GMAT Quant Practise Question Paper 3 with Solutions

Time Allowed: 2 hours 15 minutes | Maximum Marks: 100

General Instructions

Read the following instructions very carefully and strictly follow them:

- 1. The GMAT exam is 2 hours and 15 minutes long (with one optional 10-minute break) and consists of 64 questions in total.
- 2. The GMAT exam is comprised of three sections:
- 3. Quantitative Reasoning: 21 questions, 45 minutes
- 4. Verbal Reasoning: 23 questions, 45 minutes
- 5. Data Insights: 20 questions, 45 minutes
- 6. You can answer the three sections in any order. As you move through a section, you can bookmark questions that you would like to review later.
- 7. When you have answered all questions in a section, you will proceed to the Question Review & Edit screen for that section.
- 8. If there is no time remaining in the section, you will NOT proceed to the Question Review & Edit screen and you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
- 9. Each Question Review & Edit screen includes a numbered list of the questions in that section and indicates the questions you bookmarked.
- 10. Clicking a question number will take you to that specific question. You can review as many questions as you would like and can edit up to three (3) answers.
- 1. Lady Edith bought several necklaces at the jewelry store, and each necklace cost 16 dollars. Lady Mary also purchased several necklaces, at a cost of \$20 each. If the ratio of the number of necklaces Lady Edith purchased to the number of necklaces Lady Mary purchased is 3 to 2, what is the average cost of the necklaces purchased by Lady Edith and Lady Mary?
- (A) 16.7
- (B) 17.1
- (C) 17.6
- (D) 17.9
- (E) 18.2

Correct Answer: (C) 17.6

Solution:

Step 1: Understanding the Concept:

The question asks for the average cost of all necklaces purchased by Lady Edith and Lady Mary. This is a weighted average problem because the two sets of necklaces have different costs and were purchased in different quantities. The average cost is the total cost of all necklaces divided by the total number of necklaces.

Step 2: Key Formula or Approach:

The formula for the average cost is:

$$\label{eq:average_cost} \text{Average Cost} = \frac{\text{Total Cost}}{\text{Total Number of Items}}$$

The total cost is the sum of the costs of necklaces purchased by both individuals.

Step 3: Detailed Explanation:

The ratio of the number of necklaces purchased by Lady Edith to Lady Mary is 3 to 2. We can assume the simplest case where Lady Edith bought 3 necklaces and Lady Mary bought 2 necklaces. The ratio will hold true for any multiple (e.g., 6 and 4), and the average cost will remain the same.

First, calculate the total cost for Lady Edith:

Cost for Edith = (Number of necklaces) \times (Cost per necklace) = $3 \times \$16 = \48 Next, calculate the total cost for Lady Mary:

Cost for Mary = (Number of necklaces) \times (Cost per necklace) = $2 \times \$20 = \40 Now, find the total cost and total number of necklaces for both:

Total Cost = Cost for Edith + Cost for Mary =
$$$48 + $40 = $88$$

Total Number of Necklaces = $3 + 2 = 5$

Finally, calculate the average cost:

$$\label{eq:average Cost} \text{Average Cost} = \frac{\text{Total Cost}}{\text{Total Number of Necklaces}} = \frac{\$88}{5} = \$17.6$$

Step 4: Final Answer

The average cost of the necklaces is \$17.6.

Quick Tip

For problems involving ratios to find an average, you can use the simplest whole numbers from the ratio (in this case, 3 and 2) for your calculation. The average will be the same regardless of the actual number of items, as long as the ratio is maintained.

- 2. Matthew, Jared, and Richard all bought flowers. The number of flowers Matthew purchased was equal to a single digit. Of the numbers of flowers purchased by Matthew, Jared, and Richard, only one was divisible by 3. The number of flowers one of them bought was an even number. Which of the following could represent the numbers of flowers each purchased?
- (A) 3, 8, 24
- (B) 7, 9, 17
- (C) 6, 9, 12
- (D) 5, 15, 18
- (E) 9, 10, 13

Correct Answer: (E) 9, 10, 13

Solution:

Step 1: Understanding the Concept:

This question tests your understanding of basic number properties: single-digit numbers, divisibility by 3, and even/odd numbers. We need to evaluate each set of numbers against three given conditions.

Step 2: Key Formula or Approach:

We will check each option against the three conditions provided in the question:

- 1. Matthew's number is a single digit (1-9).
- 2. Exactly one of the three numbers is divisible by 3.
- 3. Exactly one of the three numbers is even.

Step 3: Detailed Explanation:

Let's analyze each option:

(A) 3, 8, 24:

- Single digit: Yes, 3.
- Divisible by 3: 3 is divisible by 3. 24 is also divisible by 3 $(24 = 3 \times 8)$. This violates the condition that only

(B) 7, 9, 17:

- Single digit: Yes, 7 and 9.
- Divisible by 3: Only 9 is divisible by 3. This condition is met.
- Even number: 7, 9, and 17 are all odd. This violates the condition that one of them bought an even number of flowers.

(C) 6, 9, 12:

• Single digit: Yes, 6 and 9.

• Divisible by 3: 6, 9, and 12 are all divisible by 3. This violates the condition that *only one* number is divisible by 3.

(D) 5, 15, 18:

- Single digit: Yes, 5.
- Divisible by 3: 15 is divisible by 3. 18 is also divisible by 3 ($18 = 3 \times 6$). This violates the condition that on

(E) 9, 10, 13:

- Single digit: Yes, 9. (This can be Matthew's number). This condition is met.
- Divisible by 3: Only 9 is divisible by 3. 10 and 13 are not. This condition is met.
- Even number: Only 10 is an even number. 9 and 13 are odd. This condition is met.

All three conditions are satisfied by the set 9, 10, 13.

Step 4: Final Answer

The set of numbers that could represent the flowers purchased is 9, 10, 13.

Quick Tip

When faced with multiple conditions, use a process of elimination. Test each option against the conditions one by one. As soon as an option fails to meet a condition, you can discard it and move to the next, saving time.

- **3.** Circle P is inside Circle Q, and the two circles share the same center X. If the circumference of Q is four times the circumference of P, and the radius of Circle P is three, what is the difference between Circle Q's diameter and Circle P's diameter?
- (A) 6
- (B) 9
- (C) 12
- (D) 18
- (E) 24

Correct Answer: (D) 18

Solution:

Step 1: Understanding the Concept:

The problem involves the relationship between the circumference, radius, and diameter of two concentric circles. The circumference of a circle is directly proportional to its radius and its

diameter.

Step 2: Key Formula or Approach:

The key formulas for a circle are:

- Circumference $C = 2\pi r$, where r is the radius.
- Diameter d = 2r.

From these, we can see that $C = \pi d$. This means that if the circumference is scaled by a factor, the diameter and radius are scaled by the same factor.

Step 3: Detailed Explanation:

Let C_P, r_P, d_P be the circumference, radius, and diameter of Circle P. Let C_Q, r_Q, d_Q be the circumference, radius, and diameter of Circle Q.

We are given:

1.
$$C_Q = 4 \times C_P$$

2.
$$r_P = 3$$

Since $C = 2\pi r$, the first condition can be written as:

$$2\pi r_Q = 4 \times (2\pi r_P)$$

We can cancel 2π from both sides:

$$r_O = 4 \times r_P$$

This shows that the radius of Circle Q is also four times the radius of Circle P.

Now, we can find the radius of Circle Q:

$$r_O = 4 \times 3 = 12$$

Next, we calculate the diameters of both circles:

$$d_P = 2 \times r_P = 2 \times 3 = 6$$

$$d_Q = 2 \times r_Q = 2 \times 12 = 24$$

Finally, we find the difference between their diameters:

Difference =
$$d_Q - d_P = 24 - 6 = 18$$

Step 4: Final Answer

The difference between Circle Q's diameter and Circle P's diameter is 18.

Quick Tip

For concentric circles, the ratio of their circumferences is the same as the ratio of their radii and the ratio of their diameters. If $C_Q/C_P = 4$, then $r_Q/r_P = 4$ and $d_Q/d_P = 4$. You can use this shortcut to find the diameter of Q $(d_Q = 4 \times d_P)$ and then compute the difference.

4. A yellow taxi cab went from Downtown to the Beachside and back at an average speed of 2/3 miles per hour. If the distance from Beachside to Downtown is 1 mile, and the trip back took half as much time as the trip there, what was the average speed of the yellow taxi cab on the way to Beachside?

- (A) 1/3
- (B) 1/2
- (C) 3/4
- (D) 2/3
- (E) 3/2

Correct Answer: (B) 1/2

Solution:

Step 1: Understanding the Concept:

This problem involves the relationship between speed, distance, and time. The key is to understand that average speed is calculated as total distance divided by total time, not as the average of the speeds of the individual parts of the journey.

Step 2: Key Formula or Approach:

The fundamental formula is:

$$Speed = \frac{Distance}{Time}$$

This can be rearranged to find time: $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$. For the entire trip: Average Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$.

Step 3: Detailed Explanation:

First, let's define our variables:

- d = 1 mile (one-way distance).
- Total Distance $D_{total} = d + d = 1 + 1 = 2$ miles.
- Average Speed $S_{avg} = 2/3$ mph.
- Let t_1 be the time from Downtown to Beachside.
- Let t_2 be the time from Beachside back to Downtown.
- We are given $t_2 = \frac{1}{2}t_1$.

Using the average speed formula, we can find the total time for the round trip:

$$T_{total} = \frac{D_{total}}{S_{avg}} = \frac{2 \text{ miles}}{2/3 \text{ mph}} = 2 \times \frac{3}{2} = 3 \text{ hours}$$

The total time is also the sum of the individual trip times:

$$T_{total} = t_1 + t_2$$

$$3 = t_1 + t_2$$

Now substitute the relationship $t_2 = \frac{1}{2}t_1$ into this equation:

$$3 = t_1 + \frac{1}{2}t_1$$

$$3 = \frac{3}{2}t_1$$

Solve for t_1 , the time taken on the way to Beachside:

$$t_1 = 3 \times \frac{2}{3} = 2 \text{ hours}$$

The question asks for the average speed on the way to Beachside (S_1) . The distance for this trip is 1 mile and we just found the time is 2 hours.

$$S_1 = \frac{\text{Distance}}{\text{Time}} = \frac{d}{t_1} = \frac{1 \text{ mile}}{2 \text{ hours}} = \frac{1}{2} \text{ mph}$$

Step 4: Final Answer

The average speed of the yellow taxi cab on the way to Beachside was 1/2 miles per hour.

Quick Tip

Be careful with average speed problems. You cannot simply average the speeds of the two legs of a journey unless the time taken for each leg is identical. Always work with total distance and total time.

- **5.** Circle B's diameter was multiplied by 1.8. By what percent, approximately, was the area increased?
- (A) 80%
- (B) 125%
- (C) 225%
- (D) 325%
- (E) 375%

Correct Answer: (C) 225%

Solution:

Step 1: Understanding the Concept:

The area of a two-dimensional shape scales with the square of its linear dimensions. This means if a linear dimension like the radius or diameter is multiplied by a factor k, the area is multiplied by a factor of k^2 .

Step 2: Key Formula or Approach:

The area of a circle is given by $A = \pi r^2$, where r is the radius. Since diameter d = 2r, we can

also write the area in terms of diameter: $A = \pi (d/2)^2 = \frac{\pi d^2}{4}$. The formula for percentage increase is:

$$\label{eq:Percentage} \text{Percentage Increase} = \left(\frac{\text{New Value} - \text{Original Value}}{\text{Original Value}}\right) \times 100\%$$

Step 3: Detailed Explanation:

Let the original diameter be d_{old} and the original area be A_{old} .

$$A_{old} = \frac{\pi d_{old}^2}{4}$$

The new diameter, d_{new} , is 1.8 times the old diameter:

$$d_{new} = 1.8 \times d_{old}$$

Now, let's find the new area, A_{new} , using the new diameter:

$$A_{new} = \frac{\pi d_{new}^2}{4} = \frac{\pi (1.8 \times d_{old})^2}{4} = \frac{\pi (1.8^2 \times d_{old}^2)}{4}$$
$$A_{new} = 1.8^2 \times \left(\frac{\pi d_{old}^2}{4}\right) = 1.8^2 \times A_{old}$$

Calculate the scaling factor for the area:

$$1.8^2 = 1.8 \times 1.8 = 3.24$$

So, the new area is 3.24 times the old area:

$$A_{new} = 3.24 \times A_{old}$$

Now, we calculate the percentage increase:

$$\begin{aligned} \text{Increase} &= \frac{A_{new} - A_{old}}{A_{old}} \times 100\% = \frac{3.24 A_{old} - A_{old}}{A_{old}} \times 100\% \\ \text{Increase} &= \frac{2.24 A_{old}}{A_{old}} \times 100\% = 2.24 \times 100\% = 224\% \end{aligned}$$

The question asks for the approximate percentage, and 224% is approximately 225%.

Step 4: Final Answer

The area was increased by approximately 225%.

Quick Tip

When a linear dimension (like radius, diameter, side length) of a 2D shape is scaled by a factor of k, its area is scaled by k^2 . The percentage increase is $(k^2 - 1) \times 100\%$. Here, k = 1.8, so the increase is $(1.8^2 - 1) \times 100\% = (3.24 - 1) \times 100\% = 224\%$.

6. What is the value of the square root of the square root of .00000256?

- (A) 0.004
- (B) 0.016
- (C) 0.04
- (D) 0.16
- (E) 0.4

Correct Answer: (C) 0.04

Solution:

Step 1: Understanding the Concept:

The question asks to evaluate a nested square root, which means we need to perform the square root operation twice. The expression can be written as $\sqrt{\sqrt{0.00000256}}$. This is equivalent to finding the fourth root of the number.

Step 2: Key Formula or Approach:

We will solve this by taking the square root two times. It is often easier to handle decimals by converting them to scientific notation or fractions.

 $0.00000256 = 256 \times 10^{-8}$

Step 3: Detailed Explanation:

Let's evaluate the expression step-by-step.

First, find the inner square root: $\sqrt{0.00000256}$.

Using scientific notation:

$$\sqrt{0.00000256} = \sqrt{256 \times 10^{-8}}$$

The square root of a product is the product of the square roots:

$$\sqrt{256}\times\sqrt{10^{-8}}$$

We know that $\sqrt{256} = 16$ and $\sqrt{10^{-8}} = 10^{-8/2} = 10^{-4}$.

So, the result of the first square root is:

$$16 \times 10^{-4} = 0.0016$$

Now, we take the square root of this result: $\sqrt{0.0016}$.

Again, using scientific notation:

$$\sqrt{0.0016} = \sqrt{16 \times 10^{-4}}$$

$$\sqrt{16} \times \sqrt{10^{-4}}$$

We know that $\sqrt{16} = 4$ and $\sqrt{10^{-4}} = 10^{-4/2} = 10^{-2}$.

So, the final result is:

$$4 \times 10^{-2} = 0.04$$

Step 4: Final Answer

The value of the square root of the square root of .00000256 is 0.04.

Quick Tip

When taking the square root of a decimal number, count the number of decimal places. If there are 2n places, the root will have n places. For 0.00000256 (8 decimal places), its square root (0.0016) has 4 decimal places. For 0.0016 (4 decimal places), its square root (0.04) has 2 decimal places.

- 7. Which of the following is NOT a possible value of $\frac{1}{4-x}$
- (A) -4
- (B) 4/17
- (C) 0
- (D) 4
- (E) 17/4

Correct Answer: (C) 0

Solution:

Step 1: Understanding the Concept:

This question deals with the properties of rational functions (fractions with variables). A key property of any fraction $\frac{a}{b}$ is that its value is zero if and only if the numerator a is zero and the denominator b is not zero. Another key property is that the denominator b cannot be zero.

Step 2: Key Formula or Approach:

Let the given expression be equal to a value y:

$$y = \frac{1}{4 - x}$$

We need to determine if there is any value that y cannot take. The expression is a fraction with a constant numerator (1). For a fraction to be equal to zero, its numerator must be zero.

Step 3: Detailed Explanation:

The numerator of the fraction $\frac{1}{4-x}$ is 1. Since the numerator is a constant value of 1, it can never be equal to 0. Therefore, the entire fraction can never be equal to 0, regardless of the value of x in the denominator.

Let's check if other values are possible by solving for x: If $y = \frac{1}{4-x}$, then y(4-x) = 1, which means $4 - x = \frac{1}{y}$, and $x = 4 - \frac{1}{y}$. This equation can be solved for x for any non-zero value of y. For example:

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- If y = -4, $x = 4 \frac{1}{-4} = 4.25$.
- If y = 4/17, $x = 4 \frac{17}{4} = -0.25$.
- If y = 4, $x = 4 \frac{1}{4} = 3.75$.
- If y = 17/4, $x = 4 \frac{4}{17} \approx 3.76$.

All these values are possible. However, if we try to set y = 0:

$$0 = \frac{1}{4 - x}$$

To solve this, we would multiply both sides by 4 - x, resulting in $0 \times (4 - x) = 1$, which simplifies to 0 = 1. This is a contradiction, which means there is no value of x that can make the expression equal to 0.

Step 4: Final Answer

The value that is NOT possible for the expression is 0.

Quick Tip

A fraction of the form $\frac{k}{f(x)}$, where k is a non-zero constant, can never be equal to zero. The only way a fraction can be zero is if its numerator is zero.

8. Which of the following numbers has the greatest number of unique digits?

- (A) 1/6
- (B) 1/4
- (C) 1/3
- (D) 3/4
- (E) 5/7

Correct Answer: (E) 5/7

Solution:

Step 1: Understanding the Concept:

The question asks to find which fraction, when converted to a decimal, uses the highest number of different (unique) digits. This requires converting each fraction to its decimal representation and then counting the unique digits that appear.

Step 2: Key Formula or Approach:

We will perform the division for each fraction to find its decimal form. Then we will list the set of unique digits for each and count them.

Step 3: Detailed Explanation:

Let's convert each fraction to a decimal and count its unique digits.

(A) 1/6

$$1 \div 6 = 0.1666... = 0.1\overline{6}$$

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The digits used are 1 and 6.

Unique digits: $\{1, 6\}$. Count = 2.

(B) 1/4

$$1 \div 4 = 0.25$$

This is a terminating decimal. The digits used are 2 and 5. Unique digits: $\{2, 5\}$. Count = 2.

(C) 1/3

$$1 \div 3 = 0.333... = 0.\overline{3}$$

The only digit used is 3.

Unique digits: $\{3\}$. Count = 1.

(D) 3/4

$$3 \div 4 = 0.75$$

This is a terminating decimal. The digits used are 7 and 5. Unique digits: $\{5, 7\}$. Count = 2.

(E) 5/7

To convert 5/7, we perform long division:

$$5 \div 7 = 0.714285714285... = 0.\overline{714285}$$

The repeating block of digits is 714285.

The digits used are 7, 1, 4, 2, 8, and 5.

Unique digits: $\{1, 2, 4, 5, 7, 8\}$. Count = 6.

Comparing the counts of unique digits:

- 1/6: 2
- 1/4: 2
- 1/3: 1
- 3/4: 2
- 5/7:6

The fraction 5/7 has the greatest number of unique digits.

Step 4: Final Answer

The number with the greatest number of unique digits is 5/7.

Quick Tip

For a fraction p/q in simplest form, the length of the repeating part of its decimal expansion is related to the prime factors of the denominator q. If q is a prime number (other than 2 or 5), the length of the repeating cycle can be up to q-1. For 5/7, the length is 6, which is 7-1. This often leads to a large number of unique digits.

9.
$$\frac{0.0027 \times 10^x}{0.09 \times 10^y} = 3 \times 10^8$$

What is the value of y less than x?

- (A) 9
- (B) 10
- (C) 11
- (D) 12
- (E) 13

Correct Answer: (B) 10

Solution:

Step 1: Understanding the Concept:

This problem involves simplifying an expression with scientific notation and solving for the difference between two exponents, x - y. The phrase "y less than x" translates to the expression x - y.

Step 2: Key Formula or Approach:

We will use the rules of exponents:

- $a^m \times a^n = a^{m+n}$
- \bullet $\frac{a^m}{a^n} = a^{m-n}$

The first step is to convert the decimal coefficients into a consistent format, preferably standard scientific notation.

Step 3: Detailed Explanation:

Let's rewrite the decimal numbers in the expression:

$$0.0027 = 2.7 \times 10^{-3}$$

$$0.09 = 9 \times 10^{-2}$$

Substitute these back into the original equation:

$$\frac{(2.7 \times 10^{-3}) \times 10^x}{(9 \times 10^{-2}) \times 10^y} = 3 \times 10^8$$

Now, group the coefficients and the powers of 10 separately:

$$\left(\frac{2.7}{9}\right) \times \left(\frac{10^{-3} \times 10^x}{10^{-2} \times 10^y}\right) = 3 \times 10^8$$

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Simplify the coefficient part:

$$\frac{2.7}{9} = \frac{27}{90} = \frac{3}{10} = 0.3$$

Simplify the powers of 10 using exponent rules:

$$\frac{10^{-3+x}}{10^{-2+y}} = 10^{(-3+x)-(-2+y)} = 10^{x-y-3+2} = 10^{x-y-1}$$

Now the equation becomes:

$$0.3 \times 10^{x-y-1} = 3 \times 10^8$$

To solve this, we need the coefficient on the left to match the coefficient on the right. Let's write 0.3 in scientific notation as 3×10^{-1} :

$$(3 \times 10^{-1}) \times 10^{x-y-1} = 3 \times 10^8$$

Combine the powers of 10 on the left side:

$$3 \times 10^{-1+(x-y-1)} = 3 \times 10^8$$

$$3 \times 10^{x-y-2} = 3 \times 10^8$$

Since the bases (3 and 10) are equal on both sides, the exponents of 10 must also be equal:

$$x - y - 2 = 8$$

Solve for x - y:

$$x - y = 8 + 2$$

$$x - y = 10$$

Step 4: Final Answer

The value of y less than x, which is x - y, is 10.

Quick Tip

To avoid errors with decimals and exponents, always convert all numbers into standard scientific notation (a single non-zero digit before the decimal point) before you begin simplifying the expression.

- 10. If x and y are positive integers, what percent of three more than y is twice the value of x?
- (A) 1/200x(y + 3)
- (B) y + 3/200x
- (C) 100(y + 3)/2x
- (D) (200x/y) + 3
- (E) 200x/(y + 3)

Correct Answer: (E) 200x/(y + 3)

Solution:

Step 1: Understanding the Concept:

This is a word problem that requires translating a sentence into a mathematical expression for percentage. The key is to correctly identify the "part" and the "whole" (or base) in the percentage relationship.

Step 2: Key Formula or Approach:

The general formula for percentage is:

$$Percentage = \left(\frac{Part}{Whole}\right) \times 100$$

The phrase "what percent of A is B" translates to:

Percentage =
$$\left(\frac{B}{A}\right) \times 100$$

Here, A is the "whole" or the base, and B is the "part".

Step 3: Detailed Explanation:

Let's break down the sentence: "what percent of three more than y is twice the value of x?"

- The quantity that follows "of" is the base or the "whole". So, Whole = "three more than y'' = y + 3.
- The quantity being compared to the base is the "part". So, Part = "twice the value of x" = 2x

Now, we plug these into the percentage formula:

$$Percentage = \left(\frac{Part}{Whole}\right) \times 100$$

Percentage =
$$\left(\frac{2x}{y+3}\right) \times 100$$

Simplifying this expression gives:

Percentage =
$$\frac{200x}{y+3}$$

This matches option (E).

Step 4: Final Answer

The expression representing the percentage is $\frac{200x}{y+3}$.

Quick Tip

In percentage word problems, the number or expression that follows the word "of" is almost always the denominator (the "whole" or "base") in the percentage calculation.

DATA-SUFFICIENCY

- 1. A new Coffee Bean & Tea Leaf coffee drink consists only of certain amounts of espresso and sugar. What is the ratio of espresso to sugar in the new drink?
- (1) There are 15 ounces of sugar in 35 ounces of the new drink.
- (2) There are 40 ounces of espresso in 70 ounces of the new drink.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (D) EACH statement ALONE is sufficient.

Solution:

Step 1: Understanding the Question

Let E be the amount of espresso and S be the amount of sugar. The total amount of the drink is T = E + S. The question asks for the ratio E:S.

Step 2: Analysis of Statement (1)

Statement (1) says that in a total drink of T = 35 ounces, the amount of sugar is S = 15 ounces. Since the drink consists only of espresso and sugar, we can find the amount of espresso:

$$E = T - S = 35 - 15 = 20$$
 ounces

Now we can find the ratio of espresso to sugar:

$$\frac{E}{S} = \frac{20}{15} = \frac{4}{3}$$

So, the ratio E:S is 4:3. This provides a specific, unique answer to the question. Therefore, Statement (1) ALONE is sufficient.

Step 3: Analysis of Statement (2)

Statement (2) says that in a total drink of T = 70 ounces, the amount of espresso is E = 40 ounces.

We can find the amount of sugar:

$$S = T - E = 70 - 40 = 30$$
 ounces

Now we can find the ratio of espresso to sugar:

$$\frac{E}{S} = \frac{40}{30} = \frac{4}{3}$$

So, the ratio E:S is 4:3. This also provides a specific, unique answer to the question. Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer

Since each statement alone is sufficient to answer the question, the correct answer is (D).

Quick Tip

In Data Sufficiency ratio problems, you don't always need the exact total amounts. As long as a statement allows you to determine the relationship between the parts, it's often sufficient. Both statements here define the composition of the drink, allowing for the calculation of the ratio.

2. How many in a group are women with blue eyes?

- (1) Of the women in the group, 5 percent have blue eyes.
- (2) Of the men in the group, 10 percent have dark-colored eyes.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Solution:

Step 1: Understanding the Question

The question asks for a specific number: the count of women with blue eyes. Let W be the total number of women in the group. We need to find the value of $0.05 \times W$.

Step 2: Analysis of Statement (1)

Statement (1) tells us that 5% of the women have blue eyes. This gives us the proportion but not the actual number. The number of women with blue eyes is $0.05 \times W$. Since we do not know the value of W (the total number of women), we cannot determine the specific number of women with blue eyes.

For example, if there are 100 women, then 5 have blue eyes. If there are 200 women, then 10 have blue eyes. We cannot find a unique answer.

Therefore, Statement (1) ALONE is not sufficient.

Step 3: Analysis of Statement (2)

Statement (2) provides information about the men in the group (10% have dark-colored eyes). This information is completely unrelated to the number of women with blue eyes.

Therefore, Statement (2) ALONE is not sufficient.

Step 4: Analysis of Statements (1) and (2) Together

Combining both statements does not provide any new information to help solve the problem.

We have a percentage for women (from statement 1) and a percentage for men (from statement 2), but we don't know the total number of women, the total number of men, or the total number of people in the group. There is no link between the two pieces of information. Therefore, Statements (1) and (2) TOGETHER are not sufficient.

Step 5: Final Answer

Since neither statement alone nor both statements together are sufficient to answer the question, the correct answer is (E).

Quick Tip

For Data Sufficiency questions that ask for a specific value (e.g., "how many," "what is the value of"), providing only a percentage or a ratio is not enough. You must be able to calculate a single numerical answer.

- 3. On a soccer team, one team member is selected at random to be the goalie. What is the probability that a substitute player will be the goalie?
- (1) One-sixth of the team members are substitute players.
- (2) 18 of the team members are not substitute players.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

Solution:

Step 1: Understanding the Question

The question asks for the probability that a randomly selected goalie is a substitute player. The formula for this probability is:

$$P(\text{goalie is a substitute}) = \frac{\text{Number of substitute players}}{\text{Total number of team members}}$$

Step 2: Analysis of Statement (1)

Statement (1) says that one-sixth of the team members are substitute players. This directly gives the ratio of substitute players to the total number of team members.

$$\frac{\text{Number of substitute players}}{\text{Total number of team members}} = \frac{1}{6}$$

So, the probability is 1/6. This provides a specific, unique answer.

Therefore, Statement (1) ALONE is sufficient.

Step 3: Analysis of Statement (2)

Statement (2) says that 18 team members are not substitute players. Let S be the number of substitutes and T be the total number of team members. This statement tells us that T - S = 18.

The probability we want to find is S/T. From the equation T - S = 18, we have T = S + 18. The probability is S/(S + 18).

The value of this probability depends on S.

- If there is S = 1 substitute player, T = 19, and the probability is 1/19.
- If there are S = 2 substitute players, T = 20, and the probability is 2/20 or 1/10.

Since we cannot find a unique value for the probability, Statement (2) ALONE is not sufficient.

Step 4: Final Answer

Since Statement (1) alone is sufficient and Statement (2) alone is not, the correct answer is (A).

Quick Tip

Probability is a ratio. If a statement provides the direct ratio needed, it is sufficient. If a statement provides absolute numbers that still leave the ratio undetermined, it is not sufficient.

- 4. A designer purchased 20 mannequins that each cost an equal amount and then sold each one at a constant price. What was the designer's gross profit on the sale of the 20 mannequins?
- (1) If the selling price per mannequin had been double what it was, the gross profit on the total would have been \$2400.
- (2) If the selling price per mannequin had been \$2 more, the store's gross profit on the total would have been \$440.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

Solution:

Step 1: Understanding the Question

Let C be the cost per mannequin and S be the selling price per mannequin. The number of mannequins is 20. The total cost is $20 \times C$. The total revenue is $20 \times S$. The gross profit (GP) is Total Revenue - Total Cost. We need to find the value of GP = 20S - 20C = 20(S - C).

Step 2: Analysis of Statement (1)

Statement (1) presents a hypothetical situation. If the selling price were 2S, the new gross profit would be \$2400. The new total revenue would be $20 \times (2S) = 40S$. The cost remains 20C. So, the equation is:

$$40S - 20C = 2400$$

Dividing by 20 gives:

$$2S - C = 120$$

This is one equation with two variables, S and C. We cannot use this single equation to find the value of 20(S-C). For example, if S=70, C=20, then 2(70)-20=120. Profit = 20(70-20)=1000. If S=80, C=40, then 2(80)-40=120. Profit = 20(80-40)=800. We don't get a unique answer. Therefore, Statement (1) ALONE is not sufficient.

Step 3: Analysis of Statement (2)

Statement (2) presents another hypothetical. If the selling price were S + \$2, the new gross profit would be \$440. The new total revenue would be $20 \times (S + 2)$. The cost remains 20C. So, the equation is:

$$20(S+2) - 20C = 440$$

Let's expand the equation:

$$20S + 40 - 20C = 440$$

We can rearrange this to isolate the expression for the actual gross profit:

$$20S - 20C = 440 - 40$$

$$20(S-C) = 400$$

This equation directly gives us the value of the designer's gross profit, which is \$400. Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer

Since Statement (2) alone is sufficient and Statement (1) alone is not, the correct answer is (B).

Quick Tip

In Data Sufficiency, always write down the algebraic expression you need to find. Then, analyze each statement to see if it allows you to solve for that specific expression, even if you can't solve for the individual variables.

5. A shopping center increased its revenues by 10% between 2010 and 2011. The shopping center's costs increased by 8% during the same period. What is the firm's

percent increase in profits over this period, if profits are defined as revenues minus costs?

- (1) The firm's initial profit is \$200,000.
- (2) The firm's initial revenues are 1.5 times its initial costs.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

Solution:

Step 1: Understanding the Question

Let R_1, C_1, P_1 be the initial revenue, costs, and profit for 2010. Let R_2, C_2, P_2 be the final revenue, costs, and profit for 2011. We are given: $P_1 = R_1 - C_1 R_2 = 1.10 \times R_1 C_2 = 1.08 \times C_1 P_2 = R_2 - C_2 = 1.10R_1 - 1.08C_1$ The question asks for the percent increase in profit, which is $\frac{P_2 - P_1}{P_1} \times 100\%$.

Let's express this in terms of R_1 and C_1 :

$$\frac{(1.10R_1 - 1.08C_1) - (R_1 - C_1)}{R_1 - C_1} = \frac{0.10R_1 - 0.08C_1}{R_1 - C_1}$$

To find a numerical answer, we need to know the relationship (ratio) between R_1 and C_1 .

Step 2: Analysis of Statement (1)

Statement (1) tells us that $P_1 = \$200,000$. This means $R_1 - C_1 = 200,000$. Substituting this into our expression:

$$\frac{0.10R_1 - 0.08C_1}{200,000}$$

We still have two variables, R_1 and C_1 , in the numerator. We can write $R_1 = C_1 + 200,000$, but the expression will still depend on the value of C_1 . We cannot find a unique numerical value for the percent increase.

Therefore, Statement (1) ALONE is not sufficient.

Step 3: Analysis of Statement (2)

Statement (2) tells us that $R_1 = 1.5C_1$. This gives the relationship between initial revenue and costs. Let's substitute this into the expression for the percent increase:

$$\frac{0.10(1.5C_1) - 0.08C_1}{1.5C_1 - C_1} = \frac{0.15C_1 - 0.08C_1}{0.5C_1}$$
$$= \frac{0.07C_1}{0.5C_1} = \frac{0.07}{0.5} = 0.14$$

The percent increase is $0.14 \times 100\% = 14\%$. This is a specific, unique numerical answer. Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer

Since Statement (2) alone is sufficient and Statement (1) alone is not, the correct answer is (B).

Quick Tip

For percent change problems involving variables (like revenue and cost), you often need the ratio between the variables, not their absolute values. Statement (2) provides this ratio, making it sufficient.

6. A certain number is not an integer. Is the number less than .4?

- (1) The number rounded to the nearest tenth is .4.
- (2) The number rounded to the nearest integer is 0.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Solution:

Step 1: Understanding the Question

Let N be the number. We are given that N is not an integer. The question is a Yes/No question: Is N; 0.4?

Step 2: Analysis of Statement (1)

Statement (1) says that when N is rounded to the nearest tenth, the result is 0.4. This means that N must be in the range:

$$0.35 \le N < 0.45$$

Within this range, we can find numbers that are less than 0.4 and numbers that are not less than 0.4.

- If N = 0.38, then N is less than 0.4. The answer is "Yes".
- If N = 0.42, then N is not less than 0.4. The answer is "No".

Since we can get both a "Yes" and a "No" answer, Statement (1) ALONE is not sufficient.

Step 3: Analysis of Statement (2)

Statement (2) says that when N is rounded to the nearest integer, the result is 0. This means that N must be in the range:

$$-0.5 \le N < 0.5$$

(Note: We are given N is not an integer, so $N \neq 0$). Within this range, we can also find numbers that satisfy both conditions.

- If N = 0.3, then N is less than 0.4. The answer is "Yes".
- If N = 0.45, then N is not less than 0.4. The answer is "No".

Since we can get both a "Yes" and a "No" answer, Statement (2) ALONE is not sufficient.

Step 4: Analysis of Statements (1) and (2) Together

Now we combine the information from both statements. From (1): $0.35 \le N < 0.45$ From (2): $-0.5 \le N < 0.5$ The intersection of these two ranges is still $0.35 \le N < 0.45$. Even with both conditions, we can still pick numbers that give different answers to the question "Is N; 0.4?".

- If we pick N = 0.39, it satisfies both conditions (rounds to 0.4 and to 0), and $N \neq 0.4$. (Answer: Yes)
- If we pick N = 0.41, it satisfies both conditions (rounds to 0.4 and to 0), and N is not ; 0.4. (Answer: No)

Since we still cannot determine a definite answer, the statements together are not sufficient.

Step 5: Final Answer

Because we can't get a definitive Yes or No answer even with both statements, the correct answer is (E).

Quick Tip

For Data Sufficiency questions about rounding, always write out the full range of possible values. A statement is insufficient if you can find one example within the range that gives a "Yes" answer and another that gives a "No" answer.

- 7. What is the area of a triangle (with vertices at FCE) that is inscribed in a hexagon with vertices at ABCDEF?
- (1) The hexagon is regular and BE = 14.
- (2) EC = 73.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

Solution:

Step 1: Understanding the Question

We need to find the area of a triangle FCE, where F, C, and E are vertices of a hexagon ABCDEF. To find the area, we need to know the dimensions and properties of the hexagon and the resulting triangle.

Step 2: Analysis of Statement (1)

Statement (1) states that the hexagon is regular and the length of the long diagonal BE is 14. In a regular hexagon, all sides are equal, and all interior angles are equal. A regular hexagon can be divided into six equilateral triangles with their common vertex at the center of the hexagon.

The long diagonal (e.g., BE) passes through the center and is equal to twice the side length of the hexagon (s). So, BE = 2s = 14, which means the side length is s = 7.

The vertices of triangle FCE form a specific shape within the regular hexagon. Let's consider the geometry. The distance from the center to any vertex is s=7. The triangle FCE is an isosceles triangle. The side FC is a short diagonal, EC is a short diagonal, and FE is a long diagonal. The length of a short diagonal is $s\sqrt{3}$. So, $FC = EC = 7\sqrt{3}$. The side FE is a long diagonal with length 2s = 14.

We can find the area of triangle FCE. It is a right-angled triangle, since the angle at C (angle FCE) subtends the diameter of the circumscribed circle. The base and height can be considered FC and EC, which are short diagonals. Wait, FCE is not right-angled. Let's use coordinates or another method.

Let's place vertex E on the x-axis at (-7, 0) and F at (7,0), with the center at (0,0). Then vertex C would be at $(-7\cos(60^\circ), 7\sin(60^\circ)) = (-3.5, 7\sqrt{3}/2)$. No, this is incorrect.

A simpler way: The triangle FCE is an isosceles triangle with base FE = 14 and equal sides FC and EC. The height of this triangle is the perpendicular distance from C to the line segment FE. This height is equal to 1.5 times the side length of the equilateral triangles that form the hexagon, which is $1.5 \times (s\sqrt{3}/2) = 1.5 \times (7\sqrt{3}/2) = 21\sqrt{3}/4$. This is getting complicated.

Let's use a simpler decomposition. The area of the regular hexagon is $\frac{3\sqrt{3}}{2}s^2 = \frac{3\sqrt{3}}{2}(7^2) = \frac{147\sqrt{3}}{2}$. The triangle FCE covers a specific fraction of this area. Triangle FCE consists of three smaller triangles: F-Center-E, F-Center-E. These are not easy to calculate.

Let's reconsider the triangle shape. Vertices F, C, E. Let's use the shoelace formula or place E at origin. Let's use the property that the area of triangle FCE is composed of triangle FDE and triangle CDE. This is also not simple.

Let's go back to the simplest observation: The line segment FE is a diagonal that passes through the center. C is another vertex. The triangle FCE is inscribed in the hexagon's circumcircle. The base is the diameter FE=14. The height is the perpendicular distance from C to the diameter FE. The height is the apothem of the hexagon, which is $s\sqrt{3}/2 = 7\sqrt{3}/2$. No, the height is the y-coordinate of C if FE is on the x-axis. This would be $s\sqrt{3}/2$. Ah, no, the vertices are F, C, E. The triangle is made of two small equilateral triangles (area $s^2\sqrt{3}/4$) and one isoceles triangle made of two radii and a 120 degree angle.

Area = Area(F-Center-E) + Area(C-Center-F). This is not correct.

Let's use coordinates: Center at (0,0), s=7. A=(7,0). B= $(3.5, 3.5\sqrt{3})$. C= $(-3.5, 3.5\sqrt{3})$. D=(-7,0).

 $E=(-3.5, -3.5\sqrt{3}). F=(3.5, -3.5\sqrt{3}).$

Vertices are $F(3.5, -3.5\sqrt{3})$, $C(-3.5, 3.5\sqrt{3})$, $E(-3.5, -3.5\sqrt{3})$.

This is a right-angled triangle with the right angle at E.

Base is CE on the vertical line x=-3.5. Length of CE = $3.5\sqrt{3} - (-3.5\sqrt{3}) = 7\sqrt{3}$.

Height is the horizontal distance from F to the line x=-3.5. Length of height = 3.5 - (-3.5) = 7.

Area =
$$\frac{1}{2}$$
 × base × height = $\frac{1}{2}$ × $7\sqrt{3}$ × $7 = \frac{49\sqrt{3}}{2}$.

Since we found a unique value for the area, statement (1) is sufficient.

Step 3: Analysis of Statement (2)

Statement (2) gives $EC = 7\sqrt{3}$. EC is a short diagonal of the hexagon. If we assume the hexagon is regular, then the length of a short diagonal is $s\sqrt{3}$. So $s\sqrt{3} = 7\sqrt{3}$, which implies s = 7. This would make it sufficient. However, the statement does not say the hexagon is regular. An irregular hexagon could have a diagonal of length $7\sqrt{3}$ without all its sides being 7. Without knowing the hexagon is regular, we do not know the coordinates or positions of the other vertices (like F), so we cannot calculate the area of triangle FCE.

Therefore, Statement (2) ALONE is not sufficient.

Step 4: Final Answer

Statement (1) alone is sufficient, but Statement (2) alone is not. The correct answer is (A).

Quick Tip

In geometry Data Sufficiency problems, be careful about assumptions. Don't assume a shape is regular unless it's explicitly stated. Statement (1) provides the crucial "regular" property, while statement (2) does not.

- 8. How many integers are there between m and n, exclusive, if m and n are themselves integers?
- (1) m n = 8
- (2) There are 5 integers between, but not including, m 1 and n 1.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (D) EACH statement ALONE is sufficient.

Solution:

Step 1: Understanding the Question

The question asks for the number of integers strictly between m and n. The formula for the number of integers between two integers m and n (exclusive) is |m - n| - 1. We need to find the value of this expression.

Step 2: Analysis of Statement (1)

Statement (1) gives us m - n = 8. This tells us the difference between m and n is 8. Using the formula, the number of integers between them is:

$$|m-n|-1=|8|-1=8-1=7$$

For example, if m=10 and n=2, then m-n=8. The integers between them are 3, 4, 5, 6, 7, 8, 9, which is a total of 7 integers. The result is always 7. This provides a unique answer. Therefore, Statement (1) ALONE is sufficient.

Step 3: Analysis of Statement (2)

Statement (2) says there are 5 integers between m-1 and n-1. Let's apply the same formula to these new endpoints. The number of integers between m-1 and n-1 is:

$$|(m-1)-(n-1)|-1$$

We are told this value is 5.

$$|(m-1) - (n-1)| - 1 = 5$$

 $|m-1-n+1| - 1 = 5$
 $|m-n| - 1 = 5$
 $|m-n| = 6$

Now we can use this result to answer the original question. The number of integers between m and n is:

$$|m-n|-1=6-1=5$$

This also provides a unique answer.

Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer

Since each statement alone is sufficient to answer the question, the correct answer is (D).

Quick Tip

The number of integers strictly between two integers 'a' and 'b' is '—a - b— - 1'. The number of integers between 'a-k' and 'b-k' is the same as the number of integers between 'a' and 'b'. Statement (2) effectively tells you the answer is 5.

- 9. For integers w, x, y, and z, is wxyz = -1?
- (1) wx/yz = -1
- (2) w = -1/x and y = 1/z
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

Solution:

Step 1: Understanding the Question

This is a Yes/No question. We are given that w, x, y, and z are integers. For their product 'wxyz' to equal -1, an odd number of them must be -1 and the rest must be 1. In other words, each variable must be either 1 or -1.

Step 2: Analysis of Statement (1)

Statement (1) gives the equation wx/yz = -1, which implies wx = -yz. Let's test some integer values that satisfy this condition.

- Case 1: Let w=1, x=1, y=1, z=-1. Here, wx = 1 and yz = -1. So, wx = -yz becomes 1 = -(-1), which is true. The product is wxyz = (1)(1)(1)(-1) = -1. The answer to the question is "Yes".
- Case 2: Let w=2, x=1, y=-2, z=1. Here, wx = 2 and yz = -2. So, wx = -yz becomes 2 = -(-2), which is true. The product is wxyz = (2)(1)(-2)(1) = -4. The answer to the question is "No".

Since we can get both a "Yes" and a "No" answer, Statement (1) ALONE is not sufficient.

Step 3: Analysis of Statement (2)

Statement (2) gives two equations: w = -1/x and y = 1/z. From w = -1/x, we can write wx = -1. Since w and x must be integers, the only possibilities are:

- w = 1 and x = -1
- w = -1 and x = 1

In both cases, the product wx is -1.

From y = 1/z, we can write yz = 1. Since y and z must be integers, the only possibilities are:

- y = 1 and z = 1
- y = -1 and z = -1

In both cases, the product yz is 1.

Now we can find the value of the product wxyz:

$$wxyz = (wx)(yz) = (-1)(1) = -1$$

This gives a definite value of -1. The answer to the question is always "Yes".

Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer

Since Statement (2) alone is sufficient, but Statement (1) alone is not, the correct answer is (B).

Quick Tip

When variables are stated to be integers, equations like ax = 1 or ax = -1 severely restrict the possible values for 'a' and 'x'. This is a powerful constraint to look for in Data Sufficiency problems.

- 10. If the product of j and k does not equal zero, is j < 0 and k > 0?
- (1) (-j, k) lies above the x-axis and to the right of the y-axis.
- (2) (j, -k) lies below the x-axis and to the left of the y-axis.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Correct Answer: (D) EACH statement ALONE is sufficient.

Solution:

Step 1: Understanding the Question

This is a Yes/No question about the signs of the variables j and k. The condition $jk \neq 0$ means that neither j nor k is zero. The question asks if j is negative AND k is positive. This corresponds to the second quadrant in a j-k plane.

Step 2: Analysis of Statement (1)

Statement (1) describes the location of the point (-j, k) in the Cartesian coordinate system.

- "lies above the x-axis" means the y-coordinate is positive. So, k > 0.
- "to the right of the y-axis" means the x-coordinate is positive. So, -j > 0.

If we have the inequality -j > 0, we can multiply both sides by -1. Remember to flip the inequality sign when multiplying by a negative number.

So, from statement (1), we have concluded that j < 0 and k > 0. This provides a definitive "Yes" to the question.

Therefore, Statement (1) ALONE is sufficient.

Step 3: Analysis of Statement (2)

Statement (2) describes the location of the point (j, -k).

- "lies below the x-axis" means the y-coordinate is negative. So, -k < 0. Multiplying by -1 and flipping the inequality sign gives k > 0.
- "to the left of the y-axis" means the x-coordinate is negative. So, j < 0.

From statement (2), we have also concluded that j < 0 and k > 0. This also provides a definitive "Yes" to the question.

Therefore, Statement (2) ALONE is sufficient.

Step 4: Final Answer

Since each statement alone is sufficient to answer the question, the correct answer is (D).

Quick Tip

Quickly translate coordinate plane locations into inequalities:

• Right of y-axis: x-coordinate >0

• Left of y-axis: x-coordinate <0

• Above x-axis: y-coordinate >0

• Below x-axis: y-coordinate <0

This allows for rapid evaluation of such statements.