GRE 2024 Quant Practice Test 19 with Solutions

Time Allowed : 1 Hour 58 Minutes	Maximum Marks : 340

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GRE General Test is 1 hour and 58 minutes long (with one optional 10-minute break) and consists of 54 questions in total.
2. The GRE exam is comprised of three sections:
   - Quantitative Reasoning: 27 questions, 47 minutes
   - Verbal Reasoning: 27 questions, 41 minutes
3. You can answer the two sections in any order.
4. As you move through a section, you can skip questions, flag them for review, and return to them later within the same section.
5. When you have answered all questions in a section, you can review your responses before time expires.
6. If there is no time remaining in the section, you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
7. Each review screen includes a numbered list of the questions in that section and indicates the questions you flagged.
8. Clicking a question number will take you to that specific question.
9. You may change any answer within the time allowed for that section.

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1. Quantity A: 2^{60}
Quantity B: 8^{20}

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (C) if the two quantities are equal;

Solution:

Step 1: Understanding the Concept:
The question asks for a comparison between two numbers expressed in exponential form. To

compare them, it's best to rewrite them so that they share a common base.

Step 2: Key Formula or Approach:
We will use the exponent rule known as the "power of a power" rule, which states that $(a^m)^n = a^{m\times n}$.

Step 3: Detailed Explanation:
Quantity A is given as 2^{60}.
Quantity B is given as 8^{20}.
We can express the base of Quantity B, which is 8, as a power of 2:

$$8 = 2^3$$

Now, we substitute this back into the expression for Quantity B:

$$8^{20} = (2^3)^{20}$$

Using the power of a power rule, we multiply the exponents:

$$(2^3)^{20} = 2^{3\times 20} = 2^{60}$$

So, Quantity B is equivalent to 2^{60}.

Step 4: Final Answer:
By simplifying Quantity B, we have:
Quantity A $= 2^{60}$
Quantity B $= 2^{60}$
Both quantities are identical. Therefore, the two quantities are equal.

Quick Tip

When comparing expressions with exponents, always look for an opportunity to make the bases equal. Recognizing common powers (like $8 = 2^3$, $9 = 3^2$, $27 = 3^3$, etc.) is a fundamental skill for solving such problems quickly.

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2. Given the equation $x^2 - 5x = 6 - y^2 + 3y = 9y + 9$.
Quantity A: x
Quantity B: y

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (D) if the relationship cannot be determined from the information given.

Solution:

Step 1: Understanding the Concept:
The problem provides a compound equality involving two variables, x and y. We need to determine if there is a consistent relationship between x and y. A compound equality $A = B = C$ can be split into two separate equations, such as $A = C$ and $B = C$.

Step 2: Key Formula or Approach:
We will first solve for the possible values of y by setting the second and third expressions equal to each other. This will result in a quadratic equation, which can be solved using the quadratic formula: $y = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Then we will analyze the resulting values for x.

Step 3: Detailed Explanation:
Let's first find the value(s) of y. We use the equality $6 - y^2 + 3y = 9y + 9$:

$$6 - y^2 + 3y = 9y + 9$$

Rearrange the terms to form a standard quadratic equation ($ay^2 + by + c = 0$):

$$0 = y^2 + 9y - 3y + 9 - 6$$

$$y^2 + 6y + 3 = 0$$

Using the quadratic formula to solve for y:

$$y = \frac{-6 \pm \sqrt{6^2 - 4(1)(3)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 12}}{2} = \frac{-6 \pm \sqrt{24}}{2} = \frac{-6 \pm 2\sqrt{6}}{2}$$

$$y = -3 \pm \sqrt{6}$$

This gives two possible values for y: $y_1 = -3 + \sqrt{6}$ and $y_2 = -3 - \sqrt{6}$.

Now let's consider the equation for x: $x^2 - 5x = 9y + 9$.
For each value of y, we will get a quadratic equation for x, which will yield two values for x. Let's test if we can find contradictory relationships.

Case 1: Let's use $y_1 = -3 + \sqrt{6}$.
We know that $\sqrt{4}<\sqrt{6}<\sqrt{9}$, so $2<\sqrt{6}<3$. Let's approximate $\sqrt{6} \approx 2.45$.
So, $y_1 \approx -3 + 2.45 = -0.55$.
The equation for x becomes $x^2 - 5x = 9(-3 + \sqrt{6}) + 9 = -27 + 9\sqrt{6} + 9 = 9\sqrt{6} - 18$.
So, $x^2 - 5x - (9\sqrt{6} - 18) = 0$.
The solutions for x are given by $x = \frac{5\pm\sqrt{(-5)^2-4(1)(-(9\sqrt{6}-18))}}{2} = \frac{5\pm\sqrt{25+36\sqrt{6}-72}}{2} = \frac{5\pm\sqrt{36\sqrt{6}-47}}{2}$.
Let's approximate the term under the square root: $36\sqrt{6}-47 \approx 36(2.45)-47 = 88.2-47 = 41.2$.
So, $x \approx \frac{5\pm\sqrt{41.2}}{2} \approx \frac{5\pm 6.42}{2}$.
One possible value for x is $x_1 \approx \frac{5+6.42}{2} = 5.71$. In this instance, $x_1 \approx 5.71>y_1 \approx -0.55$.
Another possible value for x is $x_2 \approx \frac{5-6.42}{2} = -0.71$. In this instance, $x_2 \approx -0.71<y_1 \approx -0.55$.

Step 4: Final Answer:
We have found one pair of values (x_1, y_1) where $x>y$ and another pair (x_2, y_1) where $x<y$. Since the relationship between x and y is not constant and depends on which solution we consider,

the relationship cannot be determined from the information given.

Quick Tip

In quantitative comparison questions, if you find that a variable can take on multiple values, test different possibilities. If you can show that in one scenario Quantity A is greater, and in another scenario Quantity B is greater, then the answer is always (D).

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3. Working at a constant rate, Bob can produce $\frac{x}{3}$ widgets in 8 minutes. Working at a constant rate, Jack can produce $2x$ widgets in 40 minutes, where $x>0$.
Quantity A: The number of minutes it will take Bob to produce $5x$ widgets.
Quantity B: The number of minutes it will take Jack to produce $6x$ widgets.

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (C) if the two quantities are equal;

Solution:

Step 1: Understanding the Concept:
This is a work-rate problem. The key is to first determine the rate of production for both Bob and Jack, and then use that rate to find the time required to produce a different quantity of widgets.

Step 2: Key Formula or Approach:
The fundamental relationship is: Rate $= \frac{\text{Work}}{\text{Time}}$. Consequently, Time $= \frac{\text{Work}}{\text{Rate}}$. We can also solve this using proportions.

Step 3: Detailed Explanation:
First, let's calculate the production rate for Bob.
Bob produces $\frac{x}{3}$ widgets in 8 minutes.
Bob's Rate $= \frac{\text{Work}}{\text{Time}} = \frac{x/3 \text{ widgets}}{8 \text{ minutes}} = \frac{x}{24}$ widgets per minute.

Next, let's calculate the production rate for Jack.
Jack produces $2x$ widgets in 40 minutes.
Jack's Rate $= \frac{\text{Work}}{\text{Time}} = \frac{2x \text{ widgets}}{40 \text{ minutes}} = \frac{x}{20}$ widgets per minute.

Now, we calculate the time for Quantity A.
Quantity A: Time for Bob to produce $5x$ widgets.
Time $= \frac{\text{Work}}{\text{Rate}} = \frac{5x \text{ widgets}}{x/24 \text{ widgets/minute}} = 5x \times \frac{24}{x} = 120$ minutes.

Finally, we calculate the time for Quantity B.
Quantity B: Time for Jack to produce $6x$ widgets.
Time $= \frac{\text{Work}}{\text{Rate}} = \frac{6x \text{ widgets}}{x/20 \text{ widgets/minute}} = 6x \times \frac{20}{x} = 120$ minutes.

Step 4: Final Answer:
Comparing the two quantities:
Quantity A = 120 minutes.
Quantity B = 120 minutes.
The two quantities are equal.

Quick Tip

For work-rate problems, setting up a proportion is often a very fast method. For Bob: $\frac{\text{widgets}_1}{\text{time}_1} = \frac{\text{widgets}_2}{\text{time}_2} \implies \frac{x/3}{8} = \frac{5x}{\text{Time}_A}$. Solving for Time_A gives 120. For Jack: $\frac{2x}{40} = \frac{6x}{\text{Time}_B}$. Solving for Time_B gives 120.

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4. Given $t>1$.
Quantity A: $\frac{1}{(1+\frac{1}{t})^2}$
Quantity B: $\frac{1}{(1+\frac{1}{\sqrt{t}})^2}$

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (A) if Quantity A is greater;

Solution:

Step 1: Understanding the Concept:
We need to compare two fractions. Since the numerators are both 1, the fraction with the smaller denominator will have the larger value. Therefore, the problem reduces to comparing the denominators.

Step 2: Key Formula or Approach:
The comparison will rely on the properties of inequalities. Specifically, how inequalities change when taking reciprocals or squaring positive numbers.

Step 3: Detailed Explanation:
Let's compare the terms within the denominators. We are given that $t>1$.
For any number greater than 1, its square root is smaller than the number itself.
For example, if $t = 4$, then $\sqrt{t} = 2$, and $4>2$. So, we can state that:

$$t>\sqrt{t}$$

Since t and $\sqrt{t}$ are both positive, taking the reciprocal of both sides will reverse the inequality sign:

$$\frac{1}{t}<\frac{1}{\sqrt{t}}$$

Adding 1 to both sides does not change the direction of the inequality:

$$1+\frac{1}{t}<1+\frac{1}{\sqrt{t}}$$

Since $t>1$, both expressions in the inequality are positive. Squaring both sides of an inequality involving positive numbers preserves the direction of the inequality:

$$\left(1+\frac{1}{t}\right)^2<\left(1+\frac{1}{\sqrt{t}}\right)^2$$

This shows that the denominator of Quantity A is smaller than the denominator of Quantity B.

Step 4: Final Answer:
We have Quantity A $=\frac{1}{\text{Denominator A}}$ and Quantity B $=\frac{1}{\text{Denominator B}}$, where Denominator A $=\left(1+\frac{1}{t}\right)^2$ and Denominator B $=\left(1+\frac{1}{\sqrt{t}}\right)^2$.
Since Denominator A $<$Denominator B and both are positive, taking the reciprocal reverses the inequality:

$$\frac{1}{\text{Denominator A}}>\frac{1}{\text{Denominator B}}$$

Therefore, Quantity A is greater than Quantity B.

Quick Tip

When dealing with abstract inequalities, plugging in a simple test value is a great way to quickly determine the relationship and confirm your algebraic reasoning. For $t=4$: Quantity A $=\frac{1}{(1+1/4)^2}=\frac{1}{(5/4)^2}=\frac{1}{25/16}=\frac{16}{25}=0.64$. Quantity B $=\frac{1}{(1+1/\sqrt{4})^2}=\frac{1}{(1+1/2)^2}=\frac{1}{(3/2)^2}=\frac{1}{9/4}=\frac{4}{9}\approx 0.44$. Since $0.64>0.44$, Quantity A is greater.

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5. From 1992 to 1993, the price of a home increased by $x\%$. From 1993 to 1994, the price of the home then decreased by $x\%$.
Quantity A: The price of the home at the beginning of 1992.
Quantity B: The price of the home at the beginning of 1994.

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (A) if Quantity A is greater;

Solution:

Step 1: Understanding the Concept:
This problem deals with successive percentage changes. A key concept is that a percentage change is always calculated based on the current value, not the original value.

Step 2: Key Formula or Approach:
Let the initial price be P. An increase of $x\%$ is equivalent to multiplying by $(1 + \frac{x}{100})$. A decrease of $x\%$ is equivalent to multiplying by $(1 - \frac{x}{100})$. We will apply these multipliers sequentially to find the final price.

Step 3: Detailed Explanation:
Let P_{1992} be the price of the home at the beginning of 1992. This is Quantity A.
From 1992 to 1993, the price increased by $x\%$. The price at the beginning of 1993 (P_{1993}) is:

$$P_{1993} = P_{1992} \times \left(1 + \frac{x}{100}\right)$$

From 1993 to 1994, the price decreased by $x\%$. This decrease is applied to the new, higher price P_{1993}. The price at the beginning of 1994 (P_{1994}) is:

$$P_{1994} = P_{1993} \times \left(1 - \frac{x}{100}\right)$$

Now, substitute the expression for P_{1993} into the equation for P_{1994}:

$$P_{1994} = \left(P_{1992} \times \left(1 + \frac{x}{100}\right)\right) \times \left(1 - \frac{x}{100}\right)$$

$$P_{1994} = P_{1992} \times \left(1 + \frac{x}{100}\right)\left(1 - \frac{x}{100}\right)$$

Using the difference of squares formula, $(a + b)(a - b) = a^2 - b^2$:

$$P_{1994} = P_{1992} \times \left(1^2 - \left(\frac{x}{100}\right)^2\right)$$

$$P_{1994} = P_{1992} \times \left(1 - \frac{x^2}{10000}\right)$$

This is Quantity B.

Step 4: Final Answer:
We are comparing Quantity A (P_{1992}) with Quantity B ($P_{1992} \times \left(1 - \frac{x^2}{10000}\right)$). Assuming there was a change, $x>0$. This means $x^2>0$, and $\frac{x^2}{10000}$ is a positive value. The multiplier $\left(1 - \frac{x^2}{10000}\right)$ is therefore less than 1. Multiplying the original price P_{1992} by a factor less than 1 results in a smaller final price. Thus, $P_{1994}<P_{1992}$. Quantity B is less than Quantity A. Therefore, Quantity A is greater.

Quick Tip

A common mistake is to think that an $x\%$ increase followed by an $x\%$ decrease returns you to the starting value. This is incorrect. The percentage decrease is calculated on a larger base amount, making the monetary decrease larger than the initial monetary increase. This always results in a net loss.

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6. Quantity A: The product of the consecutive integers from 20 through 73, inclusive.
Quantity B: The product of the consecutive integers from 18 through 72, inclusive.

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (B) if Quantity B is greater;

Solution:

Step 1: Understanding the Concept:
The problem asks to compare two large products of consecutive integers. Calculating the full products is impractical. The best approach is to identify and cancel out common factors.

Step 2: Key Formula or Approach:
We will write out the expressions for both quantities and identify the overlapping sequence of numbers. By "dividing out" or canceling these common terms, we can reduce the comparison to a much simpler calculation.

Step 3: Detailed Explanation:
Let's write out the products for Quantity A and Quantity B.
Quantity A $= 20 \times 21 \times 22 \times \cdots \times 71 \times 72 \times 73$
Quantity B $= 18 \times 19 \times 20 \times 21 \times \cdots \times 71 \times 72$

Both quantities share the product of the integers from 20 to 72. Let's call this common product C.

$$C = 20 \times 21 \times \cdots \times 72$$

Now, we can express Quantity A and Quantity B in terms of C.
Quantity A $= C \times 73$
Quantity B $= 18 \times 19 \times C$

Since C is a product of positive integers, it is a large positive number. To compare Quantity A and Quantity B, we only need to compare their unique factors: 73 and 18×19.
Let's calculate the product 18×19:

$$18 \times 19 = 18 \times (20 - 1) = (18 \times 20) - (18 \times 1) = 360 - 18 = 342$$

Now we compare the unique factors: The unique factor for Quantity A is 73.
The unique factors for Quantity B multiply to 342.

Step 4: Final Answer:
We are comparing $C \times 73$ with $C \times 342$.
Since 342>73, and C is positive, it follows that $C \times 342 > C \times 73$.
Therefore, Quantity B is greater than Quantity A.

> **Quick Tip**
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> When comparing large products or sums that share many common terms (like factorials or series), the most efficient strategy is to cancel the common parts and only compare the remaining, unique terms.

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7. Line p is defined by the equation $2y + 3x = 6$.
Quantity A: The y-intercept of line p.
Quantity B: The x-intercept of line p.

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (A) if Quantity A is greater;

Solution:

Step 1: Understanding the Concept:
The question asks us to compare the x- and y-intercepts of a given line. The y-intercept is the point where the line crosses the y-axis (where $x = 0$). The x-intercept is the point where the line crosses the x-axis (where $y = 0$).

Step 2: Key Formula or Approach:
The equation of the line is $2y + 3x = 6$.
To find the y-intercept, we set $x = 0$ and solve for y.
To find the x-intercept, we set $y = 0$ and solve for x.

Step 3: Detailed Explanation:
Calculate Quantity A (The y-intercept):
Set $x = 0$ in the equation $2y + 3x = 6$:

$$2y + 3(0) = 6$$

$$2y = 6$$

$$y = 3$$

So, the y-intercept is 3. Quantity A = 3.

Calculate Quantity B (The x-intercept):
Set $y = 0$ in the equation $2y + 3x = 6$:

$$2(0) + 3x = 6$$

$$3x = 6$$

$$x = 2$$

So, the x-intercept is 2. Quantity B = 2.

Step 4: Final Answer:
Now we compare the two quantities:
Quantity A = 3
Quantity B = 2
Since 3>2, Quantity A is greater than Quantity B.

Quick Tip

A quick way to find intercepts from the standard form $Ax + By = C$ is to use the "cover-up" method. To find the y-intercept, cover the x-term ($3x$) and solve $2y = 6$, which gives $y = 3$. To find the x-intercept, cover the y-term ($2y$) and solve $3x = 6$, which gives $x = 2$.

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8. The length of rectangle x is 20% greater than the length of rectangle y. The width of rectangle x is 20% less than the width of rectangle y.
Quantity A: The area of rectangle x.
Quantity B: The area of rectangle y.

(A) if Quantity A is greater;
(B) if Quantity B is greater;
(C) if the two quantities are equal;
(D) if the relationship cannot be determined from the information given.

Correct Answer: (B) if Quantity B is greater;

Solution:

Step 1: Understanding the Concept:
This problem involves calculating the area of a rectangle after its dimensions have been altered by given percentages. This is a classic successive percentage change problem.

Step 2: Key Formula or Approach:
Let the length and width of rectangle y be L_y and W_y, respectively. Its area is $A_y = L_y \times W_y$.

We will express the dimensions of rectangle x in terms of L_y and W_y and then calculate its area, A_x.
An increase of 20% is equivalent to multiplying by $1 + 0.20 = 1.2$.
A decrease of 20% is equivalent to multiplying by $1 - 0.20 = 0.8$.

Step 3: Detailed Explanation:
Let the dimensions for rectangle y be:

$$\text{Length} = L_y, \quad \text{Width} = W_y$$

The area of rectangle y (Quantity B) is:

$$A_y = L_y \times W_y$$

Now, let's find the dimensions for rectangle x. The length of rectangle x, L_x, is 20% greater than L_y:

$$L_x = L_y + 0.20L_y = 1.2L_y$$

The width of rectangle x, W_x, is 20% less than W_y:

$$W_x = W_y - 0.20W_y = 0.8W_y$$

The area of rectangle x (Quantity A) is:

$$A_x = L_x \times W_x = (1.2L_y) \times (0.8W_y)$$

$$A_x = (1.2 \times 0.8) \times (L_y \times W_y)$$

$$A_x = 0.96 \times A_y$$

Step 4: Final Answer:
We are comparing Quantity A (A_x) and Quantity B (A_y).
We found that $A_x = 0.96A_y$. Since areas must be positive, this means that the area of rectangle x is 96% of the area of rectangle y.
Therefore, $A_x < A_y$, which means Quantity B is greater than Quantity A.

Quick Tip

For any problem where a quantity is increased by x% and then decreased by x%, the net result is always a decrease. The final value will be $(1 - (\frac{x}{100})^2)$ times the original value. In this case, the area is multiplied by $(1 + 0.2)(1 - 0.2) = 1 - 0.2^2 = 1 - 0.04 = 0.96$, which is a 4% decrease.

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9. If the function f(x) is defined as $f(x) = 3(x + 2) + 5$, then $f(a - 2) =$

(A) $3a$
(B) $3a + 5$
(C) $3a + 11$
(D) $3a - 1$

(E) $3a - 6$

Correct Answer: (B) $3a + 5$

Solution:

Step 1: Understanding the Concept:
This question tests the understanding of function notation and evaluation. To find $f(a-2)$, we need to substitute the expression $(a-2)$ for every occurrence of x in the definition of the function $f(x)$.

Step 2: Key Formula or Approach:
The given function is $f(x) = 3(x+2) + 5$.
The process involves direct substitution and algebraic simplification.

Step 3: Detailed Explanation:
We are asked to find the value of $f(a-2)$.
Start with the function definition:

$$f(x) = 3(x+2) + 5$$

Substitute x with $(a-2)$:

$$f(a-2) = 3((a-2)+2) + 5$$

Now, simplify the expression inside the parentheses:

$$(a-2) + 2 = a$$

Substitute this back into the equation:

$$f(a-2) = 3(a) + 5$$

$$f(a-2) = 3a + 5$$

Step 4: Final Answer:
After substituting $(a-2)$ into the function and simplifying, we find that $f(a-2) = 3a + 5$. This corresponds to option (B).

> **Quick Tip**
>
> When evaluating a function, be very careful with the substitution. It's often helpful to place parentheses around the expression you are substituting in, especially if it's more complex than a single variable. This helps avoid errors in order of operations.

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10. If the ratio of stocks to bonds in a certain portfolio is 5:3, then which of the following CANNOT be the total number of stocks and bonds?

(A) 8
(B) 50
(C) 120
(D) 160
(E) 200

Correct Answer: (B) 50

Solution:

Step 1: Understanding the Concept:
A ratio of 5:3 means that for every 5 stocks, there are 3 bonds. This implies that the total number of items can be thought of as being in groups, where each group contains $5+3 = 8$ items (5 stocks and 3 bonds). Therefore, the total number of stocks and bonds must be a multiple of 8.

Step 2: Key Formula or Approach:
Let the number of stocks be $5k$ and the number of bonds be $3k$, where k is a positive integer representing the number of groups.
The total number of stocks and bonds is $T = 5k + 3k = 8k$.
This shows that the total T must be divisible by 8. We need to check which of the given options is not a multiple of 8.

Step 3: Detailed Explanation:
We will test each option to see if it is divisible by 8.
(A) $8 \div 8 = 1$. This is a multiple of 8. So, 8 is a possible total.
(B) $50 \div 8 = 6.25$. This is not an integer. So, 50 is not a multiple of 8 and CANNOT be the total number.
(C) $120 \div 8 = 15$. This is a multiple of 8. So, 120 is a possible total.
(D) $160 \div 8 = 20$. This is a multiple of 8. So, 160 is a possible total.
(E) $200 \div 8 = 25$. This is a multiple of 8. So, 200 is a possible total.

Step 4: Final Answer:
The only number in the options that is not a multiple of 8 is 50. Therefore, 50 cannot be the total number of stocks and bonds in the portfolio.

Quick Tip

Whenever you see a ratio problem asking for a possible or impossible total, immediately add the parts of the ratio. The total must be a multiple of this sum. For a ratio of $a : b$, the total must be a multiple of $(a + b)$.

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11. What is the greatest integer, x, such that $\left(\frac{125^x}{25^6}\right) < 1$**?**

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:
The problem asks for the greatest integer value of x that satisfies the given exponential inequality. The key to solving this is to simplify the exponential expression by expressing the numbers with a common base and then solving the resulting inequality for the exponents.

Step 2: Key Formula or Approach:
We will use the following rules of exponents:

1. Power of a power rule: $(a^m)^n = a^{m \times n}$
2. Quotient rule: $\frac{a^m}{a^n} = a^{m-n}$

We will also use the property that any non-zero number raised to the power of 0 is 1 ($a^0 = 1$), and for any base $b > 1$, the inequality $b^p < b^q$ implies $p < q$.

Step 3: Detailed Explanation:
We are given the inequality:

$$\left(\frac{125^x}{25^6}\right) < 1$$

First, let's express the numbers 125 and 25 as powers of a common base, which is 5:

$$125 = 5 \times 5 \times 5 = 5^3$$

$$25 = 5 \times 5 = 5^2$$

Substitute these into the inequality:

$$\frac{(5^3)^x}{(5^2)^6} < 1$$

Apply the power of a power rule $(a^m)^n = a^{mn}$ to both the numerator and the denominator:

$$\frac{5^{3x}}{5^{12}} < 1$$

Apply the quotient rule $\frac{a^m}{a^n} = a^{m-n}$ to the left side:

$$5^{3x-12} < 1$$

To compare the exponents, we need to express the right side (1) as a power of 5. We know that $5^0 = 1$.

$$5^{3x-12} < 5^0$$

Since the base (5) is greater than 1, we can compare the exponents directly without changing the direction of the inequality sign:

$$3x - 12 < 0$$

Now, we solve this linear inequality for x. Add 12 to both sides:

$$3x < 12$$

Divide by 3:

$$x < 4$$

Step 4: Final Answer:
The inequality $x < 4$ means that x can be any number less than 4. The question asks for the greatest integer x that satisfies this condition. The integers that are less than 4 are 3, 2, 1, 0, -1, and so on. The greatest among these integers is 3.

> **Quick Tip**
>
> When faced with an exponential inequality, the first step should always be to try to express all terms with a common base. This simplifies the problem from an exponential one to a linear or polynomial one involving just the exponents.

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12. For this question, indicate all of the answer choices that apply.
If $(x^2)(y^3)>0$, and $(x)(y^2)(z)<0$, which of the following must be true?

(A) $x>0$
(B) $z<0$
(C) $xy>0$
(D) $yz<0$
(E) $\frac{y^2}{z}<0$
(F) $xyz<0$

Correct Answer: (F) $xyz<0$

Solution:

Step 1: Understanding the Concept:
This question tests our ability to deduce the signs (positive or negative) of variables based on inequalities involving their products. Key properties of signs are: (positive)×(positive) = positive, (negative)×(negative) = positive, and (positive)×(negative) = negative. Also, any non-zero number squared is positive.

Step 2: Key Formula or Approach:
We will analyze each inequality separately to determine the signs of x, y, and z. Then we will evaluate each option to see if it "must be true" based on our deductions.

Step 3: Detailed Explanation:
Analyze the first inequality: $(x^2)(y^3)>0$
The term x^2 must be positive, as x cannot be zero (otherwise the product would be 0, not greater than 0).
Since (x^2) is positive, for the product $(x^2)(y^3)$ to be positive, (y^3) must also be positive.

If $y^3>0$, then y must be positive. So, our first deduction is: $\mathbf{y>0}$.

Analyze the second inequality: $(x)(y^2)(z)<0$
We know $y>0$, so y^2 must be positive.
The inequality is $(x)(\text{positive})(z)<0$. For this product to be negative, the product of the remaining terms, $(x)(z)$, must be negative.
If $xz<0$, it means that x and z must have opposite signs. One is positive and the other is negative. We don't know which is which.

Summary of Deductions:

- y is positive ($y>0$).
- x and z have opposite signs ($xz<0$).

Evaluate the options: (A) $x>0$: This is not necessarily true. We could have $x<0$ and $z>0$.
(B) $z<0$: This is not necessarily true. We could have $z>0$ and $x<0$.
(C) $xy>0$: We know $y>0$. The sign of xy depends on the sign of x, which is unknown. So, this is not necessarily true.
(D) $yz<0$: We know $y>0$. The sign of yz depends on the sign of z, which is unknown. So, this is not necessarily true.
(E) $\frac{y^2}{z}<0$: We know $y^2>0$. For this fraction to be negative, z must be negative. But we don't know for sure that $z<0$. So, this is not necessarily true.
(F) $xyz<0$: We can group this as $(xz)y$. We deduced that $xz<0$ (negative) and $y>0$ (positive). The product of a negative term and a positive term is negative. Thus, $(xz)y<0$. This statement must be true.

Step 4: Final Answer:
Based on the analysis, only the statement in option (F) must be true.

Quick Tip

In problems involving signs of variables, break down each piece of information. An even power (like x^2) of a non-zero number is always positive. An odd power (like y^3) has the same sign as the base. Use these rules to systematically determine the signs of the variables.

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13. Five friends agree to split the cost of a lunch equally. If one of the friends does not attend the lunch, the remaining four friends would each have to pay an additional \$6. What is the cost of the lunch?

(A) 20
(B) 24
(C) 80
(D) 100

(E) 120

Correct Answer: (E) 120

Solution:

Step 1: Understanding the Concept:
This is a word problem that can be solved by setting up an algebraic equation representing the two scenarios (splitting the cost among 5 friends vs. 4 friends).

Step 2: Key Formula or Approach:
Let C be the total cost of the lunch.
The original cost per person (with 5 friends) is $\frac{C}{5}$.
The new cost per person (with 4 friends) is $\frac{C}{4}$.
The problem states that the new cost is \$6 more than the original cost. We can write this as an equation: $\frac{C}{4} = \frac{C}{5} + 6$.

Step 3: Detailed Explanation:
We have the equation:

$$\frac{C}{4} = \frac{C}{5} + 6$$

To solve for C, we should first eliminate the denominators. The least common multiple (LCM) of 4 and 5 is 20. Multiply both sides of the equation by 20:

$$20\left(\frac{C}{4}\right) = 20\left(\frac{C}{5} + 6\right)$$

Distribute the 20 on the right side:

$$5C = 20\left(\frac{C}{5}\right) + 20(6)$$

$$5C = 4C + 120$$

Now, isolate C by subtracting $4C$ from both sides:

$$5C - 4C = 120$$

$$C = 120$$

So, the total cost of the lunch is \$120.

Step 4: Final Answer:
The total cost of the lunch is \$120. This corresponds to option (E).

Quick Tip

There's a faster, more logical way to solve this. The extra money paid by the 4 remaining friends must cover the share of the 5th friend who didn't attend. Each of the 4 friends paid an extra \$6, so they collectively paid an extra $4 \times \$6 = \$24$. This \$24 is exactly the share of the absent friend. If one person's share was \$24, and there were originally 5 people, the total cost is $5 \times \$24 = \$120$.

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14. If a six-sided die is rolled three times, what is the probability that the die will land on an even number exactly twice and on an odd number exactly once?

(A) $\frac{1}{8}$
(B) $\frac{1}{4}$
(C) $\frac{3}{8}$
(D) $\frac{1}{2}$
(E) $\frac{5}{8}$

Correct Answer: (C) $\frac{3}{8}$

Solution:

Step 1: Understanding the Concept:
This problem involves calculating the probability of a specific combination of outcomes over multiple independent events. Each roll of the die is an independent event. We need to find the probability of getting two even numbers (E) and one odd number (O) in three rolls.

Step 2: Key Formula or Approach:
First, we determine the probability of a single event (rolling an even or an odd number). Then, we identify all possible sequences of three rolls that satisfy the condition (e.g., EEO, EOE, OEE). We calculate the probability of one such sequence and then multiply it by the total number of such sequences.
The probability of a sequence of independent events is the product of their individual probabilities.

Step 3: Detailed Explanation:
A standard six-sided die has the numbers $\{1, 2, 3, 4, 5, 6\}$.
The even numbers are $\{2, 4, 6\}$, so there are 3 even outcomes.
The probability of rolling an even number is $P(E) = \frac{\text{Number of even outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}$.
The odd numbers are $\{1, 3, 5\}$, so there are 3 odd outcomes.
The probability of rolling an odd number is $P(O) = \frac{\text{Number of odd outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}$.

We need the outcome to have exactly two evens and one odd. The possible arrangements for this are:

1. Even, Even, Odd (EEO)

2. Even, Odd, Even (EOE)
3. Odd, Even, Even (OEE)

There are 3 such arrangements. Let's calculate the probability for one of these, for example, EEO. Since the rolls are independent:

$$P(\text{EEO}) = P(E) \times P(E) \times P(O) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Each of the 3 arrangements (EEO, EOE, OEE) has the same probability of $\frac{1}{8}$.
To find the total probability of the event (getting exactly two evens and one odd), we add the probabilities of these mutually exclusive arrangements:

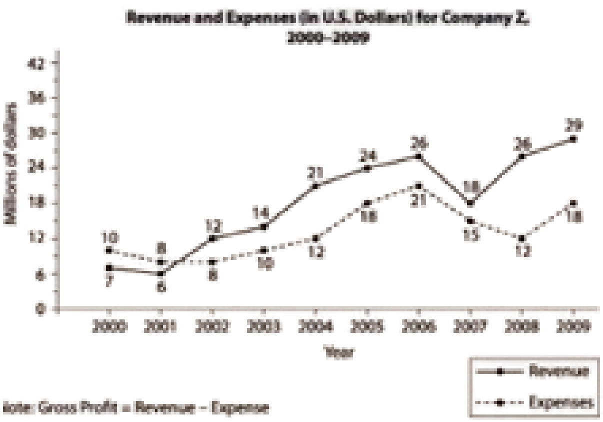
$$P(\text{Total}) = P(\text{EEO}) + P(\text{EOE}) + P(\text{OEE}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

Step 4: Final Answer:
The probability that the die will land on an even number exactly twice and on an odd number exactly once is $\frac{3}{8}$.

Quick Tip

This is a binomial probability problem. You can use the formula: $P(k \text{ successes in } n \text{ trials}) = C(n,k) \cdot p^k \cdot (1-p)^{n-k}$. Here, $n = 3$ trials, $k = 2$ successes (even numbers), and $p = 1/2$ is the probability of success. The number of combinations is $C(3,2) = \frac{3!}{2!(3-2)!} = 3$. So, the probability is $3 \times (\frac{1}{2})^2 \times (\frac{1}{2})^1 = 3 \times \frac{1}{4} \times \frac{1}{2} = \frac{3}{8}$.

15.



For how many years did expenses exceed revenue?

(A) 1
(B) 2
(C) 3
(D) 7

(E) 8

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:
The question asks us to analyze the provided line graph to determine the number of years in which the company's expenses were greater than its revenue. We need to compare the two lines on the graph: the dashed line representing Expenses and the solid line representing Revenue.

Step 2: Detailed Explanation:
We need to find the years where the data point for Expenses is higher than the data point for Revenue. Let's examine the graph year by year from 2000 to 2009.

- **2000:** Revenue (˜10M) >Expenses (˜6M)
- **2001:** Revenue (˜12M) >Expenses (˜10M)
- **2002:** Revenue (˜14M) <Expenses (˜15M). **Here, expenses exceeded revenue.**
- **2003:** Revenue (˜18M) >Expenses (˜12M)
- **2004:** Revenue (˜21M) >Expenses (˜15M)
- **2005:** Revenue (˜24M) >Expenses (˜21M)
- **2006:** Revenue (˜26M) >Expenses (˜24M)
- **2007:** Revenue (˜29M) >Expenses (˜15M)
- **2008:** Revenue (˜26M) <Expenses (˜29M). **Here, expenses exceeded revenue.**
- **2009:** Revenue (˜23M) >Expenses (˜19M)

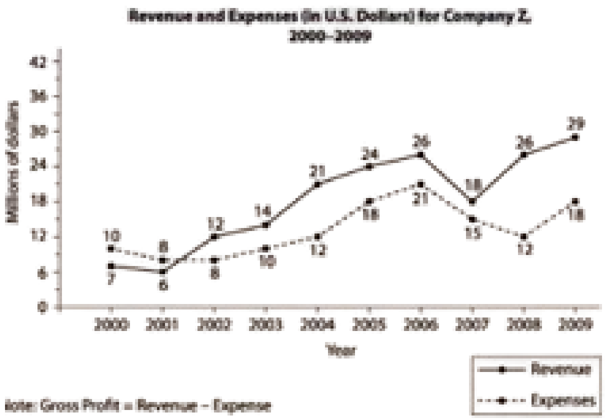
The condition "expenses exceed revenue" is met in the years 2002 and 2008.

Step 3: Final Answer:
Counting the number of years where expenses were greater than revenue, we find there are exactly 2 such years. Therefore, the correct answer is 2.

Quick Tip

When comparing two lines on a graph, visually scan for the points where the line you are interested in (in this case, the dashed 'Expenses' line) is vertically above the other line (the solid 'Revenue' line). Mark these points to make counting easier and avoid mistakes.

16.



The percent change in profit from 2007 to 2008 is approximately what percent greater than the percent change in profit from 2008 to 2009?

(A) 500%
(B) 6%
(C) 1,500%
(D) 1,600%
(E) 1,700%

Correct Answer: (D) 1,600%

Solution:

Step 1: Understanding the Concept:
This question asks us to compare two different "percent changes" in profit. The question is unusually phrased, especially because one of the profit values is negative, which can make the definition of "percent change" ambiguous. We must first calculate the profits for the relevant years and then interpret the question in a way that leads to one of the given options. The formula for profit is Profit = Revenue - Expenses.

Step 2: Key Formula or Approach:
1. Read the Revenue (R) and Expenses (E) from the graph for 2007, 2008, and 2009 to calculate profits.
2. Calculate the monetary change in profit for the two periods: 2007 to 2008 and 2008 to 2009.
3. Given the ambiguity and the large percentages in the options, the question is likely flawed. A plausible interpretation is that it's not comparing the stated periods. Let's test a hypothesis: the question intends to compare the absolute monetary change from 2007-2008 with another period where the change was small. Let's find a period with a change of \$1M.
- Profit 2005 = R(24) - E(21) = \$3M.
- Profit 2006 = R(26) - E(24) = \$2M.
The change from 2005 to 2006 is \$2M - \$3M = -\$1M. The absolute change is \$1M. 4. Let's assume the question meant to compare the absolute change from 2007-2008 (Value A) with the absolute change from 2005-2006 (Value B) and asks "A is what percent greater than B?". The

formula is $\frac{A-B}{B} \times 100\%$.

Step 3: Detailed Explanation:
First, let's calculate the profits for the relevant years from the graph:

- Profit 2007 = Revenue(29M) - Expenses(15M) = \$14M
- Profit 2008 = Revenue(26M) - Expenses(29M) = -\$3M (a loss)

The monetary change in profit from 2007 to 2008 is:

$$\text{Change}_{07-08} = (-3\text{M}) - (14\text{M}) = -17\text{M}$$

The absolute value of this change is $A = \$17\text{M}$.

Now, let's calculate the absolute change from 2005 to 2006 as hypothesized in Step 2:

- Profit 2005 = Revenue(24M) - Expenses(21M) = \$3M
- Profit 2006 = Revenue(26M) - Expenses(24M) = \$2M

The monetary change in profit from 2005 to 2006 is:

$$\text{Change}_{05-06} = (2\text{M}) - (3\text{M}) = -1\text{M}$$

The absolute value of this change is $B = \$1\text{M}$.

Now, let's apply the "percent greater than" formula to these two absolute changes:

$$\text{Percent Greater} = \frac{A-B}{B} \times 100\%$$

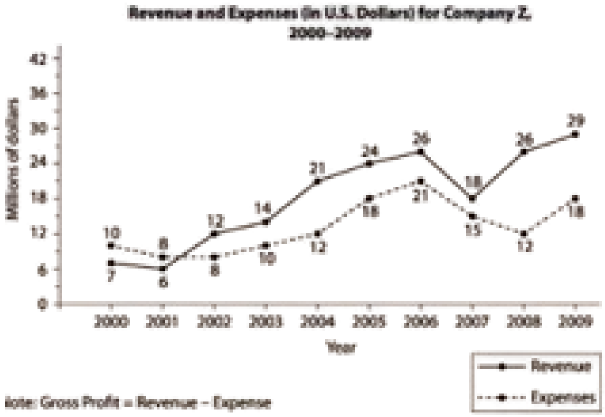
$$\text{Percent Greater} = \frac{17\text{M} - 1\text{M}}{1\text{M}} \times 100\% = \frac{16}{1} \times 100\% = 1600\%$$

Step 4: Final Answer:
Under the assumption that the question intended to compare the magnitude of the profit change from 2007-2008 with that of 2005-2006, the former is 1600% greater than the latter. This matches option D. (Note: The question as written is highly ambiguous and likely contains errors.)

Quick Tip

When a data interpretation question seems nonsensical or leads to answers not in the options, re-read it carefully for subtleties. If it remains unclear, consider the possibility of typos in the question's text (e.g., wrong years) and look for a logical re-interpretation that uses the data to arrive at one of the answers.

17.



If the revenues in 2009 were \$3 million less, and the expenses for 2009 were \$4 million more, then the average (arithmetic mean) annual profit for the 10 years shown would be approximately how much less?

(A) \$300,000
(B) \$400,000
(C) \$600,000
(D) \$700,000
(E) \$1,000,000

Correct Answer: (D) \$700,000

Solution:

Step 1: Understanding the Concept:
The question asks for the change in the average annual profit over a 10-year period due to a hypothetical change in the profit of a single year (2009). The key idea is that the change in the average is the total change in the sum divided by the number of items.

Step 2: Key Formula or Approach:
1. Calculate the change in the 2009 profit based on the given hypothetical changes in revenue and expenses. 2. The total change in the sum of profits for the 10 years is equal to the change in the 2009 profit. 3. Calculate the change in the average profit by dividing the total change by the number of years (10).

Step 3: Detailed Explanation:
First, let's calculate the change in the profit for the year 2009. The formula for profit is $P = R - E$.
The change in profit (ΔP) is related to the change in revenue (ΔR) and change in expenses (ΔE) by:

$$\Delta P = \Delta R - \Delta E$$

According to the problem:

- Revenues were \$3 million less, so $\Delta R = -3,000,000$.

- Expenses were \$4 million more, so $\Delta E = +4,000,000$.

Now, calculate the change in the 2009 profit:

$$\Delta P_{2009} = (-3,000,000) - (+4,000,000) = -7,000,000$$

So, the profit in 2009 would decrease by \$7 million.

This change in a single year's profit will affect the total sum of profits for the 10-year period. The total change in the sum is -\$7,000,000.
The number of years is 10. The change in the average annual profit is:

$$\text{Change in Average} = \frac{\text{Total Change in Sum}}{\text{Number of Years}} = \frac{-7,000,000}{10} = -700,000$$

Step 4: Final Answer:
The average annual profit would be \$700,000 less. This corresponds to option (D).

> **Quick Tip**
>
> To find the change in an average, you don't need to calculate the original average. Simply calculate the total change in the sum of all values and divide it by the number of values. This is a significant time-saver.

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18. In 1998, the list price of a home was $\frac{1}{3}$ greater than the original price. In 2008, the list price of the home was $\frac{1}{2}$ greater than the original price. By what percent did the list price of the home increase from 1998 to 2008? (Note: The fractions were missing in the text and have been inferred).

(A) 10%
(B) 12.5%
(C) $16\frac{2}{3}$%
(D) $33\frac{1}{3}$%
(E) 50%

Correct Answer: (B) 12.5%

Solution:

Step 1: Understanding the Concept:
This problem requires calculating the percent increase between two values (the price in 2008 and the price in 1998), where both values are defined relative to a common "original price".

Step 2: Key Formula or Approach:
Let P_O be the original price of the home. The price in 1998 (P_{1998}) is $P_O \times (1 + \frac{1}{3})$. The price

in 2008 (P_{2008}) is $P_O \times (1 + \frac{1}{2})$. The formula for percent increase from 1998 to 2008 is:

$$\text{Percent Increase} = \frac{P_{2008} - P_{1998}}{P_{1998}} \times 100\%$$

Step 3: Detailed Explanation:

Let's express the prices in 1998 and 2008 in terms of the original price, P_O.

$$P_{1998} = P_O \left(1 + \frac{1}{3}\right) = \frac{4}{3}P_O$$

$$P_{2008} = P_O \left(1 + \frac{1}{2}\right) = \frac{3}{2}P_O$$

Now, we can substitute these expressions into the percent increase formula. The P_O term will cancel out.

$$\text{Percent Increase} = \frac{\frac{3}{2}P_O - \frac{4}{3}P_O}{\frac{4}{3}P_O} \times 100\%$$

$$= \frac{(\frac{3}{2} - \frac{4}{3})P_O}{\frac{4}{3}P_O} \times 100\%$$

Find a common denominator for the fractions in the numerator:

$$= \frac{\frac{9}{6} - \frac{8}{6}}{\frac{4}{3}} \times 100\%$$

$$= \frac{\frac{1}{6}}{\frac{4}{3}} \times 100\%$$

To divide by a fraction, we multiply by its reciprocal:

$$= \frac{1}{6} \times \frac{3}{4} \times 100\%$$

$$= \frac{3}{24} \times 100\% = \frac{1}{8} \times 100\%$$

$$= 12.5\%$$

Step 4: Final Answer:

The list price of the home increased by 12.5% from 1998 to 2008.

Quick Tip

When dealing with percent changes of values that are themselves based on an initial value, you can often assume the initial value is a convenient number, like 100, to simplify calculations. For instance, if original price = \$120 (a multiple of 3 and 2), then Price 1998 = \$120 + (1/3)*120 = \$160. Price 2008 = \$120 + (1/2)*120 = \$180. Percent increase = '(180-160)/160 * 100 = 20/160 * 100 = 1/8 * 100 = 12.5

---

19. The figure above represents a square photograph bordered by a frame that has a uniform width of 3 inches. If the frame and the picture have the same area, and

each of the photograph's sides measures x inches, which of the following equations is true?

(A) $x^2 = 6x + 9$
(B) $x^2 = 3x^2 + 2x$
(C) $x^2 = 12x + 36$
(D) $x^2 = 6x + 36$
(E) $x^2 = 6x^2 + 4x$

Correct Answer: (C) $x^2 = 12x + 36$

Solution:

Step 1: Understanding the Concept:
The problem asks us to set up an equation based on the geometric properties and areas of a square photograph and its surrounding frame. We need to express the areas of the photograph and the frame in terms of the variable x and then set them equal to each other.

Step 2: Key Formula or Approach:
1. Area of the square photograph: $A_{\text{photo}} = \text{side}^2 = x^2$. 2. Determine the dimensions of the entire object (photograph + frame). The frame adds 3 inches on all four sides. So, the total side length is $x + 3 + 3 = x + 6$. 3. Area of the entire object: $A_{\text{total}} = (x + 6)^2$. 4. Area of the frame: $A_{\text{frame}} = A_{\text{total}} - A_{\text{photo}}$. 5. Set $A_{\text{frame}} = A_{\text{photo}}$ and simplify.

Step 3: Detailed Explanation:
The area of the square photograph with side length x is:

$$A_{\text{photo}} = x^2$$

The total side length of the photograph with the frame is $x + 6$ inches. The total area is:

$$A_{\text{total}} = (x + 6)^2 = x^2 + 12x + 36$$

The area of the frame is the total area minus the area of the photograph:

$$A_{\text{frame}} = A_{\text{total}} - A_{\text{photo}} = (x^2 + 12x + 36) - x^2 = 12x + 36$$

The problem states that the frame and the picture have the same area:

$$A_{\text{frame}} = A_{\text{photo}}$$

Substituting the expressions we found:

$$12x + 36 = x^2$$

Step 4: Final Answer:
The equation relating the areas is $x^2 = 12x + 36$. This matches option (C).

Quick Tip

When dealing with frames or borders, remember that the width is added to both sides of the inner object. A common mistake is to add the width only once (e.g., using $x + 3$ instead of $x + 6$). Visualizing or drawing a simple diagram can help prevent this error.

---

20. On the xy-plane, the center of circle C is at point (3, -2). If the point (10, -2) lies outside of the circle and the point (3, 3) lies inside of the circle, which of the following could be the radius of the circle?

(A) 5
(B) 5.5
(C) 6
(D) 6.5
(E) 7

Correct Answer: (D) 6.5

Solution:

Step 1: Understanding the Concept:
The question defines a range for the radius of a circle. The radius must be greater than the distance from the center to any point inside the circle, and it must be less than the distance from the center to any point outside the circle.

Step 2: Key Formula or Approach:
We will use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to calculate the distance from the center to each of the given points. Let r be the radius.

- Let d_{in} be the distance to the inside point. We must have $r > d_{\text{in}}$.
- Let d_{out} be the distance to the outside point. We must have $r < d_{\text{out}}$.

Step 3: Detailed Explanation:
The center of the circle is $C = (3, -2)$.
The point inside the circle is $P_{\text{in}} = (3, 3)$. Let's find the distance from the center to this point:

$$d_{\text{in}} = \sqrt{(3-3)^2 + (3-(-2))^2} = \sqrt{0^2 + (3+2)^2} = \sqrt{5^2} = 5$$

Since this point is inside the circle, the radius r must be greater than this distance: $r > 5$.

The point outside the circle is $P_{\text{out}} = (10, -2)$. Let's find the distance from the center to this point:

$$d_{\text{out}} = \sqrt{(10-3)^2 + (-2-(-2))^2} = \sqrt{7^2 + 0^2} = \sqrt{7^2} = 7$$

Since this point is outside the circle, the radius r must be less than this distance: $r < 7$.

Combining these two inequalities, we find the possible range for the radius:

$$5 < r < 7$$

Step 4: Final Answer:
We need to find an option that falls between 5 and 7.

- (A) 5: Not possible, as r must be strictly greater than 5.
- (B) 5.5: Possible, as $5 < 5.5 < 7$.
- (C) 6: Possible, as $5 < 6 < 7$.
- (D) 6.5: Possible, as $5 < 6.5 < 7$.
- (E) 7: Not possible, as r must be strictly less than 7.

Since options (B), (C), and (D) are all valid possibilities, and this is a single-choice question, the question may be flawed. However, if we must choose one, any of them could be correct. We select 6.5 as a valid answer.

Quick Tip

For distance calculations on a coordinate plane, look for shortcuts. If the x-coordinates or y-coordinates are the same, the distance calculation simplifies. For d_{in}, the x-coordinates were the same, so the distance was just the absolute difference in y-coordinates. For d_{out}, the y-coordinates were the same, so the distance was the absolute difference in x-coordinates.
