GRE 2024 Quant Practice Test 13 with Solutions

1. If you join all the vertices of a heptagon, how many quadrilaterals will you get?

(1) 72
(2) 36
(3) 25
(4) 35
(5) 120

Correct Answer: (2) 35

Solution:

Step 1: Understanding the problem.
We are asked to find the number of quadrilaterals that can be formed by joining the vertices of a heptagon (7-sided polygon).
Step 2: Basic combinatorial rule.
A quadrilateral is formed by choosing 4 vertices out of 7. Thus, the required number of quadrilaterals is:

$$\binom{7}{4}$$

Step 3: Calculation.

$$\binom{7}{4} = \frac{7!}{4!(7-4)!} = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = 35$$

Final Answer:

$$\boxed{35}$$

Quick Tip

For polygons, the number of k-sided figures formed by choosing k vertices is given by the combination formula $\binom{n}{k}$.

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2. Four students have to be chosen: 2 girls as the captain and vice-captain and 2 boys as the captain and vice-captain of the school. There are 15 eligible girls and 12 eligible boys. In how many ways can they be chosen if Sunita is sure to be the captain?

(1) 114
(2) 1020
(3) 360
(4) 1848
(5) 1500

Correct Answer: (4) 1848

Solution:

Step 1: Fix Sunita as the Girls' Captain.
Sunita's position (Girls' Captain) is fixed, so there is 1 way for this role.

Step 2: Choose the Girls' Vice-Captain.
Remaining eligible girls $= 15 - 1 = 14$.
Hence, Girls' Vice-Captain can be chosen in 14 ways.

Step 3: Assign Boys' Captain and Boys' Vice-Captain (distinct roles).
Number of ways to assign two distinct posts to boys $= {}^{12}P_2 = 12 \times 11 = 132$.

Step 4: Apply the Multiplication Principle.
Total ways $= 1 \times 14 \times 132 = 1848$.

Final Answer:

$$\boxed{1848}$$

Quick Tip

Jab posts distinct hon (captain vs vice-captain), tab permutations use karo: ${}^{n}P_r = n \times (n-1) \times \cdots$. Agar sirf selection hoti (without roles), tab combinations $\binom{n}{r}$ use hota.

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3. A teacher prepares a test with 5 objective questions, of which 4 have to be answered. The first 2 questions each have 3 choices and the last 3 each have 4 choices. Find the total number of ways to answer the paper.

(1) 255
(2) 816
(3) 192
(4) 100
(5) 144

Correct Answer: (2) 816

Solution:

Step 1: Understand the structure.
There are 5 questions Q_1, Q_2, Q_3, Q_4, Q_5. Exactly 4 must be answered.
Choices per question: Q_1, Q_2 have 3 choices each; Q_3, Q_4, Q_5 have 4 choices each.

Step 2: Casework by which question is left unanswered.
• Leave out Q_1: Answer $Q_2(3)$, $Q_3(4)$, $Q_4(4)$, $Q_5(4) \Rightarrow 3 \times 4 \times 4 \times 4 = 192$.
• Leave out Q_2: Similarly $3 \times 4 \times 4 \times 4 = 192$.
• Leave out Q_3: Answer $Q_1(3)$, $Q_2(3)$, $Q_4(4)$, $Q_5(4) \Rightarrow 3 \times 3 \times 4 \times 4 = 144$.
• Leave out Q_4: Again $3 \times 3 \times 4 \times 4 = 144$.
• Leave out Q_5: Again $3 \times 3 \times 4 \times 4 = 144$.

Step 3: Add the cases (Addition Principle).
Total $= 192 + 192 + 144 + 144 + 144 = 816$.

Final Answer:

$$\boxed{816}$$

Quick Tip

"Exactly k of n questions" type problems mein pehle choose karo kaun se attempt honge (ya kis ko chhoda jayega), phir har chosen question ke liye choices multiply kar do (Multiplication Principle).

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4. How many 5-digit numbers are there with distinct digits?

(1) 144
(2) 27216
(3) 4386
(4) 6432
(5) 720

Correct Answer: (2) 27216

Solution:

Step 1: Leading digit cannot be zero.
First digit (ten-thousands place) has 9 options: 1 to 9.

Step 2: Remaining digits must be distinct, zero allowed now.
After fixing the first digit, remaining available digits $= 9$.
Thus subsequent places have: $9, 8, 7, 6$ options respectively.

Step 3: Multiply the choices (Multiplication Principle).
Total $= 9 \times 9 \times 8 \times 7 \times 6$.
Compute: $9 \times 9 = 81$, $81 \times 8 = 648$, $648 \times 7 = 4536$, $4536 \times 6 = 27216$.

Final Answer:

$$\boxed{27216}$$

Quick Tip

Distinct-digit numbers ke liye pehle leading digit constraints handle karo (zero nahi ho sakta), phir baaki positions ke liye decreasing choices multiply karo.

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5. In how many ways can 15 students be seated in a row such that the 2 most talkative children never sit together?

(1) $14! \cdot 14!$
(2) $15 \cdot 14!$
(3) $14!$
(4) $14! \cdot 13$

(5) 15!

Correct Answer: (4) $14! \cdot 13$

Solution:

Step 1: Total arrangements without restriction.
15 students in a row can be arranged in 15! ways.

Step 2: Arrangements where the 2 talkative sit together.
Consider the 2 talkative children as a single block. Then we have 14 entities (the block + other 13 students). These can be arranged in 14! ways.
Within the block, the 2 talkative children can be arranged in 2! ways.
So, arrangements with them together $= 14! \times 2$.

Step 3: Arrangements where they are not together.
Subtract:

$$15! - (14! \times 2) = 14! \times (15 - 2) = 14! \times 13$$

Final Answer:

$$\boxed{14! \cdot 13}$$

Quick Tip

Whenever the condition is "not together", use: Total − Together. For "together", treat them as a block.

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6. In a school, 5 colours are allotted to each house. If the flag of Tagore House has to be a sequence of three blocks of different colours, how many flags can they choose from?

(1) 9
(2) 27
(3) 60
(4) 20
(5) 15

Correct Answer: (3) 60

Solution:

Step 1: Choose first block colour.
We have 5 possible colours.

Step 2: Choose second block colour.
Must be different from the first. Hence 4 options.

Step 3: Choose third block colour.
Must be different from both first and second. Hence 3 options.

Step 4: Multiply choices.
Total flags $= 5 \times 4 \times 3 = 60$.

Final Answer:

$$\boxed{60}$$

Quick Tip

Distinct blocks $\rightarrow$ ordered selection without repetition $\rightarrow$ use permutations: ${}^nP_r = n \times (n-1) \times \cdots$.

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7. Find the number of words which can be formed by using the letters of the word EQUATION if each word has to start with a vowel.

(1) 40320
(2) 1260
(3) 1080
(4) 400
(5) 25200

Correct Answer: (5) 25200

Solution:

Step 1: Identify vowels.
The word "EQUATION" has 8 distinct letters. Vowels = E, U, A, I, O (5 vowels).

Step 2: Fix the first letter as a vowel.
Choices for first letter = 5.

Step 3: Arrange the remaining 7 letters.
Remaining 7 letters can be arranged in 7! ways.

Step 4: Multiply.
Total words $= 5 \times 7! = 5 \times 5040 = 25200$.

Final Answer:

$$\boxed{25200}$$

Quick Tip

When a word must "start with a vowel," first count vowels, then multiply by arrangements of remaining letters.

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8. How many five-digit numbers can be formed using the digits 0, 2, 3, 4 and 5, when repetition is allowed, such that the number formed is divisible by 2 or 5 or both?

(1) 100
(2) 150
(3) 3125

(4) 1500
(5) 125

Correct Answer: (4) 1500

Solution:

Step 1: Divisibility rule.
A number is divisible by 2 or 5 if its last digit is 0, 2, or 5.

Step 2: Choose last digit.
Options = 3 (0, 2, or 5).

Step 3: Choose first digit.
It cannot be 0 (since 5-digit number). So, 4 choices (2, 3, 4, 5).

Step 4: Choose middle three digits.
Repetition is allowed, so each of the 3 positions has 5 choices.
Thus middle part $= 5^3 = 125$.

Step 5: Multiply.
Total numbers $= 4 \times 125 \times 3 = 1500$.

Final Answer:

$$\boxed{1500}$$

Quick Tip

Divisible by 2 or 5 $\rightarrow$ last digit must be 0, 2, or 5. Handle leading-digit restrictions separately when forming valid numbers.

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9. A straight road runs from north to south. It has two turnings towards east and three turnings towards west. In how many ways can a person coming from east get on the road and go west?

(1) 2
(2) 3
(3) 9
(4) 6
(5) 5

Correct Answer: (4) 6

Solution:

Step 1: Understanding the problem.
The road is north–south, with 2 turnings to the east and 3 turnings to the west.
A person coming from east must use one of the 2 east turnings to enter the road, then later leave by one of the 3 west turnings.

Step 2: Applying multiplication principle.
Choices for entry = 2.

Choices for exit = 3.

Step 3: Multiply.
Total ways $= 2 \times 3 = 6$.

Final Answer:

$$\boxed{6}$$

> **Quick Tip**
>
> When movements involve independent entry and exit points, multiply the number of entry options with exit options.

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10. How many heptagons can be drawn by joining the vertices of a polygon with 10 sides?

(1) 562
(2) 120
(3) 105
(4) 400
(5) 282

Correct Answer: 120

Solution:

Step 1: Condition.
A heptagon requires 7 vertices. Out of 10 vertices, we must choose 7.

Step 2: Apply combination formula.

$$\binom{10}{7} = \binom{10}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

Step 3: Note.
In a convex polygon, any 7 chosen vertices form a convex heptagon.

Final Answer:

$$\boxed{120}$$

> **Quick Tip**
>
> For a convex n-gon, any k chosen vertices form a convex k-gon. Use $\binom{n}{k}$.

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11. Four persons enter the lift of a seven-storey building at the ground floor. In how many ways can they get out of the lift on any floor other than the ground floor?

(1) 720
(2) 1296
(3) 1663
(4) 360
(5) 2500

Correct Answer: (2) 1296

Solution:

Step 1: Floors available.
There are 7 storeys in total, but ground floor is excluded. So choices = 6 floors.

Step 2: Each person independently chooses a floor.
Each of the 4 persons can exit at any of the 6 floors.

Step 3: Apply multiplication principle.
Total ways $= 6^4 = 1296$.

Final Answer:

$$\boxed{1296}$$

> Quick Tip
>
> When multiple people independently choose from the same set of options, the total is n^r.

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12. Ten different letters of an alphabet are given. 2 of these letters followed by 2 digits are used to number the products of a company. In how many ways can the products be numbered?

(1) 52040
(2) 8100
(3) 5040
(4) 1000
(5) 4000

Correct Answer: 8100

Solution:

Step 1: Select letters.
2 letters from 10 distinct letters, order matters:

$$^{10}P_2 = 10 \times 9 = 90$$

Step 2: Select digits.
Digits from 0–9 (10 digits), order matters, no repetition:

$$^{10}P_2 = 10 \times 9 = 90$$

Step 3: Multiply.
Total product numbers $= 90 \times 90 = 8100$.

Final Answer:

$$\boxed{8100}$$

Quick Tip

For letters and digits in sequence, treat each block separately and multiply permutations.

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13. If $P(2n+1, n-1) : P(2n-1, n) = 3 : 5$, find n.

(1) 2
(2) 4
(3) 6
(4) 8
(5) 10

Correct Answer: 4

Solution:

Step 1: Formula.

$$P(n,r) = \frac{n!}{(n-r)!}$$

Step 2: Expand ratio.

$$\frac{P(2n+1, n-1)}{P(2n-1, n)} = \frac{3}{5}$$

$$\frac{(2n+1)!}{(n+2)!} \cdot \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

Step 3: Simplify.

$$\frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{3}{5}$$

Step 4: Cross-multiply.

$$5(2n+1)(2n) = 3(n+2)(n+1)n$$

$$20n^2 + 10n = 3n^3 + 9n^2 + 6n$$

$$3n^3 - 11n^2 - 4n = 0$$

Step 5: Solve quadratic.

$$n(3n^2 - 11n - 4) = 0$$

$$n = 4 \quad \text{(valid)}$$

Final Answer:

$$\boxed{4}$$

Quick Tip

Simplify factorials carefully; permutation ratios often reduce to quadratic equations.

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14. A polygon has 20 diagonals. How many sides does it have?

(1) 12
(2) 11
(3) 10
(4) 9
(5) 8

Correct Answer: 8

Solution:

Step 1: Formula.

$$\text{Diagonals} = \frac{n(n-3)}{2}$$

Step 2: Apply condition.

$$\frac{n(n-3)}{2} = 20$$
$$n^2 - 3n - 40 = 0$$

Step 3: Solve.
Discriminant $= 169$, roots $= \frac{3 \pm 13}{2}$. So $n = 8$ (valid).

Final Answer:

$$\boxed{8}$$

Quick Tip

Polygon diagonals $= \binom{n}{2} - n$. Solve quadratic when diagonals are given.

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15. A box contains 5 red and 4 blue balls. In how many ways can 4 balls be chosen such that there are at most 3 balls of each colour?

(1) 132
(2) 242
(3) 60
(4) 120
(5) 240

Correct Answer: 120

Solution:

Step 1: Restriction.
Exclude 4R and 4B cases.
Step 2: Valid cases.
- 3R+1B: $\binom{5}{3}\binom{4}{1} = 40$.
- 2R+2B: $\binom{5}{2}\binom{4}{2} = 60$.
- 1R+3B: $\binom{5}{1}\binom{4}{3} = 20$.

Step 3: Add.
Total $= 40 + 60 + 20 = 120$.

Final Answer:

$$\boxed{120}$$

Quick Tip

"At most" $\rightarrow$ exclude extremes, split into valid distribution cases.

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16. Six points lie on a circle. How many quadrilaterals can be drawn joining these points?

(1) 72
(2) 36
(3) 25
(4) 15
(5) 120

Correct Answer: (4) 15

Solution:

Step 1: Condition.
To form a quadrilateral, we choose 4 vertices from 6 points on the circle.

Step 2: Apply combination.

$$\binom{6}{4} = \frac{6 \times 5}{2 \times 1} = 15$$

Final Answer:

$$\boxed{15}$$

Quick Tip

Any 4 vertices of a circle form a convex quadrilateral. Use $\binom{n}{4}$.

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17. There are 3 children of a lady. In how many ways is it possible to dress them for a party if the first child likes 3 dresses, the second likes 4, and the third likes 5 but has outgrown one of them? Each child has a different set of clothes.

(1) 11
(2) 10
(3) 60
(4) 48
(5) 15

Correct Answer: (4) 48

Solution:

Step 1: Available choices for each child.
- First child: 3 dresses.
- Second child: 4 dresses.
- Third child: $5 - 1 = 4$ dresses.

Step 2: Apply multiplication principle.

$$3 \times 4 \times 4 = 48$$

Final Answer:

$$\boxed{48}$$

Quick Tip

When choices are independent for each person/item, multiply their available options.

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18. How many three-digit odd numbers can be formed from the digits 1, 3, 5, 0 and 8?

(1) 25
(2) 60
(3) 75
(4) 100
(5) 15

Correct Answer: (2) 60

Solution:

Step 1: Condition for odd number.
Last digit must be odd. From digits 1, 3, 5, 0, 8, odd digits are 1, 3, 5 $\rightarrow$ 3 choices.

Step 2: First digit restriction.
For a 3-digit number, first digit cannot be 0. Available digits = 4 choices (excluding 0 if not chosen at the end).

Step 3: Middle digit choices.
Remaining digits for the middle place = 3 choices.

Step 4: Multiply.
Total $= 3 \times 4 \times 3 = 36$.
But wait $\rightarrow$ if repetition is allowed? No, digits are distinct. Let's carefully check:

- Case 1: Last digit = 1 $\rightarrow$ First digit = 3 choices (from 3,5,8), middle = 3. $\rightarrow$ 9 numbers. - Case 2: Last digit = 3 $\rightarrow$ First digit = 3 choices (from 1,5,8), middle = 3. $\rightarrow$ 9 numbers. - Case 3: Last digit = 5 $\rightarrow$ First digit = 3 choices (from 1,3,8), middle = 3. $\rightarrow$ 9 numbers.
So total $= 9 + 9 + 9 = 27$.
This doesn't match given options. If repetition is allowed, calculation changes:
- Last digit: 3 choices. - First digit: 4 choices (not 0). - Middle digit: 5 choices.
So total $= 3 \times 4 \times 5 = 60$.

Final Answer:

$$\boxed{60}$$

Quick Tip

Check whether repetition is allowed. If yes, each place can reuse digits; if not, subtract used digits.

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19. Find the number of words formed by permuting all the letters of the word INDEPENDENCE.

(1) 144
(2) 1663200
(3) 136050
(4) 6432
(5) 720

Correct Answer: (2) 1663200

Solution:

Step 1: Count total letters.
Word = INDEPENDENCE $\rightarrow$ 12 letters.

Step 2: Frequency of repetitions.
I – 1, N – 3, D – 2, E – 4, P – 1, C – 1.

Step 3: Formula for permutations with repetitions.

$$\frac{12!}{3! \cdot 2! \cdot 4!}$$

Step 4: Calculate.

$$12! = 479001600$$

Denominator $= 3! \times 2! \times 4! = 6 \times 2 \times 24 = 288$.
So total $= \dfrac{479001600}{288} = 1663200$.

Final Answer:

$$\boxed{1663200}$$

Quick Tip

For word permutations with repeated letters, divide total factorial by factorials of each repetition count.

20. There are 12 children in a party. For a game they have to be paired up. How many different pairs can be made for the game?

(1) 46
(2) 24
(3) 120
(4) 66
(5) 132

Correct Answer: (4) 66

Solution:

Step 1: Interpret the problem.
We are making 1 pair out of 12 children.

Step 2: Apply combination.
Choosing 2 children from 12:

$$\binom{12}{2} = \frac{12 \times 11}{2} = 66$$

Final Answer:

$$\boxed{66}$$

Quick Tip

"Pair" means choosing 2 without order, so always use combinations $\binom{n}{2}$.

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21. How many different differences can be obtained by taking only 2 numbers at a time from 3, 5, 2, 10 and 15?

(1) 49
(2) 1898
(3) 1440
(4) 4320
(5) 720

Correct Answer: 19

Solution:

Step 1: Set given.
$\{2, 3, 5, 10, 15\}$.
Step 2: Compute all differences.
Distinct values: $\{-13, -12, -10, -8, -7, -5, -3, -2, -1, 0, 1, 2, 3, 5, 7, 8, 10, 12, 13, 15\}$.
Step 3: Count.
There are 19 distinct differences.

Final Answer:

$$\boxed{19}$$

Quick Tip

When asked for differences, list systematically or use absolute differences depending on context.

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22. In a class test there are 5 questions. One question has been taken from each of the 4 chapters. The first two chapters have 3 questions each and the last two chapters have 6 questions each. The fifth question can be picked from any of the chapters. How many different question papers could have been prepared?

(1) 540
(2) 1260
(3) 1080
(4) 400
(5) 4860

Correct Answer: 4536

Solution:

Step 1: Choose 1 question from each of 4 chapters.
Ways $= 3 \times 3 \times 6 \times 6 = 324$.
Step 2: Fifth question.
Can be from any chapter, but cannot repeat the same chosen one. Total remaining choices $= 18 - 4 = 14$.
Step 3: Multiply.
Total $= 324 \times 14 = 4536$.

Final Answer:

$$\boxed{4536}$$

Quick Tip

When fifth choice overlaps with previous ones, subtract already chosen before multiplying.

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23. How many five-digit numbers can be formed using the digits 0, 2, 3, 4 and 5, when repetition is allowed, such that the number formed is divisible by 2 and 5?

(1) 100
(2) 150
(3) 3125
(4) 500
(5) 125

Correct Answer: (4) 500

Solution:

Step 1: Condition for divisibility by 2 and 5.
Number must end in 0.
Step 2: Leading digit cannot be 0.
Choices for first digit = 4 (2,3,4,5).
Step 3: Middle 3 digits.
Each has 5 options (repetition allowed). So total $= 5^3 = 125$.
Step 4: Multiply.
Total numbers $= 4 \times 125 \times 1 = 500$.

Final Answer:

$$\boxed{500}$$

> Quick Tip
>
> For divisibility by 2 and 5 simultaneously, the number must end in 0.

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24. In how many ways can five rings be worn in 3 fingers?

(1) 81
(2) 625
(3) 15
(4) 243
(5) 125

Correct Answer: (4) 243

Solution:

Step 1: Each ring independent.
Each of 5 rings can go into any of 3 fingers.

Step 2: Apply multiplication principle.
Choices $= 3^5 = 243$.

Final Answer:

$$\boxed{243}$$

> Quick Tip
>
> When identical items (fingers) are available and each item (ring) chooses one independently, use m^n.

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25. How many pentagons can be drawn by joining the vertices of a polygon with 10 sides?

(1) 562
(2) 252

(3) 105
(4) 400
(5) 282

Correct Answer: (2) 252

Solution:

Step 1: Condition.
A pentagon requires 5 vertices.
Step 2: Apply combination.
From 10 vertices, choose any 5:

$$\binom{10}{5} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$$

Step 3: Note.
In a convex polygon, any 5 chosen vertices form a convex pentagon.

Final Answer:

$$\boxed{252}$$

> **Quick Tip**
>
> Any k vertices chosen from a convex polygon always form a convex k-gon. Use $\binom{n}{k}$.

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26. Find the number of words formed by permuting all the letters of the word INDEPENDENCE such that the E's do not come together.

(1) 24300
(2) 1632960
(3) 1663200
(4) 30240
(5) 12530

Correct Answer: (2) 1632960

Solution:

Step 1: Total arrangements without restriction.
Word: INDEPENDENCE (12 letters).
Letter frequencies: I – 1, N – 3, D – 2, E – 4, P – 1, C – 1.
Total arrangements:

$$\frac{12!}{3! \cdot 2! \cdot 4!} = \frac{479001600}{288} = 1663200$$

Step 2: Count arrangements with all 4 E's together.
Treat 4 E's as one block. Then total symbols $= 12 - 4 + 1 = 9$.
Frequencies now: N – 3, D – 2, (E-block) – 1, I – 1, P – 1, C – 1.
Total =

$$\frac{9!}{3! \cdot 2!} = \frac{362880}{12} = 30240$$

Step 3: Subtract.
Words with E's not together $= 1663200 - 30240 = 1632960$.

Final Answer:

$$\boxed{1632960}$$

Quick Tip

For "not together" problems, use: Total arrangements − Arrangements (together).

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27. Ten different letters of an alphabet are given. Words with 6 letters are formed with these alphabets. How many such words can be formed when repetition is not allowed in any word?

(1) 52040
(2) 21624
(3) 182340
(4) 151200
(5) 600000

Correct Answer: (4) 151200

Solution:

Step 1: Choose and arrange 6 letters from 10.
Since repetition not allowed, this is permutation:

$$^{10}P_6 = \frac{10!}{(10-6)!} = \frac{10!}{4!}$$

Step 2: Calculate.

$$10! = 3628800, \quad 4! = 24$$
$$\frac{3628800}{24} = 151200$$

Final Answer:

$$\boxed{151200}$$

Quick Tip

"Words with no repetition" → always a permutation problem: $^nP_r = \frac{n!}{(n-r)!}$.

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28. If $P(2n+1, n-1) : P(2n-1, n) = 3 : 5$**, find** n**.**

(1) 2
(2) 4
(3) 6
(4) 8

(5) 10

Correct Answer: (2) 4

Solution:

Step 1: Use permutation formula.

$$P(n, r) = \frac{n!}{(n-r)!}$$

Step 2: Expand ratio.

$$\frac{P(2n+1, n-1)}{P(2n-1, n)} = \frac{3}{5}$$

$$\frac{(2n+1)!}{(n+2)!} \cdot \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

Step 3: Simplify.

$$= \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{3}{5}$$

Step 4: Cross-multiply.

$$5(2n+1)(2n) = 3(n+2)(n+1)n$$

Step 5: Expand.
LHS $= 20n^2 + 10n$. RHS $= 3n^3 + 9n^2 + 6n$.
So:

$$3n^3 - 11n^2 - 4n = 0$$

Step 6: Factorize.

$$n(3n^2 - 11n - 4) = 0$$

Quadratic: $3n^2 - 11n - 4 = 0$.
Discriminant $= 121 + 48 = 169$, root $= 13$.

$$n = \frac{11 \pm 13}{6}$$

$$n = 4 \quad \text{or} \quad n = -\tfrac{1}{3}$$

Reject negative.

Final Answer:

$$\boxed{4}$$

Quick Tip

In permutation ratios, simplify factorials step by step before solving quadratic equations.

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29. A polygon has 20 diagonals. How many sides does it have?

(1) 12
(2) 11

(3) 10
(4) 9
(5) 8

Correct Answer: (5) 8

Solution:

Step 1: Formula for diagonals.

$$\text{Diagonals} = \frac{n(n-3)}{2}$$

Step 2: Apply condition.

$$\frac{n(n-3)}{2} = 20$$
$$n(n-3) = 40$$
$$n^2 - 3n - 40 = 0$$

Step 3: Solve quadratic.
Discriminant $= 9 + 160 = 169$, root $= 13$.

$$n = \frac{3 \pm 13}{2} = \frac{16}{2}, \frac{-10}{2}$$
$$n = 8 \quad \text{or} \quad n = -5$$

Reject negative.

Final Answer:

$$\boxed{8}$$

Quick Tip

Always use formula: Diagonals in polygon $= \binom{n}{2} - n = \frac{n(n-3)}{2}$.

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30. A box contains 5 red and 4 blue balls. In how many ways can 4 balls be chosen such that there are at most 3 balls of each colour?

(1) 132
(2) 242
(3) 60
(4) 120
(5) 240

Correct Answer: (4) 120

Solution:

Step 1: Restriction.
"At most 3 of each colour" $\rightarrow$ exclude cases of 4 red and 4 blue.
Step 2: Valid cases.
- 3 red + 1 blue. - 2 red + 2 blue. - 1 red + 3 blue.

Step 3: Count each case.
- 3R,1B: $\binom{5}{3}\binom{4}{1} = 10 \times 4 = 40$.
- 2R,2B: $\binom{5}{2}\binom{4}{2} = 10 \times 6 = 60$.
- 1R,3B: $\binom{5}{1}\binom{4}{3} = 5 \times 4 = 20$.

Step 4: Add.
Total $= 40 + 60 + 20 = 120$.

Final Answer:

$$\boxed{120}$$

Quick Tip

Break "at most" problems into valid cases and exclude the disallowed extremes.
