GMAT Quant Practise Question Paper 2 with Solutions

Time Allowed : 2 hours 15 minutes	Maximum Marks : 100

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GMAT exam is 2 hours and 15 minutes long (with one optional 10-minute break) and consists of 64 questions in total.
2. The GMAT exam is comprised of three sections:
3. Quantitative Reasoning: 21 questions, 45 minutes
4. Verbal Reasoning: 23 questions, 45 minutes
5. Data Insights: 20 questions, 45 minutes
6. You can answer the three sections in any order. As you move through a section, you can bookmark questions that you would like to review later.
7. When you have answered all questions in a section, you will proceed to the Question Review & Edit screen for that section.
8. If there is no time remaining in the section, you will NOT proceed to the Question Review & Edit screen and you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
9. Each Question Review & Edit screen includes a numbered list of the questions in that section and indicates the questions you bookmarked.
10. Clicking a question number will take you to that specific question. You can review as many questions as you would like and can edit up to three (3) answers.

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1. If a, b, and c are distinct positive integers where a<b<c and $\sqrt{abc} = c$, what is the value of a?
1. c=8
2. The average of a, b, and c is 143

(A) EACH statement ALONE is sufficient to answer the question asked
(B) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(D) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(E) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

Correct Answer: (C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

Solution:

Step 1: Understanding the Concept:
This is a data sufficiency question involving number properties. We are given a relationship between three distinct positive integers a, b, and c and are asked to find the value of a. We need to analyze each statement to see if it provides enough information to find a unique value for a.

Step 2: Key Formula or Approach:
The given equation is $\sqrt{abc} = c$.
Since c is a positive integer, we can square both sides:

$$(\sqrt{abc})^2 = c^2$$
$$abc = c^2$$

Since $c > 0$, we can divide both sides by c:

$$ab = c$$

We are also given that a, b, c are distinct positive integers with the condition $a < b < c$.
Substituting $c = ab$ into the inequality gives $a < b < ab$.
Since a is a positive integer, $a \geq 1$.
The inequality $b < ab$ simplifies to $1 < a$ (by dividing by b, which is positive). So, a must be greater than 1, i.e., $a \geq 2$. If $a = 1$, $1 < b < b$, which is impossible. So $a \neq 1$. Wait, let's recheck. a, b, c are distinct positive integers. $ab = c$. We also need $a < b < c$. Substituting $c = ab$ into $b < c$ gives $b < ab$. Since b is positive, we can divide by b to get $1 < a$. So a must be at least 2. This is a key constraint derived from the problem statement itself.

Step 3: Detailed Explanation:
Analyzing Statement (1): c = 8
Using the derived relationship $ab = c$, we have:

$$ab = 8$$

Since a and b are distinct positive integers and $a < b$, we need to find pairs of factors of 8.
The possible pairs (a, b) are $(1, 8)$ and $(2, 4)$.
Now we check these pairs against the condition $a < b < c$. Here, $c = 8$.
Case 1: $(a, b) = (1, 8)$. The condition becomes $1 < 8 < 8$. This is false because b is not less than c.
Case 2: $(a, b) = (2, 4)$. The condition becomes $2 < 4 < 8$. This is true.
In this case, the value of a is uniquely determined as 2.
Therefore, Statement (1) ALONE is sufficient.

Analyzing Statement (2): The average of a, b, and c is 143
The average is given by $\frac{a+b+c}{3} = 143$.
This means $a + b + c = 3 \times 143 = 429$.
We also know $c = ab$. Substituting this into the sum equation:

$$a + b + ab = 429$$

To solve this, we can use a factoring trick:

$$ab + a + b + 1 = 429 + 1$$
$$(a + 1)(b + 1) = 430$$

We need to find integer factors of 430, where $a + 1$ and $b + 1$ are the factors. Since $a < b$, we must have $a + 1 < b + 1$.
The prime factorization of 430 is $2 \times 5 \times 43$. The factors are 1, 2, 5, 10, 43, 86, 215, 430.
We know from the initial analysis that $a > 1$, so $a + 1 > 2$.
Let's test the factor pairs of 430 for $(a + 1, b + 1)$:
Case 1: $a + 1 = 5$. Then $a = 4$. And $b + 1 = 86$, so $b = 85$. Let's find c: $c = ab = 4 \times 85 = 340$.
Check the condition $a < b < c$: $4 < 85 < 340$. This is a valid solution. Here, $a = 4$.
Case 2: $a+1 = 10$. Then $a = 9$. And $b+1 = 43$, so $b = 42$. Let's find c: $c = ab = 9 \times 42 = 378$.
Check the condition $a < b < c$: $9 < 42 < 378$. This is also a valid solution. Here, $a = 9$.
Since we found at least two possible values for a (4 and 9), this statement does not give a unique value for a.
Therefore, Statement (2) ALONE is not sufficient.

Step 4: Final Answer:
Statement (1) alone is sufficient to determine a unique value for a, but statement (2) alone is not. Therefore, the correct option is (C).

> **Quick Tip**
>
> In Data Sufficiency problems involving number properties, always start by simplifying the initial conditions given in the question stem. Here, simplifying $\sqrt{abc} = c$ to $ab = c$ and then deriving the constraint $a > 1$ is crucial for evaluating the statements efficiently.

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2. Line M is tangent to a circle, which is centered on point (3, 4). Does Line M run through point (6, 6)?
1. Line M runs through point (-8, 6)
2. Line M is tangent to the circle at point (3, 6)

(A) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
(B) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(C) EACH statement ALONE is sufficient to answer the question asked

(D) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

Correct Answer: (D) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

Solution:

Step 1: Understanding the Concept:
This is a data sufficiency question in coordinate geometry. We need to determine if a specific point, (6, 6), lies on a tangent line M. A line's equation must be uniquely determined to check if a point lies on it. The key property of a tangent line is that it is perpendicular to the radius at the point of tangency.

Step 2: Key Formula or Approach:
- The center of the circle is C = (3, 4).
- A line is uniquely defined by two distinct points or one point and a slope.
- The slope of a line passing through points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2-y_1}{x_2-x_1}$.
- Two lines are perpendicular if the product of their slopes is -1 (i.e., $m_1 \times m_2 = -1$), or if one is horizontal (slope=0) and the other is vertical (slope is undefined).

Step 3: Detailed Explanation:
Analyzing Statement (1): Line M runs through point (-8, 6)
This statement gives us one point on line M. However, knowing just one point is not enough to define a line. There are infinitely many lines passing through the point (-8, 6). Some of these lines could be tangent to the given circle, while others are not. Even if we consider only the lines through (-8,6) that are tangent to the circle, there would be two such lines. We don't have enough information to find the unique equation of line M.
Therefore, Statement (1) ALONE is not sufficient.

Analyzing Statement (2): Line M is tangent to the circle at point (3, 6)
This statement gives us the point of tangency, T = (3, 6).
The center of the circle is C = (3, 4).
The radius CT connects the center to the point of tangency.
Let's find the slope of the radius CT:

$$m_{radius} = \frac{6-4}{3-3} = \frac{2}{0}$$

The slope is undefined, which means the radius CT is a vertical line.
The tangent line M is perpendicular to the radius at the point of tangency. A line perpendicular to a vertical line must be a horizontal line.
A horizontal line has the equation $y =$ constant.
Since line M passes through the point of tangency (3, 6), the y-coordinate for every point on the line must be 6.
So, the equation of line M is $y = 6$.
The question is: Does Line M run through point (6, 6)?

We check if the point (6, 6) satisfies the equation $y = 6$. Yes, its y-coordinate is 6.
This gives a definitive "Yes" answer to the question.
Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer:
Statement (2) alone is sufficient to answer the question, but statement (1) alone is not. Therefore, the correct option is (D).

> Quick Tip
>
> For geometry questions on the coordinate plane, always visualize or sketch the points and lines. The fact that the radius from (3,4) to (3,6) is vertical immediately tells you the tangent must be horizontal, which simplifies the problem immensely.

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3. For nonnegative integers x and y, what is the remainder when x is divided by y?
1. $\frac{x}{y} = 13.8$
2. The numbers x and y have a combined total of less than 5 digits.

(A) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(B) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
(C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(D) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(E) EACH statement ALONE is sufficient to answer the question asked

Correct Answer: (B) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

Solution:

Step 1: Understanding the Concept:
This data sufficiency question asks for the value of the remainder when integer x is divided by integer y. The remainder is a specific value, so we need information that leads to a unique solution for x and y, or at least for the remainder. The division algorithm states $x = qy + r$, where q is the quotient and r is the remainder, with $0 \leq r < y$.

Step 2: Key Formula or Approach:
From the division algorithm, $\frac{x}{y} = q + \frac{r}{y}$. The question asks for the value of r.
The number of digits in an integer n is given by $\lfloor \log_{10}(n) \rfloor + 1$.

Step 3: Detailed Explanation:
Analyzing Statement (1): $\frac{x}{y} = 13.8$
This equation can be rewritten as $x = 13.8y$.
In the form of the division algorithm, this is $x = 13y + 0.8y$.
Here, the quotient q is 13, and the remainder r is $0.8y$.
To find the value of the remainder, we need the value of y. Since y is unknown, the remainder cannot be determined.
For example:
- If $y = 5$, then $x = 13.8 \times 5 = 69$. When 69 is divided by 5, the remainder is 4. (Note: $0.8y = 0.8 \times 5 = 4$).
- If $y = 10$, then $x = 13.8 \times 10 = 138$. When 138 is divided by 10, the remainder is 8. (Note: $0.8y = 0.8 \times 10 = 8$).
Since the remainder can be different values, Statement (1) ALONE is not sufficient.

Analyzing Statement (2): The numbers x and y have a combined total of less than 5 digits.
Let $D(n)$ be the number of digits in n. The statement says $D(x) + D(y) < 5$.
This information is too general. For example:
- Let $x = 100$ and $y = 3$. $D(100) = 3, D(3) = 1$. Combined digits $= 4 < 5$. Remainder of $100 \div 3$ is 1.
- Let $x = 90$ and $y = 4$. $D(90) = 2, D(4) = 1$. Combined digits $= 3 < 5$. Remainder of $90 \div 4$ is 2.
This statement alone is not sufficient.

Analyzing Statements (1) and (2) Together:
From Statement (1), we have $\frac{x}{y} = 13.8 = \frac{138}{10} = \frac{69}{5}$.
Since x and y are integers, the ratio $\frac{x}{y}$ must be $\frac{69}{5}$ in its simplest form. This means x and y must be integer multiples of 69 and 5, respectively.
So, we can write $x = 69k$ and $y = 5k$ for some positive integer k (since x, y are non-negative, and y cannot be 0).
Now we use Statement (2): $D(x) + D(y) < 5$.
Let's test values of k:
- If $k = 1$: $x = 69(1) = 69$, $y = 5(1) = 5$. $D(x) = D(69) = 2$. $D(y) = D(5) = 1$. Combined digits $= 2 + 1 = 3$. Since $3 < 5$, this is a valid solution.
The remainder when $x = 69$ is divided by $y = 5$ is 4.
- If $k = 2$: $x = 69(2) = 138$, $y = 5(2) = 10$. $D(x) = D(138) = 3$. $D(y) = D(10) = 2$. Combined digits $= 3 + 2 = 5$. Since 5 is not less than 5, this case is not valid.
- For any $k > 1$, the number of digits will only increase, so their sum will be 5 or greater.
The only possible value for k is 1. This gives unique values for x and y as 69 and 5.
With unique values for x and y, we can find a unique remainder.
Therefore, both statements TOGETHER are sufficient.

Step 4: Final Answer:
Neither statement alone is sufficient, but together they are sufficient to find a unique remainder.
The correct option is (B).

Quick Tip

When you see a decimal relationship like $\frac{x}{y} = 13.8$, immediately convert it to a fraction in simplest form ($\frac{69}{5}$). This reveals the fundamental ratio between the integers x and y, which is often the key to solving the problem.

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4. If x and y are positive integers, is x/y an integer?
1. Every factor of y is also a factor of x
2. Every factor of x is also a factor of y

(A) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(B) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(C) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
(D) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(E) EACH statement ALONE is sufficient to answer the question asked

Correct Answer: (D) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

Solution:
Note: The question in the image is "is xy an integer?". Since x and y are defined as positive integers, their product xy is always an integer. This would make the question trivial. We assume the intended question, common in such exams, is "is x/y an integer?".

Step 1: Understanding the Concept:
The question asks whether x is a multiple of y. This is a question about divisibility and factors. We need to analyze the conditions on the factors of x and y to determine if x/y is always an integer.

Step 2: Key Formula or Approach:
For x/y to be an integer, x must be a multiple of y. This means that for the prime factorization of $x = p_1^{a_1} p_2^{a_2} \dots$ and $y = p_1^{b_1} p_2^{b_2} \dots$, we must have $a_i \geq b_i$ for all prime factors p_i.

Step 3: Detailed Explanation:
Analyzing Statement (1): Every factor of y is also a factor of x
Let's consider the number y itself. Since any number is a factor of itself, y must be a factor of x.
If y is a factor of x, it means that x is a multiple of y.
This can be written as $x = k \cdot y$ for some positive integer k.
Then, the expression x/y becomes:

$$\frac{x}{y} = \frac{k \cdot y}{y} = k$$

Since k is an integer, x/y is an integer.
This statement guarantees that the answer to the question "is x/y an integer?" is always "Yes". Therefore, Statement (1) ALONE is sufficient.

Analyzing Statement (2): Every factor of x is also a factor of y
By the same logic as above, this statement implies that x is a factor of y.
This means that y is a multiple of x.
This can be written as $y = m \cdot x$ for some positive integer m.
Now let's evaluate x/y:

$$\frac{x}{y} = \frac{x}{m \cdot x} = \frac{1}{m}$$

The value $\frac{1}{m}$ is an integer only if $m = 1$.
- If $m = 1$, then $y = x$. In this case, $x/y = 1$, which is an integer. (Answer: Yes) - If $m = 2$, then $y = 2x$. In this case, $x/y = 1/2$, which is not an integer. (Answer: No)
Since we can get both a "Yes" and a "No" answer, this statement does not provide a definitive conclusion.
Therefore, Statement (2) ALONE is not sufficient.

Step 4: Final Answer:
Statement (1) is sufficient to answer the question, but Statement (2) is not. The correct option is (D).

Quick Tip

In factor-related Data Sufficiency questions, the most powerful tool is often the definition itself. The statement "Every factor of Y is a factor of X" directly implies that Y is a factor of X, which means X is a multiple of Y. Don't overcomplicate with prime factorizations unless necessary.

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5. What is the average of the terms in set J?
1. The sum of any three terms in Set J is 21
2. Set J consists of 12 total terms.

(A) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(B) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
(C) EACH statement ALONE is sufficient to answer the question asked
(D) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

Correct Answer: (D) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

Solution:

Step 1: Understanding the Concept:
The question asks for the average (arithmetic mean) of the terms in a set. To find the average, we need two pieces of information: the sum of the terms and the number of terms.

Step 2: Key Formula or Approach:

$$\text{Average} = \frac{\text{Sum of terms}}{\text{Number of terms}}$$

We need to evaluate if the statements, alone or together, provide the necessary information to calculate this value uniquely.

Step 3: Detailed Explanation:
Analyzing Statement (1): The sum of any three terms in Set J is 21
Let the terms in Set J be $j_1, j_2, j_3, j_4, \ldots$.
The statement says that for any three terms we choose, their sum is 21. Let's assume the set has at least four terms to test this condition.
Pick three terms: j_1, j_2, j_3. We have:

$$j_1 + j_2 + j_3 = 21$$

Now, pick another set of three terms, replacing one term, for example j_1, j_2, j_4:

$$j_1 + j_2 + j_4 = 21$$

Comparing these two equations:

$$j_1 + j_2 + j_3 = j_1 + j_2 + j_4$$

$$j_3 = j_4$$

This logic applies to any pair of terms in the set. This means all terms in Set J must be equal to each other. Let's call the value of each term j.
So, for any three terms, we have:

$$j + j + j = 21$$

$$3j = 21$$

$$j = 7$$

This means every term in Set J is 7.
The average of a set of numbers that are all identical is simply that number itself. So, the average of the terms in Set J is 7.
This statement provides a unique value for the average.
Therefore, Statement (1) ALONE is sufficient.

Analyzing Statement (2): Set J consists of 12 total terms.
This statement tells us the number of terms in the set. However, it gives no information about the values of these terms or their sum.
The average could be anything. For example, if all terms are 1, the average is 1. If all terms are 10, the average is 10.
Therefore, Statement (2) ALONE is not sufficient.

Step 4: Final Answer:
Statement (1) is sufficient to find the average, while statement (2) is not. The correct option is (D).

> **Quick Tip**
>
> In problems about sets, a condition that applies to "any" subset (like "any three terms") is extremely restrictive. It often implies that all elements of the set are identical, which simplifies the problem dramatically.

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6. Is xy > 24?
1. y - 2 < x
2. 2y > x + 8

(A) EACH statement ALONE is sufficient to answer the question asked
(B) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
(C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(D) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

Correct Answer: (B) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

Solution:

Step 1: Understanding the Concept:
This is a data sufficiency question involving inequalities. We need to determine if the product xy is definitively greater than 24. We must test if the given statements constrain x and y in such a way that $xy > 24$ is always true or always false.

Step 2: Key Formula or Approach:
We will analyze the inequalities provided in each statement. A statement is sufficient if it leads to a definite "Yes" or a definite "No" answer. If we can find examples that give both "Yes"

and "No" answers, the statement is not sufficient.

Step 3: Detailed Explanation:
Analyzing Statement (1): y - 2 < x
This inequality can be rewritten as $y < x + 2$.
Let's test some cases: - Case 1 (Answer "Yes"): If $x = 10$, then $y < 12$. Let's choose $y = 10$. Then $xy = 100$, and $100 > 24$. - Case 2 (Answer "No"): If $x = 4$, then $y < 6$. Let's choose $y = 5$. Then $xy = 20$, and 20 is not greater than 24.
Since we can get both "Yes" and "No" answers, Statement (1) ALONE is not sufficient.

Analyzing Statement (2): 2y > x + 8
This inequality can be rewritten as $y > \frac{x}{2} + 4$.
Let's test some cases: - Case 1 (Answer "Yes"): If $x = 10$, then $y > \frac{10}{2} + 4$, so $y > 9$. Let's choose $y = 10$. Then $xy = 100$, and $100 > 24$. - Case 2 (Answer "No"): If $x = 2$, then $y > \frac{2}{2} + 4$, so $y > 5$. Let's choose $y = 6$. Then $xy = 12$, and 12 is not greater than 24.
Since we can get both "Yes" and "No" answers, Statement (2) ALONE is not sufficient.

Analyzing Statements (1) and (2) Together:
We have a system of two inequalities: 1. $y < x + 2$ 2. $y > \frac{x}{2} + 4$
Combining them, we get:

$$\frac{x}{2} + 4 < y < x + 2$$

For this range of y to exist, the lower bound must be less than the upper bound:

$$\frac{x}{2} + 4 < x + 2$$

Subtract $\frac{x}{2}$ from both sides:

$$4 < \frac{x}{2} + 2$$

Subtract 2 from both sides:

$$2 < \frac{x}{2}$$

Multiply by 2:

$$4 < x$$

So, a necessary condition for both statements to hold is that x must be greater than 4. (Assuming x and y are positive, which is standard unless specified).
Now, let's look at the product xy. We can use the inequality for y:

$$xy > x\left(\frac{x}{2} + 4\right)$$

$$xy > \frac{x^2}{2} + 4x$$

Let's analyze the expression $\frac{x^2}{2} + 4x$. We know that $x > 4$. Let's see what the minimum value of this expression is for $x > 4$. Let $f(x) = \frac{x^2}{2} + 4x$. Since this is an upward-opening parabola, its value increases for $x > -4$. As our domain is $x > 4$, the expression will be minimized as x approaches 4 from the right. Let's evaluate the expression at $x = 4$:

$$f(4) = \frac{4^2}{2} + 4(4) = \frac{16}{2} + 16 = 8 + 16 = 24$$

Since we know that $x > 4$, it must be true that $\frac{x^2}{2} + 4x > 24$. Therefore, we have:

$$xy > \frac{x^2}{2} + 4x > 24$$

This proves that xy must always be greater than 24. The answer to the question is a definite "Yes".
Therefore, both statements TOGETHER are sufficient.

Step 4: Final Answer:
Neither statement is sufficient on its own, but when combined, they provide enough information to definitively answer the question. The correct option is (B).

> Quick Tip
>
> When combining two inequalities in a Data Sufficiency problem, first try to establish a relationship between the variables, as in $\frac{x}{2}+4 < y < x+2$. Then, check the condition for this relationship to be possible (here, $x > 4$). Finally, use this new constraint to evaluate the expression in the question.

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7. If xy $\neq$ 0, is $\frac{1}{x} + \frac{1}{y} = 16$?
1. x + y = 16xy
2. x = y

(A) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(B) EACH statement ALONE is sufficient to answer the question asked
(C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(D) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(E) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

Correct Answer: (C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

Solution:

Step 1: Understanding the Concept:
The question asks whether the sum of the reciprocals of x and y is equal to 16. This is a "Yes/No" data sufficiency question. A statement is sufficient if it always leads to the answer "Yes" or always leads to the answer "No".

Step 2: Key Formula or Approach:
The expression in the question is $\frac{1}{x} + \frac{1}{y}$. We can combine this into a single fraction:

$$\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{x+y}{xy}$$

So, the question is equivalent to asking: Is $\frac{x+y}{xy} = 16$?

Step 3: Detailed Explanation:
Analyzing Statement (1): x + y = 16xy
We are given the equation $x + y = 16xy$.
The question is whether $\frac{1}{x} + \frac{1}{y} = 16$, which we found is equivalent to asking if $\frac{x+y}{xy} = 16$.
From the information in the problem stem, we know that $xy \neq 0$. Therefore, we can divide the equation from Statement (1) by xy:

$$\frac{x+y}{xy} = \frac{16xy}{xy}$$

$$\frac{x+y}{xy} = 16$$

This directly answers the question with a "Yes".
Therefore, Statement (1) ALONE is sufficient.

Analyzing Statement (2): x = y
This statement tells us that x and y are equal.
Let's substitute $y = x$ into the expression from the question:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$$

The question becomes: Is $\frac{2}{x} = 16$?
This simplifies to: Is $x = \frac{2}{16} = \frac{1}{8}$?
We do not know the value of x.
- If $x = \frac{1}{8}$, then the answer is "Yes". - If $x = 1$, then $\frac{2}{1} = 2 \neq 16$, so the answer is "No". Since we can get both a "Yes" and a "No" answer, this statement is not sufficient.

Step 4: Final Answer:
Statement (1) alone provides a definitive "Yes" answer, while Statement (2) does not. Therefore, Statement (1) alone is sufficient. The correct option is (C).

Quick Tip

In algebraic data sufficiency questions, always try to manipulate the expression in the question stem first. Here, changing $\frac{1}{x} + \frac{1}{y}$ to $\frac{x+y}{xy}$ makes it immediately obvious how Statement (1) relates to the question.

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8. What is the value of x+2y?
1. $3^x \cdot 9^y = 27^{12}$

2. x=2y

(A) EACH statement ALONE is sufficient to answer the question asked
(B) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(C) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
(D) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(E) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

Correct Answer: (B) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

Solution:

Step 1: Understanding the Concept:
This is a "What is the value?" data sufficiency question. We need to determine if the given statements provide enough information to find a single, unique numerical value for the expression $x + 2y$.

Step 2: Key Formula or Approach:
The key to solving equations with exponents is to express all terms with the same base. Here, the numbers 3, 9, and 27 can all be expressed as powers of 3. - $9 = 3^2$ - $27 = 3^3$ The rule of exponents to be used is: $(a^m)^n = a^{mn}$ and $a^m \cdot a^n = a^{m+n}$.

Step 3: Detailed Explanation:
Analyzing Statement (1): $3^x \cdot 9^y = 27^{12}$
Let's convert all terms to the base 3:

$$3^x \cdot (3^2)^y = (3^3)^{12}$$

Apply the power of a power rule:

$$3^x \cdot 3^{2y} = 3^{3\times 12}$$
$$3^x \cdot 3^{2y} = 3^{36}$$

Apply the product of powers rule:

$$3^{x+2y} = 3^{36}$$

Since the bases are equal, the exponents must also be equal.

$$x + 2y = 36$$

This statement gives a unique value for the expression $x + 2y$.
Therefore, Statement (1) ALONE is sufficient.

Analyzing Statement (2): x = 2y
This statement provides a relationship between x and y.
Let's substitute $x = 2y$ into the expression we need to find:

$$x + 2y = (2y) + 2y = 4y$$

The value of the expression is $4y$. Since we do not know the value of y, we cannot find a unique numerical value for $x + 2y$.
- If $y = 1$, then $x = 2$, and $x + 2y = 4$. - If $y = 2$, then $x = 4$, and $x + 2y = 8$. The value is not unique.
Therefore, Statement (2) ALONE is not sufficient.

Step 4: Final Answer:
Statement (1) alone is sufficient to find the value of $x+2y$, but Statement (2) alone is not. The correct option is (B).

> **Quick Tip**
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> When you see an equation with different bases that are powers of the same number (like 3, 9, 27 or 2, 4, 8, 16), the first step should always be to convert everything to the smallest base. This often reveals a simple linear equation between the exponents.

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9. Is $a^2 > 3a - b^4$?
1. $3a - b^4 = -5$
2. a > 5 and b > 0

(A) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient
(B) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(D) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(E) EACH statement ALONE is sufficient to answer the question asked

Correct Answer: (E) EACH statement ALONE is sufficient to answer the question asked

Solution:

Step 1: Understanding the Concept:
This is a "Yes/No" data sufficiency question involving an inequality. We need to determine if a^2 is greater than the expression $3a - b^4$. A statement is sufficient if we can definitively answer "Yes" or "No".

Step 2: Key Formula or Approach:
We can analyze the inequality by either substituting values from the statements or by rearranging the inequality to a more telling form, such as $a^2 - 3a + b^4 > 0$. We also need to remember key properties of real numbers, such as $x^2 \geq 0$ for any real number x.

Step 3: Detailed Explanation:
Analyzing Statement (1): $3a - b^4 = -5$
This statement gives us the exact value of the right side of the inequality.
The question "Is $a^2 > 3a - b^4$? " can be rewritten by substituting the value from this statement:

$$\text{Is } a^2 > -5?$$

The square of any real number a is always non-negative, meaning $a^2 \geq 0$.
Since 0 is greater than -5, it must be true that $a^2 > -5$.
The answer to the question is always "Yes".
Therefore, Statement (1) ALONE is sufficient.

Analyzing Statement (2): a > 5 and b > 0
This statement provides constraints on the values of a and b.
Let's rearrange the original inequality:

$$\text{Is } a^2 - 3a + b^4 > 0?$$

Let's analyze the expression $a^2 - 3a$. This is a quadratic in a. The function $f(a) = a^2 - 3a$ is an upward-opening parabola. Its value increases as a moves away from the vertex, which is at $a = 1.5$.
Since we are given that $a > 5$, we are on the increasing part of the parabola, far from the vertex. The minimum value of $a^2 - 3a$ for $a > 5$ will be greater than its value at $a = 5$.

$$\text{At } a = 5, \quad a^2 - 3a = 5^2 - 3(5) = 25 - 15 = 10.$$

So, for $a > 5$, we have $a^2 - 3a > 10$.
Now let's consider the term b^4. We are given that $b > 0$. This means b^4 must be a positive number, so $b^4 > 0$.
Now combine the parts:

$$a^2 - 3a + b^4 > 10 + 0$$
$$a^2 - 3a + b^4 > 10$$

Since $10 > 0$, the expression $a^2 - 3a + b^4$ is always greater than 0.
The answer to the question is always "Yes".
Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer:
Both Statement (1) and Statement (2) are independently sufficient to answer the question. The correct option is (E).

Quick Tip

For inequalities involving quadratics like $a^2 - 3a$, remember the properties of parabolas. Knowing the vertex and direction of opening can help you find the minimum or maximum value of the expression over a given range, which is often the key to solving the inequality.

10. If xy $\neq$ 0, is $a > \frac{y}{x}$?
1. $a = \frac{1}{x} + \frac{1}{y}$
2. x and y are positive integers

(A) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(C) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(D) EACH statement ALONE is sufficient to answer the question asked
(E) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient

Correct Answer: (A) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

Solution:

Step 1: Understanding the Concept:
This is a "Yes/No" data sufficiency question comparing the value of a to the ratio $\frac{y}{x}$. We need to determine if the given statements provide enough information to always give a "Yes" or always give a "No" answer. (Note: The OCR for statement 1 was ambiguous; this solution assumes the most likely interpretation, $a = \frac{1}{x} + \frac{1}{y}$).

Step 2: Key Formula or Approach:
We will substitute the expression for a from Statement (1) into the inequality in the question and then use the constraints from Statement (2) to test if the inequality holds true for all possible values of x and y.

Step 3: Detailed Explanation:
Analyzing Statement (1): $a = \frac{1}{x} + \frac{1}{y}$
Substituting this into the question, we get:

$$\text{Is } \frac{1}{x} + \frac{1}{y} > \frac{y}{x}?$$

The variables x and y can be any non-zero real numbers. Let's test some cases. - Case 1: Let $x = 1, y = 1$. The inequality becomes: Is $\frac{1}{1} + \frac{1}{1} > \frac{1}{1}$? Is $2 > 1$? Yes. - Case 2: Let $x = 1, y = 2$. The inequality becomes: Is $\frac{1}{1} + \frac{1}{2} > \frac{2}{1}$? Is $1.5 > 2$? No. Since we can get both "Yes" and "No" answers, Statement (1) ALONE is not sufficient.

Analyzing Statement (2): x and y are positive integers
This statement gives us information about x and y, but tells us nothing about a. The value of a could be anything. - If $a = 100$, $x = 1$, $y = 1$, then $a > y/x$ because $100 > 1$. (Yes) - If $a = 0$, $x = 1$, $y = 1$, then a is not greater than y/x because 0 is not greater than 1. (No) Therefore, Statement (2) ALONE is not sufficient.

Analyzing Statements (1) and (2) Together:
We now have both pieces of information: - $a = \frac{1}{x} + \frac{1}{y}$ - x and y are positive integers. The question is: Is $\frac{1}{x} + \frac{1}{y} > \frac{y}{x}$?
Since x and y are positive, we can multiply the inequality by xy without changing the direction of the inequality sign.

$$\text{Is } (\frac{1}{x} + \frac{1}{y})xy > (\frac{y}{x})xy?$$

$$\text{Is } y + x > y^2?$$

Let's test this with allowed values for x and y. - Case 1: Let $y = 1$. The inequality becomes: Is $1 + x > 1^2$? Is $1 + x > 1$? Is $x > 0$? Since x is a positive integer, $x \geq 1$, so $x > 0$ is always true. In this case, the answer to the question is "Yes". For example, if $x = 5, y = 1$, we check if $5 + 1 > 1^2$, which is $6 > 1$, a "Yes". - Case 2: Let $y = 2$. The inequality becomes: Is $2 + x > 2^2$? Is $2 + x > 4$? Is $x > 2$? This is not always true. - If we pick $x = 3, y = 2$, then $x > 2$ is true. The answer is "Yes". - If we pick $x = 1, y = 2$, then $x > 2$ is false. The answer is "No". - If we pick $x = 2, y = 2$, then $x > 2$ is false. The answer is "No". Since we can still get both "Yes" and "No" answers even with both statements, the information is not sufficient.

Step 4: Final Answer:
Even when combined, the statements are not sufficient to determine a definitive answer. The correct option is (A).

> **Quick Tip**
>
> For "Yes/No" Data Sufficiency questions, your goal is to find a counterexample. If you can find one case that gives a "Yes" and another that gives a "No" (while satisfying the given statements), the information is not sufficient. Start with small, simple numbers like 1, 2, and -1 to test the statements.
