GMAT 2024 Quant Sample Paper Set 2 Question Paper with Solutions

Time Allowed :2 Hours 15 Minutes	Maximum Marks :205-805	Total Questions :64

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GMAT exam is 2 hours and 15 minutes long (with one optional 10-minute break) and consists of 64 questions in total.
2. The GMAT exam is comprised of three sections:
3. Quantitative Reasoning: 21 questions, 45 minutes
4. Verbal Reasoning: 23 questions, 45 minutes
5. Data Insights: 20 questions, 45 minutes
6. You can answer the three sections in any order. As you move through a section, you can bookmark questions that you would like to review later.
7. When you have answered all questions in a section, you will proceed to the Question Review & Edit screen for that section.
8. If there is no time remaining in the section, you will NOT proceed to the Question Review & Edit screen and you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
9. Each Question Review & Edit screen includes a numbered list of the questions in that section and indicates the questions you bookmarked.
10. Clicking a question number will take you to that specific question. You can review as many questions as you would like and can edit up to three (3) answers.

Quantitative Aptitude

1. Last month a certain music club offered a discount to preferred customers. After the first compact disc purchased, preferred customers paid \$3.99 for each additional compact disc purchased. If a preferred customer purchased a total of 6 compact discs and paid \$15.95 for the first compact disc, then the dollar amount that the customer paid for the 6 compact discs is equivalent to which of the following?

(A) 5(4.00) + 15.90
(B) 5(4.00) + 15.95
(C) 5(4.00) + 16.00
(D) 5(4.00 – 0.01) + 15.90
(E) 5(4.00 – 0.05) + 15.95

Correct Answer: (A) $5(4.00) + 15.90$

Solution:

Step 1: Understanding the Concept:
The problem asks for an expression that is equivalent to the total cost of purchasing 6 compact discs under a specific discount offer. This involves calculating the actual total cost first and then evaluating the given options to find which one results in the same value.

Step 2: Detailed Explanation:
First, let's calculate the total cost based on the information provided.
The customer buys a total of 6 compact discs.
The cost of the first disc is \$15.95.
The number of additional discs is $6 - 1 = 5$.
The cost for each additional disc is \$3.99.
The total cost for the additional discs is $5 \times \$3.99$.

$$5 \times 3.99 = 19.95$$

The total cost for all 6 discs is the sum of the cost of the first disc and the cost of the additional discs.

$$\text{Total Cost} = \$15.95 + \$19.95 = \$35.90$$

Step 3: Evaluate the Options:
Now, we need to find which of the given options is equivalent to \$35.90.
(A) $5(4.00) + 15.90 = 20.00 + 15.90 = 35.90$
(B) $5(4.00) + 15.95 = 20.00 + 15.95 = 35.95$
(C) $5(4.00) + 16.00 = 20.00 + 16.00 = 36.00$
(D) $5(4.00 - 0.01) + 15.90 = 5(3.99) + 15.90 = 19.95 + 15.90 = 35.85$
(E) $5(4.00 - 0.05) + 15.95 = 5(3.95) + 15.95 = 19.75 + 15.95 = 35.70$

Step 4: Final Answer:
Comparing the results, option (A) is the only expression that evaluates to \$35.90, which is the total amount the customer paid.

> **Quick Tip**
>
> In questions asking for an "equivalent" expression, first calculate the definitive numerical answer from the problem's data. Then, systematically evaluate each option to see which one matches your calculated answer. Don't assume the option will look like your initial calculation steps.

---

2. The average (arithmetic mean) of the integers from 200 to 400, inclusive, is how much greater than the average of the integers from 50 to 100, inclusive?

(A) 150
(B) 175
(C) 200
(D) 225
(E) 300

Correct Answer: (D) 225

Solution:

Step 1: Understanding the Concept:
The question requires us to find the difference between the averages of two different sets of consecutive integers. For any evenly spaced set of numbers (like consecutive integers), the average is simply the mean of the first and last terms.

Step 2: Key Formula or Approach:
For a set of consecutive integers, the average (arithmetic mean) can be calculated using the formula:

$$\text{Average} = \frac{\text{First Term} + \text{Last Term}}{2}$$

Step 3: Detailed Explanation:
First, let's calculate the average of the integers from 200 to 400, inclusive.
Here, the first term is 200 and the last term is 400.

$$\text{Average}_1 = \frac{200 + 400}{2} = \frac{600}{2} = 300$$

Next, let's calculate the average of the integers from 50 to 100, inclusive.
Here, the first term is 50 and the last term is 100.

$$\text{Average}_2 = \frac{50 + 100}{2} = \frac{150}{2} = 75$$

Finally, we need to find how much greater the first average is than the second average. This means we need to find their difference.

$$\text{Difference} = \text{Average}_1 - \text{Average}_2 = 300 - 75 = 225$$

Step 4: Final Answer:
The average of the integers from 200 to 400 is 225 greater than the average of the integers from 50 to 100.

Quick Tip

Remembering the shortcut for the average of an arithmetic progression (like consecutive integers) can save significant time. Instead of summing all the numbers and dividing by the count, just average the first and last numbers.

---

3. The sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is such that $a_n = \frac{a_{n-1}+a_{n-2}}{2}$ for all $n \geq 3$. If $a_3 = 4$ and $a_5 = 20$, what is the value of a_6?

(A) 12
(B) 16
(C) 20
(D) 24
(E) 28

Correct Answer: (E) 28

Solution:

Step 1: Understanding the Concept:
The problem describes a sequence where each term (from the third term onwards) is the arithmetic mean of the two preceding terms. We are given two terms in the sequence and asked to find a later term.

Step 2: Key Formula or Approach:
The recursive formula for the sequence is given:

$$a_n = \frac{a_{n-1} + a_{n-2}}{2}$$

We will use this formula to work our way to the desired term, a_6.

Step 3: Detailed Explanation:
We are given $a_3 = 4$ and $a_5 = 20$. Our goal is to find a_6.
The formula for a_6 is:

$$a_6 = \frac{a_5 + a_4}{2}$$

We know $a_5 = 20$, but we don't know a_4. We need to find a_4 first.
We can use the formula for a_5, since we know a_5 and a_3.

$$a_5 = \frac{a_4 + a_3}{2}$$

Now, substitute the known values into this equation:

$$20 = \frac{a_4 + 4}{2}$$

To solve for a_4, we multiply both sides by 2:

$$40 = a_4 + 4$$

Subtract 4 from both sides:

$$a_4 = 40 - 4 = 36$$

Now that we have the value of a_4, we can find a_6.
Substitute the values of a_5 and a_4 into the formula for a_6:

$$a_6 = \frac{20 + 36}{2} = \frac{56}{2} = 28$$

Step 4: Final Answer:
The value of a_6 is 28.

> **Quick Tip**
>
> For recursive sequence problems, identify the target term and see which preceding terms you need. Work backwards or forwards from the given information to find the necessary terms step by step.

---

4. Among a group of 2,500 people, 35 percent invest in municipal bonds, 18 percent invest in oil stocks, and 7 percent invest in both municipal bonds and oil stocks. If 1 person is to be randomly selected from the 2,500 people, what is the probability that the person selected will be one who invests in municipal bonds but NOT in oil stocks?

(A) $\frac{9}{50}$
(B) $\frac{7}{25}$
(C) $\frac{7}{20}$
(D) $\frac{21}{50}$
(E) $\frac{27}{50}$

Correct Answer: (B) $\frac{7}{25}$

Solution:

Step 1: Understanding the Concept:
This is a probability problem involving overlapping sets (people investing in bonds, stocks, or both). We need to find the probability of selecting a person who belongs to one set (municipal bonds) but not the other (oil stocks).

Step 2: Key Formula or Approach:
Let M be the set of people who invest in municipal bonds, and O be the set of people who invest in oil stocks. We are looking for the percentage of people who invest only in municipal bonds. This can be found by taking the total percentage of people who invest in municipal bonds and subtracting the percentage of people who invest in both.

$$P(\text{Only M}) = P(\text{M}) - P(\text{M and O})$$

The probability of an event is equal to the proportion (or percentage) of the group that satisfies the condition.

Step 3: Detailed Explanation:
We are given the following percentages:
Percentage investing in municipal bonds, $P(\text{M}) = 35\%$.
Percentage investing in oil stocks, $P(\text{O}) = 18\%$.

Percentage investing in both, $P(\text{M and O}) = 7\%$.
We want to find the percentage of people who invest in municipal bonds but not in oil stocks. This corresponds to the $P(\text{Only M})$.

$$P(\text{Only M}) = 35\% - 7\% = 28\%$$

The probability of selecting a person from this group is simply this percentage expressed as a fraction.

$$\text{Probability} = 28\% = \frac{28}{100}$$

Now, we simplify the fraction:

$$\frac{28}{100} = \frac{14}{50} = \frac{7}{25}$$

Step 4: Final Answer:
The probability that the selected person invests in municipal bonds but NOT in oil stocks is $\frac{7}{25}$. Note that the total number of people (2,500) is extra information and not needed to solve the problem since the data is given in percentages.

> **Quick Tip**
>
> When dealing with overlapping sets or Venn diagram problems, the key is to use the formula: Total = Group1 + Group2 - Both + Neither. To find the "Only Group1" portion, calculate Group1 - Both.

---

5. A closed cylindrical tank contains 36π cubic feet of water and is filled to half its capacity. When the tank is placed upright on its circular base on level ground, the height of the water in the tank is 4 feet. When the tank is placed on its side on level ground, what is the height, in feet, of the surface of the water above the ground?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 9

Correct Answer: (B) 3

Solution:
(Note: The OCR of the question likely omitted a π symbol from the volume, which is a common issue. The solution proceeds assuming the volume of water is 36π cubic feet, as this leads to one of the integer answers provided.)

Step 1: Understanding the Concept:
The problem involves a cylinder and its properties (volume, radius, height). We need to use

the information from when the tank is upright to find its radius. Then, we use this radius to determine the water level when the tank is on its side.

Step 2: Key Formula or Approach:
The volume of a cylinder is given by the formula:

$$V = \pi r^2 h$$

where r is the radius of the circular base and h is the height.

Step 3: Detailed Explanation:
Part 1: Find the radius of the tank.
We are given that when the tank is upright, the volume of the water is 36π cubic feet and the height of the water is 4 feet.
Using the volume formula for the water in the tank:

$$V_{\text{water}} = \pi r^2 h_{\text{water}}$$

Substitute the known values:

$$36\pi = \pi r^2(4)$$

To solve for r, we can first divide both sides by 4π:

$$\frac{36\pi}{4\pi} = r^2$$

$$9 = r^2$$

$$r = 3 \text{ feet}$$

So, the radius of the cylindrical tank is 3 feet.

Part 2: Determine the water height when the tank is on its side.
The problem states that the tank is filled to half its capacity.
When a cylinder is placed on its side, its total "height" from the ground is its diameter ($2r$).
If the cylinder is exactly half-full, the water level will be a flat surface passing through the central axis of the cylinder.
Therefore, the height of the water surface above the ground is equal to the radius of the cylinder.

$$\text{Height of water surface} = r = 3 \text{ feet}$$

Step 4: Final Answer:
When the tank is placed on its side, the height of the surface of the water above the ground is 3 feet.

Quick Tip

In geometry problems on standardized tests, if your calculation results in an irrational number (like $\sqrt{9/\pi}$) but the options are all clean integers, double-check the problem statement. It's highly likely that a π was intended to be part of a given value to allow for cancellation.

6. A marketing firm determined that, of 200 households surveyed, 80 used neither Brand A nor Brand B soap, 60 used only Brand A soap, and for every household that used both brands of soap, 3 used only Brand B soap. How many of the 200 households surveyed used both brands of soap?

(A) 15
(B) 20
(C) 30
(D) 40
(E) 45

Correct Answer: (A) 15

Solution:

Step 1: Understanding the Concept:
This problem involves analyzing survey data using set theory. The total number of surveyed households is composed of four distinct, non-overlapping groups: those who use only Brand A, those who use only Brand B, those who use both, and those who use neither.

Step 2: Key Formula or Approach:
Let's define the groups with variables.
Let x be the number of households that used both brands of soap.
From the problem, for every household that used both, 3 used only Brand B. So, the number of households that used only Brand B is $3x$.
The total number of households is the sum of the four groups:

$$\text{Total} = (\text{Only A}) + (\text{Only B}) + (\text{Both}) + (\text{Neither})$$

Step 3: Detailed Explanation:
We are given the following information:
Total households = 200
Used only Brand A = 60
Used neither Brand A nor Brand B = 80
Used both brands = x
Used only Brand B = $3x$
Now, we can set up an equation using the formula from Step 2:

$$200 = 60 + 3x + x + 80$$

First, combine the constant terms and the x terms:

$$200 = 140 + 4x$$

Next, isolate the term with x by subtracting 140 from both sides:

$$200 - 140 = 4x$$

$$60 = 4x$$

Finally, solve for x by dividing both sides by 4:

$$x = \frac{60}{4} = 15$$

Step 4: Final Answer:
The value of x represents the number of households that used both brands of soap. Therefore, 15 households used both brands.

Quick Tip

For survey problems, drawing a Venn diagram can be very helpful to visualize the different groups. Alternatively, setting up an algebraic equation based on the sum of the distinct parts is a quick and reliable method.

---

7. A certain club has 10 members, including Harry. One of the 10 members is to be chosen at random to be the president, one of the remaining 9 members is to be chosen at random to be the secretary, and one of the remaining 8 members is to be chosen at random to be the treasurer. What is the probability that Harry will be either the member chosen to be the secretary or the member chosen to be the treasurer?

(A) $\frac{1}{720}$
(B) $\frac{1}{80}$
(C) $\frac{1}{10}$
(D) $\frac{1}{9}$
(E) $\frac{1}{5}$

Correct Answer: (E) $\frac{1}{5}$

Solution:

Step 1: Understanding the Concept:
This problem asks for the probability of an event that can occur in two mutually exclusive ways (Harry can be secretary, or Harry can be treasurer, but not both). We can calculate the probability of each case and then add them together.

Step 2: Key Formula or Approach:
The probability of "Event A OR Event B" where A and B are mutually exclusive is $P(A \text{ or } B) = P(A) + P(B)$.
We will calculate the probability of Harry becoming secretary and the probability of Harry becoming treasurer separately.

Step 3: Detailed Explanation:
Method 1: Direct Logic
There are 3 distinct positions (President, Secretary, Treasurer). Each of the 10 members has

an equal chance of being selected for any specific position.
The probability that Harry is chosen for the President position is $\frac{1}{10}$.
The probability that Harry is chosen for the Secretary position is also $\frac{1}{10}$.
The probability that Harry is chosen for the Treasurer position is also $\frac{1}{10}$.
The question asks for the probability that Harry is chosen as secretary OR treasurer. Since these are mutually exclusive events, we add their probabilities:

$$P(\text{Harry is Secretary or Treasurer}) = P(\text{Harry is Secretary}) + P(\text{Harry is Treasurer})$$

$$P = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$

Method 2: Step-by-Step Probability Calculation
Case 1: Harry is chosen as Secretary.
- The president must be someone other than Harry. The probability of this is $\frac{9}{10}$.
- Given that someone else is president, there are 9 members left. The probability that Harry is chosen as secretary is $\frac{1}{9}$.
- $P(\text{Harry is Secretary}) = P(\text{Not Harry as President}) \times P(\text{Harry as Secretary}) = \frac{9}{10} \times \frac{1}{9} = \frac{1}{10}$.

Case 2: Harry is chosen as Treasurer.
- The president is not Harry (probability $\frac{9}{10}$).
- The secretary is not Harry (given the president was not Harry). There are 9 members left, 8 of whom are not Harry. Probability is $\frac{8}{9}$.
- The treasurer is Harry. There are 8 members left. Probability is $\frac{1}{8}$.
- $P(\text{Harry is Treasurer}) = \frac{9}{10} \times \frac{8}{9} \times \frac{1}{8} = \frac{1}{10}$.

Total Probability:

$$P(\text{Total}) = P(\text{Case 1}) + P(\text{Case 2}) = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$

Step 4: Final Answer:
The probability that Harry will be either the secretary or the treasurer is $\frac{1}{5}$.

> **Quick Tip**
>
> For problems involving selecting distinct roles, remember that any individual has an equal chance of landing in any specific role. The probability of a person being chosen for any one of N roles is 1/Total People. If the question asks for the probability of them being in one of K possible roles, the answer is often K/Total People.

---

8. If a certain toy store's revenue in November was $\frac{2}{5}$ of its revenue in December and its revenue in January was $\frac{1}{4}$ of its revenue in November, then the store's revenue in December was how many times the average (arithmetic mean) of its revenues in November and January?

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$

(D) 2
(E) 4

Correct Answer: (E) 4

Solution:

Step 1: Understanding the Concept:
The problem describes the relationship between a store's revenues in three different months. We need to express all revenues in terms of a single month's revenue, calculate the average of two of them, and then find the ratio of the third to this average.

Step 2: Key Formula or Approach:
Let N, D, and J represent the revenues for November, December, and January, respectively. We are given: 1. $N = \frac{2}{5}D$ 2. $J = \frac{1}{4}N$ We need to find the value of the ratio: $\frac{D}{\text{Average}(N,J)}$, which is $\frac{D}{(N+J)/2}$.

Step 3: Detailed Explanation:
First, let's express all revenues in terms of a single variable, D, since D is the reference in the first relationship and the final ratio.
We already have $N = \frac{2}{5}D$.
Now, let's express J in terms of D by substituting the expression for N into the second equation:

$$J = \frac{1}{4}N = \frac{1}{4}\left(\frac{2}{5}D\right) = \frac{2}{20}D = \frac{1}{10}D$$

Next, calculate the average of the revenues in November and January:

$$\text{Average}(N, J) = \frac{N + J}{2} = \frac{\frac{2}{5}D + \frac{1}{10}D}{2}$$

To add the fractions in the numerator, we find a common denominator (10):

$$\frac{2}{5}D = \frac{4}{10}D$$

$$\text{Average}(N, J) = \frac{\frac{4}{10}D + \frac{1}{10}D}{2} = \frac{\frac{5}{10}D}{2} = \frac{\frac{1}{2}D}{2} = \frac{1}{4}D$$

Finally, we find the ratio of the December revenue to this average:

$$\text{Ratio} = \frac{D}{\text{Average}(N, J)} = \frac{D}{\frac{1}{4}D}$$

The variable D cancels out:

$$\text{Ratio} = \frac{1}{\frac{1}{4}} = 1 \times 4 = 4$$

Step 4: Final Answer:
The store's revenue in December was 4 times the average of its revenues in November and January.

Quick Tip

In problems involving ratios and relationships, choosing one variable as a "base" (like D in this case) and expressing all other variables in terms of that base simplifies the calculations significantly. The base variable will almost always cancel out in the final ratio.

---

9. A researcher computed the mean, the median, and the standard deviation for a set of performance scores. If 5 were to be added to each score, which of these three statistics would change?

(A) The mean only
(B) The median only
(C) The standard deviation only
(D) The mean and the median
(E) The mean and the standard deviation

Correct Answer: (D) The mean and the median

Solution:

Step 1: Understanding the Concept:
This question tests the understanding of how basic statistical measures (mean, median, standard deviation) are affected when a constant value is added to every data point in a set.

Step 2: Detailed Explanation:
Let's analyze the effect of adding 5 to each score on each statistic:
Mean: The mean is the sum of all scores divided by the number of scores. If we add 5 to each of the n scores, the total sum increases by $5n$. The new mean will be $(\text{Old Sum} + 5n)/n = (\text{Old Sum}/n) + (5n/n) = \text{Old Mean} + 5$. Therefore, **the mean changes**.

Median: The median is the middle value of a sorted dataset. When 5 is added to every score, the relative order of the scores does not change. The score that was in the middle will still be in the middle, but its value will have increased by 5. For example, if the scores are 10, 20, 30, the median is 20. If we add 5, the scores become 15, 25, 35, and the new median is 25. Therefore, **the median changes**.

Standard Deviation: The standard deviation measures the spread or dispersion of the data points from the mean. When we add a constant to every score, the entire dataset shifts along the number line, but the distances between the scores remain the same. The mean also shifts by the same constant, so the distance of each score from the mean $(x_i - \text{mean})$ does not change. Since the standard deviation is calculated based on these distances, it remains unchanged. Therefore, **the standard deviation does not change**.

Step 3: Final Answer:
Both the mean and the median would change, while the standard deviation would not. Thus,

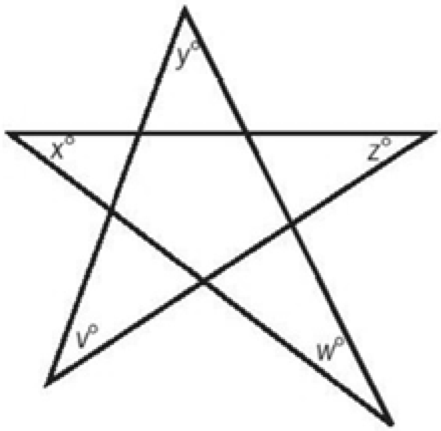
the correct option is the mean and the median.

Quick Tip

Remember these simple rules for data transformation: - Adding/subtracting a constant changes measures of center (mean, median) but not measures of spread (range, standard deviation). - Multiplying/dividing by a constant changes both measures of center and measures of spread.

---

10. In the figure shown, what is the value of $v + x + y + z + w$?



(A) 45
(B) 90
(C) 180
(D) 270
(E) 360

Correct Answer: (C) 180

Solution:

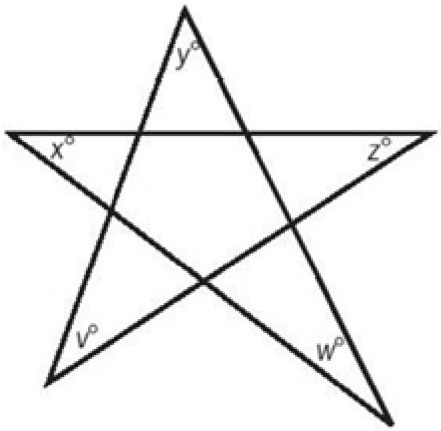
Step 1: Understanding the Concept:
The problem asks for the sum of the angles at the five vertices (tips) of a five-pointed star (a pentagram). This is a classic result in Euclidean geometry.

Step 2: Key Formula or Approach:
We can solve this using the exterior angle theorem, which states that the measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Step 3: Detailed Explanation:
Let's label two of the intersection points in the star to create a triangle that contains the angle v. Let these points be P and Q, such that the triangle at the top is $\triangle vPQ$. The sum of angles

in this triangle is:

$$v + \angle vPQ + \angle vQP = 180^\circ$$



Now, let's analyze the angles $\angle vPQ$ and $\angle vQP$.
1. The angle $\angle vPQ$ is an exterior angle to the triangle containing the vertices with angles x and z. According to the exterior angle theorem, the measure of $\angle vPQ$ is the sum of the two remote interior angles, x and z.

$$\angle vPQ = x + z$$

2. Similarly, the angle $\angle vQP$ is an exterior angle to the triangle containing the vertices with angles y and w. According to the exterior angle theorem, the measure of $\angle vQP$ is the sum of the two remote interior angles, y and w.

$$\angle vQP = y + w$$

Now substitute these two expressions back into the equation for the sum of angles in $\triangle vPQ$:

$$v + (x + z) + (y + w) = 180^\circ$$

Rearranging the terms, we get:

$$v + x + y + z + w = 180^\circ$$

Step 4: Final Answer:
The sum of the angles at the vertices of the five-pointed star is 180 degrees.

Quick Tip

Memorize the result that the sum of the angles at the tips of a five-pointed star is always 180°. This is a frequently tested concept in geometry problems on competitive exams. The general formula for the sum of the vertex angles of an n-pointed star is $(n-4) \times 180^\circ$.

---

11. Of the three-digit integers greater than 700, how many have two digits that are equal to each other and the remaining digit different from the other two?

(A) 90
(B) 82
(C) 80
(D) 45
(E) 36

Correct Answer: (C) 80

Solution:

Step 1: Understanding the Concept:
We need to count the number of integers between 701 and 999 (inclusive) that have exactly two identical digits. We can break this problem down by considering the possible hundreds digit (7, 8, or 9) and the pattern of the digits (e.g., AAB, ABA, BAA).

Step 2: Detailed Explanation:
We will analyze the count for each hundred's range separately. The condition is that two digits are equal, and one is different.
Case 1: Numbers in the 700s (from 701 to 799)
The first digit is fixed as 7.

- **Pattern 77X:** The repeated digit is 7. The third digit, X, can be any digit from 0 to 9 except 7 (to avoid 777). So, X can be $\{0, 1, 2, 3, 4, 5, 6, 8, 9\}$. This gives 9 numbers.
- **Pattern 7X7:** The repeated digit is 7. The middle digit, X, can be any digit except 7. So, X can be $\{0, 1, 2, 3, 4, 5, 6, 8, 9\}$. This gives 9 numbers.
- **Pattern 7XX:** The repeated digit is X, which cannot be 7. X can also not be 0, as the number must be greater than 700 (700 is not included). So X can be $\{1, 2, 3, 4, 5, 6, 8, 9\}$. This gives 8 numbers.

Total for the 700s $= 9 + 9 + 8 = 26$.

Case 2: Numbers in the 800s (from 800 to 899)
The first digit is fixed as 8.

- **Pattern 88X:** X can be any digit except 8. This gives 9 numbers.
- **Pattern 8X8:** X can be any digit except 8. This gives 9 numbers.
- **Pattern 8XX:** X can be any digit except 8. This gives 9 numbers (X can be 0 here).

Total for the 800s $= 9 + 9 + 9 = 27$.

Case 3: Numbers in the 900s (from 900 to 999)
The first digit is fixed as 9.

- **Pattern 99X:** X can be any digit except 9. This gives 9 numbers.
- **Pattern 9X9:** X can be any digit except 9. This gives 9 numbers.
- **Pattern 9XX:** X can be any digit except 9. This gives 9 numbers.

Total for the 900s $= 9 + 9 + 9 = 27$.

Step 3: Final Answer:
The total number of such integers is the sum of the counts from all cases:

$$\text{Total} = 26 + 27 + 27 = 80$$

Quick Tip

For counting problems with multiple constraints, break the problem into smaller, non-overlapping cases. Handle each case carefully, paying close attention to boundary conditions like "greater than 700," which may affect one case differently than others.

---

12. Positive integer y is 50 percent of 50 percent of positive integer x, and y percent of x equals 100. What is the value of x?

(A) 50
(B) 100
(C) 200
(D) 1,000
(E) 2,000

Correct Answer: (C) 200

Solution:

Step 1: Understanding the Concept:
This problem requires translating two English statements about percentages into algebraic equations and then solving the system of equations.

Step 2: Key Formula or Approach:
We will set up two equations based on the given information.
1. "y is 50 percent of 50 percent of x" translates to:

$$y = 0.50 \times (0.50 \times x)$$

2. "y percent of x equals 100" translates to:

$$\frac{y}{100} \times x = 100$$

Step 3: Detailed Explanation:
First, simplify the first equation:

$$y = 0.25x$$

Now, simplify the second equation:

$$yx = 100 \times 100$$

$$yx = 10000$$

We have a system of two equations: (1) $y = 0.25x$ (2) $yx = 10000$ Substitute the expression for y from equation (1) into equation (2):

$$(0.25x)x = 10000$$

$$0.25x^2 = 10000$$

To solve for x^2, divide both sides by 0.25 (which is the same as multiplying by 4):

$$x^2 = \frac{10000}{0.25} = 10000 \times 4 = 40000$$

Now, take the square root of both sides. Since x is a positive integer:

$$x = \sqrt{40000} = 200$$

Step 4: Final Answer:
The value of x is 200. We can check this: if $x = 200$, then $y = 0.25 \times 200 = 50$. Then, y percent of x is 50% of 200, which is $0.50 \times 200 = 100$. This matches the given information.

> Quick Tip
>
> When translating "percent" into math, remember that "P percent" is equivalent to the decimal $P/100$. Be systematic in converting each phrase into its mathematical equivalent before trying to solve.

---

13. If s and t are positive integers such that $\frac{s}{t} = 64.12$, which of the following could be the remainder when s is divided by t?

(A) 2
(B) 4
(C) 8
(D) 20
(E) 45

Correct Answer: (E) 45

Solution:

Step 1: Understanding the Concept:
The division of integer s by integer t can be expressed as $s = qt + r$, where q is the quotient and r is the remainder. Dividing this equation by t gives $\frac{s}{t} = q + \frac{r}{t}$. We can use this relationship to find the remainder.

Step 2: Key Formula or Approach:
From the given equation:

$$\frac{s}{t} = 64.12 = 64 + 0.12$$

Comparing this to $\frac{s}{t} = q + \frac{r}{t}$, we can identify: The integer part is the quotient, $q = 64$. The fractional part is the ratio of the remainder to the divisor, $\frac{r}{t} = 0.12$.

Step 3: Detailed Explanation:
We have the equation for the remainder r and the divisor t:

$$\frac{r}{t} = 0.12$$

Let's convert the decimal to a fraction and simplify it:

$$\frac{r}{t} = \frac{12}{100} = \frac{3}{25}$$

This gives us the relationship $25r = 3t$.
Since r and t must be integers, this equation tells us that r must be a multiple of 3, and t must be a multiple of 25.
Now we check the given options to see which one could be a value for r. The value must be a multiple of 3. (A) 2 is not a multiple of 3.
(B) 4 is not a multiple of 3.
(C) 8 is not a multiple of 3.
(D) 20 is not a multiple of 3.
(E) 45 is a multiple of 3 ($45 = 3 \times 15$).
Let's verify if $r = 45$ is possible. If $r = 45$, then $t = \frac{25r}{3} = \frac{25 \times 45}{3} = 25 \times 15 = 375$. The condition for a remainder is $0 \leq r < t$. Here, $0 \leq 45 < 375$, which is true. Thus, 45 is a possible remainder.

Step 4: Final Answer:
The only possible value for the remainder among the choices is 45.

Quick Tip

When you see an equation like $\frac{s}{t}$ = decimal, immediately separate the decimal into its integer and fractional parts. The integer part is the quotient, and the fractional part is the remainder divided by the divisor (r/t).

---

14. Of the 84 parents who attended a meeting at a school, 35 volunteered to supervise children during the school picnic and 11 volunteered both to supervise children during the picnic and to bring refreshments to the picnic. If the number of parents who volunteered to bring refreshments was 1.5 times the number of parents who neither volunteered to supervise children during the picnic nor volunteered to bring refreshments, how many of the parents volunteered to bring refreshments?

(A) 25
(B) 36
(C) 38
(D) 42
(E) 45

Correct Answer: (B) 36

Solution:

Step 1: Understanding the Concept:
This is a set theory problem that can be solved using a Venn diagram or the principle of inclusion-exclusion. We are given information about a total group and various overlapping subgroups.

Step 2: Key Formula or Approach:
Let S be the set of parents who volunteered to supervise, and R be the set of parents who volunteered to bring refreshments.
The formula for two sets is:

$$\text{Total} = |S| + |R| - |S \cap R| + |\text{Neither}|$$

Alternatively, a more intuitive formula is:

$$\text{Total} = |\text{Only S}| + |\text{Only R}| + |\text{Both}| + |\text{Neither}|$$

We are given:

- Total = 84
- $|S| = 35$
- $|S \cap R|$ (Both) = 11
- $|R| = 1.5 \times |\text{Neither}|$

Step 3: Detailed Explanation:
Let's use the second formula. First, find the number of parents who only supervised.

$$|\text{Only S}| = |S| - |S \cap R| = 35 - 11 = 24$$

Let $N = |\text{Neither}|$. Then the total number who brought refreshments is $|R| = 1.5N$. The number of parents who only brought refreshments is:

$$|\text{Only R}| = |R| - |S \cap R| = 1.5N - 11$$

Now, plug all the parts into the total formula:

$$\text{Total} = |\text{Only S}| + |\text{Only R}| + |\text{Both}| + |\text{Neither}|$$

$$84 = 24 + (1.5N - 11) + 11 + N$$

Simplify the equation:

$$84 = 24 + 1.5N + N$$

$$84 = 24 + 2.5N$$

Subtract 24 from both sides:

$$60 = 2.5N$$

Solve for N:

$$N = \frac{60}{2.5} = \frac{600}{25} = 24$$

The question asks for the number of parents who volunteered to bring refreshments, which is $|R|$.

$$|R| = 1.5 \times N = 1.5 \times 24 = 36$$

Step 4: Final Answer:
36 parents volunteered to bring refreshments.

Quick Tip

For problems with two overlapping sets, it's often easiest to break the total down into the four distinct regions: Only Group A, Only Group B, Both, and Neither. Set up an equation where the sum of these four regions equals the total.

---

15. The product of all the prime numbers less than 20 is closest to which of the following powers of 10?

(A) 10^9
(B) 10^8
(C) 10^7
(D) 10^6
(E) 10^5

Correct Answer: (C) 10^7

Solution:

Step 1: Understanding the Concept:
We need to identify all prime numbers less than 20, calculate their product, and then determine which power of 10 is the closest approximation to this product.

Step 2: Detailed Explanation:
First, list the prime numbers less than 20:

$$2, 3, 5, 7, 11, 13, 17, 19$$

Next, calculate their product (P). We can group numbers to make the calculation easier:

$$P = (2 \times 5) \times (3 \times 7) \times 11 \times 13 \times 17 \times 19$$

$$P = 10 \times 21 \times 11 \times 13 \times 17 \times 19$$

Now let's continue multiplying:

$$P = 210 \times 11 \times 13 \times 17 \times 19$$

$$210 \times 11 = 2310$$

$$P = 2310 \times 13 \times 17 \times 19$$

$$2310 \times 13 = 30030$$
$$P = 30030 \times 17 \times 19$$
$$30030 \times 17 = 510510$$
$$P = 510510 \times 19$$

Let's approximate $510510 \times 19 \approx 510000 \times 20 = 10200000$, which is 1.02×10^7. This suggests the answer is close to 10^7. Let's do the exact calculation:

$$510510 \times 19 = 510510 \times (20 - 1) = 10210200 - 510510 = 9699690$$

The product is 9,699,690.

Step 3: Comparison with Powers of 10:
We need to see if 9,699,690 is closer to 10^6 or 10^7.

$$10^6 = 1,000,000$$
$$10^7 = 10,000,000$$

Distance from 10^7: $|10,000,000 - 9,699,690| = 300,310$.
Distance from 10^6: $|9,699,690 - 1,000,000| = 8,699,690$.
The product is much closer to 10,000,000 (10^7).

Step 4: Final Answer:
The product is closest to 10^7.

> **Quick Tip**
>
> In estimation problems, strategically group numbers to create easy multiples (like 10 from 2 and 5). You can often get close enough to the answer with good approximations without needing to perform the full, precise calculation.

---

16. If $\sqrt{3 - 2x} = \sqrt{2x + 1}$, then $4x^2 =$

(A) 1
(B) 4
(C) 2 – 2x
(D) 4x – 2
(E) 6x – 1

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:
We are given an equation involving square roots and need to solve for an expression involving

the variable x. The key first step is to eliminate the square roots by squaring both sides of the equation.

Step 2: Key Formula or Approach:
Assuming the equation is $\sqrt{3-2x} = \sqrt{2x+1}$.
To solve, we will square both sides:

$$(\sqrt{3-2x})^2 = (\sqrt{2x+1})^2$$

This simplifies the equation to a linear one, which we can then solve for x.

Step 3: Detailed Explanation:
Squaring both sides of the equation gives:

$$3 - 2x = 2x + 1$$

Now, solve for x. Add 2x to both sides:

$$3 = 4x + 1$$

Subtract 1 from both sides:

$$2 = 4x$$

Divide by 4:

$$x = \frac{2}{4} = \frac{1}{2}$$

The problem asks for the value of $4x^2$, not x. Substitute the value of x we found:

$$4x^2 = 4\left(\frac{1}{2}\right)^2 = 4\left(\frac{1}{4}\right) = 1$$

We should also check if the solution $x = 1/2$ is valid by plugging it back into the original equation's radicals. For $\sqrt{3-2x}$: $\sqrt{3-2(1/2)} = \sqrt{3-1} = \sqrt{2}$. (Valid)
For $\sqrt{2x+1}$: $\sqrt{2(1/2)+1} = \sqrt{1+1} = \sqrt{2}$. (Valid)
Since $\sqrt{2} = \sqrt{2}$, the solution is correct.

Step 4: Final Answer:
The value of $4x^2$ is 1.

Quick Tip

When solving radical equations, always remember to check your solutions by plugging them back into the original equation. Squaring both sides can sometimes introduce extraneous solutions that are not valid. Also, be sure to answer what the question is asking for (e.g., $4x^2$, not just x).

---

17. If $n = \frac{16}{81}$, what is the value of $\sqrt{n}$?
(Note: The OCR included a typo, which has been corrected to the most logical question based on the options.)

(A) $\frac{1}{9}$
(B) $\frac{1}{4}$
(C) $\frac{4}{9}$
(D) $\frac{2}{3}$
(E) $\frac{9}{2}$

Correct Answer: (C) $\frac{4}{9}$

Solution:

Step 1: Understanding the Concept:
The problem asks to find the square root of a given fraction. The square root of a fraction is the square root of the numerator divided by the square root of the denominator.

Step 2: Key Formula or Approach:
The property of square roots we use is:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Step 3: Detailed Explanation:
We are given $n = \frac{16}{81}$ and we need to find $\sqrt{n}$.
Substitute the value of n:

$$\sqrt{n} = \sqrt{\frac{16}{81}}$$

Apply the square root property:

$$\sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}}$$

Calculate the square roots of the numerator and the denominator:

$$\sqrt{16} = 4$$

$$\sqrt{81} = 9$$

Combine the results:

$$\sqrt{n} = \frac{4}{9}$$

Step 4: Final Answer:
The value of $\sqrt{n}$ is $\frac{4}{9}$.

Quick Tip

Memorizing perfect squares (like $4^2 = 16$, $9^2 = 81$, etc.) is essential for saving time on arithmetic and algebra problems on standardized tests.

---

18. If n is the product of the integers from 1 to 8, inclusive, how many different prime factors greater than 1 does n have?

(A) Four
(B) Five
(C) Six
(D) Seven
(E) Eight

Correct Answer: (A) Four

Solution:

Step 1: Understanding the Concept:
The problem asks for the number of distinct prime factors of the number $n = 8!$ (8 factorial). A prime factor of a product of integers must be a prime factor of at least one of those integers.

Step 2: Key Formula or Approach:
We need to find all the unique prime numbers that are less than or equal to 8. The product $1 \times 2 \times \cdots \times 8$ will have these primes as its factors.

Step 3: Detailed Explanation:
The number n is the product:

$$n = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$$

To find the distinct prime factors of n, we just need to list all the prime numbers that appear in the prime factorization of any of the numbers from 1 to 8. The numbers from 1 to 8 are: 1, 2, 3, 4, 5, 6, 7, 8. Let's find the prime numbers in this list or the prime factors of the composite numbers in this list.

- 2 is a prime number.
- 3 is a prime number.
- 4 is not prime, its prime factor is 2.
- 5 is a prime number.
- 6 is not prime, its prime factors are 2 and 3.
- 7 is a prime number.
- 8 is not prime, its prime factor is 2.

The set of all unique prime factors we have found is $\{2, 3, 5, 7\}$.
There are four distinct prime factors. Prime numbers are by definition greater than 1.

Step 4: Final Answer:
The number n has four different prime factors greater than 1.

Quick Tip

To find the number of distinct prime factors of a factorial, $k!$, you simply need to count the number of prime numbers less than or equal to k.

---

19. If k is an integer and $2 < k < 7$, for how many different values of k is there a triangle with sides of lengths 2, 7, and k?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

Correct Answer: (A) One

Solution:

Step 1: Understanding the Concept:
This problem requires the application of the Triangle Inequality Theorem, which defines the relationship between the lengths of the sides of a triangle.

Step 2: Key Formula or Approach:
The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. For sides a, b, and c, the following three conditions must be met:

1. $a + b > c$
2. $a + c > b$
3. $b + c > a$

Step 3: Detailed Explanation:
First, identify the possible integer values for k. The condition $2 < k < 7$ means k can be 3, 4, 5, or 6.
Now, let's apply the Triangle Inequality Theorem with sides $a = 2$, $b = 7$, and $c = k$.

1. $2 + 7 > k \implies 9 > k$
2. $2 + k > 7 \implies k > 5$
3. $7 + k > 2 \implies k > -5$ (This is always true for positive k)

We must satisfy both $k < 9$ and $k > 5$. So, the valid range for k is $5 < k < 9$.
Now we check which of the possible integer values for k ($k \in \{3, 4, 5, 6\}$) fall into this valid range.

- Is $k = 3$ in the range $5 < k < 9$? No.
- Is $k = 4$ in the range $5 < k < 9$? No.

- Is $k = 5$ in the range $5 < k < 9$? No (it's not strictly greater).
- Is $k = 6$ in the range $5 < k < 9$? Yes.

Only one value, $k = 6$, satisfies the conditions.

Step 4: Final Answer:
There is only one possible integer value for k.

> Quick Tip
>
> A shortcut for the Triangle Inequality Theorem is that the third side 'c' must be between the difference and the sum of the other two sides: $|a - b| < c < a + b$. For sides 2 and 7, we have $|7 - 2| < k < 7 + 2$, which simplifies to $5 < k < 9$.

---

20. A right circular cone is inscribed in a hemisphere so that the base of the cone coincides with the base of the hemisphere. What is the ratio of the height of the cone to the radius of the hemisphere?

(A) $\sqrt{3} : 1$
(B) 1:1
(C) $\frac{1}{2} : 1$
(D) $\sqrt{2} : 1$
(E) 2:1

Correct Answer: (B) 1:1

Solution:

Step 1: Understanding the Concept:
This problem requires visualizing the geometric setup described. A hemisphere is half of a sphere. A cone is inscribed within it, sharing the same circular base.

Step 2: Detailed Explanation:
Let's define the dimensions based on the description.

- Let the radius of the hemisphere be R. The base of the hemisphere is a circle with radius R.
- The problem states that the base of the cone coincides with the base of the hemisphere. This means the cone's base is also a circle with radius R.
- For the cone to be "inscribed in a hemisphere," its vertex must lie on the surface of the hemisphere. Since the bases coincide, the vertex of the cone must be at the "top" point of the hemisphere, which is directly above the center of the base.
- The height of the cone (h) is the perpendicular distance from its vertex to the center of its base.

- This distance, from the top point of the hemisphere down to the center of its circular base, is precisely the radius of the hemisphere, R.
- Therefore, the height of the cone is equal to the radius of the hemisphere: $h = R$.

Step 3: Final Answer:
The question asks for the ratio of the height of the cone (h) to the radius of the hemisphere (R).

$$\text{Ratio} = h : R = R : R = 1 : 1$$

Quick Tip

For geometry problems involving inscribed shapes, drawing a simple 2D cross-section can make the relationships between dimensions much clearer. In this case, a cross-section would show a semicircle with an isosceles triangle inscribed in it, making it obvious that the height of the triangle equals the radius of the semicircle.

---

21. John deposited \$10,000 to open a new savings account that earned 4 percent annual interest, compounded quarterly. If there were no other transactions in the account, what was the amount of money in John's account 6 months after the account was opened?

(A) \$10,100
(B) \$10,101
(C) \$10,200
(D) \$10,201
(E) \$10,400

Correct Answer: (D) \$10,201

Solution:

Step 1: Understanding the Concept:
This is a compound interest problem. We need to calculate the future value of an investment with interest compounded more than once per year.

Step 2: Key Formula or Approach:
The formula for compound interest is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where:

- A = the future value of the investment/loan, including interest
- P = the principal amount (\$10,000)
- r = the annual interest rate (4% or 0.04)

- n = the number of times that interest is compounded per year (quarterly = 4)
- t = the time the money is invested for in years (6 months = 0.5 years)

Step 3: Detailed Explanation:
First, identify all the variables from the problem:

- P = 10,000
- r = 0.04
- n = 4
- t = 0.5

The interest rate per period is $\frac{r}{n} = \frac{0.04}{4} = 0.01$.
The total number of compounding periods is $nt = 4 \times 0.5 = 2$.
Now, substitute these values into the formula:

$$A = 10000\left(1 + 0.01\right)^2$$

$$A = 10000(1.01)^2$$

Calculate $(1.01)^2$:

$$1.01 \times 1.01 = 1.0201$$

Finally, calculate the total amount A:

$$A = 10000 \times 1.0201 = 10201$$

Step 4: Final Answer:
The amount of money in the account after 6 months is \$10,201.

> **Quick Tip**
>
> For simple cases of compound interest (like two periods), you can often calculate it step-by-step without the full formula. Period 1 (3 months): Interest = 1% of \$10,000 = \$100. New balance = \$10,100. Period 2 (6 months): Interest = 1% of \$10,100 = \$101. New balance = \$10,100 + \$101 = \$10,201.

---

22. A container in the shape of a right circular cylinder is $\frac{1}{2}$ full of water. If the volume of water in the container is 36 cubic inches and the height of the container is 9 inches, what is the diameter of the base of the cylinder, in inches?
(Note: The fraction in the image is blurry but is assumed to be 1/2 to match one of the answer choices.)
(A) $\frac{16}{9\pi}$
(B) $\frac{4}{\sqrt{\pi}}$
(C) $\sqrt{\frac{12}{\pi}}$

(D) $\sqrt{\frac{2}{\pi}}$

(E) $4\sqrt{\frac{2}{\pi}}$

Correct Answer: (E) $4\sqrt{\frac{2}{\pi}}$

Solution:

Step 1: Understanding the Concept:
The problem relates the partial volume of a cylinder to its total volume and dimensions (height, radius). We need to find the total volume first, then use the volume formula to solve for the radius, and finally find the diameter.

Step 2: Key Formula or Approach:
The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height.

Step 3: Detailed Explanation:
We are given that the container is $\frac{1}{2}$ full and the volume of the water is 36 cubic inches. Let V_{total} be the total volume of the cylinder.

$$\frac{1}{2}V_{total} = 36$$

First, find the total volume of the cylinder:

$$V_{total} = 36 \times 2 = 72 \text{ cubic inches}$$

Now use the cylinder volume formula with the total volume and the given height $h = 9$ inches.

$$V_{total} = \pi r^2 h$$

$$72 = \pi r^2 (9)$$

To solve for r, first divide both sides by 9:

$$8 = \pi r^2$$

Isolate r^2:

$$r^2 = \frac{8}{\pi}$$

Take the square root of both sides to find the radius r:

$$r = \sqrt{\frac{8}{\pi}} = \sqrt{\frac{4 \times 2}{\pi}} = 2\sqrt{\frac{2}{\pi}}$$

The question asks for the diameter, which is twice the radius ($d = 2r$).

$$d = 2 \times \left(2\sqrt{\frac{2}{\pi}}\right) = 4\sqrt{\frac{2}{\pi}}$$

Step 4: Final Answer:
The diameter of the base of the cylinder is $4\sqrt{\frac{2}{\pi}}$ inches.

Quick Tip

Be careful to distinguish between radius and diameter. Many geometry problems will ask for the diameter after you've calculated the radius, so always double-check what the question is asking for before selecting an answer.

---

23. If the positive integer x is a multiple of 4 and the positive integer y is a multiple of 6, then xy must be a multiple of which of the following?

I. 8
II. 12
III. 18
(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

Correct Answer: (B) I and II only

Solution:

Step 1: Understanding the Concept:
This problem tests understanding of multiples and factors. We are given properties of two integers, x and y, and asked to determine a necessary property of their product, xy.

Step 2: Key Formula or Approach:
We can express x and y algebraically. Since x is a multiple of 4, we can write $x = 4a$ for some positive integer a. Since y is a multiple of 6, we can write $y = 6b$ for some positive integer b. The product is $xy = (4a)(6b) = 24ab$.
This means that xy must be a multiple of 24. We need to check which of the given options are factors of 24. Alternatively, we can use a counterexample to disprove a statement.

Step 3: Detailed Explanation:
The product $xy = 24ab$. Let's check each statement. **I. 8**
Is 24ab always a multiple of 8? Yes, because $24ab = 8 \times (3ab)$. Since 3ab is an integer, xy is always a multiple of 8. Statement I is true.

II. 12
Is 24ab always a multiple of 12? Yes, because $24ab = 12 \times (2ab)$. Since 2ab is an integer, xy is always a multiple of 12. Statement II is true.

III. 18
Is 24ab always a multiple of 18? Not necessarily. We can test this with the smallest possible values for x and y. Let $a = 1$ and $b = 1$. Then $x = 4$ and $y = 6$. The product is $xy = 4 \times 6 = 24$. Is 24 a multiple of 18? No. Since we found a case where xy is not a multiple of 18, this statement is not always true. Statement III is false.

Step 4: Final Answer:
Only statements I and II must be true.

> **Quick Tip**
>
> To check if a statement about multiples "must be true," try to find a counterexample using the smallest possible values. If you can find one case where the statement fails, it's false. For x being a multiple of 4 and y a multiple of 6, the simplest case is x=4, y=6.

---

24. Aaron will jog from home at x miles per hour and then walk back home by the same route at y miles per hour. How many miles from home can Aaron jog so that he spends a total of t hours jogging and walking?

(A) $\frac{xt}{y}$
(B) $\frac{x+t}{xy}$
(C) $\frac{xyt}{x+y}$
(D) $\frac{x+y+t}{xy}$
(E) $\frac{y+t}{x} - \frac{t}{y}$

Correct Answer: (C) $\frac{xyt}{x+y}$

Solution:

Step 1: Understanding the Concept:
This is a classic distance, rate, and time problem. The key relationship is Distance = Rate × Time, which can be rearranged as Time = Distance / Rate.

Step 2: Key Formula or Approach:
Let d be the distance from home that Aaron jogs (in miles). This is the value we need to find. The total time spent is the sum of the time jogging out and the time walking back.

$$t_{total} = t_{jog} + t_{walk}$$

Step 3: Detailed Explanation:
Let's express the time for each part of the journey in terms of d, x, and y.

- Distance jogging out = d
- Speed jogging out = x mph
- Time jogging out (t_{jog}) = $\frac{\text{Distance}}{\text{Rate}} = \frac{d}{x}$
- Distance walking back = d (same route)
- Speed walking back = y mph

- Time walking back (t_{walk}) = $\frac{\text{Distance}}{\text{Rate}} = \frac{d}{y}$

We are given that the total time is t. So we can set up the equation:

$$t = \frac{d}{x} + \frac{d}{y}$$

Now, we need to solve this equation for d. Factor out d from the right side:

$$t = d\left(\frac{1}{x} + \frac{1}{y}\right)$$

To combine the fractions in the parenthesis, find a common denominator (xy):

$$\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{x+y}{xy}$$

Substitute this back into the equation:

$$t = d\left(\frac{x+y}{xy}\right)$$

To isolate d, multiply both sides by the reciprocal of the fraction:

$$d = t \times \left(\frac{xy}{x+y}\right)$$

$$d = \frac{xyt}{x+y}$$

Step 4: Final Answer:
The distance from home Aaron can jog is $\frac{xyt}{x+y}$ miles.

Quick Tip

This formula for average speed problems is very common. The total distance is $2d$, total time is t. The average speed is $\frac{2xy}{x+y}$. The distance from home, d, can be found by (Average Speed $\times$ Total Time) / 2. This gives $\frac{1}{2} \times \frac{2xyt}{x+y} = \frac{xyt}{x+y}$.
