GMAT 2024 Quant Sample Paper Set 3 Question Paper with Solutions

Time Allowed : 2 Hours 15 Minutes	**Maximum Marks :** 205-805	**Total Questions :** 64

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GMAT exam is 2 hours and 15 minutes long (with one optional 10-minute break) and consists of 64 questions in total.
2. The GMAT exam is comprised of three sections:
3. Quantitative Reasoning: 21 questions, 45 minutes
4. Verbal Reasoning: 23 questions, 45 minutes
5. Data Insights: 20 questions, 45 minutes
6. You can answer the three sections in any order. As you move through a section, you can bookmark questions that you would like to review later.
7. When you have answered all questions in a section, you will proceed to the Question Review & Edit screen for that section.
8. If there is no time remaining in the section, you will NOT proceed to the Question Review & Edit screen and you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
9. Each Question Review & Edit screen includes a numbered list of the questions in that section and indicates the questions you bookmarked.
10. Clicking a question number will take you to that specific question. You can review as many questions as you would like and can edit up to three (3) answers.

Quantitative Aptitude

1. If the units digit of integer n is greater than 2, what is the units digit of n?
(1) The units digit of n is the same as the units digit of n^2.
(2) The units digit of n is the same as the units digit of n^3.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.

(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Solution:

Step 1: Understanding the Concept:
We are given an integer n whose units digit is greater than 2. This means the units digit can be 3, 4, 5, 6, 7, 8, or 9. We need to find the specific units digit of n. Let U(x) denote the units digit of x.

Step 2: Detailed Explanation:
Analyze Statement (1): The units digit of n is the same as the units digit of n^2. This means $\text{U}(n) = \text{U}(n^2)$. Let's test the possible units digits:

- If $\text{U}(n) = 3$, $\text{U}(n^2) = \text{U}(9) = 9$. (No match)
- If $\text{U}(n) = 4$, $\text{U}(n^2) = \text{U}(16) = 6$. (No match)
- If $\text{U}(n) = 5$, $\text{U}(n^2) = \text{U}(25) = 5$. (Match)
- If $\text{U}(n) = 6$, $\text{U}(n^2) = \text{U}(36) = 6$. (Match)
- If $\text{U}(n) = 7$, $\text{U}(n^2) = \text{U}(49) = 9$. (No match)
- If $\text{U}(n) = 8$, $\text{U}(n^2) = \text{U}(64) = 4$. (No match)
- If $\text{U}(n) = 9$, $\text{U}(n^2) = \text{U}(81) = 1$. (No match)

From statement (1), the units digit of n could be 5 or 6. Since there are two possibilities, this statement is not sufficient.

Analyze Statement (2): The units digit of n is the same as the units digit of n^3. This means $\text{U}(n) = \text{U}(n^3)$. Let's test the possible units digits:

- If $\text{U}(n) = 3$, $\text{U}(n^3) = \text{U}(27) = 7$. (No match)
- If $\text{U}(n) = 4$, $\text{U}(n^3) = \text{U}(64) = 4$. (Match)
- If $\text{U}(n) = 5$, $\text{U}(n^3) = \text{U}(125) = 5$. (Match)
- If $\text{U}(n) = 6$, $\text{U}(n^3) = \text{U}(216) = 6$. (Match)
- If $\text{U}(n) = 7$, $\text{U}(n^3) = \text{U}(343) = 3$. (No match)
- If $\text{U}(n) = 8$, $\text{U}(n^3) = \text{U}(512) = 2$. (No match, and also 2 is not ¿ 2)
- If $\text{U}(n) = 9$, $\text{U}(n^3) = \text{U}(729) = 9$. (Match)

From statement (2), the units digit of n could be 4, 5, 6, or 9. Since there are multiple possibilities, this statement is not sufficient.

Analyze Both Statements Together:
From statement (1), the possible units digits are 5, 6.
From statement (2), the possible units digits are 4, 5, 6, 9.
The common possibilities that satisfy both statements are 5, 6. Since there are still two possible values for the units digit of n, the statements together are not sufficient.

Step 3: Final Answer:
Since combining both statements still does not yield a unique units digit for n, the information is insufficient.

> **Quick Tip**
>
> For questions involving units digits, focus on the cyclicity of the last digit for powers of numbers. For example, the units digits of powers of 4 are (4, 6, 4, 6, ...), and for 5, they are always 5. This can save time compared to calculating the full power.

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2. What is the value of the integer p?
(1) Each of the integers 2, 3, and 5 is a factor of p.
(2) Each of the integers 2, 5, and 7 is a factor of p.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Solution:

Step 1: Understanding the Concept:
The question asks for a specific value of an integer p. To determine the value of p, we need enough information to uniquely identify it. Factors of a number are integers that divide it without a remainder.

Step 2: Key Formula or Approach:
If a number has several integers as its factors, it must be a multiple of their Least Common Multiple (LCM).

Step 3: Detailed Explanation:
Analyze Statement (1): Each of the integers 2, 3, and 5 is a factor of p.
This means p must be a multiple of the LCM of 2, 3, and 5.
Since 2, 3, and 5 are prime numbers, their LCM is their product: $2 \times 3 \times 5 = 30$.
So, p is a multiple of 30. Possible values for p are 30, 60, 90, 120, and so on.
Since p can have multiple values, statement (1) is not sufficient.

Analyze Statement (2): Each of the integers 2, 5, and 7 is a factor of p.
This means p must be a multiple of the LCM of 2, 5, and 7.
Since 2, 5, and 7 are prime numbers, their LCM is their product: $2 \times 5 \times 7 = 70$.
So, p is a multiple of 70. Possible values for p are 70, 140, 210, and so on.
Since p can have multiple values, statement (2) is not sufficient.

Analyze Both Statements Together:
From statement (1), p is a multiple of 30.
From statement (2), p is a multiple of 70.
Combining them, p must be a multiple of the LCM of 30 and 70.
$\text{LCM}(30, 70) = \text{LCM}(3 \times 10, 7 \times 10) = 10 \times \text{LCM}(3, 7) = 10 \times 21 = 210$.
So, p is a multiple of 210. Possible values for p are 210, 420, 630, and so on.
Even with both statements, we cannot determine a unique value for p.

Step 4: Final Answer:
The combined information is not sufficient to find a single value for p.

Quick Tip

In "what is the value" questions on Data Sufficiency, you need information that leads to one, and only one, possible answer. If the information suggests that the variable could be any multiple of a certain number, it is not sufficient.

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3. If the length of Wanda's telephone call was rounded up to the nearest whole minute by her telephone company, then Wanda was charged for how many minutes for her telephone call?
(1) The total charge for Wanda's telephone call was $6.50.
(2) Wanda was charged $0.50 more for the first minute of the telephone call than for each minute after the first.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.

(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Solution:

Step 1: Understanding the Concept:
We need to find the number of minutes Wanda was charged for. Let's denote this by M. The charging structure involves a rate for the first minute and a different rate for subsequent minutes.

Step 2: Key Formula or Approach:
Let C_1 be the charge for the first minute and C_s be the charge for each subsequent minute. The total charge T for M minutes is given by:

$$T = C_1 + (M - 1)C_s$$

Step 3: Detailed Explanation:
Analyze Statement (1): The total charge for Wanda's telephone call was $6.50.
This gives us the equation: $6.50 = C_1 + (M - 1)C_s$.
We have one equation with three unknowns (M, C_1, C_s). We cannot solve for M. For example, if the call was 1 minute ($M = 1$), the cost for the first minute would be $6.50. If the call was 2 minutes ($M = 2$), we have $6.50 = C_1 + C_s$, which is possible for many different rate combinations. Statement (1) is not sufficient.

Analyze Statement (2): Wanda was charged $0.50 more for the first minute of the telephone call than for each minute after the first.
This gives us the relationship: $C_1 = C_s + 0.50$.
This statement provides a relationship between the rates but gives no information about the total cost or the number of minutes. Statement (2) is not sufficient.

Analyze Both Statements Together:
We have two equations: 1) $6.50 = C_1 + (M - 1)C_s$ 2) $C_1 = C_s + 0.50$ Substitute the second equation into the first:

$$6.50 = (C_s + 0.50) + (M - 1)C_s$$
$$6.50 = C_s + 0.50 + MC_s - C_s$$
$$6.50 = MC_s + 0.50$$
$$6.00 = MC_s$$

We are left with one equation, $MC_s = 6$, with two unknowns (M and C_s). We cannot find a unique value for M. For example:

- If $C_s = \$1.00$, then $M = 6$ minutes.
- If $C_s = \$0.50$, then $M = 12$ minutes.
- If $C_s = \$0.75$, then $M = 8$ minutes.

Since we cannot find a unique value for M, the statements together are not sufficient.

Step 4: Final Answer:
Even with both pieces of information, the number of minutes charged cannot be uniquely determined.

> Quick Tip
>
> In word problems involving variables, count the number of independent equations you can form and the number of unknown variables. If the number of variables is greater than the number of equations, you generally cannot find a unique solution.

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4. The only gift certificates that a certain store sold yesterday were worth either \$100 each or \$10 each. If the store sold a total of 20 gift certificates yesterday, how many gift certificates worth \$10 each did the store sell yesterday?
(1) The gift certificates sold by the store yesterday were worth a total of between \$1,650 and \$1,800.
(2) Yesterday the store sold more than 15 gift certificates worth \$100 each.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is a system of equations problem. We are asked to find the number of \$10 gift certificates sold. Let x be the number of \$100 certificates.
Let y be the number of \$10 certificates.
We are given that the total number of certificates is 20, so $x + y = 20$.
We need to find the value of y.

Step 2: Key Formula or Approach:
The total value V of the certificates sold is given by $V = 100x + 10y$. We can express x in terms of y using the first equation: $x = 20 - y$.

Substituting this into the value equation gives:

$$V = 100(20 - y) + 10y$$

$$V = 2000 - 100y + 10y$$

$$V = 2000 - 90y$$

Step 3: Detailed Explanation:
Analyze Statement (1): The total value was between \$1,650 and \$1,800.

$$1650 < V < 1800$$

Substitute our expression for V:

$$1650 < 2000 - 90y < 1800$$

Subtract 2000 from all parts of the inequality:

$$1650 - 2000 < -90y < 1800 - 2000$$

$$-350 < -90y < -200$$

Divide all parts by -90. Remember to flip the inequality signs when dividing by a negative number:

$$\frac{-200}{-90} > y > \frac{-350}{-90}$$

$$\frac{20}{9} > y > \frac{35}{9}$$

Let's write it in the standard order:

$$3.88... > y > 2.22...$$

Since y must be an integer (as it's the number of certificates), the only integer value for y in this range is 3. This gives a unique value for y. Therefore, statement (1) is sufficient.

Analyze Statement (2): The store sold more than 15 certificates worth \$100 each. This means $x > 15$.
Since $x + y = 20$, we have $x = 20 - y$.

$$20 - y > 15$$

Subtract 20 from both sides:

$$-y > -5$$

Multiply by -1 and flip the inequality sign:

$$y < 5$$

This tells us that the number of \$10 certificates is less than 5. So, y could be 0, 1, 2, 3, or 4. Since there are multiple possible values for y, statement (2) is not sufficient.

Step 4: Final Answer:
Statement (1) alone is sufficient to determine the number of \$10 certificates, while statement

(2) alone is not.

> **Quick Tip**
>
> When a Data Sufficiency problem provides a range for a value, check if only one integer solution is possible within that range. This is a common pattern for sufficiency.

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5. Is the standard deviation of the set of measurements $x_1, x_2, x_3, ..., x_{20}$ less than 3?
(1) The variance for the set of measurements is 4.
(2) For each measurement, the difference between the mean and that measurement is 2.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (D) EACH statement ALONE is sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
The question asks whether the standard deviation (SD) of a set of 20 measurements is less than 3. This is a "Yes/No" question. A statement is sufficient if it allows us to answer with a definitive "Yes" or a definitive "No".

Step 2: Key Formula or Approach:
The relationship between standard deviation and variance is fundamental:

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

The variance is the average of the squared differences from the mean (μ):

$$\text{Variance} = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

Step 3: Detailed Explanation:
The question is: Is SD < 3?

Analyze Statement (1): The variance for the set of measurements is 4.
Using the formula, we can calculate the standard deviation:

$$\text{SD} = \sqrt{\text{Variance}} = \sqrt{4} = 2$$

Now we can answer the question: Is $2 < 3$? Yes. Since we get a definitive "Yes", statement (1) is sufficient.

Analyze Statement (2): For each measurement, the difference between the mean and that measurement is 2.
This means that for every x_i in the set, the absolute difference $|x_i - \mu|$ is 2.
This implies that the squared difference $(x_i - \mu)^2$ is $2^2 = 4$ for every measurement.
Now we can calculate the variance for the 20 measurements ($N = 20$):

$$\text{Variance} = \frac{1}{20}\sum_{i=1}^{20}(x_i - \mu)^2 = \frac{1}{20}\sum_{i=1}^{20} 4$$

$$\text{Variance} = \frac{1}{20}(20 \times 4) = 4$$

With the variance being 4, we can find the standard deviation:

$$\text{SD} = \sqrt{4} = 2$$

Again, we can answer the question: Is $2 < 3$? Yes. Since we get a definitive "Yes", statement (2) is sufficient.

Step 4: Final Answer:
Both statements, independently, provide enough information to definitively answer the question.

Quick Tip

Recognize the direct relationship between standard deviation and variance. If one is given, the other is determined. Also, understand that standard deviation is a measure of the spread or dispersion of data points around the mean. Statement (2) gives a very specific description of this spread.

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6. Is $\frac{5^{x+2}}{25} < 1$?
(1) $5^x < 1$
(2) $x < 0$

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.

(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (D) EACH statement ALONE is sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is an inequality problem involving exponents. The best approach is to simplify the inequality in the question first.

Step 2: Key Formula or Approach:
We will use the rules of exponents: 1. $\frac{a^m}{a^n} = a^{m-n}$ 2. $a^0 = 1$ for any non-zero a. 3. For a base $b > 1$, the inequality $b^p < b^q$ is equivalent to $p < q$.

Step 3: Detailed Explanation:
First, let's simplify the question's inequality:

$$\frac{5^{x+2}}{25} < 1$$

Since $25 = 5^2$, we can rewrite this as:

$$\frac{5^{x+2}}{5^2} < 1$$

Using the exponent rule for division:

$$5^{(x+2)-2} < 1$$

$$5^x < 1$$

Since $1 = 5^0$, the inequality becomes:

$$5^x < 5^0$$

Because the base (5) is greater than 1, we can compare the exponents directly:

$$x < 0$$

So, the question "Is $\frac{5^{x+2}}{25} < 1$" is equivalent to asking "Is $x < 0$?".

Analyze Statement (1): $5^x < 1$.
As shown in the simplification above, this is equivalent to $5^x < 5^0$, which means $x < 0$.
This directly answers our rephrased question with a "Yes". Statement (1) is sufficient.

Analyze Statement (2): $x < 0$.
This is the rephrased question itself. It gives a definitive "Yes". Statement (2) is sufficient.

Step 4: Final Answer:
Each statement alone is sufficient to answer the question.

> **Quick Tip**
>
> Always try to simplify the question in a Data Sufficiency problem before evaluating the statements. Often, the simplified question is identical or directly related to one of the statements, making the evaluation much faster.

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7. Of the companies surveyed about the skills they required in prospective employees, 20 percent required both computer skills and writing skills. What percent of the companies surveyed required neither computer skills nor writing skills?
(1) Of those companies surveyed that required computer skills, half required writing skills.
(2) 45 percent of the companies surveyed required writing skills but not computer skills.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.

Solution:

Step 1: Understanding the Concept:
This is a set theory or Venn diagram problem. Let C be the percentage of companies requiring computer skills, and W be the percentage requiring writing skills. We are given:

- Percentage requiring both (C and W) = 20%

We need to find:

- Percentage requiring neither (Neither C nor W)

Step 2: Key Formula or Approach:
The formula for the union of two sets is:

$$\text{Total \%} = \%(\text{C only}) + \%(\text{W only}) + \%(\text{Both}) + \%(\text{Neither})$$

Or, using the principle of inclusion-exclusion:

$$\%(\text{C or W}) = \%(\text{C}) + \%(\text{W}) - \%(\text{C and W})$$

And,

$$\%(\text{Neither}) = 100\% - \%(\text{C or W})$$

To solve the problem, we need to find %(C or W), which requires us to find %C and %W.

Step 3: Detailed Explanation:
Analyze Statement (1): Of those companies surveyed that required computer skills, half required writing skills.
This is a conditional probability statement. It means that of the group C, 50% of them are also in group W.

$$0.50 \times \%(\text{C}) = \%(\text{C and W})$$

We know %(C and W) = 20%, so:

$$0.50 \times \%(\text{C}) = 20\%$$

$$\%(\text{C}) = \frac{20\%}{0.50} = 40\%$$

Now we know the total percentage requiring computer skills is 40%. However, we still don't know %(W). So we can't find %(C or W). Statement (1) is not sufficient.

Analyze Statement (2): 45 percent of the companies surveyed required writing skills but not computer skills.
This gives us the value for the "W only" region.

$$\%(\text{W only}) = 45\%$$

We know that %(W) = %(W only) + %(Both).

$$\%(\text{W}) = 45\% + 20\% = 65\%$$

Now we know the total percentage requiring writing skills is 65%. However, we don't know %(C). So we can't find %(C or W). Statement (2) is not sufficient.

Analyze Both Statements Together:
From statement (1), we found %(C) = 40%.
From statement (2), we found %(W) = 65%.
We were given %(C and W) = 20%.
Now we can calculate the percentage of companies requiring at least one skill:

$$\%(\text{C or W}) = \%(\text{C}) + \%(\text{W}) - \%(\text{C and W})$$

$$\%(\text{C or W}) = 40\% + 65\% - 20\% = 85\%$$

The percentage requiring neither skill is:

$$\%(\text{Neither}) = 100\% - \%(\text{C or W}) = 100\% - 85\% = 15\%$$

Since we can find a unique value, the statements together are sufficient.

Step 4: Final Answer:
Combining both statements provides enough information to calculate the required percentage.

Quick Tip

For overlapping sets problems, drawing a Venn diagram can be very helpful. Label the two circles "Computer" and "Writing". Fill in the intersection ("Both") with 20%. Statement (1) allows you to find the total for the "Computer" circle. Statement (2) gives you the "Writing only" part, which allows you to find the total for the "Writing" circle. With all parts of the circles filled, you can find the value for "Neither".

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8. What is the value of $w + q$?
(1) $3w = 3 - 3q$
(2) $5w + 5q = 5$

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (D) EACH statement ALONE is sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
The question asks for the value of a specific expression, $w + q$, not the individual values of w and q. We need to determine if the given statements can be manipulated to find the value of $w + q$.

Step 2: Detailed Explanation:
Analyze Statement (1): $3w = 3 - 3q$.
Our goal is to find the value of $w + q$. Let's rearrange the equation to group the w and q terms together. Add $3q$ to both sides of the equation:

$$3w + 3q = 3$$

Now, we can factor out the common term, 3, from the left side:

$$3(w + q) = 3$$

Divide both sides by 3:

$$w + q = 1$$

This statement provides a unique value for the expression $w + q$. Therefore, statement (1) is sufficient.

Analyze Statement (2): $5w + 5q = 5$.
Again, our goal is to find the value of $w + q$. We can factor out the common term, 5, from the left side of the equation:

$$5(w + q) = 5$$

Divide both sides by 5:

$$w + q = 1$$

This statement also provides a unique value for the expression $w + q$. Therefore, statement (2) is sufficient.

Step 3: Final Answer:
Each statement, by itself, is sufficient to determine the value of $w + q$.

> **Quick Tip**
>
> In Data Sufficiency, be alert for questions that ask for the value of an expression (like $x+y$) rather than individual variables. Often, a single equation can be rearranged to solve for the expression directly, even if it's impossible to solve for the individual variables.

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9. If X and Y are points in a plane and X lies inside the circle C with center O and radius 2, does Y lie inside circle C?
(1) The length of line segment XY is 3.
(2) The length of line segment OY is 1.5.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
We are given a circle C with center O and radius 2.
A point P is inside the circle if its distance from the center, OP, is less than the radius.
We are given that point X is inside the circle, which means the distance OX < 2.
The question asks: Is point Y inside the circle? This is equivalent to asking: Is the distance

OY < 2?

Step 2: Detailed Explanation:
Analyze Statement (1): The length of line segment XY is 3.
This means the distance between X and Y is 3. We can use the triangle inequality for points O, X, and Y: OY $\leq$ OX + XY. Since we know OX < 2 and XY = 3, we have:

$$\text{OY} < 2 + 3 = 5$$

This tells us that OY is less than 5, but it doesn't tell us if OY is less than 2. Let's test two scenarios:

- **Scenario A (Y is outside):** Let O be at the origin (0,0). Let X be at (1,0). OX = 1, which is < 2. If Y is at (4,0), then the distance XY = $|4 - 1| = 3$. The distance OY = 4, which is not < 2. In this case, Y is outside the circle. The answer to the question is "No".
- **Scenario B (Y is inside):** Let O be at (0,0). Let X be at (1.5, 0). OX = 1.5, which is < 2. If Y is at (-1.5, 0), then the distance XY = $|1.5 - (-1.5)| = 3$. The distance OY = 1.5, which is < 2. In this case, Y is inside the circle. The answer to the question is "Yes".

Since we can get both "Yes" and "No" answers, statement (1) is not sufficient.

Analyze Statement (2): The length of line segment OY is 1.5.
This statement directly gives us the distance of point Y from the center O. We need to determine if Y is inside the circle, which means we need to know if OY < 2.
The statement says OY = 1.5. Is 1.5 < 2? Yes. This gives us a definitive "Yes" answer to the question. Therefore, statement (2) is sufficient.

Step 3: Final Answer:
Statement (2) alone is sufficient, but statement (1) alone is not.

> **Quick Tip**
>
> For geometry-based Data Sufficiency questions, especially those involving inequalities, always try to construct counterexamples. If you can find one case where the answer is "Yes" and another where it's "No", the statement is not sufficient.

---

10. Is $x > y$?
(1) $x = y + 2$
(2) $\frac{x}{2} = y - 1$

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked,

but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
The question asks whether x is greater than y. This is equivalent to asking if the expression $x - y$ is positive. So, the rephrased question is: Is $x - y > 0$? This is a "Yes/No" Data Sufficiency question.

Step 2: Detailed Explanation:
Analyze Statement (1): $x = y + 2$.
We can rearrange this equation to find the value of $x - y$. Subtract y from both sides:

$$x - y = 2$$

Now we evaluate the rephrased question: Is $x - y > 0$? Is $2 > 0$? Yes. Since the answer is always "Yes", regardless of the specific values of x and y, this statement is sufficient.

Analyze Statement (2): $\frac{x}{2} = y - 1$.
Let's rearrange this equation to find an expression for $x - y$. First, multiply both sides by 2 to clear the fraction:

$$x = 2(y - 1)$$

$$x = 2y - 2$$

Now, subtract y from both sides to get an expression for $x - y$:

$$x - y = 2y - 2 - y$$

$$x - y = y - 2$$

Now we evaluate the rephrased question: Is $x-y > 0$? This is equivalent to asking: Is $y-2 > 0$? Or, Is $y > 2$? The answer to this question depends on the value of y.

- If $y = 3$, then $x - y = 3 - 2 = 1$, which is greater than 0. The answer is "Yes".
- If $y = 1$, then $x - y = 1 - 2 = -1$, which is not greater than 0. The answer is "No".

Since we can get both "Yes" and "No" answers, statement (2) is not sufficient.

Step 3: Final Answer:
Statement (1) alone is sufficient to answer the question, but statement (2) alone is not.

Quick Tip

For inequality questions like "Is $x > y$?", a useful strategy is to rephrase the question as "Is $x - y > 0$?". Then, use the information in the statements to find the value or the sign of the expression $x - y$.

---

11. If Paula drove the distance from her home to her college at an average speed that was greater than 70 kilometers per hour, did it take her less than 3 hours to drive this distance?
(1) The distance that Paula drove from her home to her college was greater than 200 kilometers.
(2) The distance that Paula drove from her home to her college was less than 205 kilometers.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This problem involves the relationship between distance, speed (rate), and time. Let D be the distance, S be the speed, and T be the time.
We are given that Paula's average speed $S > 70$ km/h.
The question asks: Is $T < 3$ hours?

Step 2: Key Formula or Approach:
The formula connecting the variables is $D = S \times T$, which can be rearranged to $T = \frac{D}{S}$.
The question is: Is $\frac{D}{S} < 3$?

Step 3: Detailed Explanation:
Analyze Statement (1): The distance was greater than 200 kilometers ($D > 200$).
We have $D > 200$ and $S > 70$. Let's test some values to see if we get a consistent answer.

- **Case 1 (Answer is "Yes"):** Let the distance $D = 201$ km and the speed $S = 75$ km/h. Both conditions are met. Then the time is $T = \frac{201}{75} = 2.68$ hours. Since $2.68 < 3$, the answer is "Yes".

- **Case 2 (Answer is "No"):** Let the distance $D = 215$ km and the speed $S = 71$ km/h. Both conditions are met. Then the time is $T = \frac{215}{71} \approx 3.02$ hours. Since $3.02 > 3$, the answer is "No".

Since we can get both "Yes" and "No" answers, statement (1) is not sufficient.

Analyze Statement (2): The distance was less than 205 kilometers ($D < 205$).
We have $D < 205$ and $S > 70$. To determine if $T < 3$, we should look at the maximum possible value of T. The time $T = \frac{D}{S}$ is maximized when the numerator (D) is as large as possible and the denominator (S) is as small as possible.

- The maximum possible value for D is just under 205.

- The minimum possible value for S is just over 70.

Let's calculate the value at the boundary: $T_{max} = \frac{205}{70} \approx 2.928$ hours.
Since $D < 205$ and $S > 70$, the actual time T must be less than this value: $T < \frac{205}{70}$.
So, $T < 2.928$. Since any possible value for T is less than 2.928, it must also be less than 3. The answer to the question is always "Yes". Therefore, statement (2) is sufficient.

Step 4: Final Answer:
Statement (2) alone is sufficient to answer the question, while statement (1) is not.

> **Quick Tip**
>
> When dealing with inequalities in rate-time-distance problems, test the boundary conditions. To find the maximum or minimum value of a fraction like Time = Distance/Speed, consider the maximum value of the numerator and the minimum value of the denominator, and vice-versa.

---

12. In the xy-plane, if line k has negative slope and passes through the point (-5,r), is the x-intercept of line k positive?
(1) The slope of line k is -5.
(2) r > 0

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Solution:

Step 1: Understanding the Concept:
We need to determine if the x-intercept of a line is positive. The x-intercept is the point where the line crosses the x-axis (i.e., where y=0).
We are given that the line has a negative slope ($m < 0$) and passes through the point $(-5, r)$.

Step 2: Key Formula or Approach:
Using the point-slope form of a linear equation, $y - y_1 = m(x - x_1)$, we have:

$$y - r = m(x - (-5)) \implies y - r = m(x + 5)$$

To find the x-intercept, we set $y = 0$:

$$0 - r = m(x + 5)$$

$$-r = mx + 5m$$

$$-r - 5m = mx$$

$$x = \frac{-r - 5m}{m} = -\frac{r}{m} - 5$$

The question is: Is $-\frac{r}{m} - 5 > 0$?

Step 3: Detailed Explanation:
Analyze Statement (1): The slope of line k is -5 ($m = -5$).
Substituting $m = -5$ into our question: Is $-\frac{r}{-5} - 5 > 0$?
Is $\frac{r}{5} - 5 > 0$?
Is $\frac{r}{5} > 5$?
Is $r > 25$?
We have no information about r, so we cannot answer this. Statement (1) is not sufficient.

Analyze Statement (2): $r > 0$.
We know $m < 0$. The question is: Is $-\frac{r}{m} - 5 > 0$?
Since $r > 0$ and $m < 0$, the term $\frac{r}{m}$ is negative, and $-\frac{r}{m}$ is positive. Let's test some values.

- **Case 1 (Answer is "Yes"):** Let $r = 10$ and $m = -1$. The conditions are met. The x-intercept is $x = -\frac{10}{-1} - 5 = 10 - 5 = 5$. Since $5 > 0$, the answer is "Yes".

- **Case 2 (Answer is "No"):** Let $r = 10$ and $m = -3$. The conditions are met. The x-intercept is $x = -\frac{10}{-3} - 5 = \frac{10}{3} - 5 = \frac{10-15}{3} = -\frac{5}{3}$. Since $-\frac{5}{3} < 0$, the answer is "No".

Since we can get both "Yes" and "No" answers, statement (2) is not sufficient.

Analyze Both Statements Together:
From statement (1), $m = -5$. From statement (2), $r > 0$. The question simplifies to: Is $r > 25$? We only know that r is a positive number. It could be $r = 10$ (in which case the answer is

"No") or it could be $r = 30$ (in which case the answer is "Yes"). The information is still not sufficient.

Step 4: Final Answer:
Even with both statements, we cannot definitively determine if the x-intercept is positive.

> Quick Tip
>
> For coordinate geometry problems, it can be helpful to visualize. The point $(-5, r)$ is in quadrant I if $r > 0$ or quadrant IV if $r < 0$. A line with a negative slope passing through a point in quadrant I could cross the x-axis on either the positive or negative side. The statements provide details that constrain the line, but you must check if the constraint is sufficient to guarantee only one outcome.

---

13. If $5,000 invested for one year at p percent simple annual interest yields $500, what amount must be invested at k percent simple annual interest for one year to yield the same number of dollars?
(1) k = 0.8p
(2) k = 8

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (D) EACH statement ALONE is sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This problem involves the simple interest formula. First, we need to use the given information to find the value of the interest rate p. Then, we need to determine the principal amount for a second investment.

Step 2: Key Formula or Approach:
The formula for simple interest is $I = P \cdot r \cdot t$, where I is the interest, P is the principal, r is the annual interest rate as a decimal, and t is the time in years.

Step 3: Detailed Explanation:
Part 1: Find the value of p.

For the first investment, we are given: $P_1 = \$5,000$, $I_1 = \$500$, $t_1 = 1$ year, and the rate is p percent.

$$I_1 = P_1 \cdot \frac{p}{100} \cdot t_1$$
$$500 = 5000 \cdot \frac{p}{100} \cdot 1$$
$$500 = 50p$$
$$p = 10$$

So, the first interest rate was 10%.

Part 2: Answer the main question.
The question asks for the amount (P_2) that must be invested at k percent for one year to yield the same interest ($I_2 = \$500$).

$$I_2 = P_2 \cdot \frac{k}{100} \cdot t_2$$
$$500 = P_2 \cdot \frac{k}{100} \cdot 1$$

To find P_2, we need the value of k. So the question is essentially "What is the value of k?".

Analyze Statement (1): $k = 0.8p$.
Since we found that $p = 10$, we can calculate k:

$$k = 0.8 \times 10 = 8$$

Now that we know $k = 8$, we can find P_2:

$$500 = P_2 \cdot \frac{8}{100} \implies P_2 = \frac{500 \cdot 100}{8} = \$6,250$$

Since we found a unique value for the amount, statement (1) is sufficient.

Analyze Statement (2): $k = 8$.
This statement directly gives us the value of k. We can find P_2:

$$500 = P_2 \cdot \frac{8}{100} \implies P_2 = \frac{500 \cdot 100}{8} = \$6,250$$

Since we found a unique value for the amount, statement (2) is sufficient.

Step 4: Final Answer:
Each statement alone provides enough information to determine the value of k and thus solve for the required investment amount.

> **Quick Tip**
>
> In some Data Sufficiency questions, the prompt itself contains enough information to solve for one or more variables. Always process this information first. Here, calculating $p = 10$ from the prompt is the key first step before looking at the statements.

14. If $\frac{x+y}{z} > 0$, is $x < 0$?
(1) $x < y$
(2) $z < 0$

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.

Solution:

Step 1: Understanding the Concept:
We are given an inequality and asked to determine the sign of x.
The condition $\frac{x+y}{z} > 0$ implies that the numerator $(x + y)$ and the denominator (z) must have the same sign. This leads to two possible cases:

- **Case A:** $x + y > 0$ and $z > 0$
- **Case B:** $x + y < 0$ and $z < 0$

The question is: Is $x < 0$?

Step 2: Detailed Explanation:
Analyze Statement (1): $x < y$.
This statement alone is not sufficient.

- **Example 1 (Answer "Yes"):** Let $y = 5$ and $x = -2$. Then $x < y$ is true. Let $z = 1$. Then $\frac{x+y}{z} = \frac{-2+5}{1} = 3 > 0$. Here, $x = -2$, so $x < 0$. The answer is "Yes".

- **Example 2 (Answer "No"):** Let $y = 5$ and $x = 2$. Then $x < y$ is true. Let $z = 1$. Then $\frac{x+y}{z} = \frac{2+5}{1} = 7 > 0$. Here, $x = 2$, so x is not less than 0. The answer is "No".

Statement (1) is not sufficient.

Analyze Statement (2): $z < 0$.
If $z < 0$, we must be in Case B from our initial analysis. This means we must have $x + y < 0$. The question now becomes: If $x + y < 0$, is it necessary that $x < 0$?

- **Example 1 (Answer "Yes"):** Let $y = -1$ and $x = -2$. Then $x + y = -3 < 0$. Here, $x = -2$, so $x < 0$. The answer is "Yes".

- **Example 2 (Answer "No"):** Let $y = -5$ and $x = 2$. Then $x + y = -3 < 0$. Here, $x = 2$, so x is not less than 0. The answer is "No".

Statement (2) is not sufficient.

Analyze Both Statements Together:
From statement (2), we know $z < 0$, which implies $x + y < 0$. From statement (1), we know $x < y$. We have a system of two inequalities: 1) $x + y < 0$ 2) $x < y$ Let's manipulate the second inequality. We can add x to both sides: $x + x < y + x$ $2x < x + y$ Now we can combine this with the first inequality: $2x < x + y$ and $x + y < 0$. This gives us a transitive relation: $2x < 0$. If $2x < 0$, then dividing by 2 gives $x < 0$. The answer to the question "Is $x < 0$" is definitively "Yes". Therefore, the statements together are sufficient.

Step 3: Final Answer:
Neither statement alone is sufficient, but both statements together are sufficient.

Quick Tip

When combining inequalities, look for opportunities to link them. Here, manipulating one inequality ($x < y$) to include a term from the other inequality ($x + y$) was the key to solving the problem.

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15. Does the integer k have at least three different positive prime factors?
(1) $\frac{k}{15}$ is an integer.
(2) $\frac{k}{10}$ is an integer.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.

Solution:

Step 1: Understanding the Concept:
This question is about the properties of integers, specifically their prime factors. A prime factor is a prime number that divides an integer exactly. The question is a Yes/No question: does the prime factorization of k contain at least three distinct prime numbers?

Step 2: Detailed Explanation:
Analyze Statement (1): $\frac{k}{15}$ is an integer.
This means k is a multiple of 15. The prime factorization of 15 is 3×5. So, the prime factorization of k must include at least 3 and 5. This guarantees two different prime factors. But does it guarantee a third?

- **Case 1 (Answer "No"):** If $k = 15$, its prime factors are 3 and 5. This is only two different prime factors. The answer is "No".

- **Case 2 (Answer "Yes"):** If $k = 30$ (which is 15×2), its prime factors are 2, 3, and 5. This is three different prime factors. The answer is "Yes".

Since we can get both "Yes" and "No" answers, statement (1) is not sufficient.

Analyze Statement (2): $\frac{k}{10}$ is an integer.
This means k is a multiple of 10. The prime factorization of 10 is 2×5. So, the prime factorization of k must include at least 2 and 5. This guarantees two different prime factors. But does it guarantee a third?

- **Case 1 (Answer "No"):** If $k = 10$, its prime factors are 2 and 5. This is only two different prime factors. The answer is "No".

- **Case 2 (Answer "Yes"):** If $k = 30$ (which is 10×3), its prime factors are 2, 3, and 5. This is three different prime factors. The answer is "Yes".

Since we can get both "Yes" and "No" answers, statement (2) is not sufficient.

Analyze Both Statements Together:
From statement (1), k is a multiple of 15. From statement (2), k is a multiple of 10. If k is a multiple of both 10 and 15, it must be a multiple of their least common multiple (LCM). $\text{LCM}(10, 15) = \text{LCM}(2 \times 5, 3 \times 5) = 2 \times 3 \times 5 = 30$. So, k must be a multiple of 30. The prime factorization of 30 is $2 \times 3 \times 5$. Any multiple of 30 will have at least 2, 3, and 5 as its prime factors. For example, if $k = 60 = 2^2 \times 3 \times 5$, the distinct prime factors are still 2, 3, and 5. Therefore, k is guaranteed to have at least three different positive prime factors. The answer is always "Yes". The statements together are sufficient.

Step 3: Final Answer:
Neither statement alone is sufficient, but combined they are sufficient.

Quick Tip

When a number is a multiple of two other numbers, it must be a multiple of their LCM. Finding the prime factors of the LCM is often the key to solving problems like this one.

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16. In City X last April, was the average (arithmetic mean) daily high temperature greater than the median daily high temperature?
(1) In City X last April, the sum of the 30 daily high temperatures was 2,160°.
(2) In City X last April, 60 percent of the daily high temperatures were less than the average daily high temperature.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This question compares the mean and median of a set of data.

- **Mean (Average):** The sum of the values divided by the number of values.
- **Median:** The middle value of a data set when it is sorted. If there is an even number of values, it's the average of the two middle values. The median represents the 50th percentile: 50% of the data is at or below the median.

The question is: Is Mean ¿ Median? This often happens in a right-skewed distribution, where a few high values pull the mean up.

Step 2: Detailed Explanation:
The data set consists of 30 daily high temperatures. **Analyze Statement (1):** The sum of the 30 daily high temperatures was 2,160°.
We can calculate the mean from this information:

$$\text{Mean} = \frac{\text{Sum}}{\text{Count}} = \frac{2160}{30} = 72^\circ$$

This gives us the exact value of the mean. However, we have no information about the individual data points, so we cannot determine the median. The median could be less than, equal to, or greater than 72. Statement (1) is not sufficient.

Analyze Statement (2): 60 percent of the daily high temperatures were less than the average daily high temperature.
Let the 30 temperatures be sorted in increasing order: $t_1, t_2, ..., t_{30}$. The median is the value that splits the data set in half. For 30 values, 50% of the data (15 values) are less than or equal to the median, and 50% (15 values) are greater than or equal to the median. The median is calculated as $(t_{15} + t_{16})/2$. The median is the 50th percentile. The statement says that 60% of the temperatures are less than the mean. This means that the mean is greater than at least 60% of the data points. Since the median is the point where only 50% of the data is smaller, and the mean is a point where 60% of the data is smaller, the mean must be greater than the median. Mean ¿ 60th percentile value. Median = 50th percentile value. Therefore, Mean ¿ Median. The answer to the question is definitively "Yes". Statement (2) is sufficient.

Step 3: Final Answer:
Statement (2) alone is sufficient to answer the question.

Quick Tip

Understand the conceptual relationship between mean and median. The median is the 50th percentile mark. If a statement tells you what percentage of the data falls below the mean, you can directly compare the mean to the median. If more than 50

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17. If m and n are positive integers, is $(\sqrt{m})^n$ an integer?
(1) $(\sqrt{m})$ is an integer.
(2) $(\sqrt{n})$ is an integer.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
The question asks whether the expression $(\sqrt{m})^n$ results in an integer, given that m and n are positive integers. The expression can also be written as $m^{n/2}$. For this to be an integer, we

generally need either m to be a perfect square or n to be an even number.

Step 2: Detailed Explanation:
Analyze Statement (1): $(\sqrt{m})$ is an integer.
Let's say $\sqrt{m} = k$, where k is an integer.
The expression in the question becomes k^n.
Since k is an integer and n is a positive integer, the result of raising an integer to a positive integer power (k^n) will always be an integer.
For example, if $m = 9$ ($\sqrt{m} = 3$) and $n = 5$, then $(\sqrt{9})^5 = 3^5 = 243$, which is an integer.
The answer to the question is always "Yes". Therefore, statement (1) is sufficient.

Analyze Statement (2): $(\sqrt{n})$ is an integer.
This means that n is a perfect square. So, n can be 1, 4, 9, 16, etc. The question is whether $(\sqrt{m})^n$ is an integer.

- **Case 1 (Answer "Yes"):** Let $n = 4$ (a perfect square) and $m = 5$. The expression is $(\sqrt{5})^4 = (5^{1/2})^4 = 5^2 = 25$. This is an integer. The answer is "Yes". (Note: This works for any even value of n, and perfect squares can be even).
- **Case 2 (Answer "No"):** Let $n = 9$ (a perfect square) and $m = 2$. The expression is $(\sqrt{2})^9 = 2^{9/2} = 2^{4.5}$. This is not an integer. The answer is "No". (Note: This shows that if n is an odd perfect square, the result depends on m).

Since we can get both "Yes" and "No" answers, statement (2) is not sufficient.

Step 3: Final Answer:
Statement (1) alone is sufficient, but statement (2) alone is not.

Quick Tip

When dealing with exponents and roots, rewrite the expression in fractional exponent form (e.g., $\sqrt{m} = m^{1/2}$). This often makes the rules of exponents easier to apply and the conditions for integer results clearer.

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18. Of the 66 people in a certain auditorium, at most 6 people have their birthdays in any one given month. Does at least one person in the auditorium have a birthday in January?
(1) More of the people in the auditorium have their birthday in February than in March.
(2) Five of the people in the auditorium have their birthday in March.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked,

but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (D) EACH statement ALONE is sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is a logic and combinatorics problem that can be approached using the Pigeonhole Principle or by testing extreme scenarios.
Total people = 66.
Number of months = 12.
Maximum people per month ≤ 6.
The question is: Is the number of people with a birthday in January ≥ 1? This is a "Yes/No" question.
To prove sufficiency, we must show that the number for January cannot be 0. Let's test the "No" case: assume there are 0 people with a birthday in January.

Step 2: Detailed Explanation:
If there are 0 birthdays in January, then all 66 people must have birthdays in the remaining 11 months.
The maximum number of people that can be accommodated in these 11 months is 11 months $\times$ 6 people/month = 66 people.
This means that for the "No" case (0 birthdays in January) to be possible, every single one of the other 11 months must have exactly 6 birthdays. So, the "No" case requires: Jan=0, Feb=6, Mar=6, Apr=6, ..., Dec=6.

Analyze Statement (1): More of the people in the auditorium have their birthday in February than in March.
This means the number of birthdays in February ¿ the number of birthdays in March.
The only scenario that gives a "No" answer to the main question requires Feb=6 and Mar=6. This contradicts the statement that Feb ¿ Mar.
Therefore, the "No" scenario is impossible under this condition. The number of birthdays in January must be greater than 0. The answer is definitively "Yes".
Statement (1) is sufficient.

Analyze Statement (2): Five of the people in the auditorium have their birthday in March.
This means the number of birthdays in March = 5.
The only scenario that gives a "No" answer to the main question requires Mar=6. This contradicts the statement that Mar=5.
Therefore, the "No" scenario is impossible under this condition. The number of birthdays in January must be greater than 0. The answer is definitively "Yes".
Statement (2) is sufficient.

Step 3: Final Answer:

Each statement alone is sufficient to answer the question.

> **Quick Tip**
>
> For "Yes/No" questions, especially in logic problems, a powerful strategy is to assume the opposite of what the question asks for and look for a contradiction. Here, assuming "No" (zero birthdays in January) leads to a very specific scenario that is contradicted by both statements.

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19. Last year the average (arithmetic mean) salary of the 10 employees of Company X was $42,800. What is the average salary of the same 10 employees this year?
(1) For 8 of the 10 employees, this year's salary is 15 percent greater than last year's salary.
(2) For 2 of the 10 employees, this year's salary is the same as last year's salary.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Solution:

Step 1: Understanding the Concept:
To find the average salary for this year, we need to calculate the total sum of salaries for this year and divide by the number of employees (10).
From the prompt, we can calculate the total sum of salaries for last year:
Total Salary (Last Year) = Average Salary $\times$ Number of Employees
Total Salary (Last Year) = $42,800 $\times$ 10 = $428,000.

Step 2: Detailed Explanation:
Analyze Statement (1): For 8 of the 10 employees, this year's salary is 15 percent greater than last year's salary.
This statement tells us about the change for 8 employees but provides no information about the remaining 2 employees. Their salaries could have increased, decreased, or stayed the same, and we don't know their initial salaries. We cannot calculate this year's total salary. Statement (1) is not sufficient.

Analyze Statement (2): For 2 of the 10 employees, this year's salary is the same as last year's salary.
This statement tells us about the change for 2 employees but provides no information about the remaining 8 employees. We cannot calculate this year's total salary. Statement (2) is not sufficient.

Analyze Both Statements Together:
We know that 8 employees received a 15% raise, and 2 employees' salaries remained the same. Let S_A be the sum of last year's salaries for the 8 employees who got a raise, and S_B be the sum of last year's salaries for the 2 employees who did not.
We know $S_A + S_B = \$428,000$.
This year's total salary is calculated as: Total Salary (This Year) $= 1.15 \times S_A + 1 \times S_B$
The problem is that we don't know the values of S_A and S_B. The distribution of the $428,000 between the two groups affects the final total.

- **Scenario 1:** Assume the 8 employees with raises had low salaries, e.g., $S_A = \$80,000$. Then $S_B = \$428,000 - \$80,000 = \$348,000$. This Year's Total $= 1.15(\$80,000) + \$348,000 = \$92,000 + \$348,000 = \$440,000$. Average = $44,000.
- **Scenario 2:** Assume the 8 employees with raises had high salaries, e.g., $S_A = \$400,000$. Then $S_B = \$428,000 - \$400,000 = \$28,000$. This Year's Total $= 1.15(\$400,000) + \$28,000 = \$460,000 + \$28,000 = \$488,000$. Average = $48,800.

Since we can get different values for this year's average salary, the combined statements are not sufficient.

Step 3: Final Answer:
Even with both statements, the average salary for this year cannot be uniquely determined.

Quick Tip

In problems involving percentage changes on a total sum, remember that the change in the total depends on what portion of the sum the percentage is applied to. If the split is unknown, a unique answer cannot be found.

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20. In a certain classroom, there are 80 books, of which 24 are fiction and 23 are written in Spanish. How many of the fiction books are written in Spanish?
(1) Of the fiction books, there are 6 more that are not written in Spanish than are written in Spanish.
(2) Of the books written in Spanish, there are 5 more nonfiction books than fiction books.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.

(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (D) EACH statement ALONE is sufficient to answer the question asked.

Solution:

Step 1: Understanding the Concept:
This is an overlapping sets problem. We can organize the information in a 2x2 table.
Let F be Fiction, NF be Non-Fiction.
Let S be Spanish, NS be Not Spanish.
We are given:

- Total Books = 80
- Total Fiction (F) = 24
- Total Spanish (S) = 23

The question asks for the number of books that are both Fiction and Spanish, which is the intersection of F and S. Let's call this number x.

Step 2: Key Formula or Approach:
We can use variables to represent the quantities. Let x = number of Fiction books in Spanish. The total number of Fiction books is 24. So, the number of Fiction books not in Spanish is $24 - x$. The total number of Spanish books is 23. So, the number of Non-Fiction books in Spanish is $23 - x$.

Step 3: Detailed Explanation:
Analyze Statement (1): Of the fiction books, there are 6 more that are not written in Spanish than are written in Spanish.
This can be written as an equation: (Number of Fiction books not in Spanish) = (Number of Fiction books in Spanish) + 6
Using our variables:

$$24 - x = x + 6$$

Now, we solve for x:

$$18 = 2x$$

$$x = 9$$

Since we found a unique value for x, statement (1) is sufficient.

Analyze Statement (2): Of the books written in Spanish, there are 5 more nonfiction books than fiction books.
This can be written as an equation: (Number of Non-Fiction books in Spanish) = (Number of Fiction books in Spanish) + 5
Using our variables:

$$23 - x = x + 5$$

Now, we solve for x:

$$18 = 2x$$

$$x = 9$$

Since we found a unique value for x, statement (2) is sufficient.

Step 4: Final Answer:
Each statement alone is sufficient to determine the number of fiction books written in Spanish.

> Quick Tip
>
> For 2x2 overlapping sets problems, defining a single variable for the quantity you need to find (x) and then expressing the other quantities in the set in terms of x and the given totals is a very effective and systematic approach.

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21. If p is the perimeter of rectangle Q, what is the value of p?
(1) Each diagonal of rectangle Q has length 10.
(2) The area of rectangle Q is 48.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked.
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked.
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.
(D) EACH statement ALONE is sufficient to answer the question asked.
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

Correct Answer: (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient.

Solution:

Step 1: Understanding the Concept:
We need to find the perimeter of a rectangle. Let the length of the rectangle be l and the width be w.
The perimeter p is given by the formula $p = 2(l + w)$. To find a unique value for p, we need to find a unique value for the sum $(l + w)$.

Step 2: Key Formula or Approach:
For a rectangle:

- Area: $A = l \times w$
- Diagonal (by Pythagorean theorem): $d^2 = l^2 + w^2$

- Algebraic identity: $(l+w)^2 = l^2 + w^2 + 2lw$

Step 3: Detailed Explanation:

Analyze Statement (1): Each diagonal of rectangle Q has length 10.
This gives us the equation $l^2 + w^2 = 10^2 = 100$. This single equation has two variables, l and w, so it can have multiple solutions.

- If $l = 8$ and $w = 6$, then $8^2+6^2 = 64+36 = 100$. The perimeter would be $p = 2(8+6) = 28$.
- If $l = \sqrt{50}$ and $w = \sqrt{50}$ (a square), then $(\sqrt{50})^2+(\sqrt{50})^2 = 50+50 = 100$. The perimeter would be $p = 2(\sqrt{50}+\sqrt{50}) = 4\sqrt{50} = 20\sqrt{2} \approx 28.28$.

Since the perimeter can have different values, statement (1) is not sufficient.

Analyze Statement (2): The area of rectangle Q is 48.
This gives us the equation $l \times w = 48$. This single equation also has multiple possible solutions for l and w.

- If $l = 8$ and $w = 6$, the area is $8 \times 6 = 48$. The perimeter would be $p = 2(8+6) = 28$.
- If $l = 12$ and $w = 4$, the area is $12 \times 4 = 48$. The perimeter would be $p = 2(12+4) = 32$.

Since the perimeter can have different values, statement (2) is not sufficient.

Analyze Both Statements Together:
We now have a system of two equations with two variables: 1) $l^2 + w^2 = 100$ 2) $lw = 48$ We want to find $p = 2(l+w)$. We can use the algebraic identity $(l+w)^2 = l^2+w^2+2lw$. Substitute the values from our two equations into this identity:

$$(l+w)^2 = (100) + 2(48)$$
$$(l+w)^2 = 100 + 96 = 196$$

Taking the square root of both sides:

$$l + w = \sqrt{196} = 14$$

(We take the positive root since length and width must be positive). Now we can find the perimeter:

$$p = 2(l+w) = 2(14) = 28$$

Since we have found a unique value for the perimeter, the statements together are sufficient.

Step 4: Final Answer:
Neither statement alone is sufficient, but both statements together are sufficient.

Quick Tip

For geometry problems asking for perimeter or area, look for algebraic identities that connect the given information. The identity $(l+w)^2 = l^2 + w^2 + 2lw$ is very useful for rectangle problems when the diagonal ($l^2 + w^2$) and area (lw) are involved.
