods.

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8. Let P be a reflection (or projection) from R^3 to R^3 through a two-dimensional subspace. Then find the value of $2\,\mathrm{tr}(P) - 3\det(P)$.

Correct Answer: The value is 4 (for projection) or 5 (for reflection).

Solution:

Step 1: Understanding the Concept:
In linear algebra, the trace (tr) is the sum of eigenvalues and the determinant (det) is the product of eigenvalues. These values are invariant properties of linear transformations based on the subspace they interact with.

Step 2: Key Formula or Approach:
For a linear operator T on R^3 with eigenvalues $\lambda_1, \lambda_2, \lambda_3$:

$$\text{tr}(T) = \lambda_1 + \lambda_2 + \lambda_3$$

$$\det(T) = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$$

Step 3: Detailed Explanation:
Case A: Projection onto a 2D subspace. The eigenvalues are $1, 1, 0$.

$$\text{tr}(P) = 1 + 1 + 0 = 2$$

$$\det(P) = 1 \cdot 1 \cdot 0 = 0$$

$$\text{Value} = 2(2) - 3(0) = 4$$

Case B: Reflection through a 2D subspace. The eigenvalues are $1, 1, -1$.

$$\text{tr}(P) = 1 + 1 - 1 = 1$$

$$\det(P) = 1 \cdot 1 \cdot (-1) = -1$$

$$\text{Value} = 2(1) - 3(-1) = 2 + 3 = 5$$

Step 4: Final Answer:
The value is 4 (for projection) or 5 (for reflection).

Quick Tip

A projection matrix P always satisfies $P^2 = P$, meaning its eigenvalues are only 0 or 1. A reflection matrix R satisfies $R^2 = I$, so its eigenvalues are only 1 or -1.

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9. Let $f : D \to D$ where D is the open unit disc in C, and $f(0) = 0$. Then possible values of $f'(0)$?

(A) $i/10$
(B) $5/(2i)$
(C) $-5/(2i)$
(D) $3/(2i)$

Correct Answer: (A) $i/10$

Solution:

Step 1: Understanding the Concept:
This problem applies the **Schwarz Lemma**, which restricts the growth and derivative of a holomorphic function that maps the unit disc to itself and fixes the origin.

Step 2: Key Formula or Approach:
According to the Schwarz Lemma, if $f : D \to D$ is holomorphic and $f(0) = 0$, then:

$$|f'(0)| \leq 1$$

Step 3: Detailed Explanation:
We must find which option has a modulus less than or equal to 1: (A) $|i/10| = 0.1 \leq 1$ (Valid)
(B) $|5/(2i)| = 2.5 > 1$ (Invalid)
(C) $|-5/(2i)| = 2.5 > 1$ (Invalid)
(D) $|3/(2i)| = 1.5 > 1$ (Invalid)

Step 4: Final Answer:
The possible value is (A) $i/10$.

> **Quick Tip**
>
> The Schwarz Lemma essentially states that such functions are "contractions" or rotations; they cannot "stretch" the area around the origin.

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10. Let $f(z) = |z|^2 - 5\bar{z} + 2$. Then $f(z)$ is differentiable at

(A) $z = 5$
(B) $z = -5i$
(C) $z = 5i$
(D) $z = -5$

Correct Answer: (A) $z = 5$

Solution:

Step 1: Understanding the Concept:
For a function involving $\bar{z}$ to be complex-differentiable at a point, it must satisfy the Cauchy-Riemann equations, which in complex notation requires the Wirtinger derivative with respect to $\bar{z}$ to vanish.

Step 2: Key Formula or Approach:
Use the property $|z|^2 = z\bar{z}$ and solve:

$$\frac{\partial f}{\partial \bar{z}} = 0$$

Step 3: Detailed Explanation:
Express the function as:

$$f(z, \bar{z}) = z\bar{z} - 5\bar{z} + 2$$

Differentiating partially with respect to $\bar{z}$:

$$\frac{\partial f}{\partial \bar{z}} = z - 5$$

Setting this to zero for differentiability:

$$z - 5 = 0 \implies z = 5$$

Step 4: Final Answer:
The function is differentiable at (A) $z = 5$.

> **Quick Tip**
>
> If a function is differentiable only at a point (and not in a neighborhood), it is not "analytic" or "holomorphic" there.

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11. Consider the power series $\sum a_n(z-2)^n$. It converges at $z = 5$ and diverges at $z = -1$. Find the radius of convergence (ROC).

Correct Answer: (ROC) is 3

Solution:

Step 1: Understanding the Concept:
The Radius of Convergence (R) of a power series defines the distance from the center within which the series must converge and beyond which it must diverge.

Step 2: Key Formula or Approach:
For a series centered at c: 1. If it converges at z_1, then $R \geq |z_1 - c|$. 2. If it diverges at z_2, then $R \leq |z_2 - c|$.

Step 3: Detailed Explanation:

The center of the series is $c = 2$. Distance to convergence point $z = 5$: $|5 - 2| = 3$. Thus, $R \geq 3$. Distance to divergence point $z = -1$: $|-1-2| = 3$. Thus, $R \leq 3$. Combining these, we get $3 \leq R \leq 3$.

Step 4: Final Answer:
The radius of convergence (ROC) is 3.

Quick Tip

On the boundary of the circle ($|z - c| = R$), the series might either converge or diverge; the ROC tells us the behavior everywhere else.
