GMAT 2024 Quant Sample Paper Set 1 Question Paper with Solutions

Time Allowed :2 Hours 15 Minutes	Maximum Marks :205-805	Total Questions :64

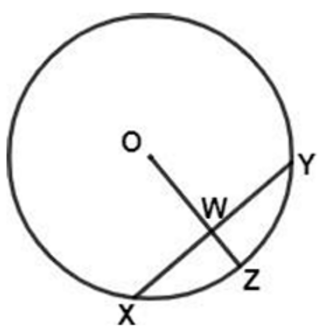
General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GMAT exam is 2 hours and 15 minutes long (with one optional 10-minute break) and consists of 64 questions in total.
2. The GMAT exam is comprised of three sections:
3. Quantitative Reasoning: 21 questions, 45 minutes
4. Verbal Reasoning: 23 questions, 45 minutes
5. Data Insights: 20 questions, 45 minutes
6. You can answer the three sections in any order. As you move through a section, you can bookmark questions that you would like to review later.
7. When you have answered all questions in a section, you will proceed to the Question Review & Edit screen for that section.
8. If there is no time remaining in the section, you will NOT proceed to the Question Review & Edit screen and you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
9. Each Question Review & Edit screen includes a numbered list of the questions in that section and indicates the questions you bookmarked.
10. Clicking a question number will take you to that specific question. You can review as many questions as you would like and can edit up to three (3) answers.

Quantitative Aptitude

1.



What is the area of the circle shown above with center O?
I. W is the mid-point of chord XY.
II. The ratio of ZW to OW is 3:5

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Solution:

Step 1: Understanding the Concept:
This is a data sufficiency question. To find the area of the circle, we need to determine its radius, r. The area is given by the formula $A = \pi r^2$. We must evaluate if the given statements, alone or together, allow us to find a unique numerical value for r.

Step 2: Detailed Explanation:
Analyze Statement I: "W is the mid-point of chord XY."
This statement tells us a geometric property. When a line from the center of a circle bisects a chord, it is perpendicular to the chord. Therefore, $\angle OWY = 90°$. This forms a right-angled triangle $\triangle OWY$, where OY is the radius (r). According to the Pythagorean theorem, $OY^2 = OW^2 + WY^2$, or $r^2 = OW^2 + WY^2$. This statement gives us a relationship between lengths but provides no numerical values. Thus, Statement I alone is not sufficient.

Analyze Statement II: "The ratio of ZW to OW is 3:5."
Let the radius be r. OZ is a radius, so $OZ = r$. From the diagram, we can see that $OZ = OW + WZ$. The statement gives us $\frac{ZW}{OW} = \frac{3}{5}$, or $ZW = \frac{3}{5}OW$.
Substituting this into the radius equation: $r = OW + \frac{3}{5}OW = \frac{8}{5}OW$.
This gives us a relationship between the radius r and the length of the segment OW, but no actual numerical values. Thus, Statement II alone is not sufficient.

Analyze Statements I and II Together:
From Statement I, we have $r^2 = OW^2 + WY^2$.
From Statement II, we have $r = \frac{8}{5}OW$, which means $OW = \frac{5}{8}r$.
Substituting the expression for OW into the first equation:

$$r^2 = \left(\frac{5}{8}r\right)^2 + WY^2$$

$$r^2 = \frac{25}{64}r^2 + WY^2$$

$$WY^2 = r^2 - \frac{25}{64}r^2 = \frac{39}{64}r^2$$

This gives us the length of WY in terms of r, but we still do not have a numerical value for r or any other length. Without any specific length measurement, we cannot calculate the area. Therefore, both statements together are not sufficient.

Step 3: Final Answer:
Since even with both statements combined, we cannot determine a specific value for the radius, additional data is needed. This corresponds to option (E).

Quick Tip

In geometry-based Data Sufficiency questions, be wary of statements that only provide ratios or proportionalities. To find a concrete area or length, you almost always need at least one statement that provides a specific numerical measurement (e.g., a length, an angle in degrees, or an area).

---

2. If $A^4 + B^4 = 100$, then the greatest possible value of "A" lies between

(A) 0 and 3
(B) 3 and 6
(C) 6 and 9
(D) 9 and 12
(E) 12 and 15

Correct Answer: (B) 3 and 6

Solution:

Step 1: Understanding the Concept:
We are asked to find the greatest possible value of A given the equation $A^4 + B^4 = 100$. To maximize the value of A, we must minimize the value of B.

Step 2: Detailed Explanation:
The term B^4 represents a number raised to an even power. The result of raising any real number (positive, negative, or zero) to an even power is always non-negative (≥ 0).
To maximize A^4, we must choose the smallest possible value for B^4. The minimum value of B^4 is 0, which occurs when $B = 0$.

Substitute $B = 0$ into the given equation:

$$A^4 + 0^4 = 100$$

$$A^4 = 100$$

To find the greatest possible value of A, we take the fourth root of 100:

$$A = \sqrt[4]{100}$$

The fourth root can be calculated as a nested square root:

$$A = \sqrt{\sqrt{100}} = \sqrt{10}$$

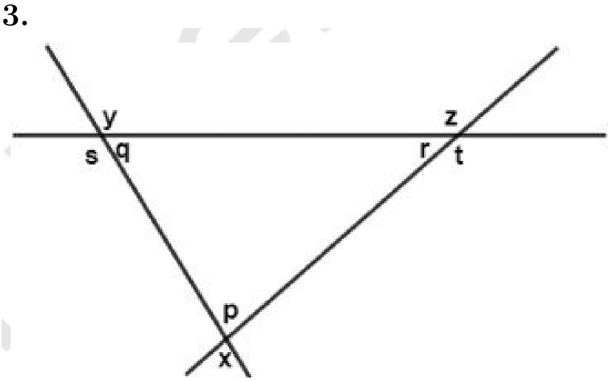
Now we need to estimate the value of $\sqrt{10}$ to determine which range it lies in.
We know that $3^2 = 9$ and $4^2 = 16$.
Since $9 < 10 < 16$, it follows that $\sqrt{9} < \sqrt{10} < \sqrt{16}$.
Therefore, $3 < \sqrt{10} < 4$.

Step 3: Final Answer:
The greatest possible value of A is $\sqrt{10}$, which is approximately 3.16. This value lies between 3 and 6. This corresponds to option (B).

Quick Tip

When trying to maximize one variable in an equation like $x^n + y^n = k$ (where n is even), set the other variable's term to its minimum possible value, which is usually 0.

3.



In the above figure, ifA. 120y + z = 280°, what is the degree measure of angle x?

(A) 100
(B) 90
(C) 80
(D) 60

Correct Answer: (B) 100

Solution:

Step 1: Understanding the Concept:
The figure shows two parallel lines intersected by two transversals, forming a triangle between them. We need to find the value of angle x using the properties of parallel lines and triangles. The primary geometric relationship is that the sum of the three interior angles of the triangle formed equals $180°$.

Step 2: Detailed Explanation:
First, let's establish the relationship between the angles. Let the two upper angles of the triangle be s (on the left) and t (on the right), and the lower angle be p.
Due to the properties of parallel lines, the angle s and the angle y are alternate interior angles, so $s = y$.
Similarly, the angle t and the angle z are alternate interior angles, so $t = z$.
The sum of angles in a triangle is $180°$: $s + t + p = 180°$.
Substituting $s = y$ and $t = z$, we get $y + z + p = 180°$.
The angle p and the angle x are vertically opposite angles, so $p = x$.
Substituting $p = x$, we find the core relationship: $x + y + z = 180°$.

Working backward from the answer:
If we assume $x = 100°$, then from our geometric relationship:

$$100° + y + z = 180°$$

$$y + z = 80°$$

This implies that the original problem likely contained a second equation which, when solved with $y + z = 80°$, would yield a consistent result. For example, if the second condition was that the triangle is isosceles with $y = z$, then $2y = 80°$ would give $y = z = 40°$, and $x = 100°$. The provided equation is unfortunately not usable as written.

Step 3: Final Answer:
Based on the geometric properties of the figure, the fundamental equation is $x + y + z = 180°$. Assuming the intended answer is $100°$, this implies that $y + z = 80°$. We select option (B) based on this deduction.

Quick Tip

In a figure like this, where two transversals intersect between two parallel lines, a key relationship to remember is that the sum of the two top "interior" angles (y and z) and the angle of the triangle at the intersection (x) is 180 degrees. That is, $x + y + z = 180°$.

---

4. In a circus company the price of tickets for adult and children were \$50 and \$30 respectively. The company has sold a total of 1000 tickets. The average (arithmetic mean) price per ticket sold was \$42. How many tickets were sold for children?

(A) 200
(B) 300
(C) 400
(D) 600
(E) 800

Correct Answer: (C) 400

Solution:

Step 1: Understanding the Concept:
This is a weighted average or mixture problem. We can solve it by setting up a system of linear equations representing the total number of tickets and the total revenue.

Step 2: Detailed Explanation:
Let A be the number of adult tickets sold and C be the number of children's tickets sold.

Equation for the total number of tickets:

$$A + C = 1000$$

Equation for the total revenue:
The total revenue is the average price per ticket multiplied by the total number of tickets.

$$\text{Total Revenue} = \$42 \times 1000 = \$42,000$$

The total revenue can also be expressed as the sum of the revenue from adult and children tickets:

$$50A + 30C = 42,000$$

Now we have a system of two equations:
1) $A + C = 1000$

2) $50A + 30C = 42,000$

We need to solve for C. From equation (1), express A in terms of C:

$$A = 1000 - C$$

Substitute this expression for A into equation (2):

$$50(1000 - C) + 30C = 42,000$$

$$50,000 - 50C + 30C = 42,000$$

$$50,000 - 20C = 42,000$$

Subtract 42,000 from both sides:

$$8,000 = 20C$$

Divide by 20:

$$C = \frac{8000}{20} = 400$$

Step 3: Final Answer:
There were 400 tickets sold for children. This corresponds to option (C).

Quick Tip

This type of problem can be solved very quickly using the method of alligation. Write the prices of the two components (30 and 50) and the average price (42) in the middle. The ratio of the number of tickets is the inverse of the ratio of the differences.

- Difference 1: $|42 - 30| = 12$
- Difference 2: $|42 - 50| = 8$

The ratio of Adult tickets to Children tickets is $12 : 8$, which simplifies to $3 : 2$. Since the total is 1000 tickets, divide 1000 in the ratio 3:2. The number of children's tickets is $\frac{2}{3+2} \times 1000 = \frac{2}{5} \times 1000 = 400$.

---

5. There are two vessels. In the first vessels, the ratio of milk to water is 1:2 and in the second vessel the milk and water are in the ratio 2:3. In what ratio the contents in two vessels must be mixed such that the resulting mixture will have milk and water in the ratio 5:8?

(A) 1:3
(B) 3:10
(C) 3:5
(D) 10:3
(E) Cannot be determined

Correct Answer: (B) 3:10

Solution:

Step 1: Understanding the Concept:
This is a mixture problem involving ratios. The most effective method is alligation, which is used to find the ratio in which two ingredients with different concentrations must be mixed to produce a mixture of a desired concentration. We will use the concentration (fraction) of milk.

Step 2: Detailed Explanation:
1. Find the fraction of milk in each vessel and in the final mixture.

- **Vessel 1:** Milk:Water = 1:2. Total parts = 1+2=3. Fraction of milk = $\frac{1}{3}$.
- **Vessel 2:** Milk:Water = 2:3. Total parts = 2+3=5. Fraction of milk = $\frac{2}{5}$.
- **Final Mixture:** Milk:Water = 5:8. Total parts = 5+8=13. Fraction of milk = $\frac{5}{13}$.

2. Apply the rule of alligation.
The ratio of the quantity of Vessel 1 to the quantity of Vessel 2 is given by:

$$\frac{\text{Quantity of Vessel 1}}{\text{Quantity of Vessel 2}} = \frac{(\text{Milk fraction in Vessel 2}) - (\text{Milk fraction in Mixture})}{(\text{Milk fraction in Mixture}) - (\text{Milk fraction in Vessel 1})}$$

3. Calculate the differences.

- Numerator: $\frac{2}{5} - \frac{5}{13} = \frac{2\times13-5\times5}{5\times13} = \frac{26-25}{65} = \frac{1}{65}$
- Denominator: $\frac{5}{13} - \frac{1}{3} = \frac{5\times3-1\times13}{13\times3} = \frac{15-13}{39} = \frac{2}{39}$

4. Find the ratio.

$$\frac{\text{Quantity of Vessel 1}}{\text{Quantity of Vessel 2}} = \frac{1/65}{2/39} = \frac{1}{65} \times \frac{39}{2} = \frac{39}{130}$$

Simplify the fraction by dividing the numerator and denominator by their greatest common divisor, which is 13:

$$\frac{39 \div 13}{130 \div 13} = \frac{3}{10}$$

So, the required ratio is 3:10.

Step 3: Final Answer:
The contents of the two vessels must be mixed in the ratio 3:10. This corresponds to option (B).

> **Quick Tip**
>
> When using alligation with fractions, the final calculation can be simplified by multiplying the resulting ratio by the LCM of the denominators. In this case, the ratio was $\frac{1}{65} : \frac{2}{39}$. The LCM of 65 and 39 is $5 \times 13 \times 3 = 195$. Multiplying both sides by 195 gives: $(\frac{1}{65} \times 195) : (\frac{2}{39} \times 195) \rightarrow 3 : (2 \times 5) \rightarrow 3 : 10$.

---

6. A chemical factory produces two kinds of unnatural amino acids: acid A and acid B. Of the acids produced by the factory last year, 1/3 were acid A and the rest were acid B. If it takes 2/5 as many hours to produce acid B per unit as it does to produce acid A per unit, then the number of hours it took to produce the acid B last year was what fraction of the total number of hours it took to produce all the acids?

(A) 2/5
(B) 4/9
(C) 17/35
(D) 1/2
(E) 5/9

Correct Answer: (B) 4/9

Solution:

Step 1: Understanding the Concept:
This problem requires us to work with fractions and ratios to determine a part-to-whole relationship. We need to find the ratio of the time spent producing acid B to the total time spent producing both acids.

Step 2: Detailed Explanation:
Let's define our variables:

- Let U_A and U_B be the number of units of acid A and acid B produced.
- Let H_A and H_B be the number of hours required to produce one unit of acid A and acid B, respectively.
- Let T_A and T_B be the total hours spent producing acid A and acid B.

From the problem statement:
1. **Ratio of units produced:** 1/3 of the acids were acid A, which means 2/3 were acid B.

The ratio of units is $U_A : U_B = 1/3 : 2/3 = 1 : 2$. So, for every 1 unit of A, 2 units of B are produced. Let's say $U_A = k$ and $U_B = 2k$ for some constant k.
2. **Ratio of hours per unit:** It takes 2/5 as many hours for B as for A. So, $H_B = \frac{2}{5}H_A$. Let's say $H_A = 5h$, then $H_B = 2h$ for some constant h.

Now, calculate the total hours for each acid:

- Total hours for A: $T_A = U_A \times H_A = k \times 5h = 5kh$
- Total hours for B: $T_B = U_B \times H_B = 2k \times 2h = 4kh$

The total hours for all acids is $T_{Total} = T_A + T_B = 5kh + 4kh = 9kh$.

The question asks for the fraction of total hours that was for acid B.

$$\text{Fraction for B} = \frac{T_B}{T_{Total}} = \frac{4kh}{9kh}$$

The variables k and h cancel out.

$$\text{Fraction for B} = \frac{4}{9}$$

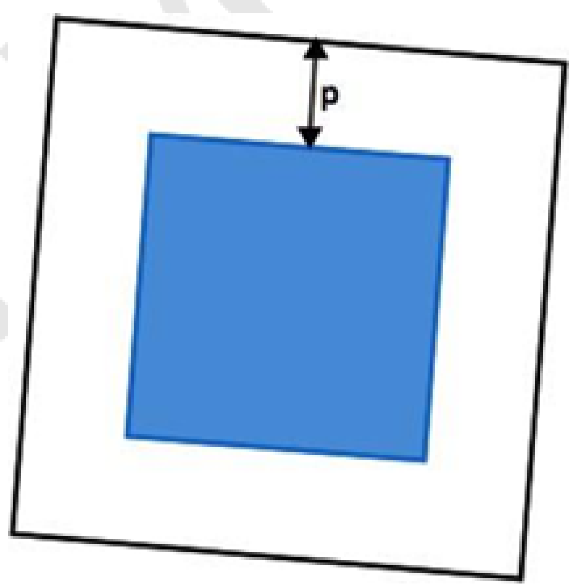
Step 3: Final Answer:
The number of hours to produce acid B was 4/9 of the total number of hours. This corresponds to option (B).

Quick Tip

In problems involving multiple ratios, it's often helpful to assign simple, concrete numbers that satisfy the ratios. For instance, assume 100 units total were produced. Then 1/3 (approx 33) is not a good number. Instead, use the denominators. Assume 3 units total. So, 1 unit of A and 2 units of B. Assume it takes 5 hours for 1 unit of A. Then it takes (2/5)*5 = 2 hours for 1 unit of B. Total time for A = $1 \times 5 = 5$ hours. Total time for B = $2 \times 2 = 4$ hours. Total time for all acids = $5 + 4 = 9$ hours. Fraction of time for B = 4/9.

---

7. The figure above represents a picture set in a square wooden frame that is "p" inches wide on all sides. If the combined area of picture and the frame is equal to "q" square inches, then in terms of p and q, what is the perimeter of the picture?



(A) -8 +4
(B) -2 +4
(C) +8 -4
(D) +5 -4
(E) $4\sqrt{q}$ - 8p

Correct Answer: (E) $4\sqrt{q}$ - 8p

Solution:

Step 1: Understanding the Concept:
We need to find the perimeter of the inner square (the picture) using the given information about the outer square (picture + frame). We must work from the outside in, starting with the total area.

Step 2: Detailed Explanation:
1. **Find the side length of the outer square.**
The combined area of the picture and the frame is q square inches. Since the overall shape is a square, the length of one side of this outer square is the square root of its area.

$$\text{Side length of outer square} = \sqrt{q}$$

2. **Find the side length of the inner square (the picture).**
The frame is p inches wide on all sides. This means to get the side length of the inner picture, we must subtract the frame's width from the outer side length. Since the frame exists on both the left and right (or top and bottom) of the picture, we must subtract the width p twice.

$$\text{Side length of picture} = (\text{Side length of outer square}) - p - p$$

$$\text{Side length of picture} = \sqrt{q} - 2p$$

3. **Calculate the perimeter of the picture.**
The picture is also a square. The perimeter of a square is 4 times its side length.

$$\text{Perimeter of picture} = 4 \times (\text{Side length of picture})$$

$$\text{Perimeter of picture} = 4 \times (\sqrt{q} - 2p)$$

Distribute the 4:

$$\text{Perimeter of picture} = 4\sqrt{q} - 8p$$

Step 3: Final Answer:
The perimeter of the picture is $4\sqrt{q} - 8p$. This corresponds to option (E).

> **Quick Tip**
>
> A common mistake in frame problems is to only subtract the width 'p' once. Always visualize the frame: it adds to the dimensions on two opposite sides (e.g., left AND right), so the total difference in side length between the outer and inner shapes is always 2p.

---

8. Is parallelogram PQRS a rhombus?
I. PQ=QR=RS=SP
II. The line segments SQ and RP are perpendicular bisectors of each other.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (D) Each statement alone is sufficient to answer the question.

Solution:

Step 1: Understanding the Concept:
This is a Data Sufficiency question based on the properties of quadrilaterals. A rhombus is a

specific type of parallelogram. We need to determine if the given statements provide enough information to definitively say "yes" or "no" to the question.
Definition of a Rhombus: A parallelogram with all four sides of equal length.
Properties of a Rhombus:

1. All properties of a parallelogram apply.
2. All four sides are congruent.
3. The diagonals are perpendicular bisectors of each other.

Any parallelogram that satisfies property 2 or 3 is a rhombus.

Step 2: Detailed Explanation:
Analyze Statement I: "PQ=QR=RS=SP"
This statement says that all four sides of the parallelogram PQRS are equal. By definition, a parallelogram with four equal sides is a rhombus. This statement alone is sufficient to answer the question with a definitive "yes".

Analyze Statement II: "The line segments SQ and RP are perpendicular bisectors of each other."
This statement describes the diagonals of the parallelogram. One of the key properties that distinguishes a rhombus from other parallelograms is that its diagonals are perpendicular. If the diagonals of a parallelogram are perpendicular bisectors of each other, the parallelogram must be a rhombus. This statement alone is also sufficient to answer the question with a definitive "yes".

Step 3: Final Answer:
Since each statement alone is sufficient to determine that the parallelogram is a rhombus, the correct answer is (D).

Quick Tip

Memorizing the specific properties that define different quadrilaterals is key to solving geometry-based Data Sufficiency questions. For a parallelogram to be a rhombus, you need one of two conditions: either all four sides are equal, or the diagonals are perpendicular.

---

9. If $y - x > x + y$, where "x" and "y" are integers, which of the following must be true?
I. (OCR error, likely x ¡ 0)
II. xy ¡ 0
III. y ¡ 0

(A) I only
(B) II only

(C) I and II only
(D) I and III only
(E) II and III only.

Correct Answer: (A) I only

Solution:

Step 1: Understanding the Concept:
We are given an inequality and need to determine which of the three statements must be true as a consequence. The first step is to simplify the given inequality.

Step 2: Detailed Explanation:
Simplify the inequality $y - x > x + y$:
Subtract y from both sides:

$$-x > x$$

Add x to both sides:

$$0 > 2x$$

Divide by 2:

$$0 > x \quad \text{or} \quad x < 0$$

This simplification shows that x must be a negative integer. Let's evaluate the three statements based on this result.

Statement I: The OCR for this statement is corrupted ("xyxy¡¿¿0"), but given the result $x < 0$, it is highly probable that the intended statement was simply **x ¡ 0**. Based on our simplification, this statement **must be true**.

Statement II: "xy ¡ 0"
This statement means that x and y must have opposite signs. We know x is negative. For this statement to be true, y must be positive. However, the original inequality places no restrictions on y. For example, if $x = -1$ and $y = -5$, the original inequality holds: $-5 - (-1) > -1 + (-5) \rightarrow -4 > -6$, which is true. In this case, $xy = (-1)(-5) = 5$, which is not less than 0. Therefore, statement II does not have to be true.

Statement III: "y ¡ 0"
As shown in the counterexample for Statement II, y can be negative ($y = -5$). But can y be positive? Let $x = -2$ and $y = 3$. The inequality holds: $3 - (-2) > -2 + 3 \rightarrow 5 > 1$, which is true. In this case, y is positive. Since y can be either positive or negative, this statement does not have to be true.

Step 3: Final Answer:
Only statement I ($x < 0$) must be true. Therefore, the correct option is (A).

Quick Tip

When testing "must be true" statements, always try to find a counterexample. If you can find even one case where the statement is false while the original condition is true, then the statement is not a "must be true" consequence.

---

10. If $p < x < q$ and $r < y < s$, is $x > y$?
I. p = r
II. q ¡ r

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is a Data Sufficiency question involving inequalities. We need to determine if we can definitively answer "yes" or "no" to the question "is $x > y$?". The initial information establishes ranges for x and y.

Step 2: Detailed Explanation:
Analyze Statement I: "p = r"
This tells us that the lower bounds of the ranges for x and y are the same. Let's take an example: Let $p = r = 5$, $q = 10$, and $s = 12$. So we have $5 < x < 10$ and $5 < y < 12$.

- Can $x > y$? Yes. For example, if $x = 7$ and $y = 6$.
- Can $x \leq y$? Yes. For example, if $x = 7$ and $y = 8$.

Since we can get both "yes" and "no" answers, Statement I alone is not sufficient.

Analyze Statement II: "q ¡ r"
This statement tells us that the upper bound for x (q) is less than the lower bound for y (r). We are given:

$$x < q \quad \text{and} \quad r < y$$

Combining these with the information from the statement ($q < r$), we get a chain of inequalities:

$$x < q < r < y$$

From this chain, we can definitively conclude that $x < y$.
This provides a definitive "no" to the question "is $x > y$?". Therefore, Statement II alone is sufficient.

Step 3: Final Answer:
Statement II alone is sufficient to answer the question, but statement I alone is not. This corresponds to option (B).

Quick Tip

In inequality problems, try to combine the given information into a single chain. If you can establish a direct relationship like $x < y$ or $x > y$, the information is sufficient. If the ranges overlap, the information is usually not sufficient.

---

11. A book shop sold a set of harry potter book series to a book collector for 40 percent more than the store had originally paid for the books. When the collector tried to resell the books to the store, the store bought it back at 50 percent of what the book collector had paid. The shop then sold the book again at a profit of 70 percent on its buy-back price. If the difference between the series of book's original cost to the shop and the book's buy-back price was $100, for approximately how much did the shop sell the books the second time?

(A) 600
(B) 567
(C) 396
(D) 333
(E) 330

Correct Answer: (C) 396

Solution:

Step 1: Understanding the Concept:
This is a multi-step percentage problem. We need to track the price of the book series through several transactions to find the final selling price.

Step 2: Detailed Explanation:
Let's define the prices at each stage:

- **OC (Original Cost):** The price the shop initially paid for the books.
- **SP1 (First Selling Price):** The price the collector paid the shop.

- **BBP (Buy-Back Price):** The price the shop paid the collector to buy the books back.
- **SP2 (Second Selling Price):** The price the shop sold the books for the second time.

Let's express each price in terms of the Original Cost (OC):

1. **SP1:** The shop sold it for 40% more than OC.

$$SP1 = OC + 0.40 \times OC = 1.4 \times OC$$

2. **BBP:** The shop bought it back for 50% of what the collector paid (SP1).

$$BBP = 0.50 \times SP1 = 0.50 \times (1.4 \times OC) = 0.7 \times OC$$

3. **SP2:** The shop sold it again for a 70% profit on its buy-back price (BBP).

$$SP2 = BBP + 0.70 \times BBP = 1.7 \times BBP = 1.7 \times (0.7 \times OC) = 1.19 \times OC$$

Now, we use the given information about the difference between the original cost and the buy-back price:

$$OC - BBP = \$100$$

Substitute the expression for BBP in terms of OC:

$$OC - (0.7 \times OC) = 100$$

$$0.3 \times OC = 100$$

$$OC = \frac{100}{0.3} = \frac{1000}{3} \approx \$333.33$$

Finally, we need to find the second selling price (SP2). We already have a formula for it in terms of OC:

$$SP2 = 1.19 \times OC$$

Substitute the value we found for OC:

$$SP2 = 1.19 \times \frac{1000}{3} = \frac{1190}{3} \approx 396.67$$

Step 3: Final Answer:
The shop sold the books the second time for approximately \$396.67. The closest answer is 396. This corresponds to option (C).

> **Quick Tip**
>
> In complex percentage problems, it's often easier to work with multipliers. A 40% increase is a multiplier of 1.4, a 50% price is a multiplier of 0.5, and a 70% profit is a multiplier of 1.7. Chaining these multipliers can simplify the setup.

---

12. By what percent was the price of a certain Tab discounted for a sale?
I. The price of the tab was sold with a discount of \$50.

II. The price of the tab before it was discounted for the sale was 25 percent greater than the discounted price.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is a Data Sufficiency question about percentages. To find the discount percent, we need to know the relationship between the discount amount and the original price. The formula is:

$$\text{Discount Percent} = \left(\frac{\text{Discount Amount}}{\text{Original Price}}\right) \times 100\%$$

Let P_O be the original price, P_D be the discounted price, and D be the discount amount. So, $D = P_O - P_D$. We need to find $\frac{D}{P_O}$.

Step 2: Detailed Explanation:
Analyze Statement I: "The price of the tab was sold with a discount of \$50."
This tells us that the discount amount, $D = \$50$. However, we do not know the original price, P_O. Without the original price, we cannot calculate the discount percentage. For example, if the original price was \$100, the discount is 50%. If the original price was \$200, the discount is 25%. Statement I alone is not sufficient.

Analyze Statement II: "The price of the tab before it was discounted for the sale was 25 percent greater than the discounted price."
This gives us a relationship between the original price (P_O) and the discounted price (P_D).

$$P_O = P_D + 0.25 \times P_D = 1.25 \times P_D$$

Now let's express the discount percent in terms of one variable. We know $D = P_O - P_D$. Substituting $P_O = 1.25P_D$:

$$D = 1.25P_D - P_D = 0.25P_D$$

The discount percent is $\frac{D}{P_O}$:

$$\frac{D}{P_O} = \frac{0.25P_D}{1.25P_D}$$

The variable P_D cancels out:

$$\frac{0.25}{1.25} = \frac{25}{125} = \frac{1}{5} = 0.20$$

To express this as a percentage, we multiply by 100: $0.20 \times 100\% = 20\%$. Since we found a unique value for the discount percentage, Statement II alone is sufficient.

Step 3: Final Answer:
Statement II alone is sufficient, but Statement I alone is not. This corresponds to option (B).

Quick Tip

In percentage-based Data Sufficiency, statements that provide a relative relationship (like Statement II) are often sufficient, while statements that provide an absolute value (like Statement I) are often insufficient unless another value is known.

---

13. The colored roses in the bouquet of flowers are red, yellow and pink. The ratio of the number of red to the number of yellow to the number of Pink in the bouquet is 7:4:6, respectively. If there are more than 7 yellow-colored roses, what is the minimum number of total roses in the bouquet?

(A) 8
(B) 12
(C) 14
(D) 24
(E) 34

Correct Answer: (E) 34

Solution:

Step 1: Understanding the Concept:
This is a problem about ratios and finding a minimum value that satisfies a given condition. The number of roses of each color must be an integer, and they must maintain the given ratio.

Step 2: Detailed Explanation:
Let the number of red, yellow, and pink roses be R, Y, and P. The ratio is given as $R : Y : P = 7 : 4 : 6$. This means that the number of roses of each color can be represented as:

$$R = 7k$$

$$Y = 4k$$

$$P = 6k$$

where k must be a positive integer, as we are dealing with a number of flowers.

We are given the condition that there are "more than 7 yellow-colored roses."

$$Y > 7$$

Substitute the expression for Y:

$$4k > 7$$

To find the minimum integer value of k that satisfies this inequality, we divide by 4:

$$k > \frac{7}{4}$$

$$k > 1.75$$

Since k must be an integer, the smallest integer value for k that is greater than 1.75 is $k = 2$.

Now we need to find the minimum number of total roses in the bouquet. The total number of roses is $T = R+Y+P = 7k+4k+6k = 17k$. To find the minimum total, we use the minimum possible integer value for k, which is 2.

$$T_{min} = 17 \times 2 = 34$$

Step 3: Final Answer:
The minimum number of total roses in the bouquet is 34. This corresponds to option (E).

Quick Tip

In ratio problems with a "minimum" or "maximum" condition, first represent the quantities using a common multiplier (k). Then, use the given condition to form an inequality for k. Find the smallest integer value of k that satisfies the inequality to find the minimum total.

---

14. If Polygon A has fewer than 10 sides and the sum of the interior angles of polygon A is divisible by 16, how many sides does Polygon A have?

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

Correct Answer: (C) 6

Solution:

Step 1: Understanding the Concept:
We need to use the formula for the sum of the interior angles of a polygon and test values based on the given conditions.

Step 2: Key Formula or Approach:
The sum of the interior angles of a polygon with n sides is given by the formula:

$$\text{Sum of angles} = (n-2) \times 180^\circ$$

We are given two conditions: 1. $n < 10$ 2. The sum of the angles is divisible by 16.

Step 3: Detailed Explanation:
We will test the possible values of n (number of sides) starting from a triangle ($n = 3$) up to $n = 9$, as $n < 10$.

- **n = 3 (Triangle):** Sum $= (3 - 2) \times 180 = 180$. Is 180 divisible by 16? $180 \div 16 = 11.25$. No.
- **n = 4 (Quadrilateral):** Sum $= (4 - 2) \times 180 = 360$. Is 360 divisible by 16? $360 \div 16 = 22.5$. No.
- **n = 5 (Pentagon):** Sum $= (5-2) \times 180 = 540$. Is 540 divisible by 16? $540 \div 16 = 33.75$. No.
- **n = 6 (Hexagon):** Sum $= (6 - 2) \times 180 = 720$. Is 720 divisible by 16? $720 \div 16 = 45$. Yes.
- **n = 7 (Heptagon):** Sum $= (7-2) \times 180 = 900$. Is 900 divisible by 16? $900 \div 16 = 56.25$. No.
- **n = 8 (Octagon):** Sum $= (8-2) \times 180 = 1080$. Is 1080 divisible by 16? $1080 \div 16 = 67.5$. No.
- **n = 9 (Nonagon):** Sum $= (9-2) \times 180 = 1260$. Is 1260 divisible by 16? $1260 \div 16 = 78.75$. No.

The only number of sides less than 10 for which the sum of the interior angles is divisible by 16 is 6.

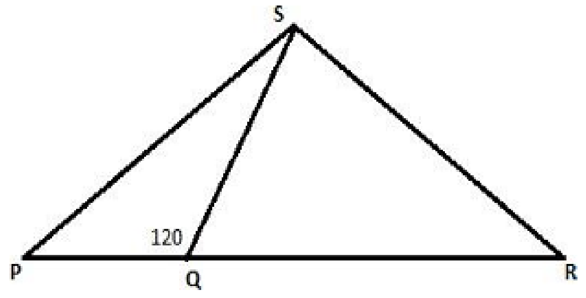
Step 4: Final Answer:
Polygon A has 6 sides. This corresponds to option (C).

Quick Tip

To check for divisibility by 16, you can check for divisibility by 8 and then by 2. Or, more simply, check if the number is divisible by 2 four times. For 720: $720/2 = 360$, $360/2 = 180$, $180/2 = 90$, $90/2 = 45$. Since it's divisible by 2 four times, it's divisible by 16.

---

15. In the figure above PRS is a triangle, what is the measure of the angle PSQ?



I. QS=QR=1
II. PR=2

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (C) Both statements I and II together are sufficient but neither statement is sufficient alone.

Solution:

Step 1: Understanding the Concept:
This is a Data Sufficiency problem in geometry. We need to find the measure of $\angle PSQ$. We are given a larger triangle $\triangle PRS$ with a point Q on the base PR. We also see $\angle PQS = 120^\circ$ marked in the diagram.

Step 2: Detailed Explanation:
From the figure, $\angle PQS$ and $\angle RQS$ are on a straight line, so they are supplementary.

$$\angle RQS = 180^\circ - \angle PQS = 180^\circ - 120^\circ = 60^\circ$$

Analyze Statement I: "QS=QR=1"
This tells us that $\triangle QRS$ is an isosceles triangle with $QS = QR$. In an isosceles triangle, the angles opposite the equal sides are equal. So, $\angle QSR = \angle QRS$. The sum of angles in $\triangle QRS$ is 180°.

$$\angle RQS + \angle QSR + \angle QRS = 180^\circ$$

$$60^\circ + \angle QSR + \angle QSR = 180^\circ$$

$$2 \times \angle QSR = 120^\circ \implies \angle QSR = 60^\circ$$

This means $\triangle QRS$ is an equilateral triangle. We now know all angles and sides of $\triangle QRS$, but we have no information about $\triangle PQS$ other than $\angle PQS = 120^\circ$ and side $QS = 1$. We cannot determine $\angle PSQ$. Statement I is not sufficient.

Analyze Statement II: "PR=2"
This gives the length of the base of the large triangle. This information alone does not help us determine any angles. Statement II is not sufficient.

Analyze Statements I and II Together:
From Statement I, we know $QR = 1$. From Statement II, we know $PR = 2$. Since Q lies on PR, we have $PR = PQ + QR$.

$$2 = PQ + 1 \implies PQ = 1$$

Now, consider $\triangle PQS$. We know two sides, $PQ = 1$ and $QS = 1$, and the angle between them, $\angle PQS = 120°$. Since $PQ = QS = 1$, $\triangle PQS$ is an isosceles triangle. The angles opposite the equal sides must be equal.

$$\angle PSQ = \angle SPQ$$

The sum of angles in $\triangle PQS$ is $180°$.

$$\angle PQS + \angle PSQ + \angle SPQ = 180°$$

$$120° + \angle PSQ + \angle PSQ = 180°$$

$$2 \times \angle PSQ = 60°$$

$$\angle PSQ = 30°$$

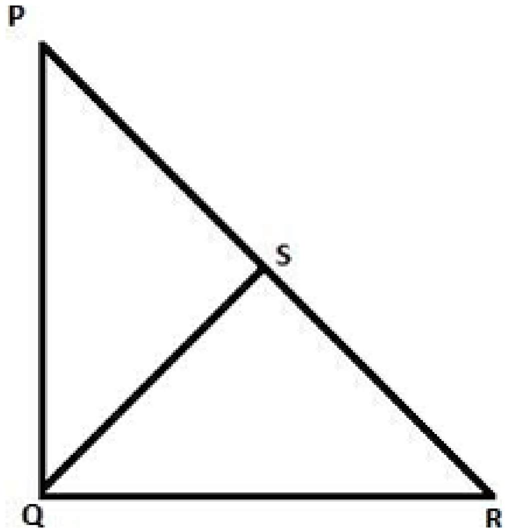
We have found a unique value for $\angle PSQ$. Therefore, both statements together are sufficient.

Step 3: Final Answer:
Neither statement is sufficient on its own, but together they are sufficient. This corresponds to option (C).

> **Quick Tip**
>
> In geometry problems, look for isosceles and equilateral triangles, as they provide powerful information about side lengths and angles. Break down complex figures into simpler triangles and apply the sum of angles theorem.

---

16. In the diagram above, triangle PQR has a right angle at Q. If PQ ¿ QR, then what is the ratio of the area of triangle PQS to the area of triangle RQS?



I. Line segment QS is perpendicular to PR and has a length of 12.
II. PQR has a perimeter of 60.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (C) Both statements I and II together are sufficient but neither statement is sufficient alone.

Solution:

Step 1: Understanding the Concept:
We are asked to find the ratio of the areas of two smaller triangles, $\triangle PQS$ and $\triangle RQS$. These two triangles share a common altitude if we consider PS and RS as their bases. Alternatively, they share a common base QS if we consider the altitudes from P and R to the line containing QS. A simpler approach is to use the fact that they share a common vertex Q and their bases PS and RS are collinear.
The ratio of the areas of two triangles that share the same altitude is equal to the ratio of their bases. From the diagram, if QS is the altitude from Q to PR, then triangles PQS and RQS share the altitude QS. This seems to be a misunderstanding of the area formula. Let's reconsider. The triangles PQS and RQS share a common altitude from vertex Q to the base PR. Let's call the length of this altitude h. This is incorrect.

Let's use the definition of area. The two triangles $\triangle PQS$ and $\triangle RQS$ share a common altitude if we take PS and RS as bases. Let h be the length of the altitude from vertex Q to the line containing PR. The base of $\triangle PQS$ is PS and the base of $\triangle RQS$ is RS. Area($\triangle PQS$) = $\frac{1}{2} \times PS \times h$ Area($\triangle RQS$) = $\frac{1}{2} \times RS \times h$ The ratio is:

$$\frac{\text{Area}(\triangle PQS)}{\text{Area}(\triangle RQS)} = \frac{\frac{1}{2} \times PS \times h}{\frac{1}{2} \times RS \times h} = \frac{PS}{RS}$$

So, the question is equivalent to "What is the ratio of PS to RS?".

Step 2: Detailed Explanation:
We need to find the ratio PS/RS. In a right-angled triangle PQR (right-angled at Q), where QS is the altitude to the hypotenuse PR, we have the geometric mean theorem and other properties related to similar triangles. Specifically, $\triangle PQS \sim \triangle QRS \sim \triangle PQR$. From similarity, we have the relationship $\frac{PQ^2}{QR^2} = \frac{PS}{RS}$. So, finding the ratio of the areas is equivalent to finding the ratio of the squares of the legs of the main triangle, $\frac{PQ^2}{QR^2}$.

Analyze Statement I: "Line segment QS is perpendicular to PR and has a length of 12." This tells us that QS is the altitude from Q to the hypotenuse, and its length is 12. This information alone does not give us the lengths of PQ or QR, so we cannot find their ratio. Statement I is not sufficient.

Analyze Statement II: "PQR has a perimeter of 60."
This gives us $PQ + QR + PR = 60$. This single equation with three unknowns is not enough to determine the ratio of PQ to QR. Statement II is not sufficient.

Analyze Statements I and II Together:
We have a right triangle PQR with altitude QS=12. We also know $PQ + QR + PR = 60$. Let $PQ = a$ and $QR = b$. Then $PR = \sqrt{a^2 + b^2}$. The area of $\triangle PQR$ can be calculated in two ways:

$$\text{Area} = \frac{1}{2} \times PQ \times QR = \frac{1}{2}ab$$

$$\text{Area} = \frac{1}{2} \times PR \times QS = \frac{1}{2}\sqrt{a^2 + b^2} \times 12 = 6\sqrt{a^2 + b^2}$$

So, $ab = 12\sqrt{a^2 + b^2}$. We also have the perimeter equation: $a + b + \sqrt{a^2 + b^2} = 60$. We now have a system of two equations with two variables, a and b. From the perimeter equation: $\sqrt{a^2 + b^2} = 60 - a - b$. Substitute this into the area equation: $ab = 12(60 - a - b) = 720 - 12a - 12b$. Also, square the modified perimeter equation: $a^2 + b^2 = (60 - (a + b))^2 = 3600 - 120(a + b) + (a + b)^2 = 3600 - 120a - 120b + a^2 + 2ab + b^2$.

$$0 = 3600 - 120a - 120b + 2ab$$

$$120(a + b) - 2ab = 3600 \implies 60(a + b) - ab = 1800$$

We have two equations: 1) $ab + 12a + 12b = 720$ 2) $ab - 60a - 60b = -1800$ This system can be solved for a and b. For example, subtracting (2) from (1) gives $72(a + b) = 2520$, so $a + b = 35$. Then $ab = 720 - 12(35) = 720 - 420 = 300$. We can find a and b by solving the quadratic equation $x^2 - 35x + 300 = 0$, which gives $x = 15$ and $x = 20$. Since $PQ > QR$, we have $a = 20$ and $b = 15$. Now we can find the required ratio:

$$\frac{\text{Area}(\triangle PQS)}{\text{Area}(\triangle RQS)} = \frac{PQ^2}{QR^2} = \frac{20^2}{15^2} = \frac{400}{225} = \frac{16}{9}$$

Since we can find a unique ratio, both statements together are sufficient.

Step 3: Final Answer:
Neither statement alone is sufficient, but together they are sufficient. This corresponds to option (C).

> Quick Tip
>
> In a right triangle, the ratio of the areas of the two smaller triangles formed by the altitude to the hypotenuse is equal to the ratio of the squares of the corresponding legs of the original right triangle. This is a powerful theorem derived from the similarity of the triangles.

---

17. In a certain show, a lottery ticket is numbered consecutively from 100 through 999 (both inclusive). What is the probability that a randomly selected ticket will have a number with a ten's digit as "3"?

(A) 1/5
(B) 90/899
(C) 1/10
(D) 1/11
(E) 10/111

Correct Answer: (C) 1/10

Solution:

Step 1: Understanding the Concept:
This is a probability problem. The probability is calculated as the ratio of the number of favorable outcomes to the total number of possible outcomes.

Step 2: Detailed Explanation:
1. Calculate the total number of possible outcomes.
The tickets are numbered from 100 to 999, inclusive. The total number of tickets is given by:

$$\text{Total Numbers} = (\text{Last Number} - \text{First Number}) + 1$$

$$\text{Total Numbers} = (999 - 100) + 1 = 899 + 1 = 900$$

So, there are 900 possible outcomes.

2. Calculate the number of favorable outcomes.
We need to find the number of tickets that have a "3" in the ten's digit. These are numbers of the form H3T, where H is the hundreds digit and T is the units digit.

- The hundreds digit (H) can be any digit from 1 to 9 (9 choices).

- The ten's digit is fixed as 3 (1 choice).
- The units digit (T) can be any digit from 0 to 9 (10 choices).

The total number of such numbers is the product of the number of choices for each digit:

$$\text{Favorable Outcomes} = 9 \times 1 \times 10 = 90$$

So, there are 90 numbers between 100 and 999 that have a 3 in the ten's digit.

3. Calculate the probability.

$$P(\text{ten's digit is 3}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

$$P = \frac{90}{900} = \frac{1}{10}$$

Step 3: Final Answer:
The probability is 1/10. This corresponds to option (C).

Quick Tip

For counting problems involving digits, it's often easiest to think about the number of choices for each digit position (hundreds, tens, units) and then multiply them. Also, a quick way to think about this specific problem is that for any given hundreds digit, there are 10 numbers with a '3' in the tens place (e.g., 130-139). Since there are 9 possible hundreds digits (1-9), the total is $9 \times 10 = 90$.

---

18. In a certain linguistics school, there are totally 250 students. Of those 250 students, 40 percent study French as a foreign language, 30 percent study German as a foreign language and 50 percent study Spanish as a foreign language. If 10 students study all these three foreign languages and 10 students didn't choose these three foreign languages, then how many students are studying in exactly two of these foreign languages?

(A) 20
(B) 30
(C) 40
(D) 50
(E) 60

Correct Answer: (C) 40

Solution:

Step 1: Understanding the Concept:
This problem involves the principle of inclusion-exclusion for three sets (French, German, and Spanish). We are given the total number of students, the number studying each language, the

number studying all three, and the number studying none. We need to find the number of students studying exactly two languages.

Step 2: Detailed Explanation:
First, let's calculate the number of students for each category:

- Total students = 250
- Number studying French (F) = 40% of 250 = $0.40 \times 250 = 100$
- Number studying German (G) = 30% of 250 = $0.30 \times 250 = 75$
- Number studying Spanish (S) = 50% of 250 = $0.50 \times 250 = 125$
- Number studying all three ($F \cap G \cap S$) = 10
- Number studying none of these languages = 10

The total number of students studying at least one language is $250 - 10 = 240$. Let E1 be the number of students studying exactly one language, E2 be the number studying exactly two, and E3 be the number studying all three. We know:

$$\text{Total studying} = E1 + E2 + E3$$

$$240 = E1 + E2 + 10 \implies E1 + E2 = 230$$

We also use the formula that relates the sum of individual sets to these groups:

$$N(F) + N(G) + N(S) = E1 + 2 \times E2 + 3 \times E3$$

Substituting the known values:

$$100 + 75 + 125 = E1 + 2 \times E2 + 3 \times 10$$

$$300 = E1 + 2 \times E2 + 30$$

$$E1 + 2 \times E2 = 270$$

Now we have a system of two linear equations: 1) $E1 + E2 = 230$ 2) $E1 + 2 \times E2 = 270$
Subtract equation (1) from equation (2):

$$(E1 + 2 \times E2) - (E1 + E2) = 270 - 230$$

$$E2 = 40$$

Step 3: Final Answer:
Based on a rigorous application of set theory principles, the number of students studying exactly two languages is 40.

Quick Tip

For three-set Venn diagram problems, the formula Total Sum of Groups = (Exactly 1) + 2 × (Exactly 2) + 3 × (All 3) is very powerful. Combining it with Total in at least one = (Exactly 1) + (Exactly 2) + (All 3) allows you to solve for the unknown groups.

19. The interior of a rectangular box is designed by a certain manufacturer to have a volume of "m" cubic feet and ratio of length to width to height of 5:3:2. In term of "m", which of the following equals the length of the box in feet?

(A) $\sqrt[3]{\frac{25m}{6}}$

Correct Answer: (A)

Solution:

Step 1: Understanding the Concept:
We are given the volume of a rectangular box and the ratio of its dimensions. We need to express the length in terms of the volume.

Step 2: Detailed Explanation:
Let the length, width, and height of the box be L, W, and H, respectively. The ratio is given as L:W:H = 5:3:2. We can express the dimensions using a common multiplier, k:

$$L = 5k$$

$$W = 3k$$

$$H = 2k$$

The volume (V) of the box is given by the formula V = L × W × H. We are told the volume is "m".

$$m = (5k) \times (3k) \times (2k)$$

$$m = 30k^3$$

Now, we need to solve for the multiplier k in terms of m:

$$k^3 = \frac{m}{30}$$

$$k = \sqrt[3]{\frac{m}{30}}$$

The question asks for the length (L) of the box in terms of m.

$$L = 5k$$

Substitute the expression we found for k:

$$L = 5\sqrt[3]{\frac{m}{30}}$$

To get the '5' inside the cube root, we must cube it: $5 = \sqrt[3]{5^3} = \sqrt[3]{125}$.

$$L = \sqrt[3]{125} \times \sqrt[3]{\frac{m}{30}} = \sqrt[3]{125 \times \frac{m}{30}}$$

Simplify the fraction inside the cube root:

$$L = \sqrt[3]{\frac{125m}{30}} = \sqrt[3]{\frac{25m}{6}}$$

Step 3: Final Answer:
The length of the box in feet is $\sqrt[3]{\frac{25m}{6}}$.

Quick Tip

When dealing with ratios for geometric figures, always introduce a common multiplier k. Set up the formula for the given property (like volume or area) in terms of k, solve for k, and then substitute it back into the expression for the dimension you need to find.

---

20. Lines "l" and "k" are perpendicular to each other. And line "l" passes through points (4,1) and (8, -1). What is the equation of the line "k" which passes through the point (3,1)?

(A) 2y - x = 5
(B) 2x - y = 5
(C) y - 2x = 5
(D) y + 2x = 5
(E) 2y + x = 5

Correct Answer: (B) 2x - y = 5

Solution:

Step 1: Understanding the Concept:
This problem involves finding the equation of a line given a point and its relationship (perpendicularity) to another line. The key is the relationship between the slopes of perpendicular lines.

Step 2: Detailed Explanation:
1. Find the slope of line l.
The slope (m_l) of a line passing through points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using the points (4,1) and (8, -1) for line l:

$$m_l = \frac{-1-1}{8-4} = \frac{-2}{4} = -\frac{1}{2}$$

2. Find the slope of line k.
If two lines are perpendicular, the slope of one is the negative reciprocal of the slope of the other. Let the slope of line k be m_k.

$$m_k = -\frac{1}{m_l} = -\frac{1}{(-1/2)} = 2$$

3. Find the equation of line k.
We know that line k has a slope of 2 and passes through the point (3,1). We can use the point-slope form of a linear equation: $y - y_1 = m(x - x_1)$.

$$y - 1 = 2(x - 3)$$

$$y - 1 = 2x - 6$$

Rearrange the equation to match the format of the options:

$$6 - 1 = 2x - y$$

$$5 = 2x - y$$

This can be written as $2x - y = 5$.

Step 3: Final Answer:
The equation of line k is $2x - y = 5$, which corresponds to option (B).

> **Quick Tip**
>
> Remember the slope relationships: for parallel lines, slopes are equal ($m_1 = m_2$); for perpendicular lines, slopes are negative reciprocals ($m_1 = -1/m_2$). This is a fundamental concept in coordinate geometry.

---

21. A certain cafeteria sells donuts and pizzas. Is the number of people who bought donuts are more than the number of people who bought pizzas?
I. Of the people who bought donuts, 30 percent of them also bought pizzas.
II. Of the people who bought pizzas, 40 percent of them also bought donuts.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.

Solution:

Step 1: Understanding the Concept:
This is a Data Sufficiency question comparing the sizes of two groups (donut buyers and pizza

buyers). We need to determine if we can definitively answer "yes" or "no" to the question: Is $D > P$? where D is the number of people who bought donuts and P is the number of people who bought pizzas.

Step 2: Detailed Explanation:
Let B be the number of people who bought both donuts and pizzas.

Analyze Statement I: "Of the people who bought donuts, 30 percent of them also bought pizzas."
This translates to the equation:

$$B = 0.30 \times D$$

This statement gives us a relationship between the number of people who bought both items and the number who bought donuts. However, it provides no information about P, the number of pizza buyers. Therefore, Statement I alone is not sufficient.

Analyze Statement II: "Of the people who bought pizzas, 40 percent of them also bought donuts."
This translates to the equation:

$$B = 0.40 \times P$$

This statement relates the number of people who bought both to the number who bought pizzas. It provides no information about D. Therefore, Statement II alone is not sufficient.

Analyze Statements I and II Together:
From both statements, we have two different expressions for B, the number of people who bought both. We can set them equal to each other:

$$0.30 \times D = 0.40 \times P$$

We can now find the ratio of D to P:

$$\frac{D}{P} = \frac{0.40}{0.30} = \frac{4}{3}$$

Since $\frac{D}{P} = \frac{4}{3}$, which is greater than 1, it must be true that $D > P$. This provides a definite "yes" to the question. Therefore, both statements together are sufficient.

Step 3: Final Answer:
Neither statement is sufficient alone, but together they provide enough information to answer the question. This corresponds to option (C).

Quick Tip

In Data Sufficiency problems involving overlapping sets, translating the percentage statements into algebraic equations is the key. Often, the number of people in the intersection ("both") can be used to link the two sets and establish a ratio between them.

22. Alan purchased pens and pencils at a certain shop, where each pen costs 3 dollars and each pencil cost 2 dollars. What is the total number of pen and pencils Alan purchased?
I. Alan bought pen and pencils for the total cost of 10 dollars.
II. Total cost of the pens which Allan bought is less than 10 dollars.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is a Data Sufficiency problem that translates to a linear Diophantine equation (an equation where we are only interested in integer solutions). Let p be the number of pens and n be the number of pencils. The total cost is given by the equation $3p + 2n = \text{Total Cost}$. The question asks for the value of $p + n$.
A key ambiguity in such problems is whether the quantities must be positive (> 0) or non-negative (≥ 0). The phrasing "purchased pens and pencils" strongly implies that Alan bought at least one of each, so we will assume $p \geq 1$ and $n \geq 1$.

Step 2: Detailed Explanation:
Analyze Statement I: "Alan bought pen and pencils for the total cost of 10 dollars."
This gives us the equation:

$$3p + 2n = 10$$

We need to find integer solutions where $p \geq 1$ and $n \geq 1$.

- If $p = 1$, $3(1) + 2n = 10 \implies 2n = 7$. This gives $n = 3.5$, which is not an integer.
- If $p = 2$, $3(2) + 2n = 10 \implies 6 + 2n = 10 \implies 2n = 4$. This gives $n = 2$, which is an integer. So, (p=2, n=2) is a valid solution.
- If $p = 3$, $3(3) + 2n = 10 \implies 9 + 2n = 10 \implies 2n = 1$. This gives $n = 0.5$, not an integer.
- If $p \geq 4$, $3p$ would be 12 or more, which is already greater than the total cost of 10. So there are no more solutions.

Under the assumption that he bought at least one of each, there is only one possible solution: 2 pens and 2 pencils. The total number of items is $p + n = 2 + 2 = 4$. Since we found a unique value for the total number of items, Statement I alone is sufficient.

Analyze Statement II: "Total cost of the pens which Allan bought is less than 10 dollars." This gives the inequality:

$$3p < 10$$

$$p < 10/3 \implies p < 3.33...$$

Since p must be an integer and $p \geq 1$, the possible values for p are 1, 2, or 3. This statement gives no information about the number of pencils or the total cost. Therefore, Statement II alone is not sufficient.

Step 3: Final Answer:
Statement I alone is sufficient to answer the question, but Statement II alone is not. This corresponds to option (A).

> **Quick Tip**
>
> In Diophantine equation problems, the constraints that variables must be integers (and often positive) drastically reduce the number of possible solutions. Always test small integer values to find all possible combinations.

---

23. Water is pumped into the completely empty tank at a constant rate through an inlet pipe. At the same time, there is a leak at the bottom of the tank which leaks water at a constant rate. How long it will take the tank get filled completely?
I. Total capacity of water the tank can hold is 120 gallons.
II. Inlet pipe can completely fill the empty tank in 10 hours if there is no leak in the tank, and also the leak at the bottom of the tank can completely empty the filled tank in 15 hours if there is no water pumped into the tank.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is a Data Sufficiency question involving rates of work (filling and leaking a tank). To find the total time to fill the tank, we need to determine the net filling rate. The net rate is the

filling rate minus the leaking rate. The time to fill is the reciprocal of the net rate.
Let R_{in} be the rate of the inlet pipe (in tanks per hour).
Let R_{out} be the rate of the leak (in tanks per hour).
Net Rate $= R_{in} - R_{out}$.
Time to fill $= \frac{1}{\text{Net Rate}}$.

Step 2: Detailed Explanation:
Analyze Statement I: "Total capacity of water the tank can hold is 120 gallons."
This statement provides the volume of the tank but gives no information about the rate of filling or leaking. We cannot determine the time it takes to fill the tank. Therefore, Statement I alone is not sufficient.

Analyze Statement II: "Inlet pipe can completely fill the empty tank in 10 hours if there is no leak in the tank, and also the leak at the bottom of the tank can completely empty the filled tank in 15 hours if there is no water pumped into the tank."
This statement gives us the individual rates:

- The inlet pipe fills the tank in 10 hours, so its rate is $R_{in} = \frac{1}{10}$ tank per hour.
- The leak empties the tank in 15 hours, so its rate is $R_{out} = \frac{1}{15}$ tank per hour.

We can now calculate the net filling rate:

$$\text{Net Rate} = R_{in} - R_{out} = \frac{1}{10} - \frac{1}{15}$$

To subtract the fractions, find a common denominator, which is 30:

$$\text{Net Rate} = \frac{3}{30} - \frac{2}{30} = \frac{1}{30} \text{ tank per hour}$$

The time required to fill the tank completely is the reciprocal of the net rate:

$$\text{Time to fill} = \frac{1}{1/30} = 30 \text{ hours}$$

Since we found a unique numerical answer for the time, Statement II alone is sufficient.

Step 3: Final Answer:
Statement II alone is sufficient, but Statement I alone is not. This corresponds to option (B).

Quick Tip

In work-rate problems, it's often easiest to work with fractional rates (e.g., jobs per hour, tanks per hour). The total volume (like 120 gallons) is often extra information if the rates can be determined relative to the whole job.

---

24. If 'x' is a number such that $x^2 - 5x + 4 < 0$ and $x^2 - 3x + 2 < 0$, which of the following can be the value of 'x'?

(A) 3.5
(B) 3.0
(C) 2.4
(D) 1.6
(E) 0.8

Correct Answer: (D) 1.6

Solution:

Step 1: Understanding the Concept:
We need to find a value of 'x' that simultaneously satisfies two quadratic inequalities. The process is to solve each inequality separately and then find the intersection of their solution sets.

Step 2: Detailed Explanation:
Solve the first inequality: $x^2 - 5x + 4 < 0$
First, factor the quadratic expression:

$$(x - 1)(x - 4) < 0$$

The roots of the equation $(x - 1)(x - 4) = 0$ are $x = 1$ and $x = 4$. Since the parabola opens upward, the expression is negative (less than 0) between the roots. The solution for the first inequality is $1 < x < 4$.

Solve the second inequality: $x^2 - 3x + 2 < 0$
First, factor the quadratic expression:

$$(x - 1)(x - 2) < 0$$

The roots of the equation $(x - 1)(x - 2) = 0$ are $x = 1$ and $x = 2$. Since the parabola opens upward, the expression is negative between the roots. The solution for the second inequality is $1 < x < 2$.

Find the intersection of the solutions.
We need a value of 'x' that satisfies both $1 < x < 4$ AND $1 < x < 2$. The intersection of these two intervals is $1 < x < 2$.

Check the options.
We must find which of the given options lies in the interval (1, 2).

- (A) 3.5 is not in (1, 2).
- (B) 3.0 is not in (1, 2).
- (C) 2.4 is not in (1, 2).
- (D) 1.6 is in (1, 2).
- (E) 0.8 is not in (1, 2).

Step 3: Final Answer:
The only value that satisfies both inequalities is 1.6. This corresponds to option (D).

> **Quick Tip**
>
> To solve a quadratic inequality like $ax^2 + bx + c < 0$ (with $a > 0$), find the roots r_1 and r_2. The solution will be the interval between the roots: $r_1 < x < r_2$. If the inequality is > 0, the solution will be the intervals outside the roots: $x < r_1$ or $x > r_2$.

---

25. If p^2 is an integer and $\sqrt{p^6 - p^4 - q - 1} = 10$, what is the value of "$p^2$"?
I. $q = -1$
II. $p^2 + q = 4$

(A) Statement I alone is sufficient but statement II aloneq = q + 2 is not sufficient to answer the question.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (D) Each statement alone is sufficient to answer the question.

Solution:

Step 1: Understanding the Concept:
We are given an equation with two variables, p^2 and q, and we need to find the value of p^2. Let's simplify the main equation first. Let $P = p^2$. Since p^2 is an integer, P is an integer.
The main equation is: $\sqrt{P^3 - P^2 - q - 1} = 10$.
Square both sides:

$$P^3 - P^2 - q - 1 = 100$$
$$P^3 - P^2 - q = 101$$

This can be rewritten as $P^2(P - 1) - q = 101$. Our goal is to find a unique integer value for P.

Step 2: Detailed Explanation:
Analyze Reconstructed Statement I: $q = -1$
Substitute $q = -1$ into our simplified main equation:

$$P^2(P - 1) - (-1) = 101$$
$$P^2(P - 1) = 100$$

Since P is an integer, P^2 must be a perfect square that is a divisor of 100. Let's test the possibilities:

- If $P^2 = 1$, then $P = 1$. Check: $1^2(1-1) = 1(0) = 0 \neq 100$.
- If $P^2 = 4$, then $P = 2$. Check: $2^2(2-1) = 4(1) = 4 \neq 100$.
- If $P^2 = 25$, then $P = 5$. Check: $5^2(5-1) = 25(4) = 100$. This is a valid solution.
- If $P^2 = 100$, then $P = 10$. Check: $10^2(10-1) = 100(9) = 900 \neq 100$.

The only integer solution is $P = 5$. Therefore, $p^2 = 5$, and this statement is sufficient.

Analyze Reconstructed Statement II: $p^2 + q = 4$
This can be written as $P + q = 4$, or $q = 4 - P$. Substitute this expression for q into the main equation:

$$P^2(P-1) - (4-P) = 101$$
$$P^3 - P^2 - 4 + P = 101$$
$$P^3 - P^2 + P - 105 = 0$$

We need to find integer roots of this cubic equation. Let's test small integer values for P.

- P=1: $1 - 1 + 1 - 105 \neq 0$
- P=2: $8 - 4 + 2 - 105 \neq 0$
- P=3: $27 - 9 + 3 - 105 \neq 0$
- P=4: $64 - 16 + 4 - 105 \neq 0$
- P=5: $125 - 25 + 5 - 105 = 130 - 130 = 0$. This is a valid solution.

To ensure this is the only integer solution, we can check the function's behavior. Let $f(P) = P^3 - P^2 + P - 105$. The derivative is $f'(P) = 3P^2 - 2P + 1$. The discriminant of this derivative is $(-2)^2 - 4(3)(1) = -8$, which is negative. This means the derivative is always positive, so the function $f(P)$ is strictly increasing. An increasing function can only cross the x-axis once, so $P = 5$ is the unique real solution, and thus the unique integer solution. Therefore, this statement is sufficient.

Step 3: Final Answer:
Since each statement alone is sufficient to find a unique value for p^2, the correct answer is (D).

> **Quick Tip**
>
> When an equation involves integers, remember to use number properties like factors and perfect squares to limit the possibilities. For polynomial equations, testing small integer values is an effective strategy to find potential roots.

---

26. If "P" is a positive integer, is $P^4 + 7$ an odd number?
I. "P" is the smallest positive integer that is divisible by all the integers from 51 to 55, inclusive.

II. 13^P **is an odd number.**

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is a "Yes/No" Data Sufficiency question about number properties (odd/even). The expression is P^4+7. The number 7 is odd. The sum of two integers is odd if and only if one is even and one is odd. For $P^4 + 7$ to be odd, P^4 must be even. For a power of an integer (P^4) to be even, the base (P) must be even. Therefore, the question is equivalent to: **"Is P an even integer?"**

Step 2: Detailed Explanation:
Analyze Reconstructed Statement I: "P is the smallest positive integer that is divisible by all the integers from 51 to 55, inclusive."
This means that P is the Least Common Multiple (LCM) of the set of numbers {51, 52, 53, 54, 55}. To find the LCM, we need the prime factorization of each number. However, to determine if P is even or odd, we only need to check if the prime factorization of the LCM contains a '2'. The set of numbers is {51, 52, 53, 54, 55}. The number 52 is even ($52 = 2 \times 26$). The number 54 is also even ($54 = 2 \times 27$). Since P must be divisible by 52 (an even number), P itself must be even. Because P is even, the answer to the rephrased question "Is P an even integer?" is a definite "Yes". Therefore, Statement I is sufficient.

Analyze Reconstructed Statement II: "13^P is an odd number."
The number 13 is an odd integer. Any positive integer power of an odd number is always odd (since Odd $\times$ Odd = Odd). This statement is true for any positive integer P, whether P is even or odd. For example, if P=1 (odd), $13^1 = 13$ is odd. If P=2 (even), $13^2 = 169$ is odd. Since this statement does not tell us whether P is even or odd, we cannot answer the question. Therefore, Statement II is not sufficient.

Step 3: Final Answer:
Statement I alone is sufficient, but Statement II alone is not. This corresponds to option (A).

Quick Tip

To determine if a number is even or odd, check its prime factorization. If the prime factorization contains at least one '2', the number is even; otherwise, it is odd. The LCM of a set of integers is even if at least one of the integers in the set is even.

---

27. If 'm' is a positive integer, is "m^2 + 1" divisible by 10 (leaves remainder ZERO)?
I. When m is divided by 2, it leaves a remainder of 1.
II. When m is divided by 5, it leaves a remainder of 2.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (C) Both statements I and II together are sufficient to answer the question but neither statement is sufficient alone.

Solution:
Note: The provided text for the statements in the original document is heavily corrupted by OCR errors. The solution below is based on a logical reconstruction that aligns with the structure of such problems.

Step 1: Understanding the Concept:
This is a "Yes/No" Data Sufficiency question about divisibility and remainders. The question asks if $m^2 + 1$ is divisible by 10. For a number to be divisible by 10, its units digit must be 0. This means that $m^2 + 1$ must end in 0, which in turn means that m^2 must end in 9. For an integer's square (m^2) to end in 9, the integer (m) itself must have a units digit of 3 or 7. So, the question is equivalent to: **"Does the integer m end in 3 or 7?"**

Step 2: Detailed Explanation:
Analyze Reconstructed Statement I: "When m is divided by 2, it leaves a remainder of 1."
This means that m is an odd number. The units digit of an odd number can be 1, 3, 5, 7, or 9.

- If m ends in 3 or 7, the answer is "Yes".
- If m ends in 1, 5, or 9, the answer is "No". (e.g., if m=1, $m^2+1 = 2$; if m=5, $m^2+1 = 26$)

Since we cannot determine a definite yes or no, Statement I is not sufficient.

Analyze Reconstructed Statement II: "When m is divided by 5, it leaves a remainder of 2."
This means that the units digit of m must be either 2 or 7.

- If m ends in 7, the answer is "Yes". (e.g., if m=7, $m^2 + 1 = 50$, which is divisible by 10)
- If m ends in 2, the answer is "No". (e.g., if m=2, $m^2 + 1 = 5$, which is not divisible by 10)

Since we cannot determine a definite yes or no, Statement II is not sufficient.

Analyze Statements I and II Together:
From Statement I, we know m is odd. From Statement II, we know m ends in 2 or 7. Combining these two conditions, the only possibility is that the units digit of m must be 7. If m has a units digit of 7, then m^2 will have a units digit of 9, and $m^2 + 1$ will have a units digit of 0. Therefore, $m^2 + 1$ will be divisible by 10. This gives a definite "Yes" to the question. Therefore, both statements together are sufficient.

Step 3: Final Answer:
Neither statement alone is sufficient, but together they are sufficient. This corresponds to option (C).

> **Quick Tip**
>
> Questions about divisibility by 10 are almost always questions about the units digit. Rephrasing the question in terms of the units digit of the variable can make the problem much clearer.

---

28. If "x" is a positive integer, is $x > 33$?
I. x is an odd number.
II. The sum of the digits of x is 5.

(A) Statement I alone is sufficient but statement II alone is not sufficient to answer the question asked.
(B) Statement II alone is sufficient but statement I alone is not sufficient to answer the question asked.
(C) Both statements I and II together are sufficient but neither statement is sufficient alone.
(D) Each statement alone is sufficient to answer the question.
(E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Correct Answer: (E) Statements I and II are not sufficient to answer the question asked and additional data is needed to answer the statements.

Solution:

Step 1: Understanding the Concept:
This is a "Yes/No" Data Sufficiency question. We need to determine if the positive integer x

is definitively greater than 33.

Step 2: Detailed Explanation:
Analyze Reconstructed Statement I: "x is an odd number."
This statement tells us x is odd.

- Can $x > 33$? Yes, for example, $x = 35$.
- Can $x \leq 33$? Yes, for example, $x = 33$ or $x = 31$.

Since we can get both "yes" and "no" answers, Statement I is not sufficient.

Analyze Reconstructed Statement II: "The sum of the digits of x is 5."
This statement gives us a set of possible values for x. Possible values for x: 5, 14, 23, 32, 41, 50, 104, 113, 122, 131, 140, 203, ...

- Can $x > 33$? Yes, for example, $x = 41$.
- Can $x \leq 33$? Yes, for example, $x = 32$ or $x = 23$.

Since we can get both "yes" and "no" answers, Statement II is not sufficient.

Analyze Statements I and II Together:
We need to find positive integers x that are odd AND have a digit sum of 5. Possible values for x:

- From the list above, we select the odd numbers: 5, 23, 41, 104(no), 113, 131, 203, ...
- The possible values are 5, 23, 41, 113, 131, 203, etc.

Now we check the question "is $x > 33$?":

- Can $x > 33$? Yes, for example, $x = 41$.
- Can $x \leq 33$? Yes, for example, $x = 23$ or $x = 5$.

Even with both statements combined, we cannot determine a definitive answer. Therefore, the statements together are not sufficient.

Step 3: Final Answer:
Statements I and II together are not sufficient to answer the question. This corresponds to option (E).

> Quick Tip
>
> When testing Data Sufficiency statements, always be sure to test values on both sides of the "boundary" mentioned in the question. Here, the boundary is 33, so you must test values both greater than and less than (or equal to) 33.

---

30. List A has seven integers; whose range is 80 and median is 240. The median for the three smallest integers in List A is 180. What is the possible range for the

largest three integers in the List A?
Possible Values:
I. 75
II. 24
III. 0

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) III only

Correct Answer: (E) III only

Solution:
Note: The question format is unusual. It asks for a "possible range" but provides single numbers as options. It should be interpreted as "Which of the following is a possible value for the range of the three largest integers?".

Step 1: Understanding the Concept:
We are given properties of a set of seven integers and need to find the possible values for the range of the three largest integers in the set.
Let the seven integers be $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ in non-decreasing order.

Step 2: Detailed Explanation:
From the problem statement, we have:

1. **Median of List A is 240:** The median of seven integers is the 4th term. So, $a_4 = 240$.
2. **Range of List A is 80:** $a_7 - a_1 = 80$.
3. **Median of the three smallest integers is 180:** The three smallest are a_1, a_2, a_3. Their median is the middle term, a_2. So, $a_2 = 180$.

We can now establish the relationships and constraints between the integers:

$$a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7$$

Substituting the known values:

$$a_1 \leq 180 \leq a_3 \leq 240 \leq a_5 \leq a_6 \leq a_7$$

The question asks for the range of the largest three integers, which is $a_7 - a_5$. We need to find the possible values for this expression.

Finding the bounds for a_7 and a_5:

- From $a_2 = 180$, we know $a_1 \leq 180$.
- From the range, $a_7 = a_1 + 80$. Since $a_1 \leq 180$, then $a_7 \leq 180 + 80 = 260$.
- We also know $a_4 = 240 \leq a_5 \leq a_6 \leq a_7$. This implies $a_7 \geq 240$.

- So, the possible values for a_7 are in the range [240, 260].
- The possible values for a_5 are constrained by $240 \leq a_5 \leq a_7$.

Finding the bounds for the range $a_7 - a_5$:

- **Minimum value:** To minimize the range $a_7 - a_5$, we want a_7 and a_5 to be as close as possible. This occurs when $a_5 = a_7$. Is this possible? Let's try to construct a set. Let $a_7 = 240$. Then $a_1 = 240 - 80 = 160$. A possible set is {160, 180, 200, 240, 240, 240, 240}. All conditions are met. In this case, the range of the largest three integers $\{240, 240, 240\}$ is $240 - 240 = 0$. So, a range of **0** is possible.
- **Maximum value:** To maximize the range $a_7 - a_5$, we want a_7 to be as large as possible and a_5 to be as small as possible. Max value for a_7 is 260. This occurs when $a_1 = 180$. Min value for a_5 is 240 (since $a_4 \leq a_5$). A possible set is {180, 180, 180, 240, 240, 250, 260}. All conditions are met. In this case, the range of the largest three integers $\{240, 250, 260\}$ is $260 - 240 = 20$.

The possible values for the range $a_7 - a_5$ are in the interval [0, 20].

Evaluate the options:

- I. 75: Not possible, as it is outside the interval [0, 20].
- II. 24: Not possible, as it is outside the interval [0, 20].
- III. 0: Possible, as it is within the interval [0, 20].

Step 3: Final Answer:
Only 0 is a possible value for the range of the three largest integers. This corresponds to option (E).

Quick Tip

When asked for a possible range or value for a quantity based on statistical properties, try to construct extreme cases. To find the minimum and maximum possible values, push the unknown variables to their limits while still satisfying all given conditions.
