

# GMAT Quant Practise Question Paper 1 with Solutions

**Time Allowed :** 2 hours 15 minutes

**Maximum Marks :** 100

## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. The GMAT exam is 2 hours and 15 minutes long (with one optional 10-minute break) and consists of 64 questions in total.
2. The GMAT exam is comprised of three sections:
3. Quantitative Reasoning: 21 questions, 45 minutes
4. Verbal Reasoning: 23 questions, 45 minutes
5. Data Insights: 20 questions, 45 minutes
6. You can answer the three sections in any order. As you move through a section, you can bookmark questions that you would like to review later.
7. When you have answered all questions in a section, you will proceed to the Question Review & Edit screen for that section.
8. If there is no time remaining in the section, you will NOT proceed to the Question Review & Edit screen and you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
9. Each Question Review & Edit screen includes a numbered list of the questions in that section and indicates the questions you bookmarked.
10. Clicking a question number will take you to that specific question. You can review as many questions as you would like and can edit up to three (3) answers.

**1. Set T is a finite set of positive consecutive multiples of 14. How many of these integers are also multiples of 21?**

**1. Set T consists of 30 integers.**

**2. The smallest integer in Set T is a multiple of 21.**

(A) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(B) EACH statement ALONE is sufficient to answer the question asked

(C) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

(D) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(E) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

**Correct Answer:** (D) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

**Solution:**

**Step 1: Understanding the Concept:**

The question asks for the number of integers in a set T that are multiples of both 14 and 21. An integer that is a multiple of both 14 and 21 must be a multiple of their least common multiple (LCM).

**Step 2: Key Formula or Approach:**

First, we find the LCM of 14 and 21.

The prime factorization of 14 is  $2 \times 7$ .

The prime factorization of 21 is  $3 \times 7$ .

The LCM is the product of the highest powers of all prime factors that appear in either factorization.

$$\text{LCM}(14, 21) = 2^1 \times 3^1 \times 7^1 = 42$$

So, the question is equivalent to asking: "How many multiples of 42 are in Set T?"

Set T consists of positive consecutive multiples of 14. We can represent the elements of T as  $14k, 14(k+1), 14(k+2), \dots$  for some positive integer  $k$ .

For an element  $14m$  to be a multiple of 42, we must have  $14m = 42j$  for some integer  $j$ .

Dividing by 14, we get  $m = 3j$ . This means that a member of set T is a multiple of 42 if and only if its corresponding multiplier is a multiple of 3.

**Step 3: Detailed Explanation:**

**Analyzing Statement (1): Set T consists of 30 integers.**

The set T can be written as  $\{14k, 14(k+1), \dots, 14(k+29)\}$  for some positive integer  $k$ .

We need to find how many of these elements are multiples of 42. This is equivalent to finding how many of the consecutive integers  $\{k, k+1, \dots, k+29\}$  are multiples of 3.

In any set of 30 consecutive integers, there will always be exactly  $30 \div 3 = 10$  multiples of 3.

For example, if  $k = 1$ , the multipliers are  $\{1, 2, \dots, 30\}$ , which contains multiples of 3: 3, 6, 9, ..., 30 (10 numbers).

If  $k = 2$ , the multipliers are  $\{2, 3, \dots, 31\}$ , which contains multiples of 3: 3, 6, 9, ..., 30 (10 numbers).

If  $k = 3$ , the multipliers are  $\{3, 4, \dots, 32\}$ , which contains multiples of 3: 3, 6, 9, ..., 30 (10 numbers).

Thus, there are always 10 integers in the set T that are multiples of 42.

Statement (1) alone is sufficient to answer the question.

**Analyzing Statement (2): The smallest integer in Set T is a multiple of 21.**

Let the smallest integer be  $14k$ . We are given that  $14k$  is a multiple of 21.

This means  $14k$  must be a multiple of  $\text{LCM}(14, 21)$ , which is 42.

So, the first element of the set is a multiple of 42. The set looks like  $\{42j, 42j + 14, 42j + 28, 42j + 42, \dots\}$ .

The multiples of 42 appear every 3 terms (the 1st term, 4th term, 7th term, etc.).

However, we do not know the size of the set T. If the set has 3 elements, there is 1 multiple of

42. If the set has 5 elements, there is still 1 multiple of 42. If the set has 6 elements, there are 2 multiples of 42.

Since we cannot determine a unique number of multiples of 42, statement (2) alone is not sufficient.

#### Step 4: Final Answer:

Statement (1) alone is sufficient, but statement (2) alone is not sufficient.

#### Quick Tip

In data sufficiency questions involving multiples, always find the LCM of the given numbers. For consecutive items, the number of multiples of 'n' in a set of 'k' consecutive items is either  $\lfloor k/n \rfloor$  or  $\lceil k/n \rceil$ . If k is a multiple of n, the count is exactly  $k/n$ .

2. If  $yz \neq 0$ , is  $x - y + z < xz - yz - xy$ ?

1.  $\frac{x}{y} < -\frac{1}{2}$

2.  $xy < 0$

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(B) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

(C) EACH statement ALONE is sufficient to answer the question asked

(D) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

**Correct Answer:** (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

#### Solution:

##### Step 1: Understanding the Concept:

This is a data sufficiency question involving an inequality with three variables x, y, and z. We need to determine if the given statements provide enough information to definitively answer "yes" or "no" to the inequality  $x - y + z < xz - yz - xy$ .

##### Step 2: Key Formula or Approach:

The best approach for complex inequalities in data sufficiency is often to test cases. We will choose numbers that satisfy the given conditions and see if we can get both "yes" and "no" answers to the main question. If we can, the information is not sufficient.

The inequality can be rearranged: Is  $x - y + z - (xz - yz - xy) < 0$ ?

Is  $x - y + z - xz + yz + xy < 0$ ?

Let's analyze the statements.

Statement 2:  $xy < 0$  means  $x$  and  $y$  have opposite signs.

Statement 1:  $\frac{x}{y} < -\frac{1}{2}$ . This is a stronger condition than statement 2. If  $\frac{x}{y}$  is negative, then  $xy$  must also be negative. So, statement 1 implies statement 2.

### Step 3: Detailed Explanation:

#### Analyzing Statement (2): $xy < 0$ .

This means  $x$  and  $y$  have opposite signs. We don't know anything about  $z$ , except that  $z \neq 0$ .

Let's test some numbers.

Case 1: Let  $x = 1, y = -1$ . Then  $xy = -1 < 0$ . Let  $z = 1$ .

Is  $1 - (-1) + 1 < 1(1) - (-1)(1) - 1(-1)$ ?

Is  $3 < 1 + 1 + 1$ ? Is  $3 < 3$ ? This is false. So the answer is "No".

Case 2: Let  $x = 2, y = -1$ . Then  $xy = -2 < 0$ . Let  $z = 1$ .

Is  $2 - (-1) + 1 < 2(1) - (-1)(1) - 2(-1)$ ?

Is  $4 < 2 + 1 + 2$ ? Is  $4 < 5$ ? This is true. So the answer is "Yes".

Since we can get both "Yes" and "No" answers, statement (2) alone is not sufficient.

#### Analyzing Statement (1): $\frac{x}{y} < -\frac{1}{2}$ .

This condition implies  $xy < 0$ . Let's test cases that satisfy this stricter condition. The variable  $z$  is still unconstrained (other than  $z \neq 0$ ). The sign of  $z$  can significantly impact the inequality.

Let's use a case that satisfies statement (1) and see how changing  $z$  affects the result.

Let  $x = 2, y = -1$ . Then  $\frac{x}{y} = \frac{2}{-1} = -2$ , which is less than  $-\frac{1}{2}$ . This case satisfies the condition.

Case 1.1: Let  $z = 1$ .

Is  $2 - (-1) + 1 < 2(1) - (-1)(1) - 2(-1)$ ?

Is  $4 < 2 + 1 + 2$ ? Is  $4 < 5$ ? Yes.

Case 1.2: Let  $z = -1$ .

Is  $2 - (-1) + (-1) < 2(-1) - (-1)(-1) - 2(-1)$ ?

Is  $2 < -2 - 1 + 2$ ? Is  $2 < -1$ ? No.

Since we can get both "Yes" and "No" answers with statement (1), it alone is not sufficient.

#### Analyzing Both Statements Together:

As noted earlier, statement (1) implies statement (2). Therefore, combining the statements provides no new information beyond what statement (1) already gives us. Since statement (1) alone was not sufficient, both statements together are also not sufficient. Our test with  $x = 2, y = -1$  satisfied both statements, and we found that the answer to the question depended on the value of  $z$ .

### Step 4: Final Answer:

Statements (1) and (2) together are not sufficient to answer the question asked.

### Quick Tip

In GMAT inequality problems, especially those in Data Sufficiency, be very mindful of variables whose signs are not determined. Here, the variable 'z' could be positive or negative, which dramatically changes the inequality. Testing both a positive and a negative value for an unconstrained variable is a key strategy.

### 3. Is $x > 9$ ?

1.  $x^2 + 3x = 28$

2.  $9x - 5x = 28$

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(B) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

(C) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(D) EACH statement ALONE is sufficient to answer the question asked

(E) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

**Correct Answer:** (D) EACH statement ALONE is sufficient to answer the question asked

### Solution:

#### Step 1: Understanding the Concept:

This is a "Yes/No" data sufficiency question. We need to determine if each statement provides enough information to give a definitive "Yes" or a definitive "No" answer to the question "Is  $x > 9$ ?". A statement is sufficient if it leads to a single, unambiguous answer.

#### Step 2: Key Formula or Approach:

We will solve the equation in each statement to find the possible value(s) for  $x$ . Then, for each possible value, we will check if it is greater than 9.

#### Step 3: Detailed Explanation:

**Analyzing Statement (1):**  $x^2 + 3x = 28$ .

This is a quadratic equation. To solve it, we first set it to zero.

$$x^2 + 3x - 28 = 0$$

Now, we can factor the quadratic expression. We are looking for two numbers that multiply to -28 and add to +3. These numbers are +7 and -4.

$$(x + 7)(x - 4) = 0$$

This gives two possible values for  $x$ :  $x = -7$  or  $x = 4$ .

Now we check these values against the question "Is  $x > 9$ ?".

For  $x = -7$ : Is  $-7 > 9$ ? The answer is "No".

For  $x = 4$ : Is  $4 > 9$ ? The answer is "No".

In both possible cases, the answer to the question is a definite "No". Since we get a single, consistent answer, statement (1) is sufficient.

**Analyzing Statement (2):**  $9x - 5x = 28$ .

This is a linear equation. We can solve for  $x$  directly.

$$9x - 5x = 28$$

$$4x = 28$$

$$x = \frac{28}{4}$$

$$x = 7$$

Now we check this value against the question "Is  $x > 9$ ?"

Is  $7 > 9$ ? The answer is a definite "No".

Since we get a single, consistent answer, statement (2) is sufficient.

**Step 4: Final Answer:**

Since each statement alone is sufficient to answer the question, the correct choice is (D).

#### Quick Tip

Remember that for a "Yes/No" data sufficiency question, a statement is sufficient if it gives a definite "Yes" or a definite "No". It is not sufficient only if it can lead to both a "Yes" and a "No" answer depending on the possibilities it allows.

**4. A beer company spent \$100,000 last year on hops, yeast, and malt. How much of the total expenditure was for hops?**

**1. The expenditure for yeast was 20% greater than the expenditure for malt.**

**2. The total expenditure for yeast and malt was equal to the expenditure for hops.**

(A) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

(B) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(C) EACH statement ALONE is sufficient to answer the question asked

(D) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

(E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

**Correct Answer:** (E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

**Solution:****Step 1: Understanding the Concept:**

This problem asks for a specific value (expenditure on hops). It is a data sufficiency question involving a system of linear equations. Let H, Y, and M represent the expenditures on hops, yeast, and malt, respectively.

From the question stem, we have the equation:

$$H + Y + M = 100,000$$

We need to find the value of H.

**Step 2: Key Formula or Approach:**

We will analyze each statement to see if it provides enough information, when combined with the initial equation, to solve for H. We need to have as many independent linear equations as we have variables to find a unique solution.

**Step 3: Detailed Explanation:**

**Analyzing Statement (1): The expenditure for yeast was 20% greater than the expenditure for malt.**

This can be written as an equation:

$$Y = M + 0.20M = 1.2M$$

Now, let's substitute this into our main equation:

$$H + (1.2M) + M = 100,000$$

$$H + 2.2M = 100,000$$

This is a single equation with two unknown variables, H and M. We cannot solve for a unique value of H. For example, if M = \$10,000, then H = \$78,000. If M = \$20,000, then H = \$56,000. Therefore, statement (1) alone is not sufficient.

**Analyzing Statement (2): The total expenditure for yeast and malt was equal to the expenditure for hops.**

This can be written as an equation:

$$Y + M = H$$

Now, let's substitute this into our main equation:

$$H + (Y + M) = 100,000$$

$$H + (H) = 100,000$$

$$2H = 100,000$$

$$H = 50,000$$

This gives us a unique value for H.

Therefore, statement (2) alone is sufficient.

**Step 4: Final Answer:**

Statement (2) alone is sufficient to answer the question, but statement (1) alone is not.

**Quick Tip**

In data sufficiency problems that can be modeled with equations, count your variables and your independent equations. To find a unique value for a variable, you generally need as many independent equations as variables. Statement (2) provides a key relationship that simplifies the original equation perfectly.

**5. What is the value of  $j + k$ ?**

1.  $mj + mk = 2m$

2.  $5j + 5k = 10$

(A) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

(B) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

(C) EACH statement ALONE is sufficient to answer the question asked.

(D) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

**Correct Answer:** (E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

**Solution:****Step 1: Understanding the Concept:**

The question asks for the value of the expression  $j + k$ . This is a value-based data sufficiency question. A statement is sufficient if it allows us to determine a single, unique numerical value for  $j + k$ .

**Step 2: Key Formula or Approach:**

The approach is to use algebraic manipulation, specifically factoring, to simplify the equations in each statement and see if we can isolate the term  $j + k$ .

**Step 3: Detailed Explanation:**

**Analyzing Statement (1):**  $mj + mk = 2m$ .

We can factor out 'm' from the left side of the equation:

$$m(j + k) = 2m$$

Our goal is to solve for  $j + k$ . We might be tempted to divide both sides by 'm'.

If we assume  $m \neq 0$ , we can divide by m:



$$j + k = 2$$

However, the statement does not specify that  $m \neq 0$ . If  $m = 0$ , the equation becomes:

$$\begin{aligned} 0(j + k) &= 2(0) \\ 0 &= 0 \end{aligned}$$

This is true for any values of  $j$  and  $k$ . In this case,  $j + k$  could be any number. For example,  $j=1, k=4$  gives  $j+k=5$ . Or  $j=2, k=0$  gives  $j+k=2$ .

Since we cannot determine a unique value for  $j + k$  (it could be 2, or it could be any other number if  $m=0$ ), statement (1) alone is not sufficient.

**Analyzing Statement (2):**  $5j + 5k = 10$ .

We can factor out 5 from the left side of the equation:

$$5(j + k) = 10$$

Since 5 is a non-zero constant, we can safely divide both sides by 5:

$$\begin{aligned} j + k &= \frac{10}{5} \\ j + k &= 2 \end{aligned}$$

This gives a single, unique value for  $j + k$ .

Therefore, statement (2) alone is sufficient.

**Step 4: Final Answer:**

Statement (2) alone is sufficient, but statement (1) alone is not sufficient.

#### Quick Tip

Be extremely careful when dividing by a variable in algebra. You can only do so if you are certain the variable is not zero. In data sufficiency, if a variable's value is unknown, you must consider the case where it might be zero. This is a common trap.

**6.  $x$  is a positive integer less than 20. What is the value of  $x$ ?**

- 1.  $x$  is the sum of two consecutive integers.**
- 2.  $x$  is the sum of five consecutive integers.**

(A) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(B) EACH statement ALONE is sufficient to answer the question asked

(C) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

(D) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

(E) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

**Correct Answer:** (C) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

**Solution:**

**Step 1: Understanding the Concept:**

The question asks for the specific value of  $x$ , which is a positive integer less than 20. We need to use the properties described in the statements to narrow down the possibilities for  $x$ . A statement (or combination) is sufficient only if it leads to a single, unique value for  $x$ .

**Step 2: Key Formula or Approach:**

We will translate each statement into an algebraic property of  $x$  and then list all possible values of  $x$  that satisfy the given conditions ( $0 < x < 20$ ).

**Step 3: Detailed Explanation:**

From the question stem, we know that  $x \in \{1, 2, 3, \dots, 19\}$ .

**Analyzing Statement (1):  $x$  is the sum of two consecutive integers.**

Let the two consecutive integers be  $n$  and  $n + 1$ .

$$x = n + (n + 1) = 2n + 1$$

This means that  $x$  must be an odd number.

The possible values for  $x$  (positive odd integers less than 20) are:  $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ . Since there is more than one possible value for  $x$ , statement (1) alone is not sufficient.

**Analyzing Statement (2):  $x$  is the sum of five consecutive integers.**

Let the five consecutive integers be  $n, n + 1, n + 2, n + 3, n + 4$ .

$$x = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 = 5(n + 2)$$

This means that  $x$  must be a multiple of 5.

Since  $x$  must be a positive integer,  $5(n + 2) > 0$ , which means  $n + 2 > 0$ , so  $n > -2$ . The smallest possible value for  $n$  is -1.

Let's find the possible values for  $x$ :

If  $n = -1$ ,  $x = 5(-1 + 2) = 5$ . The integers are -1, 0, 1, 2, 3.

If  $n = 0$ ,  $x = 5(0 + 2) = 10$ . The integers are 0, 1, 2, 3, 4.

If  $n = 1$ ,  $x = 5(1 + 2) = 15$ . The integers are 1, 2, 3, 4, 5.

If  $n = 2$ ,  $x = 5(2 + 2) = 20$ , which is not less than 20.

So, the possible values for  $x$  are:  $\{5, 10, 15\}$ .

Since there is more than one possible value for  $x$ , statement (2) alone is not sufficient.

**Analyzing Both Statements Together:**

From statement (1),  $x$  must be odd.

From statement (2),  $x$  must be a multiple of 5.

We need to find the numbers that are in both sets of possibilities.

Possible values from (1):  $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

Possible values from (2):  $\{5, 10, 15\}$

The common values are  $\{5, 15\}$ .

Even with both statements, there are still two possible values for  $x$  (5 and 15). We cannot determine a unique value for  $x$ .

Therefore, both statements together are not sufficient.

**Step 4: Final Answer:**

Statements (1) and (2) together are not sufficient to answer the question.

**Quick Tip**

For problems involving sums of consecutive integers, it's useful to know these properties:  
- The sum of ' $k$ ' consecutive integers is a multiple of ' $k$ ' if ' $k$ ' is odd. - The sum of ' $k$ ' consecutive integers is never a multiple of ' $k$ ' if ' $k$ ' is even (it's a multiple of  $k/2$ , but not an integer multiple of  $k$ ). - The sum of two consecutive integers is always odd.

---

**7. If arc XYZ above is a semicircle, what is its length?**

1.  $q = 2$

2.  $r = 8$

(A) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

(B) EACH statement ALONE is sufficient to answer the question asked

(C) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

(D) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(E) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

**Correct Answer:** (B) EACH statement ALONE is sufficient to answer the question asked

**Solution:**

**Step 1: Understanding the Concept:**

The question asks for the length of the semicircle arc XYZ. The formula for the circumference of a full circle is  $C = 2\pi R$  or  $C = \pi D$ , where  $R$  is the radius and  $D$  is the diameter. The length of a semicircle arc is half the circumference, so  $L = \frac{1}{2}\pi D = \pi R$ . To find the length, we need to determine the diameter (or radius) of the semicircle. The diameter of the semicircle is the segment XZ, which has a length of  $q + r$ .

**Step 2: Key Formula or Approach:**

In the given figure, if we connect points X to Y and Z to Y, we form a triangle  $\triangle XYZ$ . Since  $\triangle XYZ$  is inscribed in a semicircle with diameter XZ, the angle at Y,  $\angle XYZ$ , must be a right

angle ( $90^\circ$ ). The line segment of length 4 is the altitude from the right angle to the hypotenuse. There is a geometric theorem (the altitude theorem) for right triangles which states that the square of the altitude from the right angle to the hypotenuse is equal to the product of the two segments it divides the hypotenuse into.

In this case, the altitude is 4, and the segments of the hypotenuse (the diameter) are  $q$  and  $r$ . So, we have the relationship:

$$4^2 = q \times r$$

$$16 = qr$$

Our goal is to find the value of  $q + r$  to determine the diameter.

**Step 3: Detailed Explanation:**

**Analyzing Statement (1):  $q = 2$ .**

Using the relationship  $qr = 16$ , if we substitute  $q = 2$ , we can find  $r$ :

$$2 \times r = 16$$

$$r = 8$$

The diameter  $D$  is  $q + r$ .

$$D = 2 + 8 = 10$$

The radius  $R$  is  $D/2 = 5$ .

The length of the semicircle arc is  $L = \pi R = 5\pi$ .

Since we found a unique value for the length, statement (1) is sufficient.

**Analyzing Statement (2):  $r = 8$ .**

Using the relationship  $qr = 16$ , if we substitute  $r = 8$ , we can find  $q$ :

$$q \times 8 = 16$$

$$q = 2$$

The diameter  $D$  is  $q + r$ .

$$D = 2 + 8 = 10$$

The radius  $R$  is  $D/2 = 5$ .

The length of the semicircle arc is  $L = \pi R = 5\pi$ .

Since we found a unique value for the length, statement (2) is also sufficient.

**Step 4: Final Answer:**

Both statements, independently, provide enough information to find the length of the arc. Therefore, each statement alone is sufficient.

### Quick Tip

Recognize key geometric configurations. An angle inscribed in a semicircle is always a right angle. The altitude to the hypotenuse of a right triangle creates similar triangles and leads to the geometric mean theorem ( $h^2 = p_1p_2$ ), which is fundamental in solving this problem.

### 8. What is the value of $x$ ?

(1)  $(x)(x + 1) = (2013)(2014)$

(2)  $x$  is odd

- (A) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
- (B) EACH statement ALONE is sufficient to answer the question asked
- (C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (D) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
- (E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

**Correct Answer:** (A) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

### Solution:

#### Step 1: Understanding the Concept:

This is a "value" data sufficiency question. We need to determine if the given statements, alone or together, can narrow down the possibilities for  $x$  to a single, unique value.

#### Step 2: Key Formula or Approach:

Statement (1) provides a quadratic equation. We need to find all possible solutions for  $x$ . Statement (2) provides a property of  $x$ . We will then combine the information to see if a unique solution emerges.

#### Step 3: Detailed Explanation:

**Analyzing Statement (1):**  $(x)(x + 1) = (2013)(2014)$ .

This equation is in the form of a product of two consecutive numbers. By simple inspection, one obvious solution is  $x = 2013$ , because if  $x = 2013$ , then  $x + 1 = 2014$ , and the equation becomes  $(2013)(2014) = (2013)(2014)$ , which is true.

However, since this is a quadratic equation ( $x^2 + x - (2013)(2014) = 0$ ), there might be another solution. Let's consider the case with negative numbers. Let  $y = -x$ . Then the equation is  $(-y)(-y + 1) = (2013)(2014)$ , which simplifies to  $y(y - 1) = (2013)(2014)$ . Alternatively, let's look at the structure  $x(x + 1)$ . The product of two consecutive negative integers is positive. Let  $x + 1 = -2013$ . Then  $x = -2014$ . Let's check this solution: If  $x = -2014$ , then  $x + 1 = -2013$ . The product is  $(-2014)(-2013) = (2014)(2013)$ , which is also true. So, from statement (1),  $x$

could be 2013 or -2014. Since there are two possible values, statement (1) alone is not sufficient.

**Analyzing Statement (2):  $x$  is odd.**

This statement tells us that  $x$  belongs to the set  $\{\dots, -3, -1, 1, 3, \dots\}$ . This information on its own is clearly not enough to find a unique value for  $x$ . So, statement (2) alone is not sufficient.

**Analyzing Both Statements Together:**

From statement (1), we found that  $x = 2013$  or  $x = -2014$ . From statement (2), we know that  $x$  must be an odd number. Let's check our two possible values: - Is 2013 odd? Yes, its last digit is 3. - Is -2014 odd? No, it is an even number. By combining both statements, we eliminate  $x = -2014$ , leaving only one possible value:  $x = 2013$ . Since we have found a unique value for  $x$ , both statements together are sufficient.

**Step 4: Final Answer:**

Neither statement is sufficient on its own, but together they provide enough information to determine a unique value for  $x$ .

**Quick Tip**

When you see a quadratic equation in a data sufficiency problem, always look for both solutions. A common trap is to find one obvious solution by inspection and forget to look for the other, especially if it's negative.

**9. If Alyssa is twice as old as Brandon, by how many years is Brandon older than Clara?**

**(1) Four years ago, Alyssa was twice as old as Clara is now.**

**(2) Alyssa is 8 years older than Clara.**

(A) EACH statement ALONE is sufficient to answer the question asked

(B) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(C) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(D) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

(E) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

**Correct Answer:** (B) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

**Solution:**

**Step 1: Understanding the Concept:**

This is an age-related word problem presented in a data sufficiency format. We need to find

the value of the expression  $B - C$ , where B is Brandon's current age and C is Clara's current age. Let A be Alyssa's current age.

**Step 2: Key Formula or Approach:**

We will translate the information from the question stem and each statement into algebraic equations. Then we will check if we can solve for a unique value of  $B - C$ .

From the question stem, we get our first equation:

Equation (i):  $A = 2B$

**Step 3: Detailed Explanation:**

**Analyzing Statement (1): Four years ago, Alyssa was twice as old as Clara is now.**

Four years ago, Alyssa's age was  $A - 4$ . Clara's current age is C. This statement gives us:

Equation (ii):  $A - 4 = 2C$

Now we have a system of two equations with three variables: 1.  $A = 2B$  2.  $A - 4 = 2C$  We want to find  $B - C$ . Let's express B and C in terms of A and then subtract. From (1), we can write  $B = \frac{A}{2}$ .

From (2), we can write  $C = \frac{A-4}{2}$ .

Now, let's compute  $B - C$ :

$$B - C = \frac{A}{2} - \frac{A-4}{2}$$

$$B - C = \frac{A - (A-4)}{2}$$

$$B - C = \frac{A - A + 4}{2}$$

$$B - C = \frac{4}{2} = 2$$

We found a unique numerical value for  $B - C$ . Therefore, Brandon is 2 years older than Clara. Statement (1) alone is sufficient.

**Analyzing Statement (2): Alyssa is 8 years older than Clara.**

This statement gives us the equation:

Equation (iii):  $A = C + 8$

Now we combine this with the equation from the stem: 1.  $A = 2B$  2.  $A = C + 8$  We want to find  $B - C$ . Again, let's express B and C in terms of A. From (1), we have  $B = \frac{A}{2}$ .

From (2), we have  $C = A - 8$ .

Now, let's compute  $B - C$ :

$$B - C = \frac{A}{2} - (A - 8)$$

$$B - C = \frac{A}{2} - A + 8$$

$$B - C = 8 - \frac{A}{2}$$

The value of  $B - C$  depends on the value of A. Since we don't know A's age, we cannot find a unique value for  $B - C$ . For example, if A=20, B=10, C=12, and  $B - C = -2$ . If A=30, B=15, C=22, and  $B - C = -7$ . Statement (2) alone is not sufficient.

**Step 4: Final Answer:**

Statement (1) is sufficient to find the value of  $B - C$ , but statement (2) is not.

**Quick Tip**

In data sufficiency, you don't always need to find the values of all the variables. Focus only on the target value or expression the question asks for. In this case, even though we couldn't find the individual ages, we could find the difference between them.

**10. In the first hour of a bake sale, students sold either chocolate chip cookies, which sold for \$1.30, or brownies, which sold for \$1.50. What was the ratio of chocolate chip cookies sold to brownies sold during that hour?**

- 1. The average price for the items sold during that hour was \$1.42**
- 2. The total price for all items sold during that hour was \$14.20**

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) EACH statement ALONE is sufficient to answer the question asked
- (C) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
- (D) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
- (E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

**Correct Answer:** (B) EACH statement ALONE is sufficient to answer the question asked

**Solution:****Step 1: Understanding the Concept:**

The question asks for the ratio of the number of chocolate chip cookies to the number of brownies sold. Let  $C$  be the number of cookies and  $B$  be the number of brownies. We need to find the value of the ratio  $\frac{C}{B}$ . This is a weighted average and Diophantine equation problem.

**Step 2: Key Formula or Approach:**

For statement (1), we will use the weighted average formula:

$$\text{Average Price} = \frac{(\text{Price}_1 \times \text{Number}_1) + (\text{Price}_2 \times \text{Number}_2)}{\text{Total Number}}$$

For statement (2), we will set up an equation for the total revenue and analyze its integer solutions.

**Step 3: Detailed Explanation:**

**Analyzing Statement (1):** The average price for the items sold during that hour



was \$1.42.

Using the weighted average formula:

$$1.42 = \frac{1.30 \times C + 1.50 \times B}{C + B}$$

Now, we solve this equation for the ratio  $\frac{C}{B}$ .

$$1.42(C + B) = 1.30C + 1.50B$$

$$1.42C + 1.42B = 1.30C + 1.50B$$

Now, group the C terms and B terms:

$$1.42C - 1.30C = 1.50B - 1.42B$$

$$0.12C = 0.08B$$

To find the ratio  $\frac{C}{B}$ , we can rearrange the equation:

$$\frac{C}{B} = \frac{0.08}{0.12} = \frac{8}{12} = \frac{2}{3}$$

We have found a unique value for the ratio. Therefore, statement (1) is sufficient.

**Analyzing Statement (2): The total price for all items sold during that hour was \$14.20.**

This gives us an equation for the total revenue:

$$1.30C + 1.50B = 14.20$$

To work with integers, we can multiply the entire equation by 100:

$$130C + 150B = 1420$$

Divide by 10 to simplify:

$$13C + 15B = 142$$

Here, C and B must be non-negative integers representing the number of items sold. We need to find the non-negative integer solutions to this linear Diophantine equation. We can test values or use properties of numbers. Let's look at the units digit. The term  $15B$  will end in a 0 (if B is even) or a 5 (if B is odd). The term 142 ends in a 2. - Case 1: B is odd. Then  $15B$  ends in 5. So,  $13C$  must end in a 7 for the sum to end in 2 ( $7 + 5 = 12$ ). For  $13C$  to end in 7, C must end in 9 (since  $3 \times 9 = 27$ ). Let's test  $C=9$ :  $13(9) + 15B = 117 + 15B = 142 \implies 15B = 25$ . B is not an integer. The next value,  $C=19$ , would make  $13C$  too large. - Case 2: B is even. Then  $15B$  ends in 0. So,  $13C$  must end in a 2. For  $13C$  to end in 2, C must end in 4 (since  $3 \times 4 = 12$ ). Let's test  $C=4$ :  $13(4) + 15B = 52 + 15B = 142 \implies 15B = 90 \implies B = 6$ . This is a valid integer solution. The next value,  $C=14$ , would make  $13C = 13(14) = 182$ , which is already greater than 142. So, the only possible non-negative integer solution is  $C = 4, B = 6$ . This gives a unique ratio:  $\frac{C}{B} = \frac{4}{6} = \frac{2}{3}$ . Therefore, statement (2) is also sufficient.

**Step 4: Final Answer:**

Since both statements independently provide enough information to determine the ratio, each

statement alone is sufficient.

#### Quick Tip

Weighted average problems can often be solved quickly using an alligation or "number line" method. The ratio of the quantities is the inverse of the ratio of the distances of their individual values from the average. Here, the distance of 1.30 from 1.42 is 0.12, and the distance of 1.50 from 1.42 is 0.08. The ratio C:B is therefore 0.08:0.12, which simplifies to 2:3.

---