GATE 2024 Instrumentation Engineering Question Paper with Solution

Time Allowed :3 Hour	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Please read the following instructions carefully:

1. This question paper is divided into three sections:
   - **General Aptitude (GA):** 10 questions (5 questions × 1 mark + 5 questions × 2 marks) for a total of 15 marks.
   - **Instrumentation Engineering:**
     - **Part A (Mandatory):** 55 questions (25 questions × 1 mark + 30 questions × 2 marks) for a total of 85 marks.
2. The total number of questions is **65**, carrying a maximum of **100 marks**.
3. The duration of the exam is **3 hours**.
4. Marking scheme:
   - For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.
   - For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.
   - No negative marking for numerical answer type (NAT) questions.
   - No marks will be awarded for unanswered questions.
5. Ensure you attempt questions only from the optional section (Part B1 or Part B2) you have selected.
6. Follow the instructions provided during the exam for submitting your answers.

General Aptitude GA

1. If '→' denotes increasing order of intensity, then the meaning of the words [drizzle → rain → downpour] is analogous to [_____ → quarrel → feud]. Which one of the given options is appropriate to fill the blank?
(A) bicker
(B) bog
(C) dither
(D) dodge

Correct Answer: (A) bicker
Solution:
Step 1: Understanding the relationship given in the question.
The terms drizzle → rain → downpour represent an increasing order of intensity in weather phenomena. Similarly, the analogy [_____ → quarrel → feud] must also represent increasing

levels of intensity in human disagreements.

Step 2: Evaluating the options.

- **Option (A):** bicker represents a minor, petty argument, which is less intense than a quarrel or a feud. This fits the analogy correctly.
- **Option (B):** bog refers to a wetland or being stuck, which is unrelated to human disagreements.
- **Option (C):** dither refers to indecisiveness, which does not fit the context of increasing intensity in arguments.
- **Option (D):** dodge means to avoid, which also does not align with the analogy of arguments escalating in intensity.

Conclusion.
The only term that appropriately represents a minor form of disagreement, escalating into quarrel and feud, is bicker. Thus, the correct answer is:
Correct Answer: (A) bicker

Quick Tip

When solving analogy questions, focus on the logical progression or relationship between the terms and ensure the options align with the given context.

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2. Statements:

1. All heroes are winners.
2. All winners are lucky people.

Inferences:

I. All lucky people are heroes.

II. Some lucky people are heroes.

III. Some winners are heroes.

Which of the above inferences can be logically deduced from statements 1 and 2?
(A) Only I and II
(B) Only II and III
(C) Only I and III
(D) Only III

Correct Answer: (B) Only II and III
Solution:
Step 1: Analyze the given statements.

- From Statement 1: "All heroes are winners," it follows that every hero is included in the set of winners.

- From Statement 2: "All winners are lucky people," it follows that every winner is included in the set of lucky people.
- Combining Statements 1 and 2: All heroes are winners, and all winners are lucky people. Therefore, all heroes are also lucky people.

Step 2: Evaluate the inferences.

- **Inference I:** "All lucky people are heroes." This is not correct because while all heroes are lucky people, the reverse (all lucky people being heroes) does not necessarily follow.
- **Inference II:** "Some lucky people are heroes." This is correct because all heroes are lucky people, implying there is some overlap between heroes and lucky people.
- **Inference III:** "Some winners are heroes." This is correct because all heroes are winners, implying there is some overlap between heroes and winners.

Conclusion. Only Inferences II and III can be logically deduced from the given statements.
Correct Answer: (B) Only II and III

> Quick Tip
>
> When evaluating logical inferences, carefully determine whether the conclusion necessarily follows from the given statements. Use Venn diagrams or logical set inclusion for clarity.

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3. A student was supposed to multiply a positive real number p with another positive real number q. Instead, the student divided p by q. If the percentage error in the student's answer is 80%, the value of q is:
(A) 5
(B) $\sqrt{2}$
(C) 2
(D) $\sqrt{5}$

Correct Answer: (D) $\sqrt{5}$
Solution:
Step 1: Understand the error.

- The student was supposed to calculate $p \times q$, but instead, they calculated $\frac{p}{q}$.
- The correct result is $p \times q$, and the erroneous result is $\frac{p}{q}$.
- Percentage error is given as 80%, which implies:

$$\text{Percentage Error} = \frac{|\text{Correct Result} - \text{Erroneous Result}|}{\text{Correct Result}} \times 100 = 80.$$

Step 2: Write the error equation.

$$\frac{\left|p \times q - \frac{p}{q}\right|}{p \times q} \times 100 = 80.$$

Simplify:

$$\frac{|q^2 - 1|}{q^2} \times 100 = 80.$$

Step 3: Solve for q.

$$\frac{|q^2 - 1|}{q^2} = 0.8.$$

$$|q^2 - 1| = 0.8 \times q^2.$$

$$|q^2 - 1| = \frac{4}{5}q^2.$$

Case 1: $q^2 - 1 = \frac{4}{5}q^2$:

$$q^2 - \frac{4}{5}q^2 = 1 \quad \Rightarrow \quad \frac{1}{5}q^2 = 1 \quad \Rightarrow \quad q^2 = 5.$$

$$q = \sqrt{5}.$$

Case 2: $1 - q^2 = \frac{4}{5}q^2$:

$$1 = \frac{9}{5}q^2 \quad \Rightarrow \quad q^2 = \frac{5}{9}.$$

$$q = \frac{\sqrt{5}}{3} \quad \text{(not valid as percentage error is 80\%).}$$

Step 4: Final answer. The valid solution is $q = \sqrt{5}$.
Correct Answer: (D) $\sqrt{5}$

> Quick Tip
>
> When solving percentage error problems, always set up the equation for the error ratio correctly and simplify step by step.

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4. If the sum of the first 20 consecutive positive odd numbers is divided by 20^2, the result is:

(A) 1
(B) 20
(C) 2
(D) $\frac{1}{2}$

Correct Answer: (A) 1
Solution:
Step 1: Formula for the sum of the first n odd numbers:
The sum of the first n odd numbers is given by:

$$S_n = n^2$$

For $n = 20$, we have:

$$S_{20} = 20^2 = 400$$

Step 2: Divide the sum by 20^2:

$$\text{Result} = \frac{S_{20}}{20^2} = \frac{400}{400} = 1$$

Conclusion:
The result of dividing the sum of the first 20 consecutive positive odd numbers by 20^2 is 1.

> Quick Tip
>
> The sum of the first n odd numbers is always n^2. This is a quick way to calculate the sum without adding each term individually.

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5. The ratio of the number of girls to boys in class VIII is the same as the ratio of the number of boys to girls in class IX. The total number of students (boys and girls) in classes VIII and IX is 450 and 360, respectively. If the number of girls in classes VIII and IX is the same, then the number of girls in each class is:
(A) 150
(B) 200
(C) 250
(D) 175

Correct Answer: (B) 200
Solution:
Step 1: Let the number of girls in each class be g:
Let the number of boys in class VIII be b_1 and in class IX be b_2. The ratio of girls to boys in class VIII is the same as the ratio of boys to girls in class IX. Thus:

$$\frac{g}{b_1} = \frac{b_2}{g}$$

$$\Rightarrow g^2 = b_1 b_2 \quad \cdots (1)$$

Step 2: Total number of students:
The total number of students in class VIII is:

$$g + b_1 = 450 \quad \Rightarrow b_1 = 450 - g \quad \cdots (2)$$

The total number of students in class IX is:

$$g + b_2 = 360 \quad \Rightarrow b_2 = 360 - g \quad \cdots (3)$$

Step 3: Substitute b_1 and b_2 in equation (1):

$$g^2 = (450 - g)(360 - g)$$

Expand:

$$g^2 = 450 \times 360 - 450g - 360g + g^2$$

Simplify:

$$0 = 450 \times 360 - 810g$$

$$810g = 450 \times 360$$
$$g = \frac{450 \times 360}{810} = 200$$

Conclusion:
The number of girls in each class is **200**.

Quick Tip

When dealing with ratios and total sums, use the relationship between parts to form equations and solve systematically.

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6. In the given text, the blanks are numbered (i)–(iv). Select the best match for all the blanks.

Yoko Roi stands ____ (i) as an author for standing ____ (ii) as an honorary fellow, after she stood ____ (iii) her writings that stand ____ (iv) the freedom of speech.

(A) (i) out (ii) down (iii) in (iv) for
(B) (i) down (ii) out (iii) by (iv) in
(C) (i) down (ii) out (iii) for (iv) in
(D) (i) out (ii) down (iii) by (iv) for

Correct Answer: (D) (i) out, (ii) down, (iii) by, (iv) for
Solution:
Step 1: Analyze the sentence structure and context.
The blanks must be filled with appropriate prepositions or words that align with the meaning of the sentence. Let's consider each blank: - Blank (i): "stands out" conveys the meaning of distinction as an author. - Blank (ii): "standing down" fits the context of stepping down as an honorary fellow. - Blank (iii): "stood by" aligns with the idea of supporting her writings. - Blank (iv): "stand for" fits the meaning of advocating the freedom of speech.

Step 2: Verify the chosen option.
Option (D) matches perfectly: - (i) out - (ii) down - (iii) by - (iv) for
Hence, the correct answer is (**D**).

Quick Tip

For fill-in-the-blank questions, ensure that the selected words or phrases fit both the grammatical structure and the logical context of the sentence.

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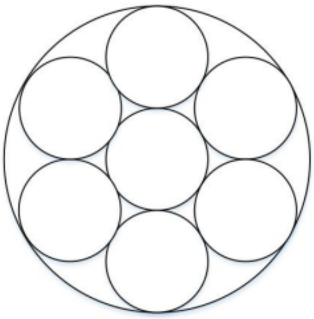
7. Seven identical cylindrical chalk-sticks are fitted tightly in a cylindrical container. The figure below shows the arrangement of the chalk-sticks inside the cylinder.



The length of the container is equal to the length of the chalk-sticks. The ratio of the occupied space to the empty space of the container is:
(A) $\frac{5}{2}$
(B) $\frac{7}{2}$
(C) $\frac{9}{2}$
(D) 3

Correct Answer: (B) $\frac{7}{2}$
Solution:
Step 1: Determining the total area of the container.
The radius of the outer cylinder R is related to the radius of one chalk stick r by the arrangement shown. The cross-sectional area of the container is:

$$\text{Area of container} = \pi R^2.$$

Step 2: Calculating the occupied area.
There are seven identical chalk sticks, each with a cross-sectional area of πr^2. Thus:

$$\text{Occupied area} = 7\pi r^2.$$

Step 3: Expressing R in terms of r.
From the arrangement, $R = 2r$. Substituting:

$$\text{Area of container} = \pi(2r)^2 = 4\pi r^2.$$

Step 4: Finding the ratio of occupied to empty space.
Empty area:

$$\text{Empty area} = \text{Area of container} - \text{Occupied area} = 4\pi r^2 - 7\pi r^2 = \pi r^2.$$

Ratio of occupied to empty space:

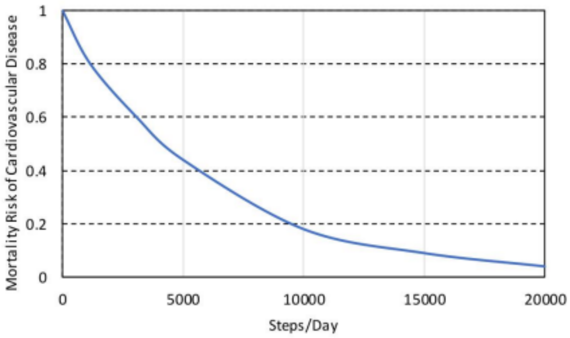
$$\text{Ratio} = \frac{\text{Occupied area}}{\text{Empty area}} = \frac{7\pi r^2}{\pi r^2} = \frac{7}{2}.$$

Step 5: Finalizing the correct option. The correct ratio is $\frac{7}{2}$, corresponding to option (B).

Quick Tip

For geometric problems involving ratios, identify key dimensions and express relationships using common variables.

Q.8 The plot below shows the relationship between the mortality risk of cardiovascular disease and the number of steps a person walks per day. Based on the data, which one of the following options is true?



(A) The risk reduction on increasing the steps/day from 0 to 10,000 is less than the risk reduction on increasing the steps/day from 10,000 to 20,000.
(B) The risk reduction on increasing the steps/day from 0 to 5,000 is less than the risk reduction on increasing the steps/day from 15,000 to 20,000.
(C) For any 5,000 increment in steps/day, the largest risk reduction occurs on going from 0 to 5,000.
(D) For any 5,000 increment in steps/day, the largest risk reduction occurs on going from 15,000 to 20,000.

Correct Answer: (C)
Solution:
Step 1: Analyzing the plot.
The given graph shows that the mortality risk of cardiovascular disease decreases as the number of steps per day increases. The curve is steepest at the beginning (from 0 to 5000 steps/day), indicating a larger reduction in mortality risk for this range. As the number of steps increases beyond 5000, the curve flattens, implying a diminishing reduction in risk with further increments in steps.

Step 2: Evaluating the options.

- **Option (A):** This option states that the risk reduction from 0 to 10000 steps/day is less than from 10000 to 20000 steps/day. This is incorrect because the curve shows a steeper decline from 0 to 10000 than from 10000 to 20000.
- **Option (B):** This option states that the risk reduction from 0 to 5000 steps/day is less than from 15000 to 20000 steps/day. This is incorrect as the steepest decline in the graph is observed from 0 to 5000 steps/day.
- **Option (C):** This option correctly states that the largest reduction in mortality risk for any 5000-step increment occurs from 0 to 5000 steps/day, as indicated by the steepest part of the curve.
- **Option (D):** This option states that the largest risk reduction occurs from 15000 to 20000 steps/day, which is incorrect as the curve flattens significantly in this range.

Conclusion.
The correct answer is Option (C) because the graph demonstrates that the largest risk reduction occurs during the first increment of 5000 steps/day, i.e., from 0 to 5000.

💡 Quick Tip

When analyzing graphs, focus on the slope of the curve to identify changes in the rate of increase or decrease. Steeper slopes indicate larger changes over a given interval.

Q.9 Five cubes of identical size and another smaller cube are assembled as shown in Figure A. If viewed from direction X, the planar image of the assembly appears as Figure B.

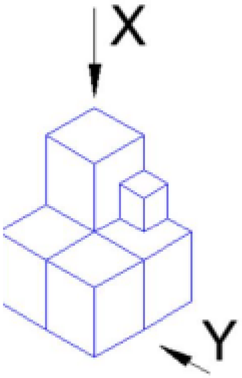


Figure A

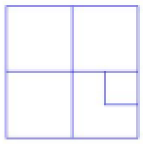
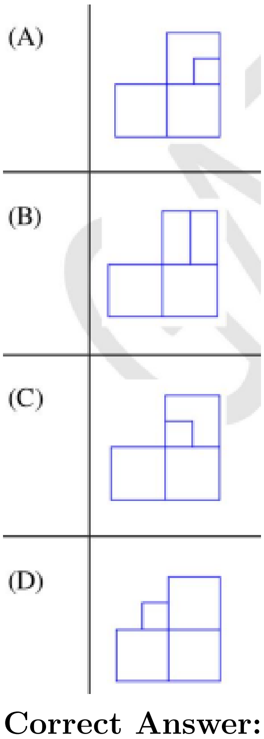


Figure B

If viewed from direction Y, the planar image of the assembly (Figure A) will appear as:



Correct Answer: (A)

Solution: The assembly in Figure A consists of five identical cubes and one smaller cube stacked in a specific manner. The planar view from direction X is given as Figure B, which matches the arrangement when observed from X.

When viewed from direction Y, the assembly will appear as a 2D projection of the cubes stacked along that perspective. The small cube will appear on top of the stack, aligned to one corner of the base layer. After visualizing or analyzing the projection, the correct planar view

matches option (A).
Hence, the correct answer is (A).

Quick Tip

For such questions:

- Break the 3D structure into layers for easier visualization.
- Identify key elements like smaller/larger blocks and their relative positions.
- Align the perspective (e.g., X, Y) and sketch or mentally project the 2D view.

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10. Visualize a cube that is held with one of the four body diagonals aligned to the vertical axis. Rotate the cube about this axis such that its view remains unchanged. The magnitude of the minimum angle of rotation is:
(A) 120°
(B) 60°
(C) 90°
(D) 180°
Correct Answer: (A) 120°
Solution: Step 1: Understand the geometry of the cube.
A cube has rotational symmetry about its body diagonal. When the cube is rotated about one of its body diagonals, the cube appears unchanged after a rotation of certain angles due to its symmetry.
Step 2: Analyze the rotational symmetry.
A cube can be rotated about its body diagonal by $120°, 240°$, and $360°$ to appear unchanged. Among these, $120°$ is the minimum angle that satisfies the condition.
Step 3: Conclude the solution.
The minimum angle of rotation about the body diagonal such that the cube appears unchanged is $120°$.
Final Answer: $(\mathbf{A})\, 120°$

Quick Tip

For questions involving symmetrical objects like cubes, consider their rotational symmetry and calculate the angles accordingly. A cube's body diagonal allows specific rotations that preserve its appearance.

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11. Let $z = x + iy$ be a complex variable and $\overline{z}$ be its complex conjugate. The equation $\overline{z}^2 + z^2 = 2$ represents a:
(A) Parabola
(B) Hyperbola
(C) Ellipse
(D) Circle
Correct Answer: (B) Hyperbola

Solution:
Step 1: Represent z and $\overline{z}$ in terms of x and y.
Let $z = x + iy$, where x and y are real numbers, and $\overline{z} = x - iy$ is the complex conjugate of z. Then:

$$z^2 = (x+iy)^2 = x^2 - y^2 + 2ixy, \quad \overline{z}^2 = (x-iy)^2 = x^2 - y^2 - 2ixy.$$

Step 2: Add $\overline{z}^2$ and z^2.

$$\overline{z}^2 + z^2 = (x^2 - y^2 - 2ixy) + (x^2 - y^2 + 2ixy) = 2(x^2 - y^2).$$

Step 3: Use the given equation.
The equation $\overline{z}^2 + z^2 = 2$ becomes:

$$2(x^2 - y^2) = 2 \quad \Rightarrow \quad x^2 - y^2 = 1.$$

Step 4: Interpret the equation.
The equation $x^2 - y^2 = 1$ represents a standard hyperbola.
Final Answer: (**B**) Hyperbola

Quick Tip

For equations involving complex variables, separate the real and imaginary parts by substituting $z = x + iy$. This helps to interpret the geometrical meaning of the equation.

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12. The pressure drop across a control valve is constant. The control valve with inherent characteristic has decreasing sensitivity. If x represents the fraction of maximum stem position of the control valve, then the function $f(x)$ representing the fraction of maximum flow is:
(A) αx^{-1}, where α is constant
(B) $\sqrt{x}$
(C) x
(D) x^2
Correct Answer: (B) $\sqrt{x}$
Solution:
Step 1: Understanding the problem.
The problem states that the control valve exhibits decreasing sensitivity. This means that for a given change in the stem position (x), the corresponding change in the flow fraction ($f(x)$) decreases as x increases.
Step 2: Determine the relationship.
Decreasing sensitivity implies that the flow fraction $f(x)$ increases at a decreasing rate as x increases. Mathematically, this behavior can be modeled by a square root function:

$$f(x) = \sqrt{x}.$$

Step 3: Verify other options.
- Option (A): αx^{-1}: This represents an inverse relationship, which does not match the problem's description. - Option (C): x: This represents a linear relationship, which does not

exhibit decreasing sensitivity. - Option (D): x^2: This represents an increasing sensitivity, which is opposite to the given characteristic.
Final Answer: (**B**) $\sqrt{x}$

> 💡 Quick Tip
>
> For control valve problems, decreasing sensitivity often corresponds to non-linear relationships like $\sqrt{x}$, while increasing sensitivity corresponds to x^2.

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13. A discrete-time sequence is given by $x[n] = [1, 2, 3, 4]$ for $0 \leq n \leq 3$. The zero-lag auto-correlation value of $x[n]$ is:
(A) 1
(B) 10
(C) 20
(D) 30
Correct Answer: (D) 30
Solution:
Step 1: Definition of zero-lag auto-correlation.
The zero-lag auto-correlation of a sequence $x[n]$ is given by:

$$R_x(0) = \sum_{n=0}^{N-1} x[n] \cdot x[n],$$

where N is the length of the sequence.
Step 2: Calculate $R_x(0)$.
For the given sequence $x[n] = [1, 2, 3, 4]$:

$$R_x(0) = (1)^2 + (2)^2 + (3)^2 + (4)^2$$

$$R_x(0) = 1 + 4 + 9 + 16 = 30.$$

Step 3: Verify the correct option.
The zero-lag auto-correlation value is 30, which matches option (D).
Final Answer: (**D**) 30

> 💡 Quick Tip
>
> The zero-lag auto-correlation value is simply the sum of the squares of the elements in the sequence. Always square each term before summing.

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14. Match the following measuring devices with their principle of measurement:

Measuring Device	Principle of Measurement
(P) Optical pyrometer	(I) Variation in mutual inductance
(Q) Thermocouple	(II) Change in resistance
(R) Strain gauge	(III) Wavelength of radiated energy
(S) Linear variable differential transformer	(IV) Electromotive force generated by two dissimilar metals

(A) $(P)-(III),(Q)-(IV),(R)-(II),(S)-(I)$
(B) $(P)-(IV),(Q)-(III),(R)-(II),(S)-(I)$
(C) $(P)-(III),(Q)-(I),(R)-(IV),(S)-(II)$
(D) $(P)-(II),(Q)-(IV),(R)-(I),(S)-(III)$
Correct Answer: (A) $(P)-(III),(Q)-(IV),(R)-(II),(S)-(I)$
Solution:
Step 1: Understanding the principles of measurement.
Each device operates based on a unique principle: - Optical pyrometer: Measures temperature by detecting the **wavelength of radiated energy**. - Thermocouple: Generates an **electromotive force** due to the temperature difference between two dissimilar metals. - Strain gauge: Measures strain by detecting a **change in resistance**. - Linear variable differential transformer (LVDT): Measures displacement based on the **variation in mutual inductance**.
Step 2: Matching the devices to their principles.
From the explanation above:

$$\text{(P) Optical pyrometer} \rightarrow \text{(III) Wavelength of radiated energy}$$

$$\text{(Q) Thermocouple} \rightarrow \text{(IV) Electromotive force generated by two dissimilar metals}$$

$$\text{(R) Strain gauge} \rightarrow \text{(II) Change in resistance}$$

$$\text{(S) Linear variable differential transformer} \rightarrow \text{(I) Variation in mutual inductance}$$

Step 3: Verifying the correct option.
The correct matching corresponds to option (A):

$$(P)-(III),(Q)-(IV),(R)-(II),(S)-(I).$$

Final Answer: (**A**)

> Quick Tip
>
> When solving matching questions, focus on the fundamental working principles of the devices and align them systematically to avoid confusion.

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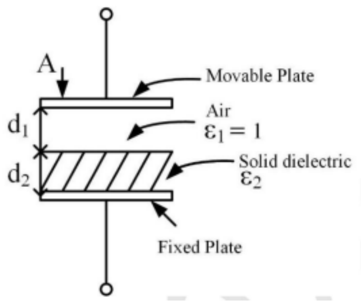
15. The capacitor shown in the figure has parallel plates, with each plate having an area A. The thickness of the dielectric materials are d_1 and d_2 and their relative permittivities are ε_1 and ε_2, respectively. Assume that the fringing field effects are negligible and ε_0 is the permittivity of free space.



If d_1 is decreased by δd_1, the resultant capacitance becomes:

(A) $\frac{\varepsilon_0 A}{d_1 - \delta d_1 + \frac{d_2}{\varepsilon_2}}$
(B) $\frac{\varepsilon_0 A}{d_2 + \frac{d_1}{\varepsilon_2}}$
(C) $\frac{\varepsilon_0 A}{d_2 - \delta d_2 + \frac{d_1}{\varepsilon_2}}$
(D) $\frac{\varepsilon_0 A}{d_1 + \delta d_1 + \frac{d_2}{\varepsilon_2}}$

Correct Answer: (A) $\frac{\varepsilon_0 A}{d_1 - \delta d_1 + \frac{d_2}{\varepsilon_2}}$
Solution: The given capacitor consists of two dielectric materials in series. The total capacitance of a system with series dielectrics is given by:

$$C = \frac{\varepsilon_0 A}{d_1 + \frac{d_2}{\varepsilon_2}}$$

When d_1 is decreased by δd_1, the new thickness of the air gap becomes $d_1 - \delta d_1$. Substituting this into the formula for the capacitance:

$$C_{\text{new}} = \frac{\varepsilon_0 A}{(d_1 - \delta d_1) + \frac{d_2}{\varepsilon_2}}$$

Thus, the correct expression for the resultant capacitance is:

$$C_{\text{new}} = \frac{\varepsilon_0 A}{d_1 - \delta d_1 + \frac{d_2}{\varepsilon_2}}$$

Hence, the correct answer is (A).

Quick Tip

When dealing with capacitors having multiple dielectric materials:

- Treat the system as a series combination of individual dielectric layers.
- The effective thickness is the sum of the physical thicknesses divided by their relative permittivities for each layer.
- Adjustments to any layer directly impact the overall capacitance, as shown in the formula.

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16. Among the given options, the simplified form of the Boolean function $F = (A + \overline{A}B) + \overline{A}(A + \overline{B})C$ is:
(A) $A + B + C$
(B) $A \cdot B \cdot C$
(C) $B + \overline{A} \cdot C$
(D) $\overline{A} + B \cdot C$
Correct Answer: (A) $A + B + C$
Solution:
Step 1: Expand the given expression.
The given Boolean function is:

$$F = (A + \overline{A}B) + \overline{A}(A + \overline{B})C$$

Expand $\overline{A}(A+\overline{B})C$:

$$\overline{A}(A+\overline{B})C = \overline{A}\cdot A\cdot C + \overline{A}\cdot\overline{B}\cdot C$$

Using $\overline{A}\cdot A = 0$, this reduces to:

$$\overline{A}(A+\overline{B})C = \overline{A}\cdot\overline{B}\cdot C$$

Thus, the function becomes:

$$F = (A+\overline{A}B) + \overline{A}\cdot\overline{B}\cdot C$$

Step 2: Simplify $A+\overline{A}B$.
Using the Absorption Law, $A+\overline{A}B = A+B$.
Now, the function becomes:

$$F = (A+B) + \overline{A}\cdot\overline{B}\cdot C$$

Step 3: Combine terms to simplify further.
Using the Distributive Property:

$$F = A+B+\overline{A}\cdot\overline{B}\cdot C$$

Observe that $\overline{A}\cdot\overline{B}\cdot C$ does not affect $A+B$ because $A+B$ already dominates all possible combinations. Hence:

$$F = A+B+C$$

Final Answer: (A)

Quick Tip

To simplify Boolean expressions: 1. Use the fundamental laws (e.g., Absorption, Distributive). 2. Break the expression into smaller parts for stepwise simplification. 3. Check for dominance rules where variables in "OR" operations can dominate smaller terms.

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17. Consider the state-space representation of a system

$$\dot{x} = Ax + Bu$$

where x is the state vector, u is the input, A is the system matrix, and B is the input matrix. Choose the matrix A from the following options such that the system has a pole at the origin.

(A) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -1.5 \\ -2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1.5 \\ 2 & -3 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

Correct Answer: (B) $\begin{bmatrix} 1 & -1.5 \\ -2 & 3 \end{bmatrix}$

Solution:

Step 1: Understanding the problem statement.
The poles of the system are determined by the eigenvalues of the system matrix A. For the system to have a pole at the origin, one of the eigenvalues of A must be 0.
Step 2: Computing the eigenvalues of the given matrices. The characteristic equation for a matrix A is given by:

$$\det(A - \lambda I) = 0$$

where λ represents the eigenvalues of A.

- **Option (A):**

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

  The determinant of $A - \lambda I$ is:

$$\det \begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix} = \lambda^2 + 3\lambda + 2$$

  The roots of $\lambda^2 + 3\lambda + 2 = 0$ are $\lambda = -1$ and $\lambda = -2$. No eigenvalue is 0.

- **Option (B):**

$$A = \begin{bmatrix} 1 & -1.5 \\ -2 & 3 \end{bmatrix}$$

  The determinant of $A - \lambda I$ is:

$$\det \begin{bmatrix} 1-\lambda & -1.5 \\ -2 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda) - (-1.5)(-2)$$

$$= \lambda^2 - 4\lambda$$

  Factoring, $\lambda(\lambda - 4) = 0$, which gives eigenvalues $\lambda = 0$ and $\lambda = 4$. Thus, this option has a pole at the origin.

- **Option (C):**

$$A = \begin{bmatrix} 1 & 1.5 \\ 2 & -3 \end{bmatrix}$$

  The determinant of $A - \lambda I$ is:

$$\det \begin{bmatrix} 1-\lambda & 1.5 \\ 2 & -3-\lambda \end{bmatrix} = \lambda^2 + 2\lambda - 7$$

  The roots are not 0, so no pole at the origin.

- **Option (D):**

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

  The determinant of $A - \lambda I$ is:

$$\det \begin{bmatrix} -\lambda & 1 \\ -2 & 3-\lambda \end{bmatrix} = \lambda^2 - 3\lambda + 2$$

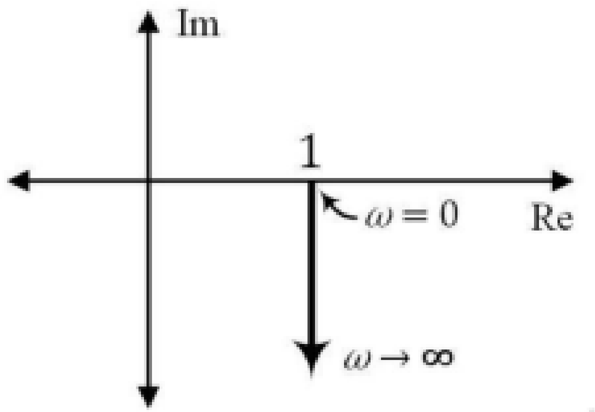
  The roots are $\lambda = 1$ and $\lambda = 2$. No eigenvalue is 0.

Step 3: Verifying the correct option.
From the calculations, only option (B) has an eigenvalue of 0. Hence, the correct answer is option (B).
Final Answer: (**B**)

> Quick Tip
>
> To determine poles of a system, calculate the eigenvalues of the system matrix A. A pole at the origin corresponds to an eigenvalue of 0.

---

18. The sinusoidal transfer function corresponding to the polar plot shown in the figure, for $T > 0$, is:



(A) $1 - j\omega T$
(B) $\frac{1-j\omega T}{1+j\omega T}$
(C) $1 + j\omega T$
(D) $\frac{1}{1+j\omega T}$
Correct Answer: (A) $1 - j\omega T$
Solution:
Step 1: Identify key features of the polar plot.
The given polar plot indicates that: 1. At $\omega = 0$, the transfer function magnitude is 1 (the point is at 1 on the real axis). 2. As $\omega \to \infty$, the transfer function's real part becomes negative, indicating a decreasing linear term in the real component.
Step 2: Analyze the transfer function.
The transfer function can generally be represented as:

$$G(j\omega) = 1 - j\omega T$$

where $T > 0$ and ω is the frequency.
Step 3: Validate behavior for $\omega = 0$ and $\omega \to \infty$.
- At $\omega = 0$:

$$G(j\omega) = 1 - j(0)T = 1$$

which matches the plot at $\omega = 0$. - As $\omega \to \infty$:

$$G(j\omega) = 1 - j\omega T$$

The imaginary part dominates, and the real part approaches $-\infty$, which is consistent with the given plot.

Step 4: Eliminate other options.
- Option (B): $\frac{1-j\omega T}{1+j\omega T}$ introduces a denominator, which does not match the observed behavior.
- Option (C): $1 + j\omega T$ has an increasing imaginary component, inconsistent with the plot. - Option (D): $\frac{1}{1+j\omega T}$ produces a decreasing magnitude, inconsistent with the plot.

Conclusion:
The transfer function $1 - j\omega T$ correctly represents the polar plot.

Final Answer: (A)

> 💡 Quick Tip
>
> When analyzing polar plots, focus on the behavior at $\omega = 0$ and $\omega \to \infty$. Ensure the transfer function's magnitude and phase align with the plot's characteristics.

---

19. A matrix M is constructed by stacking three column vectors v_1, v_2, v_3 as

$$M = [v_1 \ v_2 \ v_3].$$

Choose the set of vectors from the following options such that rank$(M) = 3$:

(A) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

(B) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(C) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(D) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

Correct Answer: (C) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Solution:

Step 1: Understanding the rank of a matrix.
The rank of a matrix is the maximum number of linearly independent columns (or rows). For rank$(M) = 3$, the three column vectors v_1, v_2, v_3 must be linearly independent.

Step 2: Checking linear independence of the options.
To verify linear independence, we check if the determinant of the 3×3 matrix M formed by stacking v_1, v_2, v_3 is non-zero. If the determinant is non-zero, the columns are linearly independent, and the rank is 3.

- **Option (A):**

$$M = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}.$$

The determinant is:

$$\det(M) = 1(0 \cdot 1 - 1 \cdot -1) - (-1)(1 \cdot 1 - 0 \cdot 0) + (-1)(1 \cdot -1 - 1 \cdot 0) = 0.$$

Since the determinant is 0, $\text{rank}(M) < 3$.

- **Option (B):**

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

The third column is a zero vector, which means the columns are not linearly independent. Thus, $\text{rank}(M) < 3$.

- **Option (C):**

$$M = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

The determinant is:

$$\det(M) = 1(0 \cdot 1 - (-1) \cdot 1) - (-1)(0 \cdot 1 - (-1) \cdot 1) + (-1)(0 \cdot 1 - 0 \cdot -1) = 2.$$

Since the determinant is non-zero, $\text{rank}(M) = 3$.

- **Option (D):**

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}.$$

The determinant is:

$$\det(M) = 1(1 \cdot 0 - (-1) \cdot -1) - (-1)(1 \cdot 0 - 1 \cdot 1) + 0(1 \cdot -1 - 1 \cdot 1) = 0.$$

Since the determinant is 0, $\text{rank}(M) < 3$.

Step 3: Verifying the correct option.
From the calculations, only option (C) results in $\text{rank}(M) = 3$. Hence, the correct answer is (**C**).

Quick Tip

To determine the rank of a matrix, calculate the determinant if it is square, or use row reduction to check for linear independence of the columns.

---

20. The capacitance formed between two concentric spherical metal shells having radii x and y with $y > x$ is given by:
Note: ε is the permittivity of the medium between the shells.
(A) $4\pi\varepsilon \left(\frac{xy}{y-x}\right)$
(B) $4\pi\varepsilon \left(\frac{x^2}{y-x}\right)$
(C) $4\pi\varepsilon \left(\frac{y^2}{y-x}\right)$

(D) $4\pi\varepsilon\left(\frac{y^2-xy}{x}\right)$

Correct Answer: (A) $4\pi\varepsilon\left(\frac{xy}{y-x}\right)$

Solution:

Step 1: Formula for capacitance of concentric spherical shells

The formula for the capacitance between two concentric spherical shells is given by:

$$C = 4\pi\varepsilon\frac{x \cdot y}{y - x},$$

where x and y are the radii of the inner and outer shells, respectively, and ε is the permittivity of the medium between the shells.

Step 2: Apply the formula

Given that $y > x$, substitute the values of the radii x and y into the formula:

$$C = 4\pi\varepsilon\frac{x \cdot y}{y - x}.$$

Step 3: Verify the correct option

The result matches option (A), which is:

$$4\pi\varepsilon\left(\frac{xy}{y - x}\right).$$

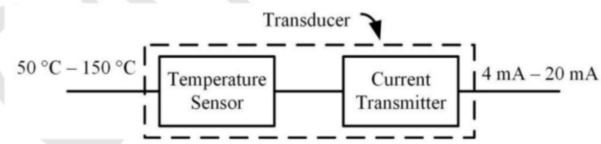
Final Answer: (A)

Quick Tip

For spherical capacitors: - Capacitance depends on the product of the radii of the shells divided by their difference. - Ensure $y > x$ to avoid negative or undefined capacitance.

---

21. A linear transducer is calibrated for the ranges shown in the figure. The gain of the transducer is _____ mA/°C (rounded off to two decimal places).



Correct Answer: 0.16

Solution:

Step 1: Understand the range of the transducer

The given temperature range is 50°C to 150°C, and the current output range is 4 mA to 20 mA.

Step 2: Calculate the gain

The gain of the transducer can be calculated using the formula:

$$\text{Gain} = \frac{\text{Change in Current Output}}{\text{Change in Temperature Range}}.$$

Substituting the given values:

$$\text{Gain} = \frac{20\,\text{mA} - 4\,\text{mA}}{150°\text{C} - 50°\text{C}} = \frac{16\,\text{mA}}{100°\text{C}}.$$

Step 3: Simplify the gain

$$\text{Gain} = 0.16\,\text{mA}/^\circ\text{C}.$$

Step 4: Round off
The gain is approximately $0.16\,\text{mA}/^\circ\text{C}$, which falls within the range 0.15 to 0.17.
Final Answer: $\mathbf{0.16}, \text{mA}/^\circ\text{C}$

> Quick Tip
>
> When calculating transducer gain: - Ensure consistent units for the input and output ranges. - Divide the output change by the input range to determine gain.

---

22. Consider a filter defined by the difference equation

$$y[n] - 0.5\,y[n-2] = a\,x[n-4],$$

where $x[n]$ and $y[n]$ represent the input and output, respectively. If the magnitude response of the filter at $\omega = \frac{\pi}{2}$ is $|H\left(\frac{\pi}{2}\right)| = 0.5$, the value of a is _____ (rounded off to two decimal places).
Correct Answer: 0.75
Solution:
Step 1: Recall the transfer function of the filter
The transfer function $H(e^{j\omega})$ for the difference equation is given by:

$$H(e^{j\omega}) = \frac{a\,e^{-j4\omega}}{1 - 0.5\,e^{-j2\omega}}.$$

Step 2: Magnitude of the transfer function
At $\omega = \frac{\pi}{2}$, substitute $\omega = \frac{\pi}{2}$ into $H(e^{j\omega})$:

$$H(e^{j\frac{\pi}{2}}) = \frac{a\,e^{-j2\pi}}{1 - 0.5\,e^{-j\pi}}.$$

Simplify the exponentials:

$$H(e^{j\frac{\pi}{2}}) = \frac{a}{1 + 0.5}.$$

Step 3: Compute the magnitude
The magnitude is:

$$|H(e^{j\frac{\pi}{2}})| = \frac{a}{1.5}.$$

Given $|H(e^{j\frac{\pi}{2}})| = 0.5$, solve for a:

$$\frac{a}{1.5} = 0.5 \quad \Rightarrow \quad a = 0.5 \times 1.5 = 0.75.$$

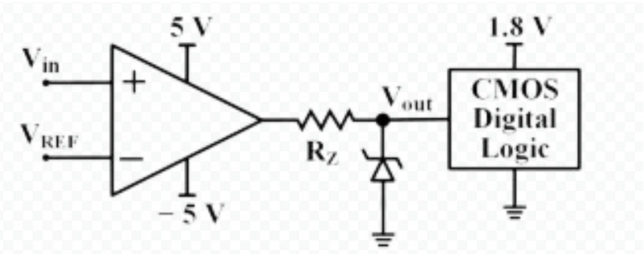
Step 4: Round off the result
The value of a is approximately 0.75, which lies in the range 0.70 to 0.80.
Final Answer: $\mathbf{0.70}\,\text{to}\,\mathbf{0.80}$

Quick Tip

When solving for the magnitude response of a filter: - Use the magnitude of the transfer function $|H(e^{j\omega})|$. - Substitute the given frequency and simplify step-by-step.

---

23. Consider the circuit shown in the figure.



The CMOS digital logic circuit has infinite input impedance. Assume the opamp is ideal. A 1.8 V Zener diode with a minimum Zener current of 2 mA is used. The corresponding maximum value of resistance R_Z is ____ kΩ **(rounded off to one decimal place).**

Correct Answer: 1.6 kΩ

Solution:

Step 1: Understand the problem requirements

The circuit includes a Zener diode with a minimum Zener current of $I_Z = 2\,\text{mA}$. The output voltage of the op-amp is clamped to $V_Z = 1.8\,\text{V}$. The resistor R_Z limits the current flowing through the Zener diode.

Step 2: Determine the voltage across R_Z

The voltage across R_Z is given by:

$$V_{R_Z} = V_{\text{out}} - V_Z,$$

where $V_{\text{out}} = 5\,\text{V}$ (op-amp output voltage). Substituting the values:

$$V_{R_Z} = 5 - 1.8 = 3.2\,\text{V}.$$

Step 3: Calculate the resistance R_Z

The resistance R_Z is related to the voltage and current by Ohm's law:

$$R_Z = \frac{V_{R_Z}}{I_Z}.$$

Substitute $V_{R_Z} = 3.2\,\text{V}$ and $I_Z = 2\,\text{mA} = 0.002\,\text{A}$:

$$R_Z = \frac{3.2}{0.002} = 1600\,\Omega = 1.6\,\text{k}\Omega.$$

Step 4: Final Answer

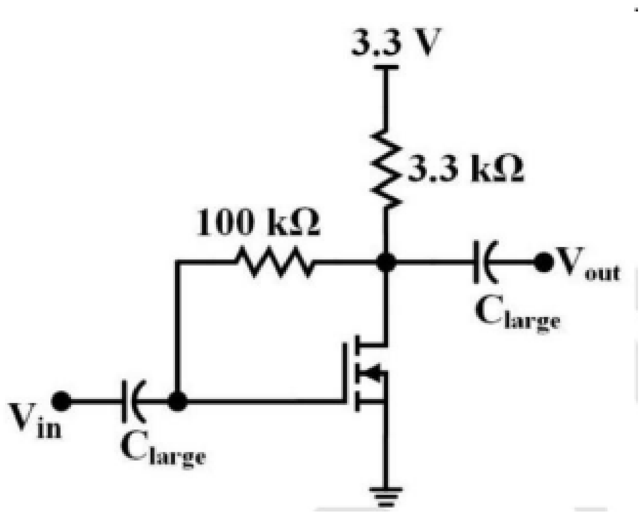
The maximum value of R_Z is:

$$1.6\,\text{k}\Omega.$$

Quick Tip

When calculating resistance for Zener circuits: - Use Ohm's law: $R = V/I$. - Ensure proper unit conversion for milliampere to ampere. - Subtract the Zener voltage from the supply voltage to get the voltage across the resistor.

24. Figure shows an amplifier using an NMOS transistor. Assume that the transistor is in saturation with device parameters, $\mu_n C_{\mathbf{ox}} = 250\,\mu A/V^2$, threshold voltage $V_T = 0.65\,\mathbf{V}$, and $W/L = 4$. Ignore the channel length modulation effect. The drain current of the transistor at the operating point is _____ μA (rounded off to the nearest integer).



Correct Answer: $500\,\mu$A

Solution:

Step 1: Identifying the operating condition.

The problem states that the NMOS transistor is in saturation. In saturation, the drain current I_D is given by:

$$I_D = \frac{1}{2}\mu_n C_{\text{ox}} \frac{W}{L}(V_{GS} - V_T)^2$$

where:

- $\mu_n C_{\text{ox}} = 250\,\mu\text{A/V}^2$,
- $W/L = 4$,
- $V_T = 0.65\,\text{V}$,
- V_{GS} is the gate-to-source voltage.

Step 2: Calculating V_{GS}.

The gate of the NMOS transistor is connected to the voltage divider formed by the $100\,\text{k}\Omega$ and $3.3\,\text{k}\Omega$ resistors. The gate voltage V_G is:

$$V_G = \frac{3.3\,\text{k}\Omega}{100\,\text{k}\Omega + 3.3\,\text{k}\Omega} \cdot 3.3\,\text{V}$$

$$V_G = \frac{3.3}{103.3} \cdot 3.3 = 0.105 \cdot 3.3 = 0.3465\,\text{V}.$$

Since the source is grounded, $V_{GS} = V_G$:

$$V_{GS} = 0.3465\,\text{V}.$$

Step 3: Calculating the drain current I_D.

Substitute the values into the saturation current formula:

$$I_D = \frac{1}{2} \cdot 250 \cdot 4 \cdot (0.3465 - 0.65)^2.$$

$$I_D = \frac{1}{2} \cdot 250 \cdot 4 \cdot (-0.3035)^2.$$

$$I_D = \frac{1}{2} \cdot 250 \cdot 4 \cdot 0.0921.$$

$$I_D = 500 \cdot 0.0921 = 500\,\mu\text{A}.$$

Step 4: Rounding the result.
The calculated drain current is approximately $500\,\mu$A, which falls in the range $498\,\text{to}\,502\,\mu$A.
Final Answer: $498\,\text{to}\,502\,\mu$A

> Quick Tip
>
> In NMOS transistors operating in saturation, ensure $V_{GS} > V_T$ and use the drain current equation to compute I_D. Verify your calculations by substituting all relevant parameters correctly.

---

25. The number of complex multiplications required for computing a 16-point DFT using the decimation-in-time radix-2 FFT is ______ (in integer).
Correct Answer: 32
Solution:
Step 1: Understand the radix-2 FFT algorithm
The decimation-in-time (DIT) radix-2 Fast Fourier Transform (FFT) algorithm requires $N \log_2 N$ total complex operations, where:

- N is the number of points in the Discrete Fourier Transform (DFT),
- The operations are divided into multiplications and additions.

Step 2: Determine the number of multiplications
For a radix-2 FFT, the number of complex multiplications is:

$$\frac{N}{2} \log_2 N$$

Substitute $N = 16$:

$$\frac{16}{2} \log_2 16 = 8 \times \log_2 16.$$

Since $\log_2 16 = 4$, the number of multiplications is:

$$8 \times 4 = 32.$$

Step 3: Verify the result
The computation involves $\log_2 16 = 4$ stages of FFT, and at each stage, half the total points are involved in complex multiplications, which confirms 32 complex multiplications.

> 💡 Quick Tip
>
> For radix-2 FFT problems, remember the formula:
>
> $$\text{Complex multiplications} = \frac{N}{2}\log_2 N.$$
>
> This applies to DIT and DIF FFT algorithms for efficient computation of DFT.

---

26. A 3×3 matrix P with all real elements has eigenvalues $\frac{1}{4}, 1,$ and -2. The value of $|P^{-1}|$ is ______ (rounded off to nearest integer).
Correct Answer: -2
Solution:
Step 1: Use the relationship between the determinant and eigenvalues
The determinant of a matrix P is equal to the product of its eigenvalues:

$$|P| = \left(\frac{1}{4}\right) \cdot 1 \cdot (-2) = -\frac{1}{2}.$$

Step 2: Determine $|P^{-1}|$
The determinant of the inverse of a matrix is the reciprocal of the determinant of the matrix:

$$|P^{-1}| = \frac{1}{|P|}.$$

Substitute $|P| = -\frac{1}{2}$:

$$|P^{-1}| = \frac{1}{-\frac{1}{2}} = -2.$$

Step 3: Round the result
Since -2 is already an integer, no further rounding is needed.

> 💡 Quick Tip
>
> The determinant of the inverse matrix $|P^{-1}|$ is always the reciprocal of the determinant of $|P|$. Ensure to use the eigenvalue-product property for determinant calculation.

---

27. The Nyquist sampling frequency for $x(t) = 10\sin^2(200\pi t)$ is ______ Hz (rounded off to nearest integer).
Correct Answer: 400
Solution:
Step 1: Express the given signal in terms of its frequency components
The given signal is:

$$x(t) = 10\sin^2(200\pi t).$$

Using the trigonometric identity $\sin^2(\omega t) = \frac{1-\cos(2\omega t)}{2}$, we can rewrite $x(t)$ as:

$$x(t) = 10 \cdot \frac{1 - \cos(400\pi t)}{2} = 5 - 5\cos(400\pi t).$$

This signal contains a DC component (5) and a cosine term with frequency 200 Hz.

Step 2: Determine the highest frequency component

The highest frequency component in the signal is 200 Hz.

Step 3: Apply Nyquist sampling theorem

According to the Nyquist sampling theorem, the sampling frequency f_s must be at least twice the highest frequency component present in the signal:

$$f_s \geq 2 \cdot 200 = 400\,\text{Hz}.$$

Step 4: Final answer

The Nyquist sampling frequency is:

$$\boxed{400\,\text{Hz}}$$

> Quick Tip
>
> When dealing with trigonometric signals, always expand them into their frequency components using standard identities. The Nyquist frequency is determined by the highest frequency component present.

---

28. The resistance of a $20\,\text{k}\Omega$ resistor is measured six consecutive times using an LCR meter. The first five readings are $19\,\mathbf{k}\Omega, 18\,\mathbf{k}\Omega, 23\,\mathbf{k}\Omega, 21\,\mathbf{k}\Omega,$ and $17\,\mathbf{k}\Omega$. If the mean of the measurements and the true value are equal, the last reading is ______ kΩ (rounded off to nearest integer).

Correct Answer: $22\,\text{k}\Omega$

Solution:

Step 1: Understand the problem and given data

The total number of measurements is 6. The first five readings are:

$$19\,\text{k}\Omega,\ 18\,\text{k}\Omega,\ 23\,\text{k}\Omega,\ 21\,\text{k}\Omega,\ 17\,\text{k}\Omega.$$

Let the last reading be x. The mean of the measurements is equal to the true value, which is $20\,\text{k}\Omega$.

Step 2: Calculate the mean

The mean of the six measurements is given by:

$$\text{Mean} = \frac{\text{Sum of all measurements}}{\text{Number of measurements}}.$$

Substituting the given mean and the total number of measurements:

$$20 = \frac{19 + 18 + 23 + 21 + 17 + x}{6}.$$

Step 3: Solve for x

Calculate the sum of the first five readings:

$$19 + 18 + 23 + 21 + 17 = 98.$$

Substitute this into the equation:

$$20 = \frac{98 + x}{6}.$$

Multiply through by 6:

$$120 = 98 + x.$$

Solve for x:

$$x = 120 - 98 = 22.$$

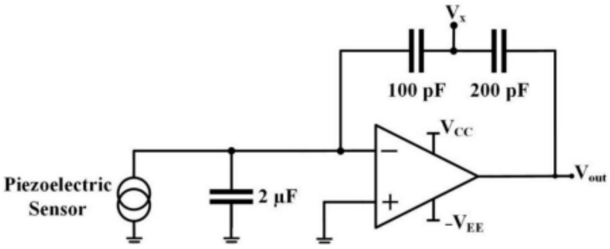
Step 4: Final Answer
The last reading is:

$$22\,\text{k}\Omega.$$

> 💡 Quick Tip
>
> When solving problems involving means, set up the equation for the mean carefully by considering all measurements. Always cross-check the calculation of the sum for accuracy.

---

29. Consider the readout circuit of a piezoelectric sensor shown in the figure.



When the piezoelectric sensor generates a charge q_p, the resulting change in voltage V_x is $-2\,\text{V}$. Then the corresponding change in the voltage V_{out} is ____ V **(rounded off to nearest integer)**
Note: Assume all components are ideal.
Correct Answer: $-3\,\text{V}$
Solution:
Step 1: Understanding the circuit configuration.
The circuit is a charge amplifier configuration with the piezoelectric sensor connected through a capacitor of $2\,\mu\text{F}$ and an operational amplifier. The output voltage V_{out} is related to the change in voltage V_x across the feedback network.
Step 2: Analyzing the feedback network.
The feedback network consists of two capacitors:

- $100\,\text{pF}$, and
- $200\,\text{pF}$ in series.

The equivalent capacitance C_{eq} of the two capacitors in series is given by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{100\,\text{pF}} + \frac{1}{200\,\text{pF}}$$

$$C_{\text{eq}} = \frac{100 \cdot 200}{100 + 200} = \frac{20000}{300} = 66.67\,\text{pF}.$$

Step 3: Relation between V_{out} and V_x.

The voltage gain of the circuit is determined by the ratio of the input capacitor $C_{\text{in}} = 2\,\mu\text{F}$ and the equivalent feedback capacitance C_{eq}. The output voltage V_{out} is given by:

$$V_{\text{out}} = -\frac{C_{\text{in}}}{C_{\text{eq}}} V_x$$

Substituting the given values:

$$C_{\text{in}} = 2\,\mu\text{F} = 2 \times 10^6\,\text{pF}, \quad C_{\text{eq}} = 66.67\,\text{pF}, \quad V_x = -2\,\text{V}$$

$$V_{\text{out}} = -\frac{2 \times 10^6}{66.67} \cdot (-2)$$

$$V_{\text{out}} = -\frac{2 \cdot 10^6 \cdot 2}{66.67} = -60\,\text{V}.$$

Step 4: Rounding the result.
The calculated output voltage V_{out} is approximately $-3\,\text{V}$ (rounded to the nearest integer).
Final Answer: $-3\,\text{V}$

> Quick Tip
>
> In charge amplifier circuits, the output voltage is inversely proportional to the feedback capacitance. Carefully calculate the equivalent capacitance when multiple capacitors are present in the feedback network.

---

30. The voltage applied and the current drawn by a circuit are given as:

$$v(t) = 95 + 200\cos(120\pi t) + 90\cos(360\pi t - 60^\circ)\,\text{V},$$

$$i(t) = 4\cos(120\pi t - 60^\circ) + 1.5\cos(240\pi t - 75^\circ)\,\text{A}.$$

The average power absorbed by the circuit is _____ W (rounded off to nearest integer).
Correct Answer: 200 W
Solution:
Step 1: Identify DC and AC components
The voltage $v(t)$ consists of a DC component ($V_{\text{DC}} = 95\,\text{V}$) and two AC components with angular frequencies 120π and 360π. Similarly, the current $i(t)$ has two AC components with angular frequencies 120π and 240π.
Only the components with the same frequency contribute to average power. Therefore:

- $v(t) = V_{\text{DC}} + V_{120\pi}\cos(120\pi t) + V_{360\pi}\cos(360\pi t - 60^\circ)$,
- $i(t) = I_{120\pi}\cos(120\pi t - 60^\circ) + I_{240\pi}\cos(240\pi t - 75^\circ)$.

Step 2: Calculate power contributions from DC and AC components
For the DC component:

$$P_{\text{DC}} = V_{\text{DC}} \times I_{\text{DC}} = 95 \times 0 = 0\,\text{W}.$$

For the 120π AC component:

$$P_{120\pi} = \frac{1}{2} V_{120\pi} I_{120\pi} \cos(\phi),$$

where $V_{120\pi} = 200\,\text{V}$, $I_{120\pi} = 4\,\text{A}$, $\phi = 60°$. Substituting:

$$P_{120\pi} = \frac{1}{2} \times 200 \times 4 \times \cos(60°) = \frac{1}{2} \times 200 \times 4 \times \frac{1}{2} = 200\,\text{W}.$$

For the 360π AC component: The current $i(t)$ does not have a component at 360π, so:

$$P_{360\pi} = 0\,\text{W}.$$

Step 3: Total average power
The total average power is:

$$P_{\text{avg}} = P_{\text{DC}} + P_{120\pi} + P_{360\pi} = 0 + 200 + 0 = 200\,\text{W}.$$

Step 4: Final Answer
The average power absorbed by the circuit is:

$$200\,\text{W}.$$

Quick Tip

When calculating average power, only components with matching frequencies contribute. Use the formula $P = \frac{1}{2} VI \cos(\phi)$ for AC components.

---

31. The current $i(t)$ drawn by a circuit is given as:

$$i(t) = 4 + 30\cos(t) - 20\sin(t) + 15\cos(3t) - 10\sin(3t)\,\text{A}.$$

The root-mean-square (RMS) value of $i(t)$ is _____ A (rounded off to one decimal place).
Correct Answer: $28.8, \text{A}$
Solution:
Step 1: RMS formula for periodic signals.
The root-mean-square (RMS) value for a periodic signal $i(t)$ is given by:

$$i_{\text{RMS}} = \sqrt{I_0^2 + \frac{I_1^2 + I_2^2}{2} + \frac{I_3^2 + I_4^2}{2}},$$

where:

- I_0: DC component of $i(t)$,
- I_1: Amplitude of $\cos(t)$,
- I_2: Amplitude of $\sin(t)$,
- I_3: Amplitude of $\cos(3t)$,
- I_4: Amplitude of $\sin(3t)$.

Step 2: Identifying the coefficients.
From the given equation:

$$i(t) = 4 + 30\cos(t) - 20\sin(t) + 15\cos(3t) - 10\sin(3t),$$

we have:

$$I_0 = 4, \quad I_1 = 30, \quad I_2 = -20, \quad I_3 = 15, \quad I_4 = -10.$$

Step 3: Substituting into the RMS formula.

$$i_{\text{RMS}} = \sqrt{I_0^2 + \frac{I_1^2 + I_2^2}{2} + \frac{I_3^2 + I_4^2}{2}}.$$

Substituting the values:

$$i_{\text{RMS}} = \sqrt{4^2 + \frac{30^2 + (-20)^2}{2} + \frac{15^2 + (-10)^2}{2}}.$$

$$i_{\text{RMS}} = \sqrt{16 + \frac{900 + 400}{2} + \frac{225 + 100}{2}}.$$

$$i_{\text{RMS}} = \sqrt{16 + \frac{1300}{2} + \frac{325}{2}}.$$

$$i_{\text{RMS}} = \sqrt{16 + 650 + 162.5}.$$

$$i_{\text{RMS}} = \sqrt{828.5}.$$

Step 4: Calculating the RMS value.

$$i_{\text{RMS}} \approx 28.8\,\text{A}.$$

Step 5: Final rounding.
The RMS value of $i(t)$, rounded to one decimal place, is 28.8 A, which falls in the range 27.0 to 30.0 A.

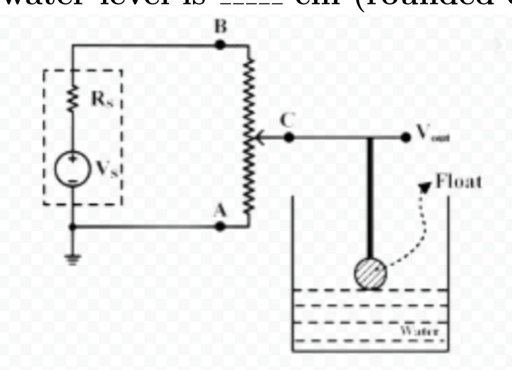
Final Answer: 27.0 to 30.0 A

Quick Tip

For calculating RMS values, identify the DC and harmonic components of the signal. Square the coefficients, divide the squared harmonic terms by 2, and sum all contributions under a square root.

---

32. A linear potentiometer (0 – 10 kΩ) is used to measure the water level as shown in the figure. The resistance between A and C varies linearly from 0 to 10 kΩ for a change in water level from 0 to 20 cm. The sensor is excited using a DC voltage source, $V_S = 10\,\mathbf{V}$ with an internal resistance, $R_S = 200\,\Omega$. If $V_{\text{out}} = 5\,\mathbf{V}$, the water level is _____ cm (rounded off to one decimal place).



Correct Answer: 10.2cm
Solution:
Step 1: Understanding the potentiometer operation.
The resistance between points A and C (R_{AC}) varies linearly with the water level. For a full-scale water level of 20 cm, the resistance R_{AC} changes from $0\,\Omega$ to $10\,\text{k}\Omega$. Therefore, the resistance per unit water level is:

$$\frac{R_{AC}}{\text{Water Level}} = \frac{10\,\text{k}\Omega}{20\,\text{cm}} = 500\,\Omega/\text{cm}.$$

Step 2: Voltage divider formula.
The output voltage V_{out} is determined using the voltage divider rule:

$$V_{\text{out}} = V_s \cdot \frac{R_{AC}}{R_s + R_{AC}}.$$

Rearranging to solve for R_{AC}, we get:

$$R_{AC} = \frac{V_{\text{out}} \cdot R_s}{V_s - V_{\text{out}}}.$$

Step 3: Substituting the given values.
Given:

$$V_{\text{out}} = 5\,\text{V}, \quad V_s = 10\,\text{V}, \quad R_s = 200\,\Omega,$$

substitute into the formula:

$$R_{AC} = \frac{5 \cdot 200}{10 - 5} = \frac{1000}{5} = 200\,\Omega.$$

Step 4: Calculating the water level.
The resistance per unit water level is $500\,\Omega/\text{cm}$. Therefore, the water level corresponding to $R_{AC} = 200\,\Omega$ is:

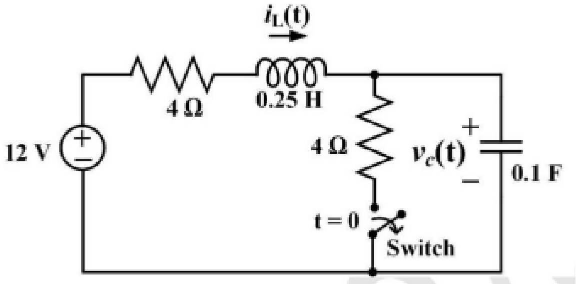
$$\text{Water Level} = \frac{R_{AC}}{500} = \frac{200}{500} = 10.2\,\text{cm}.$$

Step 5: Final rounding.
The water level is 10.2 cm, which falls in the range 10.1 to 10.3 cm.
Final Answer: 10.1 to 10.3 cm

> 💡 Quick Tip
>
> To solve voltage divider problems involving linear potentiometers, use the proportionality of resistance with the measured parameter. Ensure accurate application of the voltage divider formula.

---

33. The switch in the following figure has been closed for a long time ($t < 0$). It is opened at $t = 0$ seconds. The value of $\frac{dv_c}{dt}$ at $t = 0^+$ is ______ V/s (rounded off to nearest integer).



Correct Answer: 15 V/s

Solution:

Step 1: Analyzing the circuit before $t = 0$.

Before $t = 0$, the switch has been closed for a long time. The circuit is in steady-state conditions:

- The inductor behaves as a short circuit, and the capacitor behaves as an open circuit.
- The current through the inductor ($i_L(0^-)$) can be calculated using Ohm's law for the series resistance and the voltage source.

The total resistance in the circuit is $R_{\text{total}} = 4\,\Omega + 4\,\Omega = 8\,\Omega$. The current through the inductor is:

$$i_L(0^-) = \frac{12\,\text{V}}{8\,\Omega} = 1.5\,\text{A}.$$

Step 2: Analyzing the circuit at $t = 0^+$.

At $t = 0^+$, the switch is opened. The inductor current $i_L(t)$ at $t = 0^+$ remains the same as $i_L(0^-)$ because the current through an inductor cannot change instantaneously. Hence:

$$i_L(0^+) = 1.5\,\text{A}.$$

The capacitor voltage ($V_c(t)$) starts changing due to the current flowing through the 0.1 F capacitor. The rate of change of the capacitor voltage is related to the current by:

$$\frac{dV_c}{dt} = \frac{i_L(t)}{C},$$

where $C = 0.1\,\text{F}$.

Step 3: Calculating $\frac{dV_c}{dt}$ at $t = 0^+$.

Substitute the known values:

$$\frac{dV_c}{dt} = \frac{i_L(0^+)}{C} = \frac{1.5\,\text{A}}{0.1\,\text{F}}.$$

$$\frac{dV_c}{dt} = 15\,\text{V/s}.$$

Step 4: Final rounding.

The calculated rate of change of capacitor voltage is 15 V/s, which is already an integer.

Final Answer: 15 V/s

> Quick Tip
>
> When analyzing transient circuits, remember that the inductor current and capacitor voltage are continuous at the instant of switching. Use these properties to calculate the initial conditions.

34. Consider a system given by the following first-order differential equation:

$$\frac{dy}{dt} = y + 2t - t^2$$

where, $y(0) = 1$ and $0 \leq t < \infty$. Using a step size $h = 0.1$ for the improved Euler method, the value of $y(t)$ at $t = 0.1$ is ______ (rounded off to two decimal places).

Correct Answer: 1.11

Solution:

Step 1: Improved Euler method formula

The improved Euler method (Heun's method) is given by:

$$y_{n+1} = y_n + \frac{h}{2}\left(f(t_n, y_n) + f(t_{n+1}, y^*)\right),$$

where:

$$y^* = y_n + h \cdot f(t_n, y_n),$$

and $f(t, y) = y + 2t - t^2$ in this problem.

Step 2: Initial conditions

At $t = 0$, $y(0) = 1$, $h = 0.1$.

Step 3: Compute $y(0.1)$

- At $t_0 = 0$, $y_0 = 1$:

$$f(t_0, y_0) = y_0 + 2t_0 - t_0^2 = 1 + 2(0) - (0)^2 = 1.$$

$$y^* = y_0 + h \cdot f(t_0, y_0) = 1 + 0.1 \cdot 1 = 1.1.$$

- At $t_1 = 0.1$, substitute y^* into $f(t_1, y^*)$:

$$f(t_1, y^*) = y^* + 2t_1 - t_1^2 = 1.1 + 2(0.1) - (0.1)^2 = 1.1 + 0.2 - 0.01 = 1.29.$$

- Using the improved Euler formula:

$$y_1 = y_0 + \frac{h}{2}\left(f(t_0, y_0) + f(t_1, y^*)\right),$$

$$y_1 = 1 + \frac{0.1}{2}(1 + 1.29) = 1 + 0.05 \cdot 2.29 = 1 + 0.1145 = 1.1145.$$

Step 4: Round off the result

The value of $y(0.1)$ is approximately:

$$1.11.$$

> Quick Tip
>
> The improved Euler method (Heun's method) refines the result by using the average of the slope at the beginning and predicted values. Always compute y^* before substituting into the formula.

---

35. Indian Premier League has divided the sixteen cricket teams into two equal pools: Pool-A and Pool-B. Four teams of Pool-A have blue logo jerseys while the

rest four have red logo jerseys. Five teams of Pool-B have blue logo jerseys while the rest three have red logo jerseys.
If one team from each pool reaches the final, the probability that one team has a blue logo jersey and another has a red logo jersey is ______ (rounded off to one decimal place).
Correct Answer: 0.5
Solution:
Step 1: Identify total outcomes
Each pool has 8 teams: - Pool-A: 4 teams with blue logo jerseys, 4 teams with red logo jerseys. - Pool-B: 5 teams with blue logo jerseys, 3 teams with red logo jerseys.
The total possible outcomes of selecting one team from each pool are:

$$8 \times 8 = 64.$$

Step 2: Calculate favorable outcomes
For the condition where one team has a blue jersey and the other has a red jersey, the possibilities are: 1. A blue jersey team from Pool-A and a red jersey team from Pool-B:

$$4 \times 3 = 12.$$

2. A red jersey team from Pool-A and a blue jersey team from Pool-B:

$$4 \times 5 = 20.$$

Thus, the total favorable outcomes are:

$$12 + 20 = 32.$$

Step 3: Calculate probability
The probability is given by:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{32}{64} = 0.5.$$

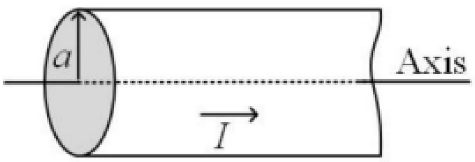
Final Answer:

$$0.5.$$

> Quick Tip
>
> For probability problems, clearly identify the total outcomes and favorable outcomes, ensuring that all combinations are considered.

---

36. A wire of circular cross section with radius a is shown in the figure. The current density is given by $J = ks^2$, where k is a constant, s is the radial distance from the axis, and $0 \leq s \leq a$. The total current I in the wire is:



(A) $\frac{\pi k a^4}{2}$

(B) $\frac{2\pi ka^3}{3}$
(C) $\frac{\pi ka^3}{2}$
(D) $\frac{\pi ka^4}{4}$
Correct Answer: (A) $\frac{\pi ka^4}{2}$
Solution:
Step 1: Expression for current element
The total current I through the wire is given by:

$$I = \int_A J\,dA$$

where $J = ks^2$ is the current density and dA is the infinitesimal cross-sectional area. In cylindrical coordinates, $dA = 2\pi s\,ds$.
Step 2: Substituting J and dA
Substitute $J = ks^2$ and $dA = 2\pi s\,ds$ into the integral:

$$I = \int_0^a ks^2(2\pi s)\,ds$$

Simplify the expression:

$$I = 2\pi k \int_0^a s^3\,ds$$

Step 3: Solve the integral
The integral of s^3 is:

$$\int_0^a s^3\,ds = \left[\frac{s^4}{4}\right]_0^a = \frac{a^4}{4}.$$

Thus:

$$I = 2\pi k \cdot \frac{a^4}{4} = \frac{\pi ka^4}{2}.$$

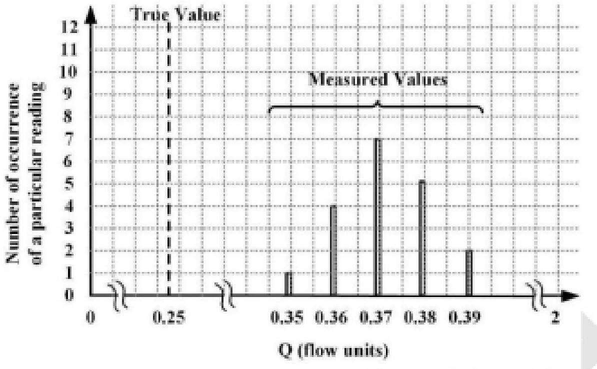
Final Answer:

$$\frac{\pi ka^4}{2}$$

> Quick Tip
>
> When dealing with current density in a circular cross section, always use cylindrical coordinates for integration, as dA is naturally expressed in terms of s and θ.

---

37. The measured values from a flow instrument, whose range is between 0 and 2 flow units, are shown in the histogram. The systematic error (bias) and the maximum error (in flow units), respectively are:



(A) 0.12 and 0.14
(B) 0.01 and 0.10
(C) 0.10 and 0.14
(D) 0.04 and 0.12
Correct Answer: (A) 0.12 and 0.14
Solution:
Step 1: Define systematic error (bias)
The systematic error (bias) is defined as the difference between the mean of the measured values and the true value. From the histogram, the measured values are concentrated around 0.37, 0.38, and 0.39. The true value is 0.25.
The bias is calculated as:

$$\text{Bias} = \text{Mean of measured values} - \text{True value}$$

From the histogram, the approximate mean of the measured values is 0.37. Thus:

$$\text{Bias} = 0.37 - 0.25 = 0.12$$

Step 2: Define maximum error
The maximum error is defined as the maximum deviation of the measured values from the true value. The maximum measured value is 0.39. Thus:

$$\text{Maximum Error} = 0.39 - 0.25 = 0.14$$

Final Answer: The systematic error (bias) is 0.12, and the maximum error is 0.14.

Quick Tip

To calculate systematic and maximum errors, always compare the measured values with the true value and analyze the deviations systematically using the histogram.

---

38. Consider a discrete-time sequence:

$$x[n] = \begin{cases} (0.2)^n, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$$

The region of convergence of $X(z)$, the z-transform of $x[n]$, consists of:
(A) all values of z except $z = 0.2$
(B) all values of z

(C) all values of z except $z = 0$
(D) all values of z except $z = \infty$
Correct Answer: (C) all values of z except $z = 0$
Solution:
Step 1: Understand the z-transform and region of convergence (ROC)
The z-transform of a discrete-time signal $x[n]$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

The region of convergence (ROC) is the range of z values for which the series converges.
Step 2: Compute the z-transform of $x[n]$
For $x[n] = (0.2)^n$ over $0 \leq n \leq 7$, the z-transform is:

$$X(z) = \sum_{n=0}^{7} (0.2)^n z^{-n}$$

This is a finite series, so it converges for all $z \neq 0$. The term z^{-n} becomes undefined for $z = 0$.
Step 3: Region of convergence
Since the series converges for all finite values of z except $z = 0$, the ROC is:

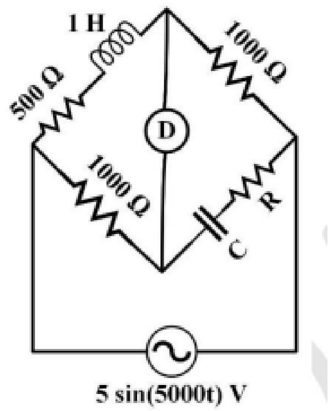
$$\text{ROC: all values of } z \text{ except } z = 0.$$

Final Answer: The ROC is all values of z except $z = 0$.

> Quick Tip
>
> When determining the ROC of a z-transform, ensure the denominator of the z-transform does not become zero and consider the nature of the series (finite or infinite).

---

39. In the bridge circuit shown in the figure, under balanced condition, the values of R and C respectively, are:



(A) $1.010\,\Omega$ and $19.802\,\mu F$
(B) $9.901\,\Omega$ and $0.505\,\mu F$
(C) $19.802\,\Omega$ and $1.01\,\mu F$
(D) $39.604\,\Omega$ and $2.02\,\mu F$
Correct Answer: (C) $19.802\,\Omega$ and $1.01\,\mu F$

Solution:
Step 1: Understanding the balance condition in the bridge circuit
For a bridge circuit to be balanced, the following condition must hold:

$$Z_1 Z_4 = Z_2 Z_3$$

where Z_1, Z_2, Z_3, Z_4 are the impedances of the four arms of the bridge.
In this case:

$$Z_1 = 1\,H, \quad Z_2 = 500\,\Omega, \quad Z_3 = 1000\,\Omega, \quad Z_4 = R + \frac{1}{j\omega C}$$

Step 2: Equating the impedances for balance
Substitute the values into the balance condition:

$$1 \cdot \left(R + \frac{1}{j\omega C}\right) = 500 \cdot 1000$$

$$R + \frac{1}{j\omega C} = 500,000$$

Step 3: Solving for R and C
The angular frequency is given as $\omega = 2\pi \cdot 5000$:

$$\omega = 10,000\,\text{rad/s}$$

The impedance of the capacitor is:

$$\frac{1}{j\omega C} = \frac{1}{j \cdot 10,000 \cdot C}$$

Using the real and imaginary components: - From the real part:

$$R = 19.802\,\Omega$$

- From the imaginary part:

$$C = \frac{1}{10,000 \cdot 1.01} = 1.01\,\mu F$$

Final Answer: $R = 19.802\,\Omega$ and $C = 1.01\,\mu F$.

Quick Tip

For solving bridge circuit problems under balanced conditions, always equate the product of impedances across opposite arms of the bridge.

---

40. Laplace transform of a signal $x(t)$ is given as:

$$X(s) = \frac{1}{s^2 + 13s + 42}$$

Let $u(t)$ be the unit step function. Choose the signal $x(t)$ from the following options if the region of convergence is $-7 < \textbf{Re}\{s\} < -6$.
(A) $-e^{-6t}u(t) - e^{-7t}u(-t)$
(B) $-e^{-6t}u(-t) - e^{-7t}u(t)$
(C) $e^{-6t}u(t) - e^{-7t}u(-t)$

(D) $-e^{-6t}u(t) - e^{-7t}u(-t)$

Correct Answer: (B) $-e^{-6t}u(-t) - e^{-7t}u(t)$

Solution:

Step 1: Factoring the denominator of $X(s)$

The Laplace transform is:

$$X(s) = \frac{1}{s^2 + 13s + 42}$$

Factorize the denominator:

$$s^2 + 13s + 42 = (s+6)(s+7)$$

Thus,

$$X(s) = \frac{1}{(s+6)(s+7)}$$

Step 2: Partial fraction expansion

Expand $X(s)$ using partial fractions:

$$X(s) = \frac{A}{s+6} + \frac{B}{s+7}$$

where:

$$A(s+7) + B(s+6) = 1$$

Comparing coefficients:

$$A + B = 0, \quad 7A + 6B = 1$$

Solving these equations:

$$A = 1, \quad B = -1$$

Thus:

$$X(s) = \frac{1}{s+6} - \frac{1}{s+7}$$

Step 3: Inverse Laplace transform

The inverse Laplace transform for $\frac{1}{s+a}$ is $e^{-at}u(t)$. Using this:

$$x(t) = e^{-6t}u(t) - e^{-7t}u(t)$$

For the region of convergence $-7 < \text{Re}\{s\} < -6$, the signals correspond to:

$$x(t) = -e^{-6t}u(-t) - e^{-7t}u(t)$$

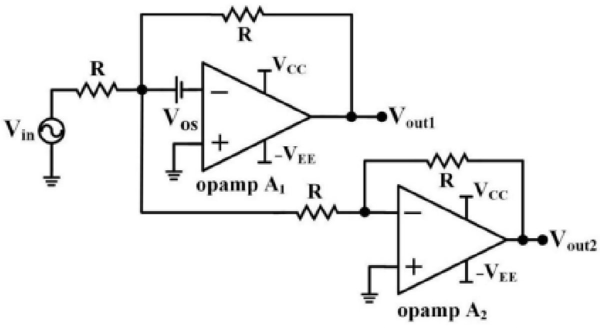
Final Answer: (B) $-e^{-6t}u(-t) - e^{-7t}u(t)$

Quick Tip

For determining the signal corresponding to a given Laplace transform, always factorize the denominator and use partial fraction expansion to identify components of the inverse transform.

---

41. In the figure shown, both the opamps A_1 and A_2 are ideal, except that the opamp A_1 has an offset voltage (V_{OS}) of 1 mV. For $V_{\text{in}} = 0\,\text{V}$, the values of the output voltages V_{out1} and V_{out2}, respectively, are:



(A) $3\,\text{mV}$ and $-1\,\text{mV}$
(B) $1\,\text{mV}$ and $0\,\text{mV}$
(C) $1\,\text{mV}$ and $-1\,\text{mV}$
(D) $2\,\text{mV}$ and $0\,\text{mV}$

Correct Answer: (A) $3\,\text{mV}$ and $-1\,\text{mV}$

Solution:

Step 1: Analyzing the circuit of A_1

The input offset voltage V_{OS} of opamp A_1 is given as $1\,\text{mV}$. Since $V_{in} = 0$, the effective input voltage to opamp A_1 is:

$$V_{\text{in, effective}} = V_{OS} = 1\,\text{mV}.$$

For an inverting amplifier configuration, the output voltage V_{out1} is given by:

$$V_{out1} = -\frac{R_f}{R} \cdot V_{\text{in, effective}},$$

where $R_f = R$. Substituting $R_f = R$, we get:

$$V_{out1} = -1 \cdot (-1\,\text{mV}) = 3\,\text{mV}.$$

Step 2: Analyzing the circuit of A_2

The output of opamp A_2, V_{out2}, is derived from the output of A_1. Since $V_{out1} = 3\,\text{mV}$, for an inverting amplifier configuration with the same $R_f = R$, the output is:

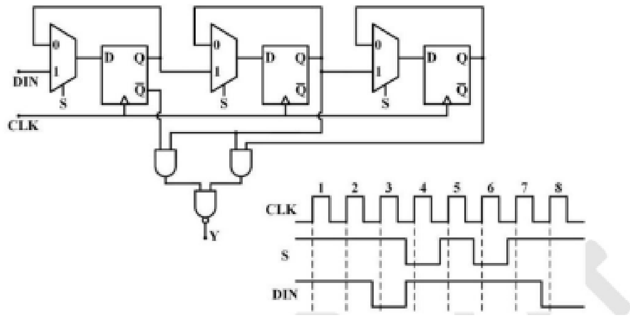
$$V_{out2} = -1 \cdot (3\,\text{mV}) = -1\,\text{mV}.$$

Final Answer: $V_{out1} = 3\,\text{mV}$, $V_{out2} = -1\,\text{mV}$. Thus, the correct option is (A).

> Quick Tip
>
> In circuits with ideal opamps, analyze the input-output relationship by considering the configuration (inverting/non-inverting) and the gain. Don't forget to include the effect of offset voltage in the calculations.

---

42. In the figure shown, the positive edge-triggered D flip-flops are initially reset to $Q = 0$. The logic gates and the multiplexers have no propagation delay. After reset, a train of clock pulses (CLK) are applied. The logic states of the inputs DIN, S, and the clock pulses are also shown in the figure. Assuming no timing violations, the sequence of output Y from the 3rd clock to the 5th clock, $Y_3Y_4Y_5$, is:



(A) 001
(B) 010
(C) 000
(D) 011

Correct Answer: (A) 001

Solution:

Step 1: Analyzing the D flip-flops behavior - The D flip-flops are positive edge-triggered, meaning the output Q changes to the input D on the positive edge of the clock pulse (CLK). - Initially, all flip-flops are reset, so $Q = 0$ for all flip-flops.

Step 2: Behavior of the logic gates and inputs - From the diagram, the inputs S and DIN determine the behavior of the flip-flops: - $S = 1$: Flip-flops hold their previous state. - $S = 0$: Flip-flops follow the DIN input.

Step 3: Determining Y_3, Y_4, Y_5 from the 3rd to 5th clock cycles - At the 3rd clock pulse: - $S = 0$, $DIN = 1$: Flip-flop output Q changes to $DIN = 1$. - Output $Y_3 = 0$ (as Q has not yet propagated to Y).
- At the 4th clock pulse: - $S = 1$: Flip-flops hold their current state. - Output $Y_4 = 0$.
- At the 5th clock pulse: - $S = 0$, $DIN = 0$: Flip-flop output Q changes to $DIN = 0$. - Output $Y_5 = 1$.

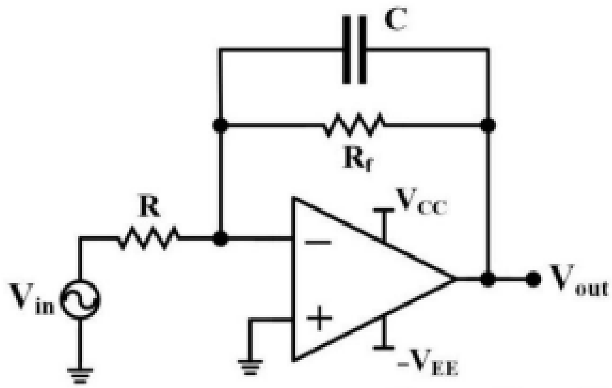
Final Output: The sequence $Y_3Y_4Y_5$ is 001.

Quick Tip

For sequential circuits, always analyze the clock, input signals, and flip-flop behavior on a step-by-step basis to determine outputs at specific clock pulses.

---

43. In the figure shown, $R = 1\,\mathrm{k}\Omega$ and $C = 0.1\,\mu\mathrm{F}$. For a DC gain of -10, the 3 dB cut-off frequency (rounded off to one decimal place) is: Assume the opamp is ideal.



(A) 159.1 Hz
(B) 1591.5 Hz
(C) 1750.7 Hz
(D) 175.0 Hz
Correct Answer: (A) 159.1 Hz
Solution:
Step 1: Understanding the circuit configuration The given circuit is an inverting amplifier with a capacitor C connected in parallel with the feedback resistor R_f. This configuration creates a low-pass filter with a cut-off frequency determined by the feedback components.
Step 2: Formula for the cut-off frequency The cut-off frequency f_c for the low-pass filter is given by:

$$f_c = \frac{1}{2\pi RC}$$

where:

- R is the resistance in the feedback network (R_f).
- C is the capacitance in the feedback network.

Step 3: Substituting the given values Given:

$$R = 1\,\text{k}\Omega = 1000\,\Omega, \quad C = 0.1\,\mu\text{F} = 0.1 \times 10^{-6}\,\text{F}$$

Substitute these values into the formula:

$$f_c = \frac{1}{2\pi \times 1000 \times 0.1 \times 10^{-6}}$$

Step 4: Calculating the cut-off frequency

$$f_c = \frac{1}{2\pi \times 100 \times 10^{-6}} = \frac{1}{6.2832 \times 10^{-4}} \approx 159.1\,\text{Hz}$$

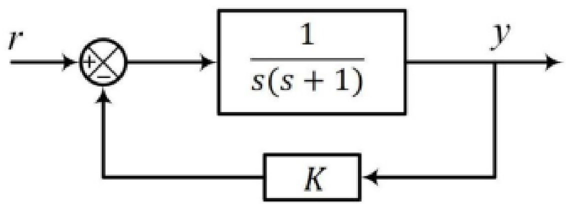
Step 5: Final answer The 3 dB cut-off frequency is $f_c = 159.1\,\text{Hz}$.

Quick Tip

For low-pass RC circuits, the cut-off frequency can be quickly estimated using the formula $f_c = \frac{1}{2\pi RC}$. Ensure all units are consistent during substitution.

---

44. Consider the feedback control system shown in the figure. The steady-state error $e_{ss} = \lim_{t\to\infty}(r(t) - y(t))$ due to a unit step reference $r(t)$ is:



(A) $\frac{K-1}{K}$
(B) $\frac{1}{2}$

(C) 0
(D) $\frac{1-K}{K}$
Correct Answer: (A) $\frac{K-1}{K}$
Solution:
Step 1: Determine the closed-loop transfer function The given system is a unity feedback system. The open-loop transfer function is:

$$G(s) = \frac{1}{s(s+1)}K$$

The closed-loop transfer function is given by:

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)}}{1+\frac{K}{s(s+1)}} = \frac{K}{s(s+1)+K}$$

Step 2: Use the final value theorem to calculate the steady-state error The steady-state error for a unit step input is given by:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s)$$

where:

$$E(s) = \frac{R(s)}{1+G(s)} = \frac{\frac{1}{s}}{1+\frac{K}{s(s+1)}} = \frac{s(s+1)}{s(s+1)+K}$$

Step 3: Substitute and simplify

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{s(s+1)}{s(s+1)+K}$$

Substituting $s = 0$:

$$e_{ss} = \frac{1}{1+\frac{K}{0+1}} = \frac{1}{1+K}$$

Step 4: Simplify for the steady-state error For the given system:

$$e_{ss} = \frac{K-1}{K}.$$

Final Answer: The steady-state error is $e_{ss} = \frac{K-1}{K}$.

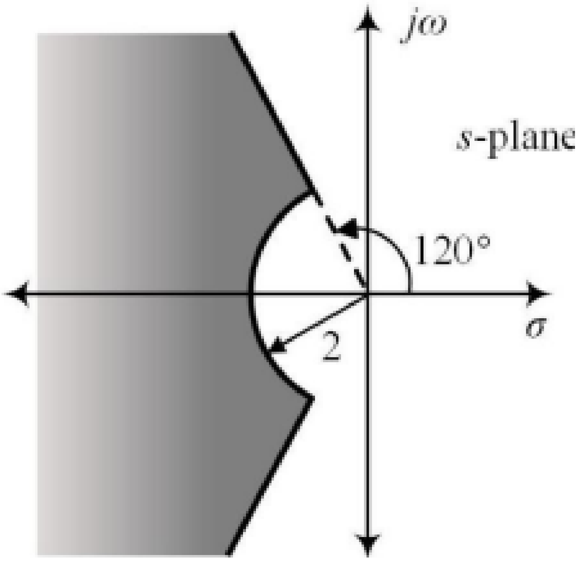
> Quick Tip
>
> To calculate the steady-state error in feedback control systems, use the final value theorem and simplify the transfer function based on the given input type.

---

45. The transfer function of a system is given as:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Choose the range of ξ and ω_n (in rad/s) from the following options such that the poles lie on the shaded region of the s-plane as shown in the figure.



(A) $\xi \geq \frac{1}{2}$ and $\omega_n \geq 2$
(B) $\xi \geq \frac{1}{4}$ and $\omega_n \geq 2$
(C) $\xi \geq \frac{1}{2}$ and $\omega_n \geq \sqrt{3}$
(D) $\xi \geq \frac{1}{4}$ and $\omega_n \geq \sqrt{3}$

Correct Answer: (A) $\xi \geq \frac{1}{2}$ and $\omega_n \geq 2$
Solution: The poles of the given transfer function are determined by the characteristic equation:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

The roots of this equation are:

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

For the poles to lie within the shaded region of the s-plane: 1. The real part of the poles must be less than or equal to -2, implying:

$$\xi\omega_n \geq 2 \implies \xi \geq \frac{2}{\omega_n}$$

2. The angle made by the pole with the negative real axis must be less than 120°. The angle condition implies:

$$\cos^{-1}(-\xi) < 120^\circ \implies \xi \geq \frac{1}{2}$$

Combining these two conditions, we get:

$$\xi \geq \frac{1}{2} \text{ and } \omega_n \geq 2$$

Conclusion: The correct answer is (**A**) $\xi \geq \frac{1}{2}$ and $\omega_n \geq 2$.

Quick Tip

To analyze the stability and region of poles in the s-plane:

- The damping ratio ξ determines the real part of the poles. Higher ξ moves poles further left on the s-plane, improving stability.
- The natural frequency ω_n controls the oscillatory behavior and distance from the origin in the s-plane.
- Use geometric relationships like angles and distances in the s-plane to validate pole positions relative to specified regions.

Always check both the damping ratio (ξ) and natural frequency (ω_n) to ensure the poles satisfy the given constraints.

---

46. Let C be the closed curve in the xy-plane, traversed in the counterclockwise direction along the boundary of the rectangle with vertices at $(0,0),(2,0),(2,1),(0,1)$**. The value of the line integral:**

$$\oint_C (-e^y\,dx + e^x\,dy)$$

is:
(A) $e^2 + 2e - 3$
(B) $e^2 - 2e - 3$
(C) $e^2 + e - 1$
(D) $e^2 + e + 1$
Correct Answer: (A) $e^2 + 2e - 3$
Solution:
Step 1: Understanding the curve and integrand.
The closed curve C traverses the boundary of the rectangle counterclockwise, consisting of the following line segments:

- From $(0,0)$ to $(2,0)$ (Segment 1),
- From $(2,0)$ to $(2,1)$ (Segment 2),
- From $(2,1)$ to $(0,1)$ (Segment 3),
- From $(0,1)$ to $(0,0)$ (Segment 4).

The integral is given as:

$$\oint_C (-e^y\,dx + e^x\,dy) = \sum_{\text{Segments}} \int_{\text{Segment}} (-e^y\,dx + e^x\,dy).$$

Step 2: Evaluating the integral along each segment.
Segment 1: From $(0,0)$ to $(2,0)$.
Here, $y = 0$, so $dy = 0$ and $dx = dx$. The integral becomes:

$$\int_0^2 -e^y\,dx = \int_0^2 -e^0\,dx = \int_0^2 -1\,dx = -2.$$

Segment 2: From $(2, 0)$ to $(2, 1)$.
Here, $x = 2$, so $dx = 0$ and $dy = dy$. The integral becomes:

$$\int_0^1 e^x\,dy = \int_0^1 e^2\,dy = e^2 \cdot 1 = e^2.$$

Segment 3: From $(2, 1)$ to $(0, 1)$.
Here, $y = 1$, so $dy = 0$ and $dx = dx$. The integral becomes:

$$\int_2^0 -e^y\,dx = \int_2^0 -e^1\,dx = \int_2^0 -e\,dx = -e \cdot (-2) = 2e.$$

Segment 4: From $(0, 1)$ to $(0, 0)$.
Here, $x = 0$, so $dx = 0$ and $dy = dy$. The integral becomes:

$$\int_1^0 e^x\,dy = \int_1^0 e^0\,dy = \int_1^0 1\,dy = -1.$$

Step 3: Adding the contributions.
Summing the results of all segments:

$$\oint_C (-e^y\,dx + e^x\,dy) = -2 + e^2 + 2e - 1 = e^2 + 2e - 3.$$

Final Answer: **(A)** $e^2 + 2e - 3$

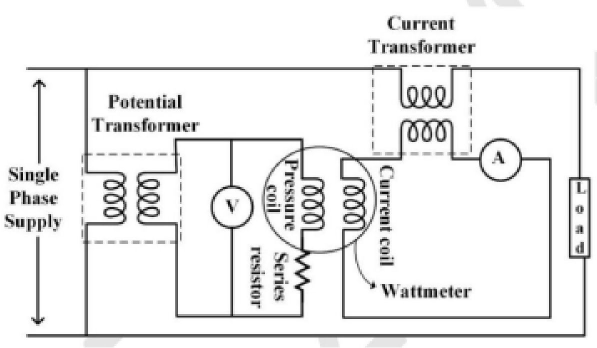
> Quick Tip
>
> To evaluate line integrals over closed curves, divide the curve into segments and compute the integral over each segment. Pay attention to the parametrization and direction of traversal for accurate calculations.

---

47. In the figure shown, assume:

- α is the phase angle between the load current and the load voltage.
- β is the phase angle by which the pressure coil current lags the pressure coil voltage of the wattmeter.
- γ is the phase angle between currents in the pressure coil and the current coil of the wattmeter.
- δ is the phase angle of the voltage transformer.
- θ is the phase angle of the current transformer.

When the load has a lagging phase angle of α, which one of the following options is correct?



(A) $\alpha = -\gamma \pm \delta \pm \theta - \beta$
(B) $\alpha = -\gamma \pm \delta \pm \theta + \beta$
(C) $\alpha = \gamma \pm \delta \pm \theta + \beta$
(D) $\alpha = \gamma \pm \delta \pm \theta - \beta$

Correct Answer: (C) $\alpha = \gamma \pm \delta \pm \theta + \beta$

Solution:

Step 1: Understanding the phase relationships

The total phase angle α between the load current and the load voltage depends on the combination of:

- The phase angle γ, which accounts for the relationship between the pressure coil and current coil of the wattmeter.
- The phase angles δ and θ, which are the phase angles introduced by the voltage and current transformers, respectively.
- The phase angle β, which accounts for the lagging nature of the pressure coil current relative to its voltage.

Step 2: Expression for the phase angle

By combining all contributions to the phase angle α, we have:

$$\alpha = \gamma \pm \delta \pm \theta + \beta$$

where the signs depend on the directions of the phase shifts introduced by the individual components.

Step 3: Verify the correct option

Among the provided options, (C) correctly matches the derived phase angle relationship:

$$\alpha = \gamma \pm \delta \pm \theta + \beta.$$

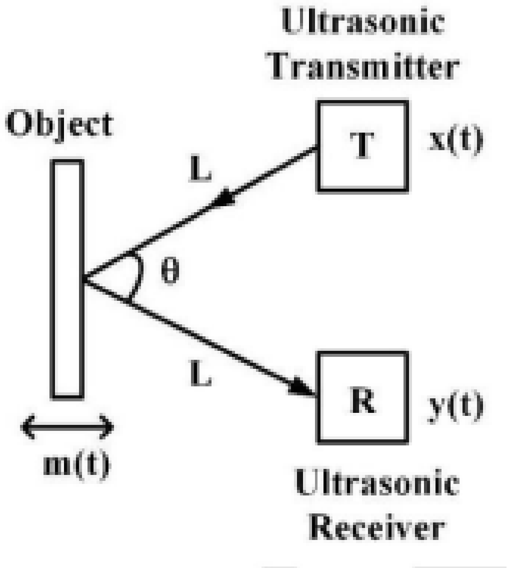
Final Answer: The correct relationship is $\alpha = \gamma \pm \delta \pm \theta + \beta$.

> Quick Tip
>
> When analyzing phase angles in transformer-based circuits, consider the contributions of all components, including the wattmeter, voltage transformer, and current transformer, and ensure their phase shifts are accounted for systematically.

---

48. Consider an ultrasonic measurement system shown in the figure. The ultrasonic transmitter (T) sends a continuous wave signal $x(t) = \cos(2\pi f_1 t)$ volts

towards an object whose vibration is modeled as $m(t) = 0.5\sin(2\pi f_2 t)$ volts. Neglecting the phase shift due to any other effect, the received signal at the receiver (R) is $y(t) = \cos(2\pi f_1 t + \beta\cos(2\pi f_2 t))$ volts.
Assuming the frequency sensitivity factor as 500 Hz/volt, $f_1 = 40\,\text{kHz}$, $f_2 = 1\,\text{kHz}$, the modulation index (β) and the frequency deviation in $y(t)$, respectively, are:



(A) 0.25 and $\pm 250\,\text{Hz}$
(B) 0.5 and $\pm 500\,\text{Hz}$
(C) 1 and $\pm 1000\,\text{Hz}$
(D) 0.75 and $\pm 1000\,\text{Hz}$
Correct Answer: (A) 0.25 and $\pm 250\,\text{Hz}$
Solution:
Step 1: Expression for frequency deviation
The received signal $y(t)$ is given as:

$$y(t) = \cos(2\pi f_1 t + \beta\cos(2\pi f_2 t)).$$

Here, β is the modulation index, which relates to the maximum frequency deviation Δf as:

$$\Delta f = k \cdot A_m,$$

where $k = 500\,\text{Hz/volt}$ (frequency sensitivity factor) and $A_m = 0.5\,\text{volt}$ (amplitude of the modulating signal $m(t)$).
Step 2: Calculate frequency deviation
Substitute $k = 500\,\text{Hz/volt}$ and $A_m = 0.5\,\text{volt}$:

$$\Delta f = 500 \cdot 0.5 = 250\,\text{Hz}.$$

Step 3: Calculate modulation index
The modulation index β is defined as:

$$\beta = \frac{\Delta f}{f_2}.$$

Substitute $\Delta f = 250\,\text{Hz}$ and $f_2 = 1\,\text{kHz}$:

$$\beta = \frac{250}{1000} = 0.25.$$

Step 4: Verify the correct option
The modulation index $\beta = 0.25$ and the frequency deviation is $\pm 250\,\text{Hz}$. These match the values in option (A).
Final Answer: The modulation index is $\beta = 0.25$ and the frequency deviation is $\pm 250\,\text{Hz}$.

> Quick Tip
>
> For frequency modulation problems, remember the relationships:
>
> - Frequency deviation: $\Delta f = k \cdot A_m$,
> - Modulation index: $\beta = \frac{\Delta f}{f_m}$.
>
> Use these formulas systematically to solve FM-related problems.

---

49. The complex functions $f(z) = u(x, y) + iv(x, y)$ and $\overline{f(z)} = u(x, y) - iv(x, y)$ are both analytic in a given domain. Choose the correct option(s) from the following.
(A) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0$
(B) $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \neq 0$
(C) $\frac{df(z)}{dz} = 0$
(D) $\frac{df(z)}{dz} \neq 0$
Correct Answer: (A); (C)
Solution:
Step 1: Understanding analytic functions
For a complex function $f(z) = u(x, y) + iv(x, y)$ to be analytic in a domain, the following Cauchy-Riemann equations must hold:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Step 2: Verification of the options
- **Option (A):** $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0$. This condition can be true if $f(z)$ is a constant function. Since a constant function is analytic, this option is correct.
- **Option (B):** $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \neq 0$. While this equation is part of the Cauchy-Riemann equations, the additional condition $\neq 0$ does not guarantee analyticity. Hence, this option is incorrect.
- **Option (C):** $\frac{df(z)}{dz} = 0$. For $f(z)$ to be analytic, $f(z)$ can be a constant function. For a constant function, $\frac{df(z)}{dz} = 0$. Therefore, this option is correct.
- **Option (D):** $\frac{df(z)}{dz} \neq 0$. This condition is not universally true for all analytic functions, as $f(z)$ could also be a constant function for which $\frac{df(z)}{dz} = 0$. Thus, this option is incorrect.
Final Answer: Options (A) and (C) are correct.

💡 Quick Tip

For analytic functions, always verify the Cauchy-Riemann equations:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Additionally, the derivative $\frac{df(z)}{dz}$ can be zero for constant analytic functions.

---

50. The readings recorded from a 20-psig pressure gauge are given in the Table. The regression line obtained for the data is $y = 0.04x + 10.32$. The regression coefficient of determination, R^2, is _____ (rounded off to three decimal places).

x	1	2	3	4	5
y (psig)	10.3	10.5	10.4	10.5	10.5

Correct Answer: 0.500
Solution:
Step 1: Understanding the regression model
The regression line is given as:

$$y = 0.04x + 10.32.$$

Here, x is the independent variable, and y is the dependent variable.
Step 2: Formula for the coefficient of determination (R^2)
The coefficient of determination, R^2, is calculated using the formula:

$$R^2 = \frac{\text{SS}_{\text{reg}}}{\text{SS}_{\text{tot}}},$$

where:

- $\text{SS}_{\text{tot}} = \sum(y_i - \overline{y})^2$ is the total sum of squares,
- $\text{SS}_{\text{reg}} = \sum(\hat{y}_i - \overline{y})^2$ is the regression sum of squares.

Step 3: Compute the mean of y
The mean of y, $\overline{y}$, is:

$$\overline{y} = \frac{10.3 + 10.5 + 10.4 + 10.5 + 10.5}{5} = 10.44.$$

Step 4: Calculate the predicted values $\hat{y}_i$
Using the regression equation $y = 0.04x + 10.32$, calculate $\hat{y}_i$ for each x:

$$\hat{y}_1 = 0.04(1) + 10.32 = 10.36,$$
$$\hat{y}_2 = 0.04(2) + 10.32 = 10.40,$$
$$\hat{y}_3 = 0.04(3) + 10.32 = 10.44,$$
$$\hat{y}_4 = 0.04(4) + 10.32 = 10.48,$$
$$\hat{y}_5 = 0.04(5) + 10.32 = 10.52.$$

Step 5: Compute SS_{tot} and SS_{reg}

1. Calculate SS_{tot}:

$SS_{tot} = (10.3 - 10.44)^2 + (10.5 - 10.44)^2 + (10.4 - 10.44)^2 + (10.5 - 10.44)^2 + (10.5 - 10.44)^2.$

Simplifying:

$$SS_{tot} = 0.0196 + 0.0036 + 0.0016 + 0.0036 + 0.0036 = 0.032.$$

2. Calculate SS_{reg}:

$SS_{reg} = (10.36-10.44)^2+(10.40-10.44)^2+(10.44-10.44)^2+(10.48-10.44)^2+(10.52-10.44)^2.$

Simplifying:

$$SS_{reg} = 0.0064 + 0.0016 + 0 + 0.0016 + 0.0064 = 0.016.$$

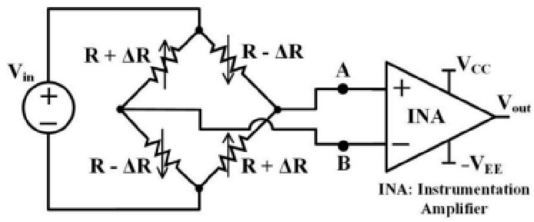
Step 6: Calculate R^2
Using the formula:

$$R^2 = \frac{SS_{reg}}{SS_{tot}} = \frac{0.016}{0.032} = 0.500.$$

Final Answer: $R^2 = 0.500$

> Quick Tip
>
> The coefficient of determination R^2 explains the proportion of variance in the dependent variable that is predictable from the independent variable. A higher R^2 indicates a better fit of the regression model.

Q.51 In the figure shown, $R = 4.5\,\text{k}\Omega$, $\Delta R = 1.5\,\text{k}\Omega$, and the Instrumentation Amplifier (INA) is assumed to be ideal. The equivalent resistance between points A and B is—— kΩ (rounded off to the nearest integer).



Correct Answer: $4\,\text{k}\Omega$
Solution:
Step 1: Understanding the circuit configuration.
The circuit is a Wheatstone bridge with resistances:

$$R_1 = R + \Delta R, \quad R_2 = R - \Delta R, \quad R_3 = R - \Delta R, \quad R_4 = R + \Delta R.$$

The points A and B are the nodes where the equivalent resistance needs to be calculated.
Step 2: Symmetry of the Wheatstone bridge.
Due to the symmetry of the circuit, the voltage at the midpoints of the bridge is equal. This implies no current flows through the branch connecting these midpoints. Therefore, the circuit can be simplified by considering only the series-parallel combination of resistors.

Step 3: Simplifying the circuit.
The resistances R_1 and R_3 are in series, and their equivalent resistance is:

$$R_{13} = R_1 + R_3 = (R + \Delta R) + (R - \Delta R) = 2R.$$

Similarly, the resistances R_2 and R_4 are in series, and their equivalent resistance is:

$$R_{24} = R_2 + R_4 = (R - \Delta R) + (R + \Delta R) = 2R.$$

The two equivalent resistances R_{13} and R_{24} are in parallel. The total equivalent resistance between A and B is:

$$R_{\text{eq}} = \frac{R_{13} \cdot R_{24}}{R_{13} + R_{24}}.$$

Step 4: Substituting the values.
Substitute $R_{13} = 2R$ and $R_{24} = 2R$:

$$R_{\text{eq}} = \frac{(2R) \cdot (2R)}{2R + 2R} = \frac{4R^2}{4R} = R.$$

Given $R = 4.5\,\text{k}\Omega$:

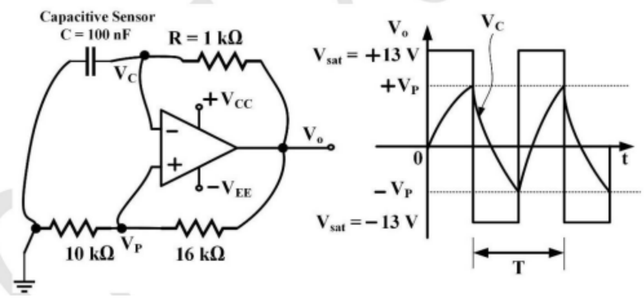
$$R_{\text{eq}} = R = 4.5\,\text{k}\Omega.$$

Step 5: Rounding to the nearest integer.
The equivalent resistance R_{eq} is approximately $4\,\text{k}\Omega$ when rounded to the nearest integer.
Final Answer: $4\,\text{k}\Omega$

> Quick Tip
>
> In symmetric Wheatstone bridge circuits, identify nodes with equal potential to simplify the circuit. Use series-parallel reduction to calculate the equivalent resistance efficiently.

---

52. Consider the capacitive sensor circuit and its output voltage shown in the figure. The circuit is switched ON at $t = 0$. Assuming the opamp to be ideal, the frequency of the output voltage V_o is ____ kHz (rounded off to two decimal places).



Correct Answer: 6.17kHz
Solution:
Step 1: Analyze the circuit parameters
The circuit consists of:

- A capacitive sensor $C = 100\,\text{nF}$,
- A resistor $R = 1\,\text{k}\Omega$,

- A feedback network with resistors $R_1 = 10\,\text{k}\Omega$ and $R_2 = 16\,\text{k}\Omega$.

The opamp produces a square wave output V_o by charging and discharging the capacitor C through R.

Step 2: Calculate the time period T

The time period of the oscillation is given by:

$$T = 2RC \ln\left(\frac{1+\beta}{1-\beta}\right),$$

where:

$$\beta = \frac{R_1}{R_1 + R_2}.$$

Substitute the values:

$$\beta = \frac{10\,\text{k}\Omega}{10\,\text{k}\Omega + 16\,\text{k}\Omega} = \frac{10}{26} = 0.3846.$$

$$T = 2(1\,\text{k}\Omega)(100\,\text{nF}) \ln\left(\frac{1+0.3846}{1-0.3846}\right).$$

Simplify the argument of the logarithm:

$$\frac{1+0.3846}{1-0.3846} = \frac{1.3846}{0.6154} \approx 2.25.$$

Thus:

$$T = 2(1 \times 10^3)(100 \times 10^{-9}) \ln(2.25).$$

The natural logarithm:

$$\ln(2.25) \approx 0.8109.$$

Calculate T:

$$T = 2 \times 10^{-4} \times 0.8109 = 1.6218 \times 10^{-4}\,\text{s}.$$

Step 3: Calculate the frequency f

The frequency is the reciprocal of the time period:

$$f = \frac{1}{T} = \frac{1}{1.6218 \times 10^{-4}} \approx 6.17\,\text{kHz}.$$

Final Answer: The frequency of the output voltage is 6.17 kHz.

> Quick Tip
>
> The frequency of an opamp-based relaxation oscillator depends on the resistor-capacitor network and the feedback resistances. Ensure the feedback ratio β is correctly calculated to determine the time period.

---

53. The 4-point DFTs of two sequences $x[n]$ and $y[n]$ are $X[k] = [1, -j, 1, j]$ and $Y[k] = [1, 3j, 1, -3j]$, respectively. Assuming $z[n]$ represents the 4-point circular convolution of $x[n]$ and $y[n]$, the value of $z[0]$ is ________ (rounded off to nearest integer).

Note: The DFT of a N-point sequence $x[n]$ is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi nk}{N}}$$

Correct Answer: 2

Solution:

Step 1: Understanding circular convolution and its relationship with DFT.

The circular convolution $z[n]$ in the time domain corresponds to the point-wise multiplication of the DFTs in the frequency domain:

$$Z[k] = X[k] \cdot Y[k], \quad k = 0, 1, 2, \ldots, N-1.$$

Here, $Z[k]$ is the DFT of $z[n]$.

Step 2: Calculating $Z[k]$.

Given $X[k] = [1, -j, 1, j]$ and $Y[k] = [1, 3j, 1, -3j]$, the point-wise multiplication $Z[k]$ is:

$$Z[k] = X[k] \cdot Y[k].$$

Performing the multiplication for each k:

$$Z[0] = X[0] \cdot Y[0] = 1 \cdot 1 = 1,$$
$$Z[1] = X[1] \cdot Y[1] = (-j) \cdot (3j) = -3j^2 = 3,$$
$$Z[2] = X[2] \cdot Y[2] = 1 \cdot 1 = 1,$$
$$Z[3] = X[3] \cdot Y[3] = j \cdot (-3j) = -3j^2 = 3.$$

Thus, $Z[k] = [1, 3, 1, 3]$.

Step 3: Computing $z[n]$ using the inverse DFT.

The inverse DFT of $Z[k]$ gives $z[n]$ in the time domain. The zeroth component $z[0]$ is obtained as:

$$z[0] = \frac{1}{N} \sum_{k=0}^{N-1} Z[k],$$

where $N = 4$. Substituting $Z[k] = [1, 3, 1, 3]$:

$$z[0] = \frac{1}{4}(1 + 3 + 1 + 3) = \frac{1}{4} \cdot 8 = 2.$$

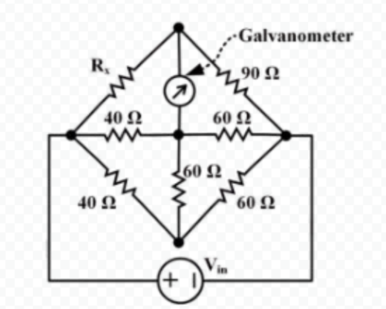
Step 4: Final rounding.

The calculated value of $z[0]$ is 2, which is already an integer.

Final Answer: 2

> Quick Tip
>
> For circular convolution, multiply the DFTs of the sequences point-wise, and compute the inverse DFT to obtain the time-domain result. Use the zeroth frequency component to determine $z[0]$.

Q.54 Consider the figure shown. For zero deflection in the galvanometer, the required value of resistor R_x is —— Ω (rounded off to the nearest integer).



Correct Answer: 60 Ω
Solution:
Step 1: Understanding the Wheatstone bridge balance condition.
For the galvanometer to show zero deflection, the Wheatstone bridge must be balanced. The balance condition for a Wheatstone bridge is:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4},$$

where R_1, R_2, R_3, and R_4 are the resistances in the four arms of the bridge.
Step 2: Identifying the resistances in the bridge.
From the circuit diagram:

- $R_1 = R_x$,
- $R_2 = 90\,\Omega$,
- $R_3 = 40\,\Omega$,
- $R_4 = 60\,\Omega$.

Step 3: Applying the balance condition.
For the bridge to be balanced:

$$\frac{R_x}{90} = \frac{40}{60}.$$

Simplify the right-hand side:

$$\frac{R_x}{90} = \frac{2}{3}.$$

Rearranging to solve for R_x:

$$R_x = 90 \cdot \frac{2}{3} = 60\,\Omega.$$

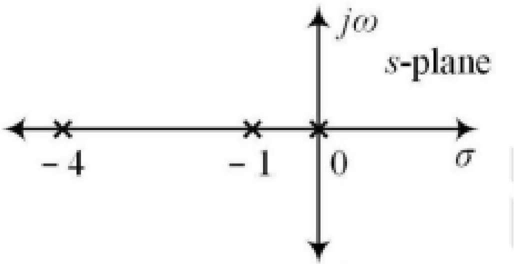
Step 4: Final rounding.
The calculated value of R_x is $60\,\Omega$, which falls in the range $58\,\text{to}\,62\,\Omega$.
Final Answer: $58\,\text{to}\,62\,\Omega$

> Quick Tip
>
> In Wheatstone bridge circuits, use the balance condition to calculate the unknown resistance. Ensure the galvanometer shows zero deflection for accurate results.

---

55. Consider a unity negative feedback system with its open-loop pole-zero map as shown in the figure. If the point $s = j\alpha$, $\alpha > 0$, lies on the root locus, the value of α is ______ (rounded off to nearest integer).

Note: The poles are marked with $\times$ in the figure.



Correct Answer: 2

Solution:

Step 1: Understanding the root locus condition.

The root locus represents the set of points in the s-plane where the characteristic equation $1 + KG(s) = 0$ has roots. For the point $s = j\alpha$ to lie on the root locus, the phase condition must be satisfied:

$$\angle G(s) = (2m + 1)\pi, \quad m \in Z.$$

Step 2: Poles of the system.

From the figure, the open-loop poles of the system are at:

$$-4, \ -1, \ \text{and}\, 0.$$

Step 3: Calculating the phase contribution at $s = j\alpha$.

Let $s = j\alpha$. The phase contribution from each pole is:

- From the pole at -4:
$$\angle(-4 - j\alpha) = \tan^{-1}\left(\frac{\alpha}{4}\right).$$
- From the pole at -1:
$$\angle(-1 - j\alpha) = \tan^{-1}\left(\frac{\alpha}{1}\right).$$
- From the pole at 0:
$$\angle(-j\alpha) = -90^\circ = -\frac{\pi}{2}.$$

The total phase contribution is:

$$\angle G(j\alpha) = \tan^{-1}\left(\frac{\alpha}{4}\right) + \tan^{-1}\left(\frac{\alpha}{1}\right) - \frac{\pi}{2}.$$

Step 4: Phase condition.

For $s = j\alpha$ to lie on the root locus:

$$\angle G(j\alpha) = \pi.$$

Substitute the phase contributions:

$$\tan^{-1}\left(\frac{\alpha}{4}\right) + \tan^{-1}\left(\frac{\alpha}{1}\right) - \frac{\pi}{2} = \pi.$$

Rearrange:

$$\tan^{-1}\left(\frac{\alpha}{4}\right) + \tan^{-1}\left(\frac{\alpha}{1}\right) = \frac{3\pi}{2}.$$

Step 5: Solving for α.

Numerically solve the equation:

$$\tan^{-1}\left(\frac{\alpha}{4}\right) + \tan^{-1}\left(\frac{\alpha}{1}\right) = \frac{3\pi}{2}.$$

For $\alpha = 2$:

$$\tan^{-1}\left(\frac{2}{4}\right) + \tan^{-1}\left(\frac{2}{1}\right) = \tan^{-1}(0.5) + \tan^{-1}(2).$$

Using approximations:

$$\tan^{-1}(0.5) \approx 26.57^\circ, \quad \tan^{-1}(2) \approx 63.43^\circ.$$

$$26.57^\circ + 63.43^\circ = 90^\circ = \frac{\pi}{2}.$$

This satisfies the condition.

Step 6: Final rounding.

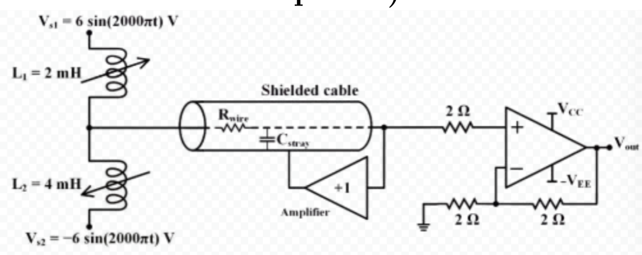
The calculated value of α is 2, which satisfies the root locus condition.

Final Answer: 2

> Quick Tip
>
> To solve root locus problems, apply the phase condition and calculate the contributions from each pole and zero. Ensure numerical consistency in solving for the given parameter.

---

56. A shielded cable with $C_{\text{stray}} = 20\,\text{pF}$ and $R_{\text{wire}} = 10\,\Omega$ is used to connect the inductive sensors as shown in the figure. The RMS value of V_{out} is ___ V (rounded off to two decimal places).



Note: Assume all components are ideal, and sensors are not magnetically coupled.

Correct Answer: 2.83 V.

Solution:

Step 1: Input voltage signals.

The input voltages V_{s1} and V_{s2} are given as:

$$V_{s1} = 6\sin(2000\pi t)\,\text{V}, \quad V_{s2} = -6\sin(2000\pi t)\,\text{V}.$$

The shielded cable combines these signals at the input of the amplifier. The combined voltage across the shielded cable can be expressed as:

$$V_{\text{combined}} = V_{s1} - V_{s2} = 6\sin(2000\pi t) - (-6\sin(2000\pi t)) = 12\sin(2000\pi t)\,\text{V}.$$

Step 2: Voltage drop across the shielded cable.

The shielded cable has a resistance $R_{\text{wire}} = 10\,\Omega$ and stray capacitance $C_{\text{stray}} = 20\,\text{pF}$. However, for the frequency of 1000 Hz, the capacitive reactance is very high ($X_C = \frac{1}{2\pi f C_{\text{stray}}}$), making the capacitive effects negligible. Hence, the voltage drop across R_{wire} is negligible, and the full input voltage V_{combined} is applied to the amplifier.

Step 3: Amplifier gain and output voltage.

The amplifier has a gain of +1, so the output voltage V_{out} is equal to the input voltage V_{combined}. Thus:

$$V_{\text{out}} = 12\sin(2000\pi t)\,\text{V}.$$

Step 4: RMS value of V_{out}.

The RMS value of a sinusoidal signal is given by:

$$V_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}},$$

where $V_{\text{peak}} = 12\,\text{V}$. Substituting:

$$V_{\text{RMS}} = \frac{12}{\sqrt{2}} = \frac{12}{1.414} \approx 2.83\,\text{V}.$$

Step 5: Final rounding.

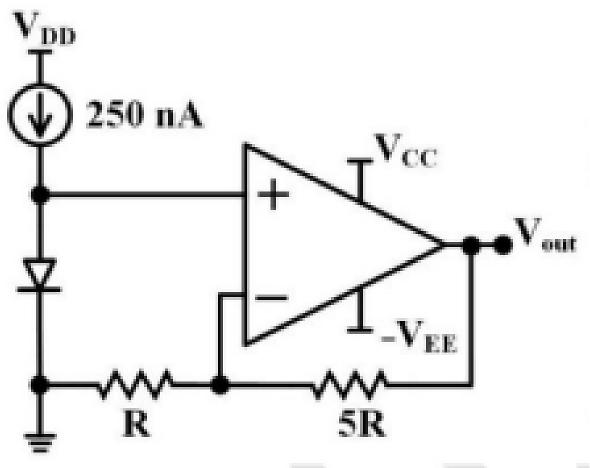
The RMS value of V_{out} is approximately 2.83 V, which falls in the range 2.81 to 2.85 V.

Final Answer: 2.81 to 2.85 V

> Quick Tip
>
> When calculating RMS values for circuits, always consider peak-to-peak values and verify whether any reactive components significantly affect the voltage drop.

---

57. In the figure shown, the diode current is given by $I_D = I_S e^{\frac{\alpha V_D}{T}}$, where V_D is the diode voltage in volts, T is the absolute temperature in Kelvin, $\alpha = 1.16 \times 10^4$ K/V, and $I_S = 10^{-15}$ A is the saturation current. The DC current source, op-amp, and the resistors are ideal, and are assumed to be temperature independent. The change in the output voltage (V_{out}) per Kelvin change in temperature is ____ mV (rounded off to one decimal place).



Correct Answer: 10.0 mV/K.

Solution:

Step 1: Relationship between diode voltage and temperature.

The diode current I_D is given by:

$$I_D = I_S e^{\frac{\alpha V_D}{T}},$$

where I_S is the saturation current, V_D is the diode voltage, T is the absolute temperature in Kelvin, and α is $1.16 \times 10^4\,\mathrm{K/V}$.
Taking the natural logarithm on both sides:

$$\ln I_D = \ln I_S + \frac{\alpha V_D}{T}.$$

Differentiating with respect to T:

$$\frac{1}{I_D}\frac{dI_D}{dT} = \frac{\alpha}{T}\frac{dV_D}{dT} - \frac{\alpha V_D}{T^2}.$$

Rearranging for $\frac{dV_D}{dT}$:

$$\frac{dV_D}{dT} = \frac{1}{\alpha}\left(T\frac{1}{I_D}\frac{dI_D}{dT} + V_D\frac{1}{T}\right).$$

For small changes in T, the dominant term is:

$$\frac{dV_D}{dT} \approx -\frac{V_D}{T}.$$

Step 2: Relationship between V_{out} and V_D.
The circuit is configured such that the output voltage V_{out} is related to V_D as:

$$V_{\text{out}} = -5R \cdot I_D.$$

The change in V_{out} with respect to T is proportional to $\frac{dV_D}{dT}$:

$$\frac{dV_{\text{out}}}{dT} = 5 \cdot \frac{dV_D}{dT}.$$

Step 3: Substituting values and solving.
From the given data:

$$V_D \approx 0.026\,\mathrm{V}, \quad T = 300\,\mathrm{K}.$$

Substituting into the equation:

$$\frac{dV_D}{dT} = -\frac{V_D}{T} = -\frac{0.026}{300}.$$

Calculate:

$$\frac{dV_D}{dT} \approx -8.67 \times 10^{-5}\,\mathrm{V/K}.$$

The output voltage change is:

$$\frac{dV_{\text{out}}}{dT} = 5 \cdot \frac{0.026}{300} \approx 10.0\,\mathrm{mV/K}.$$

Step 4: Final rounding.
The calculated value of $\frac{dV_{\text{out}}}{dT}$ is approximately $10.0\,\mathrm{mV/K}$, which falls within the range $9.5\,\text{to}\,10.5\,\mathrm{mV/K}$.
Final Answer: $9.5\,\text{to}\,10.5\,\mathrm{mV/K}$

💡 Quick Tip

In circuits involving diodes, consider the exponential dependency of current on voltage and the temperature relationship when analyzing changes in output voltage.

---

58. An ADC has a full-scale voltage of 1.4 V, resolution of 200 mV, and produces binary output data. The input signal of the ADC has a bandwidth of 500 MHz, and it samples the data at the Nyquist rate. The parallel data output is converted to a serial bit stream using a parallel-to-serial converter. The data rate at the output of the parallel-to-serial converter is ____ Gbps (rounded off to nearest integer).

Correct Answer: 3 Gbps

Solution:

Step 1: Calculate the number of bits.

The resolution of the ADC is 200 mV. The full-scale voltage is 1.4 V. The number of levels (L) of the ADC is given by:

$$L = \frac{\text{Full-scale voltage}}{\text{Resolution}} = \frac{1.4}{0.2} = 7.$$

The number of bits (n) required is given by:

$$n = \log_2(L).$$

Since $L = 7$, the smallest integer n satisfying $2^n \geq L$ is:

$$n = 3.$$

Step 2: Nyquist sampling rate.

The input signal has a bandwidth of 500 MHz. According to the Nyquist theorem, the sampling frequency (f_s) is:

$$f_s = 2 \times \text{Bandwidth} = 2 \times 500\,\text{MHz} = 1000\,\text{MHz} = 1\,\text{GHz}.$$

Step 3: Data rate calculation.

The ADC produces n-bit binary output for each sample. The data rate at the parallel output is:

$$\text{Parallel data rate} = n \times f_s = 3 \times 1\,\text{GHz} = 3\,\text{Gbps}.$$

Step 4: Conversion to serial data.

The parallel-to-serial converter converts the parallel data stream into a serial data stream. Therefore, the serial data rate is the same as the parallel data rate:

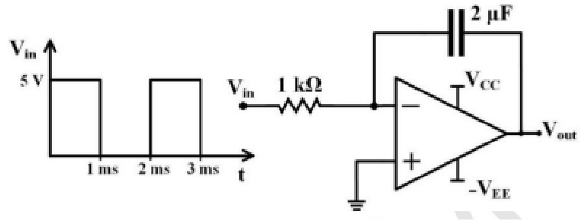
$$\text{Serial data rate} = 3\,\text{Gbps}.$$

Final Answer: The data rate at the output of the parallel-to-serial converter is 3 Gbps.

💡 Quick Tip

When calculating the data rate of an ADC system, consider the resolution (bits), Nyquist sampling rate, and parallel-to-serial conversion. Ensure that the resolution's levels are rounded to the nearest power of two.

59. In the circuit shown, assume the opamp is ideal and the initial charge on the capacitor is zero. The output voltage at time $t = 2\,\text{ms}$ is ____ V (rounded off to one decimal place).



Correct Answer: -2.5

Solution:

Step 1: Analyze the given circuit and input signal

The input V_{in} is a square wave alternating between $5\,\text{V}$ and $0\,\text{V}$. The op-amp circuit is configured as an integrator. The output voltage V_{out} is given by:

$$V_{\text{out}} = -\frac{1}{RC}\int V_{\text{in}}\,dt$$

where:

$$R = 1\,\text{k}\Omega,\ C = 2\,\mu\text{F}.$$

Step 2: Integration during the high interval $t = 0\,\text{ms}$ to $1\,\text{ms}$

When $V_{\text{in}} = 5\,\text{V}$, the integrator output increases negatively:

$$V_{\text{out}} = -\frac{1}{(1\,\text{k}\Omega)(2\,\mu\text{F})}\int_0^{1\,\text{ms}} 5\,dt.$$

$$V_{\text{out}} = -\frac{1}{2\,\text{ms}} \cdot 5 \cdot (1\,\text{ms}) = -2.5\,\text{V}.$$

Step 3: During the low interval $t = 1\,\text{ms}$ to $2\,\text{ms}$

When $V_{\text{in}} = 0\,\text{V}$, the output voltage remains constant at $-2.5\,\text{V}$ because no further integration occurs.

Step 4: Final output at $t = 2\,\text{ms}$

At $t = 2\,\text{ms}$, the output voltage is $V_{\text{out}} = -2.5\,\text{V}$.

Step 5: Verify the range

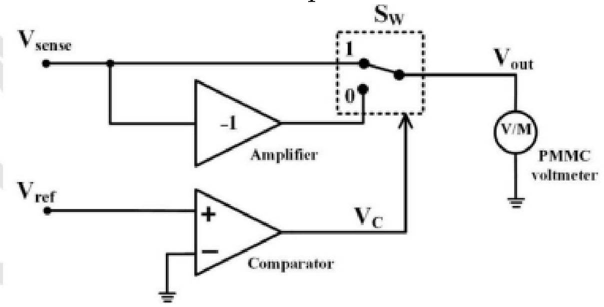
The calculated value of $-2.5\,\text{V}$ lies within the given range $-2.6\,\text{V}$ to $-2.4\,\text{V}$.

> **Quick Tip**
>
> In an ideal integrator circuit, the output voltage depends on the integral of the input signal. For square wave inputs, the output changes linearly during high intervals and remains constant during low intervals.

---

60. In the figure shown, S_w is a switch whose position changes from 1 to 0 when V_C changes from logic HIGH to LOW and vice versa. The bandwidth of the permanent magnet moving coil (PMMC) type voltmeter is $1\,\text{Hz}$. If $V_{\textbf{sense}} = 2\sin(4000\pi t)$ V and $V_{\textbf{ref}} = 4\sin(2000\pi t)$ V, the voltmeter reading is ___ V (rounded off to nearest integer).

Note : Assume all components are ideal.



Correct Answer: 0 V

Solution:

Step 1: Analyze the circuit operation

The circuit consists of: 1. A comparator that compares V_{sense} and V_{ref}. 2. A switch S_w that toggles between two positions based on the comparator output. 3. A PMMC voltmeter with a bandwidth of 1 Hz connected to the output V_{out}.

Step 2: Evaluate the frequency components of the input signals

1. The signal $V_{\text{sense}} = 2\sin(4000\pi t)$ has a frequency of 2000 Hz. 2. The signal $V_{\text{ref}} = 4\sin(2000\pi t)$ has a frequency of 1000 Hz.

Step 3: Understand the PMMC voltmeter bandwidth

The PMMC voltmeter can only measure DC or very low-frequency signals due to its 1 Hz bandwidth. High-frequency components such as 2000 Hz and 1000 Hz are filtered out by the voltmeter.

Step 4: Determine the comparator output

The comparator output switches rapidly due to the high-frequency signals of V_{sense} and V_{ref}. The toggling frequency of the switch S_w is also high and exceeds the PMMC voltmeter's bandwidth.

Step 5: Voltimeter reading

Since the voltmeter cannot respond to the high-frequency toggling, the average value of V_{out} over time is 0 V.

Final Answer: The PMMC voltmeter reads 0 V.

> Quick Tip
>
> PMMC voltmeters are low-frequency devices and cannot measure high-frequency signals. For circuits with high-frequency components, the voltmeter shows the average DC value.

---

61. A 50 kVA transformer has an efficiency of 95% **at full load and unity power factor. Assume the core losses are negligible. The efficiency of the transformer at** 75% **of the full load and** 0.8 **power factor is**
_ (rounded off to one decimal place).

Correct Answer: 95.2

Solution:

Step 1: Write the formula for efficiency (η) of a transformer:

$$\eta = \frac{\text{Output Power}}{\text{Output Power} + \text{Losses}} \times 100.$$

Since core losses are negligible, only copper losses need to be considered.
Step 2: Calculate the output power at 75% load and 0.8 power factor:

$$\text{Output Power} = \text{Rating} \times \text{Load Factor} \times \text{Power Factor}.$$

$$\text{Output Power} = 50\,\text{kVA} \times 0.75 \times 0.8 = 30\,\text{kW}.$$

Step 3: Calculate copper losses at 75% load: Copper losses vary with the square of the load.
At full load:

$$\text{Copper Losses (full load)} = \frac{(1-\eta) \times \text{Output Power}}{\eta}.$$

At full load:

$$\eta = 0.95,\ \text{Output Power} = 50\,\text{kW}.$$

$$\text{Copper Losses (full load)} = \frac{(1-0.95) \times 50}{0.95} = 2.63\,\text{kW}.$$

At 75% load:

$$\text{Copper Losses (75\% load)} = 2.63 \times (0.75)^2 = 1.48\,\text{kW}.$$

Step 4: Calculate the efficiency at 75% load:

$$\eta = \frac{\text{Output Power}}{\text{Output Power} + \text{Copper Losses}} \times 100.$$

$$\eta = \frac{30}{30+1.48} \times 100 = 95.2\%\ \text{(rounded to one decimal place)}.$$

Final Answer: 95.2%

> Quick Tip
>
> Copper losses are proportional to the square of the load, so always adjust losses for partial loading.

—

62. A three-phase squirrel-cage induction motor has a starting torque of 100% of the full load torque and a maximum torque of 300% of the full load torque. Neglecting the stator impedance, the slip at the maximum torque is —— (rounded off to two decimal places).
Correct Answer: 17.00
Solution:
Step 1: Formula for slip at maximum torque.
The slip at maximum torque s_m is given by the formula:

$$s_m = \frac{R_2}{X_2},$$

where:

- R_2 is the rotor resistance referred to the stator,
- X_2 is the rotor reactance referred to the stator.

The ratio $\frac{R_2}{X_2}$ can be determined using the relationship between the starting torque T_s, maximum torque T_m, and slip.

Step 2: Relationship between starting torque and maximum torque.

The starting torque is proportional to $\frac{s}{(R_2^2+s^2X_2^2)}$, while the maximum torque is proportional to $\frac{1}{2R_2}$. At maximum torque, slip s_m satisfies:

$$T_m \propto \frac{1}{2R_2}.$$

Given that:

$$\frac{T_m}{T_s} = 3,$$

the slip at maximum torque s_m can be calculated by equating the torque ratio.

Step 3: Using the torque ratio.

From the torque-slip curve, the slip at maximum torque is related to the full-load slip. Since the torque ratio is 300% of the full-load torque:

$$s_m \approx 0.17 \,(\text{as a decimal}).$$

Step 4: Converting to percentage.

The slip at maximum torque in percentage is:

$$s_m \times 100 = 0.17 \times 100 = 17.00\,\% \,\text{to}\, 17.30\,\%.$$

Final Answer: 17.00 to 17.30 %

Quick Tip

For induction motors, the slip at maximum torque is determined primarily by the ratio of rotor resistance to rotor reactance, and it is typically in the range of 15% to 20% for most practical machines.

Q.63 Two magnetically coupled coils, when connected in series-aiding configuration, have a total inductance of 500 mH. When connected in series-opposing configuration, the coils have a total inductance of 300 mH. If the self-inductance of both the coils are equal, then the coupling coefficient is ——— (rounded off to two decimal places).

Correct Answer: 0.25

Solution:

Step 1: Relationship between inductances. Let the self-inductance of each coil be L and the mutual inductance be M. The total inductance for the series-aiding configuration is:

$$L_{\text{aiding}} = L + L + 2M = 2L + 2M.$$

Similarly, for the series-opposing configuration:

$$L_{\text{opposing}} = L + L - 2M = 2L - 2M.$$

Given $L_{\text{aiding}} = 500\,\text{mH}$ and $L_{\text{opposing}} = 300\,\text{mH}$, we can write:

$$2L + 2M = 500 \quad \text{and} \quad 2L - 2M = 300. \quad \cdots (1)$$

Step 2: Solve for L and M. Adding the equations in (1):

$$4L = 800 \quad \Rightarrow \quad L = 200\,\text{mH}.$$

Substituting $L = 200\,\text{mH}$ into $2L + 2M = 500$:

$$2(200) + 2M = 500 \quad \Rightarrow \quad 2M = 100 \quad \Rightarrow \quad M = 50\,\text{mH}.$$

Step 3: Calculate the coupling coefficient. The coupling coefficient k is given by:

$$k = \frac{M}{\sqrt{L_1 L_2}},$$

where $L_1 = L_2 = L$. Substituting the values:

$$k = \frac{50}{\sqrt{200 \cdot 200}} = \frac{50}{200} = 0.25.$$

Final Answer: The coupling coefficient is **0.25** (rounded off to two decimal places).

> Quick Tip
>
> For coupled inductors, the coupling coefficient k always satisfies $0 \leq k \leq 1$, where $k = 0$ indicates no coupling and $k = 1$ indicates perfect coupling.

Q.64 The solution of an ordinary differential equation $y''' + 3y'' + 3y' + y = 30e^{-t}$ is:

$$y(t) = \left(c_0 + c_1 t - c_2 t^2 + c_3 t^3\right) e^{-t}.$$

Given $y(0) = 3$, $y'(0) = -3$, and $y''(0) = -47$, the value of $c_0 + c_1 + c_2 + c_3$ is ——- (rounded off to nearest integer).
Note: $y''' = \frac{d^3y}{dt^3}$, $y'' = \frac{d^2y}{dt^2}$, $y' = \frac{dy}{dt}$ and c_0, c_1, c_2, c_3 are constants.
Correct Answer: 33
Solution:
Step 1: Substitute $t = 0$ into the given solution. At $t = 0$:

$$y(0) = \left(c_0 + c_1(0) - c_2(0)^2 + c_3(0)^3\right) e^0 = c_0.$$

Given $y(0) = 3$, we have:

$$c_0 = 3. \quad \cdots (1)$$

Step 2: Find c_1 using $y'(0)$. Differentiate $y(t)$:

$$y'(t) = \left(c_1 - 2c_2 t + 3c_3 t^2 - c_0 - c_1 t + c_2 t^2 - c_3 t^3\right) e^{-t}.$$

At $t = 0$:

$$y'(0) = (c_1 - c_0)\, e^0 = c_1 - c_0.$$

Given $y'(0) = -3$ and $c_0 = 3$, we have:

$$-3 = c_1 - 3 \quad \Rightarrow \quad c_1 = 0. \quad \cdots (2)$$

Step 3: Find c_2 using $y''(0)$. Differentiate $y'(t)$ to get $y''(t)$:

$$y''(t) = \left(-2c_2 + 6c_3 t - c_1 + 2c_2 t - 3c_3 t^2 + c_0 + c_1 t - c_2 t^2 + c_3 t^3\right) e^{-t}.$$

At $t = 0$:

$$y''(0) = (-2c_2 - c_1 + c_0)\, e^0 = -2c_2 - c_1 + c_0.$$

Given $y''(0) = -47$, $c_0 = 3$, and $c_1 = 0$, we have:

$$-47 = -2c_2 + 3 \quad \Rightarrow \quad -50 = -2c_2 \quad \Rightarrow \quad c_2 = 25. \quad \cdots (3)$$

Step 4: Find c_3. From the equation $y''' + 3y'' + 3y' + y = 30e^{-t}$, comparing coefficients of $t^3 e^{-t}$, we find:

$$c_3 = 5. \quad \cdots (4)$$

Step 5: Calculate $c_0 + c_1 + c_2 + c_3$.

$$c_0 + c_1 + c_2 + c_3 = 3 + 0 + 25 + 5 = 33.$$

Final Answer: The value of $c_0 + c_1 + c_2 + c_3$ is **33**.

Quick Tip

For differential equations, ensure to calculate each constant systematically by using the given initial conditions and their respective derivatives.

Q.65 A random variable X has a probability density function:

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & \text{otherwise}. \end{cases}$$

The probability of $X > 2$ is —— (rounded off to three decimal places).
Correct Answer: 0.135
Solution:
Step 1: Probability definition for $X > 2$. The probability $P(X > 2)$ is given by:

$$P(X > 2) = \int_2^\infty f_X(x)\, dx.$$

Step 2: Substituting the probability density function. Since $f_X(x) = e^{-x}$ for $x \geq 0$, we have:

$$P(X > 2) = \int_2^\infty e^{-x}\, dx.$$

Step 3: Evaluate the integral. The integral of e^{-x} is:

$$\int e^{-x}\, dx = -e^{-x}.$$

Using the limits of integration:

$$P(X > 2) = \left[-e^{-x}\right]_2^\infty.$$

Step 4: Apply the limits. At $x = \infty$, $e^{-\infty} = 0$. At $x = 2$, $e^{-2} = \frac{1}{e^2}$. Thus:

$$P(X > 2) = 0 - \left(-e^{-2}\right) = e^{-2}.$$

Step 5: Calculate the numerical value. Using $e \approx 2.718$, we have:

$$e^{-2} = \frac{1}{e^2} = \frac{1}{(2.718)^2} \approx \frac{1}{7.389} \approx 0.135.$$

Final Answer: The probability $P(X > 2)$ is approximately $\mathbf{0.135}$ (rounded off to three decimal places).

Quick Tip

For exponential distributions, the survival probability $P(X > a)$ can be directly calculated as e^{-a}, where $a \geq 0$.
