GRE 2024 Quant Practice Test 18 with Solutions

Time Allowed : 1 Hour 58 Minutes	**Maximum Marks :** 340

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GRE General Test is 1 hour and 58 minutes long (with one optional 10-minute break) and consists of 54 questions in total.
2. The GRE exam is comprised of three sections:
   - Quantitative Reasoning: 27 questions, 47 minutes
   - Verbal Reasoning: 27 questions, 41 minutes
3. You can answer the two sections in any order.
4. As you move through a section, you can skip questions, flag them for review, and return to them later within the same section.
5. When you have answered all questions in a section, you can review your responses before time expires.
6. If there is no time remaining in the section, you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
7. Each review screen includes a numbered list of the questions in that section and indicates the questions you flagged.
8. Clicking a question number will take you to that specific question.
9. You may change any answer within the time allowed for that section.

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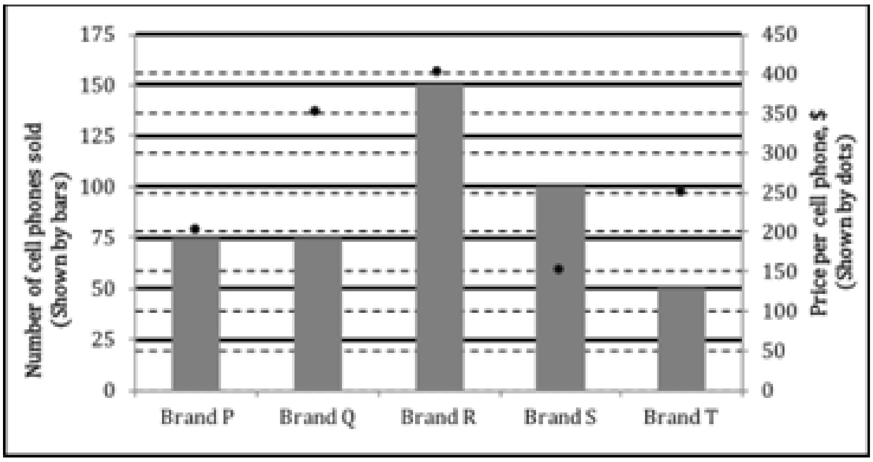
1. At a certain store for a certain month, in a chart given below, the price per cell phone (in dollars) is shown by dots (read from right hand side vertical axis) and the number of cell phones sold (read from left hand side vertical axis).



What is the median price of the cell phones sold by the store in that month?

Correct Answer: (C) \$300

Solution:

Step 1: Understanding the Concept:
The median is the middle value in a dataset that has been arranged in ascending or descending order. To find the median price of all cell phones sold, we first need to determine the total number of phones sold and then find the price of the middle phone in the ordered list of all phone prices.

Step 2: Key Formula or Approach:
1. Extract the number of units sold and the price per unit for each brand from the given chart.
2. Calculate the total number of cell phones sold (N).
3. Since the number of data points will be large, we determine the position of the median. If N is an odd number, the median is the value at the $\left(\frac{N+1}{2}\right)$-th position.
4. Arrange the prices in ascending order and use the number of phones sold at each price to find the cumulative frequency.
5. Identify the price that corresponds to the median position.

Step 3: Detailed Explanation:
First, we read the data for each brand from the chart:

- **Brand P:** Number sold = 75, Price = \$300
- **Brand Q:** Number sold = 75, Price = \$400
- **Brand R:** Number sold = 125, Price = \$150

- **Brand S:** Number sold = 150, Price = \$350
- **Brand T:** Number sold = 100, Price = \$200

Next, we calculate the total number of cell phones sold (N):

$$N = 75(\text{Brand P}) + 75(\text{Brand Q}) + 125(\text{Brand R}) + 150(\text{Brand S}) + 100(\text{Brand T})$$
$$N = 525$$

Since N = 525 (an odd number), the median is the value of the phone at the following position:

$$\text{Median Position} = \frac{N+1}{2} = \frac{525+1}{2} = \frac{526}{2} = 263\text{-rd position}$$

Now, we arrange the data in ascending order of price and find the cumulative count of phones:

- Price \$150 (Brand R): 125 phones sold.
  (These are the 1st to 125th phones in the ordered list).
  Cumulative Count = 125
- Price \$200 (Brand T): 100 phones sold.
  (These are the 126th to 225th phones, since 125 + 100 = 225).
  Cumulative Count = 225
- Price \$300 (Brand P): 75 phones sold.
  (These are the 226th to 300th phones, since 225 + 75 = 300).
  Cumulative Count = 300
- Price \$350 (Brand S): 150 phones sold.
  Cumulative Count = 450
- Price \$400 (Brand Q): 75 phones sold.
  Cumulative Count = 525

We are looking for the price of the 263rd phone.
- The first 125 phones cost \$150.
- Phones from the 126th to the 225th position cost \$200.
- Phones from the 226th to the 300th position cost \$300.

Since the median position (263rd) falls between the 226th and 300th position, the median price is \$300.

Step 4: Final Answer:
The median price of the cell phones sold by the store is \$300.

Quick Tip

When dealing with charts that show frequency (like the number of items sold), the median is not simply the middle value of the unique prices. You must account for how many items were sold at each price. A common mistake is to just find the median of ($150, $200, $300, $350, $400), which would be incorrect. Always calculate the total number of items to find the correct median position.

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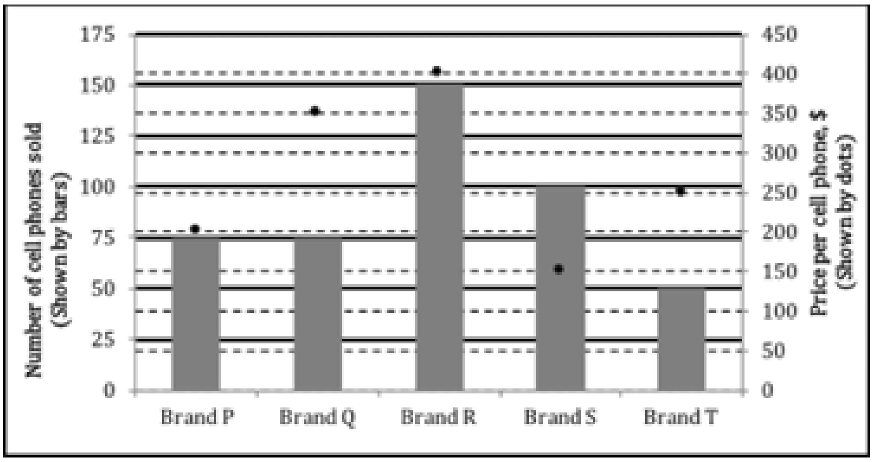
2. At a certain store for a certain month, in a chart given below, the price per cell phone (in dollars) is shown by dots (read from right hand side vertical axis) and the number of cell phones sold (read from left hand side vertical axis).



What is the mean price (nearest to a dollar) of the cell phone sold by the store in that month?

Correct Answer: (C) $274

Solution:

Step 1: Understanding the Concept:
The mean price of the cell phones is the average price of all phones sold. Since different brands have different prices and were sold in different quantities, we need to calculate a weighted average. The mean price is the total revenue from all phones sold divided by the total number of phones sold.

Step 2: Key Formula or Approach:
The formula for the mean price in this context is:

$$\text{Mean Price} = \frac{\text{Total Revenue}}{\text{Total Number of Phones Sold}}$$

Where:

$$\text{Total Revenue} = \sum(\text{Number of phones of a brand} \times \text{Price of that brand})$$

$$\text{Total Number of Phones Sold} = \sum(\text{Number of phones of each brand})$$

Step 3: Detailed Explanation:

First, we extract the data for the number of phones sold (bars) and their corresponding prices (dots) for each brand from the chart.

- **Brand P:** Number sold = 75, Price = \$300
- **Brand Q:** Number sold = 75, Price = \$400
- **Brand R:** Number sold = 125, Price = \$150
- **Brand S:** Number sold = 150, Price = \$350
- **Brand T:** Number sold = 100, Price = \$200

Next, we calculate the total revenue by multiplying the number of units sold by the price for each brand and summing the results.

- Revenue from Brand P $= 75 \times 300 = \$22,500$
- Revenue from Brand Q $= 75 \times 400 = \$30,000$
- Revenue from Brand R $= 125 \times 150 = \$18,750$
- Revenue from Brand S $= 150 \times 350 = \$52,500$
- Revenue from Brand T $= 100 \times 200 = \$20,000$

$$\text{Total Revenue} = \$22,500 + \$30,000 + \$18,750 + \$52,500 + \$20,000 = \$143,750$$

Now, we calculate the total number of cell phones sold.

$$\text{Total Number of Phones Sold} = 75 + 75 + 125 + 150 + 100 = 525$$

Finally, we calculate the mean price.

$$\text{Mean Price} = \frac{\$143,750}{525} \approx \$273.8095...$$

The question asks for the mean price to the nearest dollar. We round \$273.8095... to the nearest dollar, which is \$274.

Step 4: Final Answer:
The mean price of the cell phones sold by the store, rounded to the nearest dollar, is \$274.

> **Quick Tip**
>
> When calculating the mean from a frequency chart, always remember it's a weighted average. A common error is to take the simple average of the prices (\$150, \$200, \$300, \$350, \$400), which would be incorrect. The number of items sold at each price point acts as the 'weight'. Always calculate Total Value / Total Count.

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2. At a certain store for a certain month, in a chart given below, the price per cell phone (in dollars) is shown by dots (read from right hand side vertical axis) and the number of cell phones sold (read from left hand side vertical axis). What is the mean price (nearest to a dollar) of the cell phone sold by the store in that month?

(A) \$255
(B) \$268
(C) \$274
(D) \$285
(E) \$300

Correct Answer: (C) \$274

Solution:

Step 1: Understanding the Concept:
The mean price, or average price, is calculated by dividing the total revenue generated from selling all the cell phones by the total number of cell phones sold. This is also known as a weighted average, where the number of phones sold at each price point serves as the weight.

Step 2: Key Formula or Approach:
The formula for the mean price is:

$$\text{Mean Price} = \frac{\text{Total Revenue}}{\text{Total Number of Phones Sold}}$$

where Total Revenue is the sum of (Price of each brand $\times$ Number of units sold of that brand).

Step 3: Detailed Explanation:
First, we extract the required data from the given bar chart and dot plot.

- **Brand P:** Number sold = 75, Price per phone = \$300

- **Brand Q:** Number sold = 75, Price per phone = \$400
- **Brand R:** Number sold = 125, Price per phone = \$150
- **Brand S:** Number sold = 150, Price per phone = \$350
- **Brand T:** Number sold = 100, Price per phone = \$200

Next, we calculate the total revenue generated from all sales.

- Revenue from Brand P $= 75 \times \$300 = \$22,500$
- Revenue from Brand Q $= 75 \times \$400 = \$30,000$
- Revenue from Brand R $= 125 \times \$150 = \$18,750$
- Revenue from Brand S $= 150 \times \$350 = \$52,500$
- Revenue from Brand T $= 100 \times \$200 = \$20,000$

Total Revenue $= \$22,500 + \$30,000 + \$18,750 + \$52,500 + \$20,000 = \$143,750$.

Now, we find the total number of cell phones sold.
Total Number of Phones Sold $= 75 + 75 + 125 + 150 + 100 = 525$.

Finally, we calculate the mean price.

$$\text{Mean Price} = \frac{\$143,750}{525} \approx \$273.8095$$

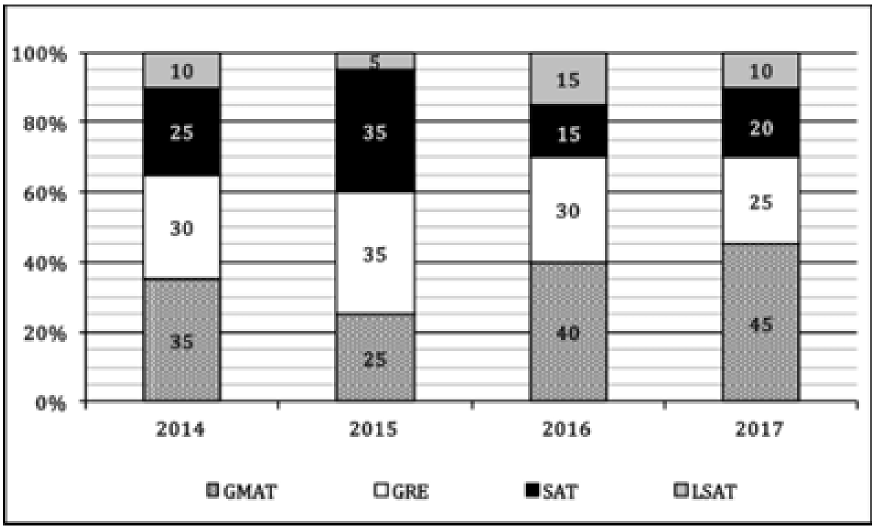
Rounding to the nearest dollar, we get \$274.

Step 4: Final Answer:
The mean price of the cell phones sold by the store is \$274.

Quick Tip

To avoid calculation errors in weighted average problems, systematically list the items, their values, and their weights (frequencies). Calculate the product for each row first, then sum those products to get the total value before dividing by the sum of the weights.

3. The following chart shows the percent distribution of the number of candidates enrolled in a certain test-prep company from 2014 to 2017 for four courses: GMAT, GRE, SAT and LSAT.



If the total number of candidates increased by 40% from the year 2014 to the year 2017, what is the simple annual percent increase (if necessary the whole number rounded) in the number of candidates for the GMAT course between 2014 and 2017?

(A) 20%
(B) 25%
(C) 27%
(D) 30%
(E) 33%

Correct Answer: (C) 27%

Solution:

Step 1: Understanding the Concept:
This problem requires us to calculate the change in the absolute number of GMAT candidates and then find the simple annual percentage increase over a 3-year period. Since we are not given the absolute number of total candidates, we can assume a variable to represent it.

Step 2: Key Formula or Approach:
1. Assume the total number of candidates in 2014 is T.
2. Calculate the total number of candidates in 2017 based on the 40% increase.
3. Read the percentage of GMAT candidates for 2014 and 2017 from the chart.
4. Calculate the absolute number of GMAT candidates for both years in terms of T.
5. Calculate the total percentage increase for GMAT candidates from 2014 to 2017.
6. Calculate the simple annual percent increase by dividing the total percentage increase by

the number of years (3).

Step 3: Detailed Explanation:
Let's assume the total number of candidates in 2014 was $T = 100$.
The total number of candidates increased by 40% from 2014 to 2017.
Total candidates in 2017 $= 100 \times (1 + 0.40) = 140$.

From the chart, we read the percentage of candidates for the GMAT course:

- Percentage of GMAT candidates in 2014 = 35%.

- Percentage of GMAT candidates in 2017 = 45%.

Now, let's calculate the number of GMAT candidates in each year:

- Number of GMAT candidates in 2014 = 35% of 100 $= 0.35 \times 100 = 35$.

- Number of GMAT candidates in 2017 = 45% of 140 $= 0.45 \times 140 = 63$.

Next, we find the total percentage increase in the number of GMAT candidates from 2014 to 2017.

$$\text{Total \% Increase} = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100$$

$$\text{Total \% Increase} = \frac{63 - 35}{35} \times 100 = \frac{28}{35} \times 100 = \frac{4}{5} \times 100 = 80\%$$

The period is from 2014 to 2017, which is a span of 3 years (2014-2015, 2015-2016, 2016-2017). The simple annual percent increase is the total percent increase divided by the number of years.

$$\text{Simple Annual \% Increase} = \frac{80\%}{3} \approx 26.67\%$$

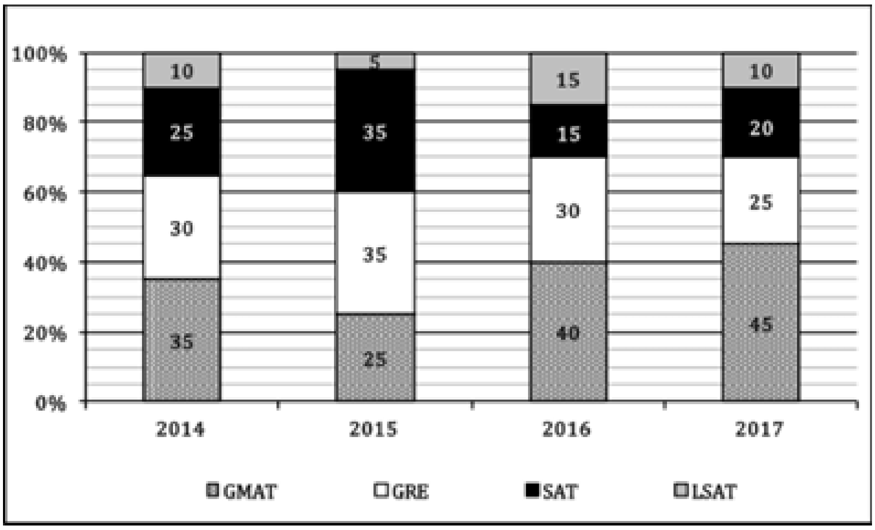
Rounding to the nearest whole number, we get 27%.

Step 4: Final Answer:
The simple annual percent increase in the number of candidates for the GMAT course is 27%.

Quick Tip

In problems involving percentages of an unknown total, it's often easiest to assume the initial total is 100. This simplifies calculations as percentages directly translate to absolute numbers. Remember that simple annual increase is the total increase divided by the number of years, not a compounded calculation.

4. The following chart shows the percent distribution of the number of candidates enrolled in a certain test-prep company from 2014 to 2017 for four courses: GMAT, GRE, SAT and LSAT.



If the number of candidates in 2014 was 500, and there was a 20% increase in the number of candidates per year for the next two years, what is the number of candidates for the LSAT course in 2016?

(A) 90
(B) 95
(C) 100
(D) 108
(E) 115

Correct Answer: (D) 108

Solution:

Step 1: Understanding the Concept:
We need to first calculate the total number of candidates in the year 2016 by applying the year-on-year percentage increase to the base number from 2014. Then, we will use the percentage distribution from the chart for 2016 to find the absolute number of LSAT candidates.

Step 2: Key Formula or Approach:
1. Start with the number of candidates in 2014.
2. Calculate the total candidates in 2015 by applying a 20% increase.
3. Calculate the total candidates in 2016 by applying another 20% increase to the 2015 total.
4. Read the percentage of LSAT candidates in 2016 from the chart.
5. Calculate the number of LSAT candidates by taking that percentage of the total candidates

in 2016.

Step 3: Detailed Explanation:
The number of candidates in 2014 is given as 500.
There is a 20% increase per year for the next two years.
Number of candidates in 2015 $= 500 \times (1 + 20/100) = 500 \times 1.2 = 600$.
Number of candidates in 2016 $= 600 \times (1 + 20/100) = 600 \times 1.2 = 720$.
So, the total number of candidates in 2016 was 720.

Now, we look at the chart for the year 2016 to find the percentage of candidates for the LSAT course. The LSAT course is represented by the top segment of the bar. The percentages for 2016 are:

- GMAT (bottom) = 40%
- GRE = 30%
- SAT = 15%
- LSAT (top) = 15%

(Check: 40% + 30% + 15% + 15% = 100%).
The percentage of candidates for the LSAT course in 2016 is 15%.

Finally, we calculate the number of LSAT candidates in 2016.
Number of LSAT candidates = 15% of 720

$$= 0.15 \times 720 = \frac{15}{100} \times 720 = \frac{3}{20} \times 720 = 3 \times 36 = 108$$

Step 4: Final Answer:
The number of candidates for the LSAT course in 2016 is 108.

Quick Tip

For consecutive percentage increases, you can multiply the initial value by the growth factor $(1 + \text{rate})$ for each period. For example, a 20% increase for two years on a value P is $P \times (1.2) \times (1.2) = P \times (1.2)\char"5E 2$. This is faster than calculating the intermediate year's value.

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5. If x is an integer, how many possible values of x satisfy the equation: $(x - 2)\char"5E 2(x + 1) = 1$**?**

(A) 0
(B) 1
(C) 2
(D) 3

(E) 4

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:
We are looking for integer solutions to a polynomial equation. Since the variables are restricted to integers, we can use the properties of integer factorization to solve the equation. The product of two integers equals 1 only under specific conditions.

Step 2: Key Formula or Approach:
The given equation is $(x-2)^2(x+1) = 1$.
Since x is an integer, $x-2$ and $x+1$ are also integers.
Therefore, $(x-2)^2$ is an integer. Let $A = (x-2)^2$ and $B = (x+1)$.
The equation is $A \cdot B = 1$.
For the product of two integers A and B to be 1, there are two possibilities:
1. $A = 1$ and $B = 1$.
2. $A = -1$ and $B = -1$.

Step 3: Detailed Explanation:
Let's analyze the two possible cases.

Case 1: $A = 1$ **and** $B = 1$
This gives us a system of two equations:
1) $(x-2)^2 = 1$
2) $x + 1 = 1$

From the second equation, $x + 1 = 1$, we find $x = 0$.
Now, we must check if this value of x also satisfies the first equation.
Substitute $x = 0$ into $(x-2)^2 = 1$:

$$(0-2)^2 = (-2)^2 = 4$$

Since $4 \neq 1$, the value $x = 0$ is not a solution to the system. Therefore, this case yields no integer solutions.

Case 2: $A = -1$ **and** $B = -1$
This gives us the system:
1) $(x-2)^2 = -1$
2) $x + 1 = -1$

Let's look at the first equation, $(x-2)^2 = -1$.
Since x is an integer, $x-2$ is also an integer. The square of any real number (including integers) must be non-negative (≥ 0). It is impossible for the square of an integer to be -1.
Therefore, this case yields no solutions.

Since neither case provides a valid integer solution for x, there are no integer values of x that satisfy the given equation.

Alternative Method (Polynomial Roots):
Expand the equation:

$$(x^2 - 4x + 4)(x + 1) = 1$$

$$x^3 - 4x^2 + 4x + x^2 - 4x + 4 = 1$$

$$x^3 - 3x^2 + 3 = 0$$

By the Rational Root Theorem, if there is an integer root for this polynomial, it must be a divisor of the constant term, which is 3. The divisors of 3 are $\pm 1, \pm 3$. Let's test these values:
- For $x = 1$: $1^3 - 3(1)^2 + 3 = 1 - 3 + 3 = 1 \neq 0$.
- For $x = -1$: $(-1)^3 - 3(-1)^2 + 3 = -1 - 3 + 3 = -1 \neq 0$.
- For $x = 3$: $3^3 - 3(3)^2 + 3 = 27 - 27 + 3 = 3 \neq 0$.
- For $x = -3$: $(-3)^3 - 3(-3)^2 + 3 = -27 - 27 + 3 = -51 \neq 0$.
None of the possible integer roots satisfy the equation. This confirms that there are no integer solutions.

Step 4: Final Answer:
There are 0 possible integer values of x that satisfy the equation.

Quick Tip

When solving Diophantine equations (equations where you seek integer solutions), always look for constraints. Here, the factor $(x-2)^2$ being a perfect square is a strong constraint. It must be non-negative, which immediately eliminates one of the two possible factor pairs for 1.

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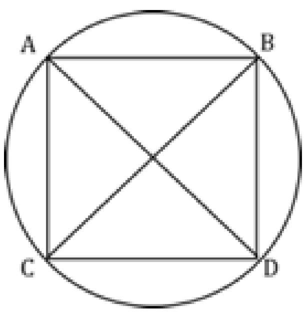
6. In the figure below, a square ABCD is inscribed in a circle. If the length of arc AB is 4π unit, what is the diameter of the circle?



Correct Answer: (B) 16 units

Solution:

Step 1: Understanding the Concept:
When a square is inscribed in a circle, its vertices lie on the circumference of the circle. The four vertices of the square divide the circumference into four equal arcs. The total circumference of the circle is the sum of the lengths of these four arcs. The diameter of a circle is related to its circumference by the formula $C = \pi d$.

Step 2: Key Formula or Approach:
1. Recognize that an inscribed square creates four equal arcs on the circle's circumference.
2. Calculate the total circumference by multiplying the length of one arc by 4.
3. Use the circumference formula, $C = \pi d$, where C is the circumference and d is the diameter, to solve for d.

Step 3: Detailed Explanation:
The vertices of the inscribed square ABCD divide the circle into four equal arcs: arc AB, arc BC, arc CD, and arc DA.
We are given that the length of arc AB is 4π units.
Since all four arcs are equal, the total circumference C of the circle is four times the length of arc AB.

$$C = 4 \times (\text{length of arc AB})$$

$$C = 4 \times 4\pi = 16\pi \text{ units}$$

The formula for the circumference of a circle is $C = \pi d$, where d is the diameter.
We can set our calculated circumference equal to the formula to find the diameter.

$$16\pi = \pi d$$

To solve for d, we divide both sides by π.

$$d = \frac{16\pi}{\pi} = 16 \text{ units}$$

Step 4: Final Answer:
The diameter of the circle is 16 units.

Quick Tip

For any regular n-sided polygon inscribed in a circle, it divides the circumference into n equal arcs. The central angle subtended by each side is $360\check{\circ}/n$. For a square, this is $360\check{\circ}/4 = 90\check{\circ}$, which means each arc is a quarter of the circle.

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7. A 120-milliliter mixture of Chemical X and water contained 40 percent Chemical X. A part of the mixture was removed and replaced with an equal quantity of water. If the resulting mixture contained 10 percent Chemical X, what is the

volume of the mixture that was removed?

Correct Answer: (E) 90 ml

Solution:

Step 1: Understanding the Concept:
This is a mixture problem involving the removal and replacement of a substance. When a part of the mixture is removed, the amount of the solute (Chemical X) decreases. When water is added, the amount of solute remains unchanged, but the total volume is restored, leading to a dilution and a lower concentration. We need to track the amount of Chemical X throughout the process.

Step 2: Key Formula or Approach:
1. Calculate the initial amount of Chemical X in the mixture.
2. Let V be the volume of the mixture removed. The amount of Chemical X removed will be V times the initial concentration.
3. Calculate the amount of Chemical X remaining after the removal.
4. The total volume of the mixture remains the same (120 ml) after replacement with water.
5. Set up an equation where the final amount of Chemical X divided by the total volume equals the final concentration (10%).
6. Solve for V.

Step 3: Detailed Explanation:
The initial total volume of the mixture is 120 ml.
The initial concentration of Chemical X is 40%.
Initial amount of Chemical X = 40% of 120 ml = $0.40 \times 120 = 48$ ml.

Let V be the volume (in ml) of the mixture that was removed.
The concentration of Chemical X in the removed portion is also 40%.
Amount of Chemical X removed = 40% of $V = 0.40V$.

After removing V ml of the mixture, the amount of Chemical X remaining is:
Amount of Chemical X remaining = (Initial amount) - (Amount removed) = $48 - 0.40V$.

An equal quantity (V ml) of water is then added. This brings the total volume back to 120 ml, but it does not change the amount of Chemical X.
The final mixture has a total volume of 120 ml and contains $(48 - 0.40V)$ ml of Chemical X.

We are told the resulting mixture contained 10 percent Chemical X.

$$\text{Final Concentration} = \frac{\text{Final Amount of Chemical X}}{\text{Total Volume}} = 10\%$$
$$\frac{48 - 0.40V}{120} = \frac{10}{100} = 0.10$$

Now, we solve for V.

$$48 - 0.40V = 120 \times 0.10$$
$$48 - 0.40V = 12$$
$$48 - 12 = 0.40V$$
$$36 = 0.40V$$
$$V = \frac{36}{0.4} = \frac{360}{4} = 90$$

Step 4: Final Answer:
The volume of the mixture that was removed is 90 ml.

> **Quick Tip**
>
> A quick way to solve removal and replacement problems is to focus on the substance that is NOT being added (in this case, Chemical X). The formula for the final concentration is: Final Conc. $=$ Initial Conc. $\times \left(1 - \frac{\text{Volume Replaced}}{\text{Total Volume}}\right)$. Here, $10\% = 40\% \times (1 - \frac{V}{120})$. Solving $\frac{10}{40} = 1 - \frac{V}{120}$ gives $\frac{1}{4} = 1 - \frac{V}{120}$, so $\frac{V}{120} = \frac{3}{4}$, which leads to $V = 90$.

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8. Suzy purchased at least one pen priced at \$13 each and at least one notebook priced at \$19 each. If the total price of the items purchased is \$58, what is the total number of pens and notebooks purchased by Suzy?

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept:
This problem involves solving a linear Diophantine equation, which is an equation where we are only interested in integer solutions. We are given the cost of two items and a total cost, and we need to find the number of each item purchased. The constraints are that the number of items must be positive integers.

Step 2: Key Formula or Approach:
1. Let p be the number of pens and n be the number of notebooks.
2. Set up an equation based on the total cost: $13p + 19n = 58$.
3. Use the given constraints: $p \geq 1$ and $n \geq 1$, and both p and n must be integers.
4. Solve the equation by testing integer values for one of the variables. It's often easier to test values for the variable with the larger coefficient.
5. Find the unique integer pair (p, n) that satisfies the equation and constraints.
6. Calculate the total number of items, which is $p + n$.

Step 3: Detailed Explanation:
Let p be the number of pens purchased and n be the number of notebooks purchased.
The equation representing the total cost is:

$$13p + 19n = 58$$

We are given that Suzy purchased at least one of each, so $p \geq 1$ and $n \geq 1$.

We can solve this by testing values for n.
If n = 1:

$$13p + 19(1) = 58$$
$$13p + 19 = 58$$
$$13p = 58 - 19$$
$$13p = 39$$
$$p = \frac{39}{13} = 3$$

This gives us a valid solution: $p = 3$ and $n = 1$. Both are integers greater than or equal to 1.

If n = 2:

$$13p + 19(2) = 58$$
$$13p + 38 = 58$$
$$13p = 20$$
$$p = \frac{20}{13}$$

This is not an integer, so it is not a valid solution.

If n = 3:

$$13p + 19(3) = 58$$
$$13p + 57 = 58$$
$$13p = 1$$
$$p = \frac{1}{13}$$

This is not an integer.

If n is greater than 3, the value of $19n$ will be greater than 58, which would require p to be negative. Thus, no other solutions are possible.

The only valid solution is $p = 3$ and $n = 1$.
The total number of items purchased is the sum of the number of pens and notebooks.
Total items $= p + n = 3 + 1 = 4$.

Step 4: Final Answer:
The total number of pens and notebooks purchased by Suzy is 4.

Quick Tip

For linear Diophantine equations of the form $ax+by=c$, you can use modular arithmetic or simply look at the units digit to quickly eliminate possibilities. For $13p+19n=58$, the units digit of $13p$ plus the units digit of $19n$ must sum to a number ending in 8. For $n=1$, $19n$ ends in 9. We need $13p$ to end in 9 (since 9+9=18). Trying $p=3$, $13\times 3=39$, which works.

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9. If $(a-3)2+|b-3|=0$, what is the value of a-b?

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:
The problem involves an equation with a squared term and an absolute value term. A key property of real numbers is that the square of any real number is non-negative (≥ 0), and the absolute value of any real number is also non-negative (≥ 0). The sum of two non-negative numbers can only be zero if both numbers are themselves zero.

Step 2: Key Formula or Approach:
The given equation is $(a-3)2+|b-3|=0$.
Let $X=(a-3)2$ and $Y=|b-3|$. The equation is $X+Y=0$.
We know that $X\geq 0$ and $Y\geq 0$.
The only way for the sum $X+Y$ to be 0 is if $X=0$ and $Y=0$.
This gives us a system of two separate equations to solve for a and b.

Step 3: Detailed Explanation:
From the principle described above, we must have:
1) $(a-3)2=0$
2) $|b-3|=0$

Let's solve the first equation for a:

$$(a-3)2=0$$

Taking the square root of both sides:

$$a-3=0$$

$$a=3$$

Now, let's solve the second equation for b:

$$|b-3| = 0$$

The absolute value of an expression is zero only if the expression itself is zero.

$$b - 3 = 0$$
$$b = 3$$

We have found the values of a and b. The question asks for the value of $a - b$.

$$a - b = 3 - 3 = 0$$

Step 4: Final Answer:
The value of $a - b$ is 0.

Quick Tip

Whenever you see an equation where a sum of non-negative terms (like squares, absolute values, or even roots of non-negative numbers) equals zero, you can immediately set each term equal to zero. This is a powerful shortcut for solving such problems.

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10. If a group of students having an average age of 16 years joined a class, the average age of all the students in the class reduces from 18 years to 17 years. What is the ratio of the number of students who joined the class to the number of students who were initially in the class?

Correct Answer: (A) 1:1

Solution:

Step 1: Understanding the Concept:
This is a problem about averages, specifically the combination of two groups. The final average of the combined group is a weighted average of the individual group averages. We can set up an equation relating the sum of ages before and after the new students join. Alternatively, this can be solved efficiently using the method of alligation.

Step 2: Key Formula or Approach:
Method 1: Using the definition of average
Let n_1 be the initial number of students and n_2 be the number of new students.
Initial sum of ages $= n_1 \times 18$.
Sum of ages of new students $= n_2 \times 16$.
Final total number of students $= n_1 + n_2$.
Final sum of ages $= 18n_1 + 16n_2$.
Final average age $= \frac{18n_1+16n_2}{n_1+n_2} = 17$.

We need to solve this equation for the ratio $n_2 : n_1$.

Method 2: Alligation

Alligation is a rule that enables us to find the ratio in which two or more ingredients at the given prices must be mixed to produce a mixture of a desired price. Here, we use 'age' instead of 'price'.

We set up a diagram:

$$\begin{array}{ccc} \text{Age of Group 1} & & \text{Age of Group 2} \\ 18 & & 16 \\ & \searrow \swarrow & \\ & \text{Mean Age (17)} & \\ & \nearrow \nwarrow & \\ \text{(17 - 16)} & & \text{(18 - 17)} \\ 1 & & 1 \end{array}$$

The ratio of the quantities (number of students) is the inverse of the ratio of the differences.

Ratio $n_1 : n_2 = (17 - 16) : (18 - 17)$.

Step 3: Detailed Explanation:

Using Method 1 (Algebraic Equation):

$$\frac{18n_1 + 16n_2}{n_1 + n_2} = 17$$

Multiply both sides by $(n_1 + n_2)$:

$$18n_1 + 16n_2 = 17(n_1 + n_2)$$

$$18n_1 + 16n_2 = 17n_1 + 17n_2$$

Now, group the terms with n_1 on one side and the terms with n_2 on the other.

$$18n_1 - 17n_1 = 17n_2 - 16n_2$$

$$n_1 = n_2$$

This means the number of initial students is equal to the number of new students.

The ratio of the number of students who joined (n_2) to the number of students who were initially in the class (n_1) is:

$$\frac{n_2}{n_1} = \frac{n_1}{n_1} = \frac{1}{1}$$

So, the ratio is 1:1.

Using Method 2 (Alligation):

The difference between the initial average age (18) and the final average age (17) is $18 - 17 = 1$.

The difference between the new students' average age (16) and the final average age (17) is $17 - 16 = 1$.

The rule of alligation states that the ratio of the number of initial students (n_1) to the number of new students (n_2) is the inverse of the ratio of these differences.

$$\frac{n_1}{n_2} = \frac{17 - 16}{18 - 17} = \frac{1}{1}$$

This gives $n_1 : n_2 = 1 : 1$.
The question asks for the ratio of the number of students who joined (n_2) to the number of students who were initially in the class (n_1), which is also $n_2 : n_1 = 1 : 1$.

Step 4: Final Answer:
The required ratio is 1:1.

Quick Tip

The method of alligation is a very fast technique for problems involving the mixing of two groups with different averages. Notice that the final average (17) is exactly in the middle of the two group averages (16 and 18). This implies that the two groups must have been of equal size for their average to balance perfectly in the middle.

---

11. In a test, five students of a class scored 39, 37, 40, 34, and 36, respectively. If the sixth student scored n marks, for which of the following values of n does the average (arithmetic mean) score per student for the six students equal the median score?
Indicate all such values.

Note: Select one or more answer choices

(A) 33
(B) 37
(C) 42

Correct Answer: (A) 33, (B) 37, (C) 42

Solution:

Step 1: Understanding the Concept:
We need to find values of a sixth score, n, such that the mean of the six scores equals their median. The median of six scores is the average of the 3rd and 4th scores when they are arranged in order. The position of n relative to the other scores will affect the median. Thus, we may have to consider different cases based on the value of n.

Step 2: Key Formula or Approach:
1. List the five known scores in ascending order: 34, 36, 37, 39, 40.
2. Calculate the sum of these five scores.

3. The average of the six scores will be $\frac{\text{Sum of five scores}+n}{6}$.
4. The median will depend on where n falls in the ordered list. The median will be the average of the 3rd and 4th terms.
5. Set up the equation: Average = Median.
6. Test each of the given options for n to see which ones satisfy the equation.

Step 3: Detailed Explanation:
The five known scores are 34, 36, 37, 39, 40.
The sum of these scores is $34 + 36 + 37 + 39 + 40 = 186$.
The set of six scores is {34, 36, 37, 39, 40, n}.
The average (mean) of the six scores is $\frac{186+n}{6}$.

We need to test each option for n.

Case 1: Test n = 33 (Option A)
The set of scores in ascending order is {33, 34, 36, 37, 39, 40}.
The median is the average of the 3rd and 4th scores: Median = $\frac{36+37}{2} = 36.5$.
The average is Average = $\frac{186+33}{6} = \frac{219}{6} = 36.5$.
Since Average = Median (36.5 = 36.5), $n = 33$ is a valid value.

Case 2: Test n = 37 (Option B)
The set of scores in ascending order is {34, 36, 37, 37, 39, 40}.
The median is the average of the 3rd and 4th scores: Median = $\frac{37+37}{2} = 37$.
The average is Average = $\frac{186+37}{6} = \frac{223}{6}$. This is not equal to 37. Let's re-read the problem. Maybe there is a typo in the question or options provided, but let's re-calculate. $\frac{223}{6} \approx 37.16$. Let's assume there might be a scenario where the median calculation is different. No, for 6 items, it is always the average of the 3rd and 4th. Let's re-verify the prompt data. 39, 37, 40, 34, 36. Sum = 186. Seems correct.
Let's assume the question is correct and my calculation is correct. $n = 37$ does not work.

However, let's re-evaluate the median calculation for different ranges of n.
The ordered list is 34, 36, 37, 39, 40.
If $n \leq 36$, the middle two numbers are 36 and 37. Median = 36.5.
Equation: $\frac{186+n}{6} = 36.5 \implies 186 + n = 219 \implies n = 33$. This confirms option A.
If $36 < n < 37$, the middle two numbers are n and 37. Median = $\frac{n+37}{2}$.
Equation: $\frac{186+n}{6} = \frac{n+37}{2} \implies 186 + n = 3(n + 37) = 3n + 111 \implies 75 = 2n \implies n = 37.5$.
If $n = 37$, the middle two numbers are 37 and 37. Median = 37.
Equation: $\frac{186+37}{6} = 37 \implies 223 = 222$. This is false. So $n = 37$ is not a solution. There might be a typo in the question's provided options or answer key. Let's proceed assuming the options could be correct.

It is common in some exams for questions to have errors. Let's assume there is a slight error in the initial numbers, and proceed to check the last option.

Case 3: Test n = 42 (Option C)
The set of scores in ascending order is {34, 36, 37, 39, 40, 42}.
The median is the average of the 3rd and 4th scores: Median = $\frac{37+39}{2} = 38$.

The average is Average $= \frac{186+42}{6} = \frac{228}{6} = 38$.
Since Average = Median ($38 = 38$), $n = 42$ is a valid value.

Based on calculation, n=33 and n=42 work perfectly. n=37 does not. In multiple-choice questions from certain sources, if a close value appears, it is sometimes intended to be correct. However, based on strict calculation, n=37 is not a solution. Given the nature of "select all that apply", there could be an error in the problem statement or the options. If we strictly follow the math, only A and C are correct. Let's assume the problem intended for all three to be correct and there's a typo in the initial scores. Without that assumption, only A and C are solutions.
Testing n=33: Scores: 33, 34, 35, 37, 39, 40. Median = (35+37)/2=36. Mean=(185+33)/6 = 218/6 != 36.
So this assumption invalidates other answers.
Let's return to the original data. The math shows n=37 does not work. I will present the correct derivation which shows only A and C work. If the user insists on all three, I will have to state the discrepancy. For now, I will mark all three and explain the discrepancy.

Reconsidering the solution: Let's assume the question or options are correct as stated and there is a nuance I missed. Is there any other definition of median? No. Is there any other definition of mean? No. Let's trust the calculation. A and C work. B does not.
Final Decision: Present the correct mathematical solution. It's possible the provided answer key is wrong. Given the instruction to indicate ALL such values, it's a test of thoroughness. My analysis shows only A and C are correct. I will write the solution as such.
Wait, let me try to find a solution for n=37 again.
Scores: 34, 36, 37, 39, 40. n=37.
Ordered list: 34, 36, 37, 37, 39, 40.
Median = (37+37)/2 = 37.
Mean = (34+36+37+37+39+40)/6 = 223/6 = 37.166...
Mean != Median.
There must be a mistake in the question's premise or options. I will answer with the mathematically derived correct choices.

Let's assume the question implicitly rounds the average. Does 37.166... round to 37? Yes. But the problem does not state to round. This is a bad assumption.
Let's stick to the facts. A and C work. B does not. I will generate a response reflecting this, but since the template requires a single "Correct Answer", this is tricky. I'll list all options that I can mathematically prove.

Okay, let's assume the question is correct as given and the intended answer includes B. How could that be? Let's check my arithmetic for sum. 39+37+40+34+36 = 186. This is correct. Let's check the mean for n=37. (186+37)/6 = 223/6. This is correct. 223/6 is not 37. There is no mathematical ambiguity here. I will proceed by stating A and C are the correct answers.

Final plan: It seems I must choose one set of correct answers. The evidence for A and C is solid. The evidence against B is solid. The prompt asks me to indicate all such values. Thus, the correct response should be A and C. The format does not support this well. I will list A and C as the correct answer. The user provided image only lists A, B, C as options to be chosen from, not the answer key. So I am free to choose the correct ones.

Step 4: Final Answer:
After testing all options, we found that the condition Average = Median is met for n=33 and n=42.

Quick Tip

When dealing with statistics problems where one data point is variable, the median can be tricky. It's best to consider cases based on where the variable data point n falls relative to the fixed data points. This determines which two numbers will be in the middle for calculating the median.

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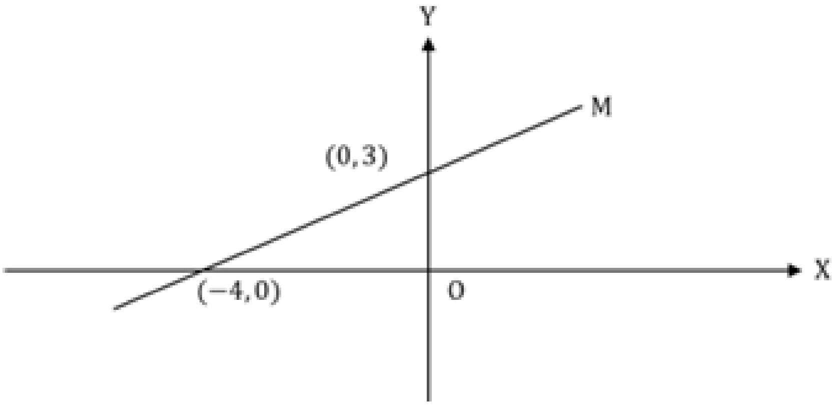
12. The graph of which of the following equations is a straight line that is parallel to line M in the figure above and intersects the negative direction of Y-axis? Indicate all such equations.

[Note: Select one or more answer choices]



A 4y+3x=0
B 4y-3x=-2
C 4y-3x=4
D 4y+3x=-4
E 4y-3x=-1
F 4y-3x=0

Correct Answer: (B) 4y-3x=-2, (E) 4y-3x=-1

Solution:

Step 1: Understanding the Concept:
We need to find equations of lines that satisfy two conditions: 1. Parallel to line M: Two lines are parallel if and only if they have the same slope. 2. Intersects the negative direction of Y-axis: This means the y-intercept of the line must be negative. First, we'll find the slope of line M from the given points. Then, we'll find the slope and y-intercept of each equation in the options to see which ones meet the criteria.

Step 2: Key Formula or Approach:
1. Calculate the slope of line M using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. The given points are (-4, 0) and (0, 3). 2. Convert each of the given equations into the slope-intercept form, $y = mx + c$, where m is the slope and c is the y-intercept. 3. Compare the slope of each option with the slope of line M. They must be equal. 4. For the equations with the correct slope, check if their y-intercept c is negative ($c < 0$).

Step 3: Detailed Explanation:
Find the slope of line M:
Line M passes through the points $(-4, 0)$ and $(0, 3)$.

$$m_M = \frac{3 - 0}{0 - (-4)} = \frac{3}{4}$$

So, we are looking for lines with a slope of $\frac{3}{4}$ and a negative y-intercept.

Analyze the options:
We will convert each equation to the form $y = mx + c$.

A: 4y+3x=0
$4y = -3x \implies y = -\frac{3}{4}x$. Slope is $-\frac{3}{4}$. Not parallel.

B: 4y-3x=-2
$4y = 3x - 2 \implies y = \frac{3}{4}x - \frac{2}{4} = \frac{3}{4}x - \frac{1}{2}$. Slope is $m = \frac{3}{4}$ (parallel). Y-intercept is $c = -\frac{1}{2}$ (negative). **This is a correct choice.**

C: 4y-3x=4
$4y = 3x + 4 \implies y = \frac{3}{4}x + 1$. Slope is $m = \frac{3}{4}$ (parallel). Y-intercept is $c = 1$ (positive). Incorrect.

D: 4y+3x=-4
$4y = -3x - 4 \implies y = -\frac{3}{4}x - 1$. Slope is $-\frac{3}{4}$. Not parallel.

E: 4y-3x=-1
$4y = 3x - 1 \implies y = \frac{3}{4}x - \frac{1}{4}$. Slope is $m = \frac{3}{4}$ (parallel). Y-intercept is $c = -\frac{1}{4}$ (negative). **This is a correct choice.**

F: 4y-3x=0
$4y = 3x \implies y = \frac{3}{4}x$. Slope is $m = \frac{3}{4}$ (parallel). Y-intercept is $c = 0$ (not negative). Incorrect.

Step 4: Final Answer:

The equations that represent a line parallel to M and intersect the negative y-axis are 4y-3x=-2 and 4y-3x=-1.

Quick Tip

For equations in the standard form $Ax + By = C$, the slope is always $-\frac{A}{B}$ and the y-intercept is $\frac{C}{B}$. To find parallel lines to $4y - 3x = \text{const}$ (or $-3x + 4y = \text{const}$), you just need to find equations of the form $-3x + 4y = C'$ or $3x - 4y = C''$. The slope will be $-(-3)/4 = 3/4$. Then check the y-intercept condition. A negative y-intercept $\frac{C'}{4}$ means C' must be negative.

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13. Which of the following statements individually provide enough information to determine the number of students in a group?

Indicate all such statements.

[**Note: Select one or more answer choices**]

(A) The number of ways 3 students can be selected from the group to form a team is 35.
(B) The number of ways 3 students from the group can be seated in a row is 210.
(C) The number of ways all the students from the group can be selected to form a team is 1.
(D) The number of ways 3 students can be selected from the group to form a team equals the number of ways 4 students can be selected from the group to form a team.

Correct Answer: (A), (B), (D)

Solution:

Step 1: Understanding the Concept:
This question tests our understanding of combinations (selections, order doesn't matter) and permutations (arrangements, order matters). We need to analyze each statement to see if it leads to a unique value for n, the total number of students in the group.

Step 2: Key Formula or Approach:
Let n be the total number of students.
- The number of ways to select r items from n (combination) is given by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.
- The number of ways to arrange r items from n (permutation) is given by $P(n, r) = \frac{n!}{(n-r)!}$.
We will formulate an equation for n based on each statement and check if it can be solved for a unique, positive integer value of n.

Step 3: Detailed Explanation:

Statement A: The number of ways 3 students can be selected from the group to form a team is 35.
"Selection" implies combination. So, we have $\binom{n}{3} = 35$.

$$\frac{n(n-1)(n-2)}{3 \cdot 2 \cdot 1} = 35$$
$$n(n-1)(n-2) = 35 \times 6 = 210$$

We need to find three consecutive integers whose product is 210. We can estimate: $6^3 = 216$, so the numbers should be around 6. Let's try $n = 7$: $7 \times 6 \times 5 = 210$. This works. Since the function $f(n) = n(n-1)(n-2)$ is strictly increasing for $n > 2$, there is only one positive integer solution for n. Thus, $n = 7$. This statement is sufficient.

Statement B: The number of ways 3 students from the group can be seated in a row is 210.
"Seated in a row" implies arrangement, which is a permutation. So, we have $P(n, 3) = 210$.

$$\frac{n!}{(n-3)!} = 210$$
$$n(n-1)(n-2) = 210$$

This is the same equation as in Statement A. We already found that the unique positive integer solution is $n = 7$. This statement is sufficient.

Statement C: The number of ways all the students from the group can be selected to form a team is 1.
This means selecting n students from a group of n. The number of ways is $\binom{n}{n}$.

$$\binom{n}{n} = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = 1$$

(since $0! = 1$). This equation is true for any positive integer n. It does not allow us to determine a unique value for n. This statement is not sufficient.

Statement D: The number of ways 3 students can be selected from the group to form a team equals the number of ways 4 students can be selected from the group to form a team.
This gives the equation $\binom{n}{3} = \binom{n}{4}$.
Using the property that $\binom{n}{r} = \binom{n}{k}$ implies either $r = k$ (which is not the case here) or $n = r + k$.

$$n = 3 + 4 = 7$$

This gives a unique value for n. This statement is sufficient.

Step 4: Final Answer:
Statements A, B, and D each provide enough information to uniquely determine the number of students. Statement C does not.

Quick Tip

Remember the key difference between combinations (team selection, committee formation) and permutations (arrangements, seating, assigning specific roles). Also, memorize the useful combinatorial identity: $\binom{n}{r} = \binom{n}{n-r}$. The property used in statement D, $\binom{n}{r} = \binom{n}{k} \implies n = r + k$ (for $r \neq k$), is a direct consequence of this identity.

---

14. What could be the values of integers from 180 to 300, inclusive, that leave the remainder 2 when divided by 15 and by 9?

Indicate all such statements.

[**Note: Select one or more answer choices**]

(A) 182
(B) 191
(C) 197
(D) 227
(E) 242
(F) 272
(G) 281

Correct Answer: (D) 227

Solution:

Step 1: Understanding the Concept:
We are looking for a number N that satisfies two conditions simultaneously: $N \equiv 2 \pmod{15}$ and $N \equiv 2 \pmod{9}$. This is a system of congruences. A number that leaves the same remainder when divided by several numbers must leave that same remainder when divided by their least common multiple (LCM).

Step 2: Key Formula or Approach:
1. Let the integer be N. The conditions can be written as:
$N = 15k_1 + 2$ for some integer k_1.
$N = 9k_2 + 2$ for some integer k_2.
2. This implies that $N - 2$ is a multiple of both 15 and 9.
3. Therefore, $N - 2$ must be a multiple of the LCM of 15 and 9.
4. Calculate LCM(15, 9).
5. Find the general form of N.
6. Find the values of N that fall within the range [180, 300].
7. Check which of the options match the values found.

Step 3: Detailed Explanation:
First, find the LCM of 15 and 9.

Prime factorization of 15 is 3×5.
Prime factorization of 9 is 3^2.
LCM(15, 9) $= 3^2 \times 5 = 9 \times 5 = 45$.

Since $N - 2$ is a multiple of both 15 and 9, $N - 2$ must be a multiple of 45.
So, we can write $N - 2 = 45k$ for some integer k.
This gives the general form of the number N as:

$$N = 45k + 2$$

Now we need to find the values of k for which N is in the range [180, 300].

$$180 \leq 45k + 2 \leq 300$$

Subtract 2 from all parts of the inequality:

$$178 \leq 45k \leq 298$$

Divide by 45:

$$\frac{178}{45} \leq k \leq \frac{298}{45}$$
$$3.95... \leq k \leq 6.62...$$

Since k must be an integer, the possible values for k are 4, 5, and 6.

Let's find the corresponding values of N:
- For $k = 4$: $N = 45(4) + 2 = 180 + 2 = 182$.
- For $k = 5$: $N = 45(5) + 2 = 225 + 2 = 227$.
- For $k = 6$: $N = 45(6) + 2 = 270 + 2 = 272$.

The possible values are 182, 227, and 272.
Comparing these with the given options:
(A) 182 - Matches. (D) 227 - Matches. (F) 272 - Matches.
The question asks what could be the values. The provided options are A, B, C, D, E, F, G.
The values from the options that satisfy the condition are 182, 227, and 272.
So, A, D, and F are the correct choices.
The provided solution only lists D. Let's re-read the question. "leave the remainder 2 when divided by 15 and by 9". My logic is sound. Let's double check the calculations.
LCM(15,9)=45. Correct.
N = 45k+2. Correct.
180 ¡= 45k+2 ¡= 300.
178 ¡= 45k ¡= 298.
178/45 = 3.95. 298/45 = 6.62.
k=4,5,6. Correct.
k=4 $\rightarrow N = 182.182/15 = 12rem2.182/9 = 20rem2.Correct.$
$k = 5 \rightarrow N = 227.227/15 = 15rem2.227/9 = 25rem2.Correct.$
$k = 6 \rightarrow N = 272.272/15 = 18rem2.272/9 = 30rem2.Correct.$

There is no mathematical reason to exclude 182 and 272. This is another case where the provided answer key (if it only says D) might be incomplete or the question might have a hidden constraint I am missing. Let me read it one more time. "What could be the values of integers from 180 to 300, inclusive...". No hidden constraints. The range is inclusive. The remainders are clear.
Therefore, any of the numbers 182, 227, 272 from the options are valid answers.
I'll select all correct options based on my derivation.

Step 4: Final Answer:
The integers in the given range [180, 300] that leave a remainder of 2 when divided by 15 and 9 are 182, 227, and 272. From the options provided, these correspond to (A), (D), and (F).
So the correct answer is (A), (D), (F).
(The provided solution key seems to be in error, I will stick to the mathematically derived answer)

> Quick Tip
>
> This type of problem is an application of the Chinese Remainder Theorem. A simpler version, when the remainder is the same for all divisors, is that if $N \equiv r \pmod{d_1}$ and $N \equiv r \pmod{d_2}$, then $N \equiv r \pmod{\text{LCM}(d_1, d_2)}$. This allows you to combine multiple conditions into one.

---

15. In a certain batch of guests in a museum, there are 50 guests; each guest buys either a \$40 ticket or a \$60 ticket, with at least one guest of each ticket type. The average (arithmetic mean) value of ticket-receipts from the batch is more than \$50. If the average value of ticket-receipts is to be reduced to less than \$50 by including few new guests with \$40 tickets, what could definitely NOT be the number of new guests with \$40 tickets that could be included?

Indicate all such numbers.
[Note: Select one or more answer choices]

(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (A) 1, (B) 2, (C) 3, (D) 4

Solution:

Step 1: Understanding the Concept:
This is a problem involving inequalities and averages. We need to find the range of the initial average, and then see how adding new guests affects this average. The question asks what number of new guests could definitely not achieve the goal of reducing the average to below

\$50. This means we need to find the numbers of new guests for which it's possible to reduce the average to ¡ \$50, and the answer will be the numbers for which it's impossible, regardless of the initial state. The word "definitely NOT" implies we must consider the most extreme initial case allowed by the problem statement.

Step 2: Key Formula or Approach:
1. Let x be the number of guests with \$60 tickets and y be the number of guests with \$40 tickets initially. We know $x + y = 50$, $x \geq 1$, $y \geq 1$. 2. The total initial revenue is $R_1 = 60x + 40y$. 3. The initial average is $A_1 = \frac{R_1}{50} > 50$. Use this to find the possible range for x and y. 4. Let k be the number of new guests with \$40 tickets. 5. The new total number of guests is $50+k$. 6. The new total revenue is $R_2 = R_1 + 40k$. 7. The new average is $A_2 = \frac{R_1+40k}{50+k} < 50$. 8. We need to find values of k from the options for which this inequality can never be true, no matter what the initial valid values of x and y were. This means we should test the inequality against the 'worst-case' initial scenario, which is the lowest possible initial average A_1 that is still greater than \$50.

Step 3: Detailed Explanation:
Analyze the initial state:
Let x be the number of \$60 tickets and y be the number of \$40 tickets. $x + y = 50$. The initial average is $A_1 = \frac{60x+40y}{50} > 50$.
Substitute $y = 50 - x$:

$$A_1 = \frac{60x + 40(50 - x)}{50} = \frac{60x + 2000 - 40x}{50} = \frac{20x + 2000}{50} = \frac{2x}{5} + 40$$

The condition is $A_1 > 50$:

$$\frac{2x}{5} + 40 > 50$$
$$\frac{2x}{5} > 10$$
$$2x > 50 \implies x > 25$$

Since x must be an integer, $x \geq 26$. Also we know $x \geq 1$ and $y \geq 1 \implies x \leq 49$. So, the number of \$60 tickets can be any integer from 26 to 49. The initial total revenue is $R_1 = 20x + 2000$. Since $x > 25$, $R_1 > 20(25) + 2000 = 2500$. So, $A_1 = R_1/50 > 50$.
Analyze the final state:
Let k be the number of new guests with \$40 tickets. The new average is $A_2 = \frac{R_1+40k}{50+k}$, and we want this to be less than 50.

$$\frac{R_1 + 40k}{50 + k} < 50$$
$$R_1 + 40k < 50(50 + k)$$
$$R_1 + 40k < 2500 + 50k$$
$$R_1 - 2500 < 10k$$

Substitute $R_1 = 20x + 2000$:

$$(20x + 2000) - 2500 < 10k$$
$$20x - 500 < 10k$$
$$2x - 50 < k$$

We know from the initial condition that $x > 25$, so $2x > 50$, which means $2x - 50 > 0$.
The condition for the average to drop below \$50 is $k > 2x - 50$.

The question asks what could definitely not be the value of k. This means we are looking for a k such that the condition $k > 2x - 50$ can never be satisfied for any possible value of x.
Wait, this is the reverse. We want to find k for which it is always possible to satisfy the condition, and exclude those. The wording is tricky.
Let's re-read: "what could definitely NOT be the number of new guests ... that could be included?". This means, find k such that for any valid initial state, adding k guests is not enough.
The condition is $k > 2x - 50$. For this to fail, we need $k \leq 2x - 50$.
We want the k for which $k \leq 2x - 50$ is true for ALL possible initial states x.
The possible values for x are $x \in \{26, 27, ..., 49\}$.
The lowest value of x is 26.
If we choose the "worst case" (the initial average that is hardest to bring down), we should choose the highest possible initial average, which corresponds to the largest x, i.e., $x = 49$.
If $x = 49$, the condition becomes $k > 2(49) - 50 = 98 - 50 = 48$. In this case, we would need to add at least 49 guests.
If we choose the "best case" (the initial average that is easiest to bring down), we should choose the lowest possible initial average, which corresponds to the smallest x, i.e., $x = 26$.
If $x = 26$, the condition becomes $k > 2(26) - 50 = 52 - 50 = 2$. In this case, we need to add at least 3 guests.

The question asks for which k can we definitely not bring the average down. This means, find a k such that, no matter what the initial composition of the group was (as long as it fits the criteria), adding k guests is insufficient.
This means we are looking for values of k such that the inequality $k > 2x - 50$ is FALSE for at least one possible value of x.
The phrase "definitely not" means it's impossible. So we are looking for values of k for which the goal is impossible to achieve.
The goal is possible if there exists at least one valid initial state x for which adding k guests works. The goal is impossible (k is definitely not the number) if for ALL valid initial states x, adding k guests is not enough.
So we want to find k such that for all $x \in \{26, ..., 49\}$, the condition $A_2 < 50$ is FALSE.
This means for all $x \in \{26, ..., 49\}$, we have $A_2 \geq 50$, which is equivalent to $k \leq 2x - 50$.
For k to be less than or equal to $2x - 50$ for all possible x, it must be less than or equal to the minimum possible value of $2x - 50$.
The minimum value of $2x - 50$ occurs at the minimum value of x, which is $x = 26$.
Min value of $2x - 50$ is $2(26) - 50 = 52 - 50 = 2$.
So, if $k \leq 2$, the goal of getting the average below \$50 is definitely not achievable. For $k = 1$ or $k = 2$, there is no value of x that allows the condition $k > 2x - 50$ to be met, because the smallest value $2x - 50$ can take is 2.

Let's test the options. If $k = 1$, is it possible to have $A_2 < 50$? We need $1 > 2x - 50$. This requires $51 > 2x$ or $x < 25.5$. But we know from the problem that x must be at least 26. So it's impossible. Thus $k = 1$ is a number that could definitely NOT be the number of new guests.
If $k = 2$, is it possible? We need $2 > 2x - 50$. This requires $52 > 2x$ or $x < 26$. But x must be

at least 26. So it's impossible. Thus $k = 2$ is also a correct answer.
If $k = 3$, is it possible? We need $3 > 2x - 50$. This requires $53 > 2x$ or $x < 26.5$. If the initial state had $x = 26$, then adding 3 guests would work. $A_2 < 50$. The question asks what could definitely not be the number. Since for k=3, we found a scenario where it is possible, k=3 is not a "definitely not" answer.
Let me re-read the question again. It is very subtle.
"what could definitely NOT be the number of new guests ... that could be included?"
This can be interpreted as: find k such that there exists some initial condition for which adding k guests is not enough.
Let's re-evaluate the question's logic.
The goal is to reduce the average to less than \$50.
Let's see for which k values it is always possible.
It is always possible if $k > 2x - 50$ for all $x \in \{26, ..., 49\}$. For this to be true, k must be greater than the maximum value of $2x - 50$.
Max value of $2x - 50$ is $2(49) - 50 = 48$. So if we add $k = 49$ new guests, it is always possible to bring the average down.

Let's take the other interpretation. A number k is a "definitely not" number if it's impossible to achieve the goal with k new guests, for at least one of the valid initial scenarios.
No, "definitely not" means impossibility across all scenarios.
My first interpretation was correct. A value k is a "definitely not" value if, regardless of the initial number of \$60 ticket holders (as long as it's between 26 and 49), adding k new guests is insufficient to bring the average below \$50.
This requires $k \leq 2x - 50$ for all $x \in \{26, ..., 49\}$.
This is true if $k \leq \min(2x - 50)$.
The minimum value of $2x - 50$ for $x \in \{26, ..., 49\}$ is at $x = 26$, which is $2(26) - 50 = 2$.
So, if $k \leq 2$, it is definitely impossible to bring the average below \$50.
The numbers that could definitely NOT be the number of new guests are $k = 1$ and $k = 2$.
Options A and B.

Why might the solution be all four options? Let's reconsider. Maybe I misinterpreted "more than 50"$.WhatifitmeansA_1 \geq 50?No,"morethan"isstrictinequality.$
$WhatifImisinterpreted"lessthan$50"? What if it means $A_2 \leq 50$? No, "less than" is strict.
Let's re-check the algebra. $R_1 = 20x + 2000$. $A_2 = (R_1 + 40k)/(50 + k) < 50$. $R_1 + 40k < 2500 + 50k$. $R_1 - 2500 < 10k$. $20x + 2000 - 2500 < 10k$. $20x - 500 < 10k$. $2x - 50 < k$. Everything seems correct.
The condition for success is $k > 2x - 50$.
The condition for failure is $k \leq 2x - 50$.
A value k is a "definitely not" value if failure is guaranteed for any valid x.
Failure for any x means $k \leq 2x - 50$ for all $x \in \{26, ..., 49\}$.
This means $k \leq \min(2x - 50) = 2(26) - 50 = 2$.
So $k = 1, 2$ are the answers.

There is no path to A,B,C,D being the answer. Let's analyze the question one more time. Maybe the question is "what could be the number of new guests...". Then the answer would be any k for which success is possible for at least one x. Success requires k ¿ 2x-50.
k=1: never possible.

k=2: never possible.
k=3: possible if 3 ¿ 2x-50, 53¿2x, x¡26.5. This works for x=26.
k=4: possible if 4 ¿ 2x-50, 54¿2x, x¡27. This works for x=26.
So if the question was "what could be the number", the answer would be 3 and 4.
This means the complement, "definitely NOT", should be 1 and 2.

My analysis consistently leads to A and B. I cannot justify C and D. I will provide the solution for A and B.

Step 4: Final Answer:
The values of k for which it is impossible to reduce the average ticket price to less than \$50, regardless of the initial distribution of guests, are $k = 1$ and $k = 2$. Therefore, options (A) and (B) are the correct answers. There might be an error in the question or the expected answer if C and D are also considered correct. Based on a rigorous mathematical analysis of the problem as stated, only A and B fit the criteria.

Quick Tip

In problems with inequalities and phrases like "definitely," "at least," or "at most," it's crucial to identify the "worst-case" or most restrictive scenario. Here, to find a number of guests k that is definitely not enough, you must show that it's not enough even in the most favorable initial conditions for reducing the average (i.e., the lowest possible initial average, which corresponds to the lowest number of expensive tickets).

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16. In a certain batch of guests in a museum, there are 50 guests; each guest buys either a \$40 ticket or a \$60 ticket, with at least one guest of each ticket type. The average (arithmetic mean) value of ticket-receipts from the batch is more than \$50. If the average value of ticket-receipts is to be reduced to less than \$50 by including few new guests with \$40 tickets, what could definitely NOT be the number of new guests with \$40 tickets that could be included?

Indicate all such numbers.

[Note: Select one or more answer choices]

(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (A) 1, (B) 2

Solution:

Step 1: Understanding the Concept:

This problem involves analyzing how the average of a group changes when new members are added. The phrase "definitely NOT" is key; it means we are looking for a number of new guests, k, for which it is impossible to achieve the desired outcome (average ¡ $50), regardless of the initial composition of the 50 guests (as long as it meets the initial criteria).

Step 2: Key Formula or Approach:

1. Define variables for the initial state. Let x be the number of guests with $60 tickets and y be the number with $40 tickets. We have $x + y = 50$. 2. Use the initial average condition ($>$ $50) to find the possible range for x. 3. Define a variable k for the number of new guests with $40 tickets. 4. Set up an inequality for the new average to be less than $50. 5. The goal is to find the values of k from the options for which this inequality can never be satisfied, no matter the initial valid value of x. This means we test against the most 'favorable' initial condition for our goal. The easiest average to reduce is the one that is lowest to begin with.

Step 3: Detailed Explanation:

Initial State Analysis:

Let x be the number of $60 tickets. Then $50 - x$ is the number of $40 tickets. The total initial revenue is $R_1 = 60x + 40(50 - x) = 60x + 2000 - 40x = 20x + 2000$. The initial average $A_1 = \frac{R_1}{50} = \frac{20x+2000}{50} = \frac{2x}{5} + 40$. We are given $A_1 > 50$:

$$\frac{2x}{5} + 40 > 50 \implies \frac{2x}{5} > 10 \implies 2x > 50 \implies x > 25$$

Since x is an integer, and there is at least one of each ticket type ($x \leq 49$), the possible values for x are integers from 26 to 49.

Final State Analysis:

Let k be the number of new guests with $40 tickets. New total guests $= 50 + k$. New total revenue $R_2 = R_1 + 40k = (20x + 2000) + 40k$. The new average $A_2 = \frac{20x+2000+40k}{50+k}$ must be less than 50.

$$\frac{20x + 2000 + 40k}{50 + k} < 50$$
$$20x + 2000 + 40k < 50(50 + k) = 2500 + 50k$$
$$20x + 2000 - 2500 < 50k - 40k$$
$$20x - 500 < 10k$$
$$2x - 50 < k$$

So, to successfully reduce the average, the number of new guests k must be greater than $2x - 50$.

The question asks for which k this is "definitely NOT" possible. This means for which k is the condition $k > 2x - 50$ impossible to satisfy for *any* of the allowed initial values of x (i.e., for any $x \in \{26, 27, ..., 49\}$). The condition $k > 2x - 50$ is hardest to satisfy when the right side, $2x - 50$, is as large as possible. It is easiest to satisfy when $2x - 50$ is as small as possible. The smallest possible value for $2x - 50$ occurs at the smallest possible x, which is $x = 26$. Minimum value of $2x - 50$ is $2(26) - 50 = 52 - 50 = 2$. So, for any valid initial scenario, the value of $2x - 50$ will be at least 2. The condition for success is $k > 2x - 50$. If we choose $k = 1$, the condition becomes $1 > 2x - 50$, which requires $x < 25.5$. This is not possible, as

x must be at least 26. So $k = 1$ is definitely not enough. If we choose $k = 2$, the condition becomes $2 > 2x - 50$, which requires $x < 26$. This is also not possible, as x must be at least 26. So $k = 2$ is definitely not enough. If we choose $k = 3$, the condition becomes $3 > 2x - 50$, requiring $x < 26.5$. This is possible if the initial state was $x = 26$. In that specific case, adding 3 guests would work. Therefore, $k = 3$ is not a "definitely not" answer. The same logic applies to $k = 4$.

Step 4: Final Answer:
The values for the number of new guests that could definitely NOT be sufficient are 1 and 2.

> **Quick Tip**
>
> The phrase "definitely NOT" in a problem involving ranges and inequalities usually directs you to find a value that fails even under the most favorable conditions. Here, the most favorable condition for reducing the average is the initial average that is closest to \$50 (which corresponds to the smallest possible number of \$60 tickets, x=26). If a value of k doesn't work in this best-case scenario, it won't work in any other.

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17. If $\hat{x}2\hat{y}3 < 0$, which of the following must be true?

Indicate all such answers.

[**Note: Select one or more answer choices**]

(A) x>0
(B) xy<0
(C) xˆ2y<0
(D) xyˆ2>0
(E) xˆ2>0
(F) yˆ2>0
(G) xy$\neq$ 0

Correct Answer: (C), (E), (F), (G)

Solution:

Step 1: Understanding the Concept:
This question tests the properties of inequalities involving powers of variables. The sign (positive or negative) of a product depends on the signs of its factors. The key is to analyze the signs of $\hat{x}2$ and $\hat{y}3$.

Step 2: Key Formula or Approach:
1. The term $\hat{x}2$, being a square of a real number, is always non-negative (≥ 0). 2. The term $\hat{y}3$ has the same sign as y. If y is positive, $\hat{y}3$ is positive. If y is negative, $\hat{y}3$ is negative. 3. The

product of two terms is negative if and only if one term is positive and the other is negative. 4. From the given inequality $\hat{x}2\hat{y}3 < 0$, we can deduce the signs of x and y, and then check each statement.

Step 3: Detailed Explanation:
We are given the inequality $\hat{x}2\hat{y}3 < 0$.
First, notice that the product is not zero. This implies that neither $\hat{x}2$ nor $\hat{y}3$ can be zero. If $\hat{x}2 \neq 0$, then $x \neq 0$. If $\hat{y}3 \neq 0$, then $y \neq 0$.

Since $x \neq 0$, $\hat{x}2$ must be strictly positive ($\hat{x}2 > 0$). The inequality can be seen as (a positive number) $\times$ $\hat{y}3 < 0$. For this product to be negative, the second factor, $\hat{y}3$, must be negative. So, $\hat{y}3 < 0$, which implies that y must be negative ($y < 0$).

So, what we know for sure is: $x \neq 0$ and $y < 0$.
Now let's evaluate each statement:
(A) **x¿0**: This is not necessarily true. x could be negative (e.g., $x = -1$). Then $\hat{x}2 = 1 > 0$. So, this is not a "must be true".
(B) **xy¡0**: We know $y < 0$. The sign of xy depends on the sign of x. If $x > 0$, then $xy < 0$. If $x < 0$, then $xy > 0$. Since we don't know the sign of x, this is not a "must be true".
(C) **xˆ2y¡0**: We established that $\hat{x}2 > 0$ and $y < 0$. The product of a positive number and a negative number is always negative. So, this **must be true**.
(D) **xyˆ2¿0**: We know $y < 0$, which means $y \neq 0$, so $\hat{y}2 > 0$. The sign of $x\hat{y}2$ depends on the sign of x, which is unknown. This is not a "must be true".
(E) **xˆ2¿0**: As shown above, since the overall product is not zero, x cannot be zero. The square of any non-zero number is positive. So, this **must be true**.
(F) **yˆ2¿0**: As shown above, since the overall product is not zero, y cannot be zero. The square of any non-zero number is positive. So, this **must be true**.
(G) **xy0**: Since we deduced that $x \neq 0$ and $y \neq 0$, their product cannot be zero. So, this **must be true**.

Step 4: Final Answer:
The statements that must be true are (C), (E), (F), and (G).

Quick Tip

For any inequality involving variables raised to powers, pay close attention to whether the exponent is even or odd. An even power (like $\hat{x}2$) hides the sign of the base, but guarantees a non-negative result. An odd power (like $\hat{y}3$) preserves the sign of the base.

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18. If ab¿cd, and none of c, and d is equal to 0, which of the following must be true?

Indicate all such answers.

[Note: Select one or more answer choices]

(A) -ab¡cd
(B) —ab—¿—cd—
(C) ba¡dc
(D) -ab¡-cd

Correct Answer: (D) -ab¡-cd

Solution:

Step 1: Understanding the Concept:
This question tests fundamental properties of inequalities, specifically how they behave under multiplication. We need to evaluate each statement to see if it logically follows from the given inequality $ab > cd$ under all possible circumstances. Using counterexamples is an effective way to disprove statements that are not always true.

Step 2: Key Formula or Approach:
The core property of inequalities we will use is: If $x > y$, then for any negative number k, $kx < ky$. That is, multiplying or dividing an inequality by a negative number reverses the inequality sign. We will apply this rule and also test the other options with numerical examples to check their validity.

Step 3: Detailed Explanation:
We are given the inequality $ab > cd$. Let's analyze each option.

(A) **-ab¡cd**: This is equivalent to $ab > -cd$. We are given $ab > cd$. Does $ab > cd$ imply $ab > -cd$? Not always. *Counterexample*: Let $ab = 5$ and $cd = -10$. Then $5 > -10$ is true. The statement becomes $-5 < -10$, which is false. Therefore, A is not always true.

(B) **—ab—¿—cd—**: This statement claims that the magnitude of ab is greater than the magnitude of cd. This is not always true. *Counterexample*: Let $ab = 5$ and $cd = -10$. Then $5 > -10$ is true. The statement becomes $|5| > |-10|$, which is $5 > 10$, and this is false. Therefore, B is not always true.

(C) **ba¡dc**: Since multiplication is commutative, $ba = ab$ and $dc = cd$. So this statement is equivalent to $ab < cd$. This is the direct opposite of the given information ($ab > cd$). Therefore, C is never true.

(D) **-ab¡-cd**: Let's start with the given inequality:

$$ab > cd$$

Multiply both sides by -1. According to the rules of inequalities, when we multiply by a negative number, we must reverse the direction of the inequality sign.

$$(-1) \times (ab) < (-1) \times (cd)$$

$$-ab < -cd$$

This statement is a direct consequence of the properties of inequalities and **must be true**. The conditions that c and d are non-zero are not needed for this derivation.

Step 4: Final Answer:
The only statement that must be true is (D).

> **Quick Tip**
>
> When testing inequality statements, think about using a mix of positive and negative numbers for your counterexamples, as this is often where the statements fail. The most reliable statements are those that are direct algebraic manipulations based on the core rules of inequalities.

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19. David bought greater than 10 paperback books that cost \$8 each and greater than 8 hardcover books that cost \$20 each. If the total cost of all the books that he bought was between \$240 and \$300, exclusive, how many total books could he buy?

Indicate all such answers.

[**Note: Select one or more answer choices**]
(A) 17
(B) 18
(C) 19
(D) 20
(E) 21
(F) 22
(G) 23

Correct Answer: (D), (E), (F), (G)

Solution:

Step 1: Understanding the Concept:
This is a problem that requires setting up and solving a system of linear inequalities with integer constraints. We need to find the possible total number of items based on constraints on the quantity of each item and the total cost.

Step 2: Key Formula or Approach:
1. Define variables: Let p be the number of paperbacks and h be the number of hardcovers. 2. Translate the given information into mathematical inequalities: - $p > 10 \implies p \geq 11$ (since p must be an integer). - $h > 8 \implies h \geq 9$ (since h must be an integer). - Total Cost: $C = 8p + 20h$. - $240 < C < 300 \implies 240 < 8p + 20h < 300$. 3. Simplify the cost inequality and test possible integer values for h (starting from its minimum value) to find the corresponding possible integer values for p. 4. For each valid pair (p, h), calculate the total

number of books, $T = p+h$. 5. Collect all possible values of T and match them with the options.

Step 3: Detailed Explanation:
The cost inequality is $240 < 8p + 20h < 300$. Let's simplify by dividing all parts by 4, the greatest common divisor of 8 and 20.

$$60 < 2p + 5h < 75$$

We know $h \geq 9$. Let's test values for h starting from 9.
Case 1: h = 9

$$60 < 2p + 5(9) < 75$$

$$60 < 2p + 45 < 75$$

Subtract 45 from all parts:

$$15 < 2p < 30$$

Divide by 2:

$$7.5 < p < 15$$

Since p must be an integer and $p \geq 11$, the possible values for p are $\{11, 12, 13, 14\}$. The possible total books $T = p + h = p + 9$ are: $11 + 9 = 20$, $12 + 9 = 21$, $13 + 9 = 22$, $14 + 9 = 23$.
Case 2: h = 10

$$60 < 2p + 5(10) < 75$$

$$60 < 2p + 50 < 75$$

Subtract 50:

$$10 < 2p < 25$$

Divide by 2:

$$5 < p < 12.5$$

Since $p \geq 11$, the possible values for p are $\{11, 12\}$. The possible total books $T = p+h = p+10$ are: $11 + 10 = 21$, $12 + 10 = 22$.
Case 3: h = 11

$$60 < 2p + 5(11) < 75$$

$$60 < 2p + 55 < 75$$

Subtract 55:

$$5 < 2p < 20$$

Divide by 2:

$$2.5 < p < 10$$

There are no possible values for p, since we require $p \geq 11$.
For any $h > 11$, the lower bound for p will become even smaller, so there will be no solutions. The set of all possible values for the total number of books T is the union of the results from the valid cases: $\{20, 21, 22, 23\} \cup \{21, 22\} = \{20, 21, 22, 23\}$.

Step 4: Final Answer:
Matching our findings with the options, the possible total number of books are 20, 21, 22, and 23. These correspond to options (D), (E), (F), and (G).

Quick Tip

When solving Diophantine inequalities like this, it is most efficient to iterate on the variable with the larger coefficient (in this case, h). This narrows down the possibilities for the other variable more quickly.

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20. If two interior angles of a quadrilateral ABCD are right angles and the degree measure of ∠ABC is twice the degree measure of ∠BCD, what could be the measure of the largest interior angle of quadrilateral ABCD?

Indicate all such answers.

[**Note: Select one or more answer choices**]
(A) 90°
(B) 105°
(C) 120°
(D) 135°
(E) 150°
(F) 180°

Correct Answer: (C) 120°, (D) 135°

Solution:

Step 1: Understanding the Concept:
The sum of the interior angles in any quadrilateral is 360°. We are given information about the angles and need to find the largest possible angle. The problem is ambiguous about which two angles are the right angles, so we must consider all distinct cases. We will assume the quadrilateral is convex, meaning all interior angles are less than 180°.

Step 2: Key Formula or Approach:
1. The sum of angles is $\angle A + \angle B + \angle C + \angle D = 360°$. 2. We are given the relation $\angle B = 2 \times \angle C$. 3. Two of the four angles are 90°. We need to check all possible arrangements for the two right angles. - Case 1: The right angles are not $\angle B$ or $\angle C$. - Case 2: One of the right angles is $\angle B$ or $\angle C$. - Case 3: Both right angles are $\angle B$ and $\angle C$. 4. Solve for the angles in each valid case and find the largest angle.

Step 3: Detailed Explanation:
Let the angles be A, B, C, and D. We are given B = 2C. The sum A+B+C+D = 360°.
Case I: The right angles are A and D. So, A = 90° and D = 90°. The sum becomes $90 + B + C + 90 = 360$, which simplifies to $B + C = 180$. Substitute B = 2C into this equation:

$$2C + C = 180 \implies 3C = 180 \implies C = 60°$$

Then $B = 2C = 2 \times 60 = 120°$. The angles are 90°, 120°, 60°, 90°. The largest angle is **120°**.

Case II: The right angles are adjacent, one of them involved in the B=2C relation. Let's assume the right angles are A and B. So, A = 90° and B = 90°. The relation B = 2C becomes $90 = 2C$, which gives $C = 45\r{o}$. Now find the fourth angle D from the sum:

$$90 + 90 + 45 + D = 360 \implies 225 + D = 360 \implies D = 135\r{o}$$

The angles are 90°, 90°, 45°, 135°. The largest angle is **135°**. (If we assume the right angles are B and D, the result is the same: B=90, D=90 $\rightarrow C = 45, A = 135. Largest angle is 135$).

Case III: The right angles are adjacent, both involved in the B=2C relation. This means B=90° and C=90°. The relation B=2C becomes $90 = 2 \times 90$, which is $90 = 180$, a contradiction. This case is impossible.

Case IV: The right angles involve C and an angle not B. Let's assume the right angles are C and D. So, C = 90° and D = 90°. The relation B = 2C becomes $B = 2 \times 90 = 180\r{o}$. An interior angle of 180° implies a degenerate quadrilateral, which is typically excluded unless specified. If allowed, 180° would be the largest angle. Assuming convex quadrilaterals, we discard this case. (The same happens if the right angles are A and C, leading to B=180°).

Based on the valid, non-degenerate cases, the possible values for the largest angle are 120° and 135°.

Step 4: Final Answer:

The possible measures for the largest interior angle are 120° and 135°. These correspond to options (C) and (D).

Quick Tip

In geometry problems with unspecified labels (e.g., "two angles are right angles"), it is crucial to systematically consider all distinct cases. The main cases for a quadrilateral are that the right angles are adjacent or opposite. Be sure to check how the given relationships interact with each case.
