

GRE 2024 Quant Practice Test 20 with Solutions

Time Allowed : 1 Hour 58 Minutes

Maximum Marks : 340

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GRE General Test is 1 hour and 58 minutes long (with one optional 10-minute break) and consists of 54 questions in total.
2. The GRE exam is comprised of three sections:
 - Quantitative Reasoning: 27 questions, 47 minutes
 - Verbal Reasoning: 27 questions, 41 minutes
3. You can answer the two sections in any order.
4. As you move through a section, you can skip questions, flag them for review, and return to them later within the same section.
5. When you have answered all questions in a section, you can review your responses before time expires.
6. If there is no time remaining in the section, you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
7. Each review screen includes a numbered list of the questions in that section and indicates the questions you flagged.
8. Clicking a question number will take you to that specific question.
9. You may change any answer within the time allowed for that section.

1. Given the equations $x^3 - 30y = 4$ and $1797 + 3y = 15x$, compare Quantity A and Quantity B.

Quantity A: y

Quantity B: 1

- (A) The relationship cannot be determined from the information given.
(B) The quantities are equal.
(C) Quantity A is greater.
(D) Quantity B is greater.

Correct Answer: (D) Quantity B is greater.

Solution:

Step 1: Understanding the Concept:

We are given a system of two equations with two variables, x and y . We need to find the value of y , or at least determine its relationship to 1, to compare Quantity A (y) and Quantity B (1).

Step 2: Key Formula or Approach:

We will use substitution to solve the system of equations. We can express one variable in terms of the other from one equation and substitute it into the second equation.

Step 3: Detailed Explanation:

The given equations are:

$$30xy = 4 \Rightarrow xy = \frac{4}{30} = \frac{2}{15} \quad (\text{Equation 1})$$

$$1797 + 3y = 15x \Rightarrow 15x - 3y = 1797$$

Dividing the second equation by 3, we get:

$$5x - y = 599 \Rightarrow y = 5x - 599 \quad (\text{Equation 2})$$

Now, we substitute the expression for y from Equation 2 into Equation 1:

$$x(5x - 599) = \frac{2}{15}$$

This gives a quadratic equation, which might be complicated to solve due to the large numbers. Let's try to analyze the possible values of y .

From Equation 1, $xy = 2/15 > 0$. This implies that x and y must have the same sign (both positive or both negative).

Case 1: x and y are both positive.

If $y \geq 0$, then from Equation 2, we must have $5x - 599 > 0$.

This means $5x > 599$, or $x > 119.8$.

Now, let's look at Equation 1 again: $y = \frac{2}{15x}$.

Since $x > 119.8$, the value of y will be:

$$y < \frac{2}{15 \times 119.8} = \frac{2}{1797}$$

In this case, y is a very small positive number, which is clearly less than 1.

Case 2: x and y are both negative.

If y is negative, it is automatically less than 1. For completeness, let's check if this case is possible. Let $x < 0$. From Equation 2, $y = 5x - 599$. Since $5x$ is negative, $5x - 599$ will be a large negative number. This is consistent with y being negative.

In both possible cases, the value of y is less than 1.

Step 4: Final Answer:

Since y is always less than 1, Quantity B (which is 1) is greater than Quantity A (y).

Quick Tip

In quantitative comparison questions, you don't always need to find the exact numerical value. Analyzing the constraints and relationships between variables can often lead to the answer more quickly. Here, checking the signs of x and y was the key.

2. John has \$50 for soda and he must buy both diet and regular sodas. His total order must have at exactly two times as many cans of diet soda as cans of regular soda. What is the greatest number of cans of diet soda John can buy if regular soda is \$0.50 per can and diet soda is \$0.75 per can?

- (A) None of the other answers
- (B) 51
- (C) 25
- (D) 75
- (E) 50

Correct Answer: (E) 50

Solution:

Step 1: Understanding the Concept:

This is a word problem that can be modeled with a system of linear equations and inequalities. We need to maximize the number of diet sodas purchased under a budget constraint and a ratio constraint.

Step 2: Key Formula or Approach:

Let d be the number of cans of diet soda and r be the number of cans of regular soda.

We can set up the following equations based on the problem statement:

1. Budget Constraint: $0.75d + 0.50r \leq 50$

2. Ratio Constraint: $d = 2r$

We need to find the maximum possible integer value for d .

Step 3: Detailed Explanation:

We have two conditions:

$$0.75d + 0.50r \leq 50$$

$$d = 2r$$

From the second equation, we can express r in terms of d : $r = d/2$.

Now, substitute this expression for r into the budget inequality:

$$0.75d + 0.50 \left(\frac{d}{2} \right) \leq 50$$

Simplify the inequality:

$$0.75d + 0.25d \leq 50$$

$$1.00d \leq 50$$

$$d \leq 50$$

This means the number of diet soda cans cannot exceed 50. The greatest possible number of cans of diet soda is 50.

Let's check if this solution is valid.

If $d = 50$, then $r = d/2 = 50/2 = 25$.

The total cost would be:

$$\text{Cost} = 0.75 \times 50 + 0.50 \times 25 = 37.50 + 12.50 = \$50.00$$

This meets the budget exactly. Also, John buys both diet (50 cans) and regular (25 cans) sodas, satisfying all conditions.

Step 4: Final Answer:

The greatest number of cans of diet soda John can buy is 50.

Quick Tip

When dealing with "greatest number" or "maximum value" problems, set up an inequality. Solve for the variable in question. The maximum integer value that satisfies the inequality is usually the answer, but always double-check it against the original problem's conditions.

3. Find the intersection of the following two equations:

$$3x + 4y = 6$$

$$15x - 4y = 3$$

- (A) (1, 0.5)
- (B) (0.2, 0)
- (C) (18, 0)
- (D) (3, 4)
- (E) (0.5, 1.125)

Correct Answer: (E) (0.5, 1.125)

Solution:

Step 1: Understanding the Concept:

The intersection of two linear equations is the point (x, y) that satisfies both equations simultaneously. We can find this point by solving the system of linear equations.

Step 2: Key Formula or Approach:

We can use the elimination method, as the coefficients of y are opposites (+4 and -4). Adding the two equations will eliminate the y variable.

Equation 1: $3x + 4y = 6$

Equation 2: $15x - 4y = 3$

Step 3: Detailed Explanation:

Add Equation 1 and Equation 2:

$$(3x + 4y) + (15x - 4y) = 6 + 3$$

$$18x = 9$$

Solve for x:

$$x = \frac{9}{18} = \frac{1}{2} = 0.5$$

Now that we have the value of x, substitute it back into either of the original equations to find y. Let's use Equation 1:

$$3x + 4y = 6$$

$$3(0.5) + 4y = 6$$

$$1.5 + 4y = 6$$

Subtract 1.5 from both sides:

$$4y = 6 - 1.5$$

$$4y = 4.5$$

Solve for y:

$$y = \frac{4.5}{4} = \frac{9/2}{4} = \frac{9}{8} = 1.125$$

The point of intersection is (x, y).

Step 4: Final Answer:

The intersection of the two equations is at the point (0.5, 1.125).

Quick Tip

When solving a system of linear equations, always look for the easiest method. If coefficients of one variable are the same or opposites, elimination is often the fastest way. If one variable is already isolated (e.g., $y = mx + b$), substitution is a good choice.

4. One of the roots of the equation $x^2 + kx - 12 = 0$ is 3, and k is a constant.

Quantity A: The value of k

Quantity B: -1

(A) The two quantities are equal.

(B) Quantity A is greater.

(C) Quantity B is greater.

(D) The relationship cannot be determined from the information given.

Correct Answer: (B) Quantity A is greater.

Solution:

Step 1: Understanding the Concept:

A root of an equation is a value that, when substituted for the variable, makes the equation true. If we know a root of a polynomial equation, we can use it to find the value of any unknown coefficients.

Step 2: Key Formula or Approach:

Substitute the given root, $x = 3$, into the equation $x^2 + kx - 12 = 0$ and solve for the constant k .

Step 3: Detailed Explanation:

The given equation is:

$$x^2 + kx - 12 = 0$$

We are told that $x = 3$ is a root. Substitute this value into the equation:

$$(3)^2 + k(3) - 12 = 0$$

$$9 + 3k - 12 = 0$$

Combine the constant terms:

$$3k - 3 = 0$$

Add 3 to both sides:

$$3k = 3$$

Divide by 3:

$$k = 1$$

So, Quantity A, the value of k , is 1.

Quantity B is -1.

Now we compare Quantity A and Quantity B.

$$1 > -1$$

Step 4: Final Answer:

Quantity A is greater than Quantity B.

Quick Tip

This problem tests the fundamental definition of a root. Remember that if 'r' is a root of a polynomial $P(x)$, then $P(r) = 0$. This is a very common type of question in algebra.

5. Two cars start 25 miles apart and drive away from each other in opposite directions at speeds of 50 and 70 miles per hour. In approximately how many minutes will they be 400 miles apart?

- (A) 200
- (B) 3.33
- (C) 187.5
- (D) None of the other answers
- (E) 3.125

Correct Answer: (C) 187.5

Solution:

Step 1: Understanding the Concept:

This is a relative speed problem. When two objects move in opposite directions, their relative speed is the sum of their individual speeds. This relative speed represents how quickly the distance between them is increasing.

Step 2: Key Formula or Approach:

1. Calculate the relative speed of the two cars.
2. Determine the additional distance the cars need to be apart.
3. Use the formula: Time = Distance / Speed.
4. Convert the time from hours to minutes.

Step 3: Detailed Explanation:

The speeds of the two cars are 50 mph and 70 mph. Since they are moving in opposite directions, their relative speed is the sum of their speeds:

$$\text{Relative Speed} = 50 \text{ mph} + 70 \text{ mph} = 120 \text{ mph}$$

The cars start 25 miles apart and need to be 400 miles apart. The additional distance they need to cover is:

$$\text{Distance} = 400 \text{ miles} - 25 \text{ miles} = 375 \text{ miles}$$

Now, we can calculate the time it will take to cover this distance at their relative speed:

$$\text{Time (in hours)} = \frac{\text{Distance}}{\text{Speed}} = \frac{375 \text{ miles}}{120 \text{ mph}}$$

$$\text{Time (in hours)} = \frac{375}{120} = \frac{75}{24} = \frac{25}{8} \text{ hours}$$

The question asks for the time in minutes. We convert hours to minutes by multiplying by 60:

$$\text{Time (in minutes)} = \frac{25}{8} \times 60 = \frac{1500}{8} = \frac{750}{4} = \frac{375}{2}$$

$$\text{Time (in minutes)} = 187.5$$

Step 4: Final Answer:

It will take 187.5 minutes for the cars to be 400 miles apart.

Quick Tip

In relative speed problems, remember: if objects move in opposite directions, add their speeds. If they move in the same direction, subtract the slower speed from the faster speed. Pay close attention to the units (hours vs. minutes) requested in the answer.

6. What is the value of $(5 + x)(10 - y)$ when $x = 3$ and $y = -3$?

- (A) 104
- (B) 108
- (C) 38
- (D) 56

Correct Answer: (A) 104

Solution:

Step 1: Understanding the Concept:

This is a straightforward question about evaluating an algebraic expression. We need to substitute the given numerical values for the variables and perform the arithmetic operations.

Step 2: Key Formula or Approach:

Substitute $x = 3$ and $y = -3$ into the expression $(5 + x)(10 - y)$.

Step 3: Detailed Explanation:

The given expression is:

$$(5 + x)(10 - y)$$

Substitute $x = 3$ and $y = -3$:

$$(5 + 3)(10 - (-3))$$

First, evaluate the expressions inside each parenthesis:

$$(8)(10 + 3)$$

$$(8)(13)$$

Now, perform the multiplication:

$$8 \times 13 = 8 \times (10 + 3) = (8 \times 10) + (8 \times 3) = 80 + 24 = 104$$

Step 4: Final Answer:

The value of the expression is 104.

Quick Tip

Be very careful with signs, especially when substituting negative numbers. A common mistake is to miscalculate $10 - (-3)$ as $10 - 3$. Remember that subtracting a negative is the same as adding a positive.

7. If $2x + y = 9$ and $y - z = 4$ then $2x + z = ?$

- (A) 5
- (B) 13
- (C) Cannot be determined
- (D) 29
- (E) 21

Correct Answer: (A) 5

Solution:

Step 1: Understanding the Concept:

We are given a system of two linear equations with three variables. We need to find the value of a specific expression involving two of these variables. This can often be done by manipulating and combining the given equations to eliminate the third variable.

Step 2: Key Formula or Approach:

We can use the substitution or elimination method. The goal is to eliminate the variable y , which appears in both equations, to find a relationship between x and z .

Equation 1: $2x + y = 9$

Equation 2: $y - z = 4$

Step 3: Detailed Explanation:

Method 1: Substitution

From Equation 1, isolate y :

$$y = 9 - 2x$$

Substitute this expression for y into Equation 2:

$$(9 - 2x) - z = 4$$

Now, we want to find the value of $2x + z$. Let's rearrange the equation:

$$9 - 4 = 2x + z$$

$$5 = 2x + z$$

Method 2: Elimination

Rearrange Equation 2 to align the variables:

$$y = 4 + z$$

Now we have:

$$2x + y = 9$$

$$y = 4 + z$$

Rewrite Equation 2 as $-y + z = -4$. Let's rewrite Equation 1 as $y = 9 - 2x$. Set the two expressions for y equal to each other:

$$9 - 2x = 4 + z$$

Rearrange to find $2x + z$:

$$9 - 4 = 2x + z$$

$$5 = 2x + z$$

Step 4: Final Answer:

The value of $2x + z$ is 5.

Quick Tip

When you are asked to find the value of an expression (like $2x + z$) rather than the individual variables, look for a way to combine the given equations directly to form that expression. This is often faster than solving for each variable separately.

8. $11/(x - 7) + 4/(7 - x) = ?$

- (A) $15/(7 - x)$
- (B) $15/(x - 7)$
- (C) $7/(7 - x)$
- (D) 15
- (E) $(-7)/(7 - x)$

Correct Answer: (E) $(-7)/(7 - x)$

Solution:

Step 1: Understanding the Concept:

This problem involves adding algebraic fractions. To add or subtract fractions, they must have a common denominator.

Step 2: Key Formula or Approach:

The key observation is that the denominators are negatives of each other: $(7 - x) = -(x - 7)$. We can use this relationship to create a common denominator.

Step 3: Detailed Explanation:

The expression is:

$$\frac{11}{x - 7} + \frac{4}{7 - x}$$

We can rewrite the second term's denominator:

$$\frac{4}{7 - x} = \frac{4}{-(x - 7)} = -\frac{4}{x - 7}$$

Now substitute this back into the original expression:

$$\frac{11}{x-7} - \frac{4}{x-7}$$

Since the fractions now have a common denominator, we can combine the numerators:

$$\frac{11-4}{x-7} = \frac{7}{x-7}$$

This answer is correct, but it is not among the options in its current form. We need to see if it's equivalent to one of the options. Let's look at option (E): $\frac{-7}{7-x}$.

We can manipulate our answer to match this form:

$$\frac{7}{x-7} = \frac{7}{-(7-x)} = -\frac{7}{7-x} = \frac{-7}{7-x}$$

This matches option (E).

Alternatively, we could have used $7-x$ as the common denominator from the start:

$$\frac{11}{x-7} = \frac{11}{-(7-x)} = -\frac{11}{7-x}$$

The original expression becomes:

$$-\frac{11}{7-x} + \frac{4}{7-x} = \frac{-11+4}{7-x} = \frac{-7}{7-x}$$

This directly gives us the answer in the form of option (E).

Step 4: Final Answer:

The simplified expression is $\frac{-7}{7-x}$.

Quick Tip

When denominators of fractions are of the form $(a-b)$ and $(b-a)$, remember that $(b-a) = -(a-b)$. You can factor out a -1 from one of the denominators to easily find a common denominator.

9. If $6h - 2g = 4g + 3h$, in terms of g , $h = ?$

- (A) g
- (B) $4g$
- (C) $2g$
- (D) $5g$
- (E) $3g$

Correct Answer: (C) $2g$

Solution:**Step 1: Understanding the Concept:**

This is a basic algebraic manipulation problem. The goal is to solve the given linear equation for the variable h , which means expressing h in terms of g .

Step 2: Key Formula or Approach:

Use standard algebraic operations (addition, subtraction, multiplication, division) to isolate the variable h on one side of the equation.

Step 3: Detailed Explanation:

The given equation is:

$$6h - 2g = 4g + 3h$$

First, we want to gather all terms containing h on one side of the equation and all terms containing g on the other side.

Subtract $3h$ from both sides:

$$6h - 3h - 2g = 4g$$

$$3h - 2g = 4g$$

Now, add $2g$ to both sides to isolate the h term:

$$3h = 4g + 2g$$

$$3h = 6g$$

Finally, divide both sides by 3 to solve for h :

$$h = \frac{6g}{3}$$

$$h = 2g$$

Step 4: Final Answer:

In terms of g , h is equal to $2g$.

Quick Tip

When solving for one variable in terms of another, the process is the same as solving a regular equation. Your goal is to get "variable = ...", where the other side contains only other variables and constants.

10. Audrey, Penelope and Clementine are all sisters. Penelope is 8 years older than Clementine and 2 years younger than Audrey. If the sum of Penelope and Clementine's age is Audrey's age, how old is Clementine's age?

- (A) 4
- (B) 2

- (C) 8
(D) 3

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

This word problem can be solved by translating the given sentences into a system of linear equations and then solving for the required variable.

Step 2: Key Formula or Approach:

Let A, P, and C represent the ages of Audrey, Penelope, and Clementine, respectively. From the problem statement, we can write the following equations:

1. Penelope is 8 years older than Clementine: $P = C + 8$
2. Penelope is 2 years younger than Audrey: $P = A - 2$
3. The sum of Penelope and Clementine's age is Audrey's age: $P + C = A$

Step 3: Detailed Explanation:

We have a system of three equations:

$$P = C + 8 \quad (1)$$

$$P = A - 2 \quad (2)$$

$$P + C = A \quad (3)$$

We want to find the value of C. Let's use substitution to solve the system. From equation (2), we can express A in terms of P:

$$A = P + 2$$

Now, substitute this expression for A into equation (3):

$$P + C = (P + 2)$$

We can now solve this equation for C. Subtract P from both sides:

$$C = 2$$

So, Clementine's age is 2.

Let's check this answer with the other equations.

If $C = 2$, then from equation (1), $P = 2 + 8 = 10$.

If $P = 10$, then from equation (2), $10 = A - 2$, which means $A = 12$.

Now check if equation (3) holds: $P + C = A \Rightarrow 10 + 2 = 12$. This is true. The solution is consistent.

Step 4: Final Answer:

Clementine's age is 2.

Quick Tip

When setting up equations from word problems, define your variables clearly first. Then, read each phrase carefully to translate it into a mathematical relationship. Often, you can solve the system by expressing all variables in terms of just one variable.

11. If $3x + y = 13$ and $x - 2y = -12$, what is the value of x ?

- (A) 3
- (B) 1
- (C) 2
- (D) $\frac{1}{3}$
- (E) $\frac{2}{3}$

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

We are given a system of two linear equations with two variables, x and y . We need to find the value of x . We can use methods such as substitution or elimination to solve the system.

Step 2: Key Formula or Approach:

The elimination method is suitable here. We can manipulate one or both equations so that the coefficients of one variable are opposites, and then add the equations together to eliminate that variable.

Equation 1: $3x + y = 13$

Equation 2: $x - 2y = -12$

Step 3: Detailed Explanation:

To eliminate the y variable, we can multiply Equation 1 by 2. This will make the coefficient of y in Equation 1 become $+2$, which is the opposite of the coefficient in Equation 2 (-2).

$$2 \times (3x + y) = 2 \times 13$$

$$6x + 2y = 26 \quad (\text{New Equation 1})$$

Now, we add this new equation to Equation 2:

$$(6x + 2y) + (x - 2y) = 26 + (-12)$$

The y -terms cancel out:

$$7x = 14$$

Finally, solve for x by dividing both sides by 7:

$$x = \frac{14}{7}$$

$$x = 2$$

Step 4: Final Answer:

The value of x is 2.

Quick Tip

When solving a system of equations, look for the quickest method. Here, since the y -coefficients already had opposite signs (one positive, one negative), the elimination method was very efficient. You only needed to multiply the first equation to match the coefficients.

12. A given university has an average professor pay of \$40,000 a year and an average administrator pay of \$45,000 per year. If the ratio of professors to administrators is 4 to 3, and the total pay for professors and administrators in a year is \$40,415,000, how many professors does the college have?

- (A) 500
- (B) 375
- (C) 411
- (D) 548
- (E) 475

Correct Answer: (D) 548

Solution:

Step 1: Understanding the Concept:

This problem involves setting up and solving a system of equations based on a given ratio and a total amount. We need to find the number of professors.

Step 2: Key Formula or Approach:

Let P be the number of professors and A be the number of administrators.

From the ratio, we can say $\frac{P}{A} = \frac{4}{3}$. A common way to handle this is to use a multiplier, k .

Let $P = 4k$ and $A = 3k$, where k is a positive integer.

The total pay is given by the equation:

$$(\text{Pay per professor} \times P) + (\text{Pay per administrator} \times A) = \text{Total Pay}$$

Step 3: Detailed Explanation:

Substitute the given values into the total pay equation:

$$40,000 \times P + 45,000 \times A = 40,415,000$$

Now substitute our expressions for P and A in terms of k :

$$40,000(4k) + 45,000(3k) = 40,415,000$$

Calculate the products:

$$160,000k + 135,000k = 40,415,000$$

Combine the terms with k:

$$295,000k = 40,415,000$$

Now, solve for k by dividing both sides by 295,000:

$$k = \frac{40,415,000}{295,000} = \frac{40,415}{295}$$

To simplify the fraction, we can divide the numerator and denominator by 5:

$$k = \frac{8083}{59}$$

Now perform the long division:

$$k = 137$$

The question asks for the number of professors, which is P.

$$P = 4k$$

Substitute the value of k we found:

$$P = 4 \times 137 = 548$$

Step 4: Final Answer:

The college has 548 professors.

Quick Tip

For problems involving ratios, using a common multiplier (like 'k') is a very powerful technique. It simplifies the setup by reducing the number of variables in your main equation, making it easier to solve.

13. Given $x > 0$.

Quantity A: $-5x + 4$

Quantity B: $8 - 2x$

- (A) The relationship cannot be determined from the information given.
- (B) The two quantities are equal.
- (C) Quantity A is greater.
- (D) Quantity B is greater.

Correct Answer: (D) Quantity B is greater.

Solution:

Step 1: Understanding the Concept:

This is a quantitative comparison question. We need to compare two algebraic expressions given a constraint on the variable x . We can do this by setting up an inequality and solving for x , or by analyzing the relationship between the two expressions.

Step 2: Key Formula or Approach:

To compare Quantity A and Quantity B, we can subtract one from the other or set up an inequality. Let's find out for which values of x Quantity B is greater than Quantity A.

$$\text{Quantity B} > \text{Quantity A}$$

$$8 - 2x > -5x + 4$$

Step 3: Detailed Explanation:

Let's solve the inequality:

$$8 - 2x > -5x + 4$$

Add $5x$ to both sides:

$$8 + 3x > 4$$

Subtract 8 from both sides:

$$3x > 4 - 8$$

$$3x > -4$$

Divide by 3:

$$x > -\frac{4}{3}$$

The inequality $8 - 2x > -5x + 4$ is true whenever $x > -4/3$.

The problem gives us the condition that $x > 0$.

Since any number greater than 0 is also greater than $-4/3$, the condition $x > -4/3$ is always satisfied for any given $x > 0$.

Therefore, for all $x > 0$, Quantity B is always greater than Quantity A.

Alternative Method (Testing Values):

Let's pick a value for x that satisfies $x > 0$, for example, $x = 1$.

$$\text{Quantity A} = -5(1) + 4 = -1$$

$$\text{Quantity B} = 8 - 2(1) = 6$$

In this case, Quantity B is greater than Quantity A.

Let's pick another value, $x = 10$.

$$\text{Quantity A} = -5(10) + 4 = -46$$

$$\text{Quantity B} = 8 - 2(10) = -12$$

In this case, Quantity B is greater than Quantity A as well. This suggests that Quantity B is always greater. The algebraic method confirms this.

Step 4: Final Answer:

Quantity B is greater.

Quick Tip

For quantitative comparisons, solving the inequality algebraically is the most rigorous method. However, testing one or two simple values that fit the given conditions can give you a strong indication of the correct answer and help you catch potential errors in your algebra.

14. A theme park charges \$10 for adults and \$5 for kids. How many kids tickets were sold if a total of 548 tickets were sold for a total of \$3750?

- (A) 431
- (B) 157
- (C) 346
- (D) 248
- (E) 269

Correct Answer: (C) 346

Solution:

Step 1: Understanding the Concept:

This problem can be modeled by a system of two linear equations with two variables. One equation represents the total number of tickets, and the other represents the total revenue.

Step 2: Key Formula or Approach:

Let a be the number of adult tickets and k be the number of kids tickets.

Equation 1 (Total tickets): $a + k = 548$

Equation 2 (Total revenue): $10a + 5k = 3750$

We need to solve this system for k .

Step 3: Detailed Explanation:

We can use the substitution method. From Equation 1, we can express a in terms of k :

$$a = 548 - k$$

Now, substitute this expression for a into Equation 2:

$$10(548 - k) + 5k = 3750$$

Distribute the 10:

$$5480 - 10k + 5k = 3750$$

Combine the k terms:

$$5480 - 5k = 3750$$

Now, we want to isolate k . Subtract 3750 from both sides and add $5k$ to both sides:

$$5480 - 3750 = 5k$$

$$1730 = 5k$$

Finally, divide by 5 to find k :

$$k = \frac{1730}{5}$$

$$k = 346$$

Step 4: Final Answer:

The number of kids tickets sold was 346.

Quick Tip

A useful check for this type of problem is to calculate the 'average' ticket price. $\$3750 / 548 \text{ tickets} \approx \6.84 . Since this is closer to the kids' price (\$5) than the adult price (\$10), you should expect there to be more kids tickets than adult tickets. $346 \text{ kids tickets and } (548 - 346) = 202 \text{ adult tickets}$ fits this expectation.

15. Given the following information:

Sally is 2 years younger than Abby.

Daisy is 5 years older than Tracy.

Abby is 6 years older than Tracy.

Quantity A: Sally's age

Quantity B: Daisy's age

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The relationship cannot be determined.
- (D) The two quantities are equal.

Correct Answer: (B) Quantity B is greater.

Solution:

Step 1: Understanding the Concept:

This is a logic problem involving relative ages. To compare Sally's age and Daisy's age, we need to express both of their ages in terms of a common reference person.

Step 2: Key Formula or Approach:

Let S , A , D , and T be the ages of Sally, Abby, Daisy, and Tracy, respectively. Translate the sentences into equations:

- $S = A - 2$
- $D = T + 5$
- $A = T + 6$

Tracy's age (T) is a common link between the relationships. We will express both S and D in terms of T.

Step 3: Detailed Explanation:

Expressing Daisy's age in terms of T:

We are directly given this relationship:

$$D = T + 5$$

Expressing Sally's age in terms of T:

We know that $S = A - 2$. We also know that $A = T + 6$. We can substitute the expression for A into the equation for S:

$$S = (T + 6) - 2$$

$$S = T + 4$$

Comparing the quantities:

Quantity A: Sally's age = $T + 4$

Quantity B: Daisy's age = $T + 5$

Since T represents a person's age, it must be a positive number. For any value of T, $T + 5$ will always be 1 greater than $T + 4$.

Therefore, Daisy's age is always greater than Sally's age.

Step 4: Final Answer:

Quantity B is greater than Quantity A.

Quick Tip

In problems with multiple relative relationships (ages, heights, etc.), the key is to find a "bridge" variable that connects the different pieces of information. Here, Tracy's age was the bridge that allowed us to compare Sally and Daisy directly.

16. Given the equations:

$$x^2 + 5x - 24 = 0$$

$$y^2 - 9y + 20 = 0$$

Quantity A: x

Quantity B: y

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

Correct Answer: (B) Quantity B is greater.

Solution:**Step 1: Understanding the Concept:**

We are asked to compare the possible values of x and y , which are the solutions (roots) of two separate quadratic equations. We must solve each equation to find the possible values for x and y .

Step 2: Key Formula or Approach:

We will solve each quadratic equation by factoring. For an equation of the form $ax^2 + bx + c = 0$, we look for two numbers that multiply to c and add to b .

Step 3: Detailed Explanation:**Solving for x :**

The equation for x is $x^2 + 5x - 24 = 0$.

We need two numbers that multiply to -24 and add to $+5$. These numbers are $+8$ and -3 .

So, we can factor the equation as:

$$(x + 8)(x - 3) = 0$$

This gives two possible values for x :

$$x = -8 \quad \text{or} \quad x = 3$$

Solving for y :

The equation for y is $y^2 - 9y + 20 = 0$.

We need two numbers that multiply to $+20$ and add to -9 . These numbers are -4 and -5 .

So, we can factor the equation as:

$$(y - 4)(y - 5) = 0$$

This gives two possible values for y :

$$y = 4 \quad \text{or} \quad y = 5$$

Comparing Quantity A and Quantity B:

The possible values for Quantity A (x) are $\{-8, 3\}$.

The possible values for Quantity B (y) are $\{4, 5\}$.

Let's compare the largest possible value of x with the smallest possible value of y .

The largest value for x is 3 .

The smallest value for y is 4 .

Since $3 < 4$, any possible value of y will be greater than any possible value of x .

Step 4: Final Answer:

Quantity B is always greater than Quantity A.

Quick Tip

In quantitative comparison questions involving variables with multiple possible values, check the extreme values. Compare the maximum possible value of one quantity with the minimum possible value of the other. If a consistent relationship holds (e.g., $\max(A) \leq \min(B)$), you have your answer. If not, the relationship cannot be determined.

17. Jen and Karen are travelling for the weekend. They both leave from Jen's house and meet at their destination 250 miles away. Jen drives 45mph the whole way. Karen drives 60mph but leaves a half hour after Jen. How long does it take for Karen to catch up with Jen?

- (A) 1.5 hours
- (B) She can't catch up.
- (C) 3 hours
- (D) 1 hour
- (E) 2 hours

Correct Answer: (A) 1.5 hours

Solution:

Step 1: Understanding the Concept:

This is a "catch-up" problem, which is a type of distance-rate-time problem. To find when Karen catches up with Jen, we need to find the time at which they have both traveled the same distance from the starting point.

Step 2: Key Formula or Approach:

The fundamental formula is $\text{Distance} = \text{Rate} \times \text{Time}$.

Let t be the time in hours that Karen has been traveling.

Since Jen left 0.5 hours earlier, Jen's travel time is $t + 0.5$ hours.

The distance traveled by Jen is $d_J = 45(t + 0.5)$.

The distance traveled by Karen is $d_K = 60t$.

To catch up, their distances must be equal: $d_J = d_K$.

Step 3: Detailed Explanation:

Set the distance equations equal to each other:

$$45(t + 0.5) = 60t$$

Distribute the 45 on the left side:

$$45t + 45(0.5) = 60t$$

$$45t + 22.5 = 60t$$

Subtract $45t$ from both sides to isolate the term with t :

$$22.5 = 60t - 45t$$

$$22.5 = 15t$$

Solve for t by dividing both sides by 15:

$$t = \frac{22.5}{15} = \frac{45/2}{15} = \frac{45}{30} = 1.5$$

The time is in hours, as the speeds were in mph.

The 250 miles information is extra, used only to confirm that they catch up before reaching the destination. At 1.5 hours, Karen travels $60 \times 1.5 = 90$ miles, which is less than 250.

Step 4: Final Answer:

It takes Karen 1.5 hours to catch up with Jen.

Quick Tip

In catch-up problems, define your time variable carefully. It's usually easiest to let ' t ' be the travel time of the person who starts later. Then the person who started earlier will have a travel time of ' $t + (\text{head start time})$ '.

18. Solve for z : $3(z + 4) - 7 = 17$

- (A) 4
- (B) 8
- (C) 2
- (D) -8
- (E) -2

Correct Answer: (A) 4

Solution:

Step 1: Understanding the Concept:

This is a basic algebra problem that requires solving a linear equation for an unknown variable, z .

Step 2: Key Formula or Approach:

We will use the order of operations in reverse to isolate z . The steps are: 1. Add or subtract constants to isolate the term with the variable. 2. Distribute multiplication over parentheses. 3. Combine like terms. 4. Divide to solve for the variable.

Step 3: Detailed Explanation:

The given equation is:

$$3(z + 4) - 7 = 17$$

First, add 7 to both sides of the equation to isolate the term with the parenthesis:

$$3(z + 4) = 17 + 7$$

$$3(z + 4) = 24$$

Next, we can either distribute the 3 or divide both sides by 3. Dividing is simpler here.

$$z + 4 = \frac{24}{3}$$

$$z + 4 = 8$$

Finally, subtract 4 from both sides to solve for z:

$$z = 8 - 4$$

$$z = 4$$

Step 4: Final Answer:

The value of z is 4.

Quick Tip

When a term in parentheses is multiplied by a constant, you have two options: distribute the constant, or divide the other side of the equation by the constant. Look ahead to see which path is easier. If the number on the other side is a multiple of the constant, dividing first can save you a step.

19. If $5(3x + y) = 15$, what is x in terms of y?

- (A) $x = 10 - \frac{y}{3}$
- (B) $x = 3 - 3y$
- (C) $x = 10 + \frac{y}{3}$
- (D) $x = 15 + \frac{5y}{3}$
- (E) $x = 1 - \frac{y}{3}$

Correct Answer: (E) $x = 1 - \frac{y}{3}$

Solution:

Step 1: Understanding the Concept:

"Expressing x in terms of y" means rearranging the given equation to isolate x on one side, with the other side containing only y and constants.

Step 2: Key Formula or Approach:

We will use algebraic manipulation to solve the equation for x.

Step 3: Detailed Explanation:

The given equation is:

$$5(3x + y) = 15$$

First, divide both sides by 5 to simplify the equation:

$$3x + y = \frac{15}{5}$$

$$3x + y = 3$$

Next, subtract y from both sides to isolate the term with x:

$$3x = 3 - y$$

Finally, divide both sides by 3 to solve for x:

$$x = \frac{3 - y}{3}$$

This can be split into two fractions:

$$x = \frac{3}{3} - \frac{y}{3}$$

$$x = 1 - \frac{y}{3}$$

Step 4: Final Answer:

In terms of y, x is $1 - \frac{y}{3}$.

Quick Tip

After you've isolated the variable, your answer might not look exactly like the multiple-choice options. Be prepared to simplify or rearrange your result, for example by splitting a fraction into two parts as shown in the solution.

20. Given:

$$y = x^2 - 10$$

$$y = 15$$

Quantity A: $y/3$

Quantity B: x

- (A) Quantity B is greater.
- (B) Quantity A is greater.
- (C) The relationship cannot be determined from the information given.
- (D) The two quantities are equal.

Correct Answer: (C) The relationship cannot be determined from the information given.

Solution:**Step 1: Understanding the Concept:**

We need to compare two quantities. First, we must use the given system of equations to find the possible values for x and y . Then we can evaluate and compare Quantity A and Quantity B.

Step 2: Key Formula or Approach:

1. Find the value of y . 2. Substitute the value of y into the first equation to find the possible values for x . 3. Calculate Quantity A. 4. Compare Quantity A with the possible values of Quantity B (x).

Step 3: Detailed Explanation:

We are given directly that $y = 15$.

Now we can calculate Quantity A:

$$\text{Quantity A} = \frac{y}{3} = \frac{15}{3} = 5$$

Next, we find the value(s) of x by substituting $y = 15$ into the first equation:

$$15 = x^2 - 10$$

Add 10 to both sides:

$$25 = x^2$$

Take the square root of both sides. Remember that the square root can be positive or negative.

$$x = \pm\sqrt{25}$$

$$x = 5 \quad \text{or} \quad x = -5$$

So, Quantity B (x) can be either 5 or -5.

Comparison:

Case 1: If $x = 5$.

Quantity A = 5 and Quantity B = 5. In this case, the quantities are equal.

Case 2: If $x = -5$.

Quantity A = 5 and Quantity B = -5. In this case, Quantity A is greater.

Since the relationship between the two quantities changes depending on the value of x , we cannot determine a single consistent relationship.

Step 4: Final Answer:

The relationship cannot be determined from the information given.

Quick Tip

Always remember that taking the square root of both sides of an equation like $x^2 = c$ (where $c \geq 0$) yields two solutions: $x = \sqrt{c}$ and $x = -\sqrt{c}$. Forgetting the negative root is a common mistake in quantitative comparison problems.

21. If $a = \frac{1}{3}b$ and $b = 4c$, then in terms of c , $a - b + c = ?$

- (A) c
- (B) $\frac{7}{3}c$
- (C) $-\frac{5}{3}c$
- (D) $-\frac{11}{3}c$

Correct Answer: (C) $-\frac{5}{3}c$

Solution:

Step 1: Understanding the Concept:

We are asked to evaluate an algebraic expression involving three variables (a , b , c) by expressing it solely in terms of one variable, c . This requires substituting the given relationships into the expression.

Step 2: Key Formula or Approach:

1. Express variable ' b ' in terms of ' c ' (this is already given). 2. Express variable ' a ' in terms of ' c ' by using the two given equations. 3. Substitute the expressions for ' a ' and ' b ' into the target expression $a - b + c$. 4. Simplify the resulting expression.

Step 3: Detailed Explanation:

We are given:

$$b = 4c$$

And:

$$a = \frac{1}{3}b$$

First, substitute the expression for b into the equation for a to find a in terms of c :

$$a = \frac{1}{3}(4c) = \frac{4c}{3}$$

Now we have both a and b in terms of c . We can substitute them into the expression $a - b + c$:

$$a - b + c = \left(\frac{4c}{3}\right) - (4c) + c$$

To combine these terms, we need a common denominator, which is 3.

$$= \frac{4c}{3} - \frac{12c}{3} + \frac{3c}{3}$$

Now, combine the numerators:

$$\begin{aligned} &= \frac{4c - 12c + 3c}{3} \\ &= \frac{-8c + 3c}{3} \\ &= \frac{-5c}{3} \end{aligned}$$

Step 4: Final Answer:

The expression $a - b + c$ is equal to $-\frac{5}{3}c$.

Quick Tip

When performing a series of substitutions, be organized. Write down what each variable is in terms of your target variable (in this case, 'c') before plugging them into the final expression. This helps avoid confusion and errors.

22. If $14x - 16y = 16$ and $yz = 12$, then what is the value of $3x - z$?

- (A) 1
- (B) 2
- (C) 4
- (D) 6
- (E) 3

Correct Answer: (D) 6

Solution:

Step 1: Understanding the Concept:

We are given a system of two equations with three variables. Typically, such a system does not have a unique solution. However, in the context of competitive exams, this often implies that either the expression we need to find can be derived directly through algebraic manipulation, or there is an intended integer solution. As stated, the problem does not yield a single value for the expression $3x - z$. There is likely a typo in the first equation. A common typo for $14x - 16y = 16$ could be $3x - 4y = 4$, which keeps the equation's structure similar while allowing for a clean integer solution. We will solve the problem assuming this correction.

Corrected Equations:

$$3x - 4y = 4 \quad (\text{Equation 1})$$

$$yz = 12 \quad (\text{Equation 2})$$

Step 2: Key Formula or Approach:

We will test integer factors of 12 for y and z from Equation 2 and see which pair leads to an integer solution for x in Equation 1.

Step 3: Detailed Explanation:

From Equation 2, $yz = 12$, we can test possible integer pairs for (y, z) :

- If $y = 1, z = 12$. In Eq 1: $3x - 4(1) = 4 \Rightarrow 3x = 8$. No integer x .
- If $y = 2, z = 6$. In Eq 1: $3x - 4(2) = 4 \Rightarrow 3x - 8 = 4 \Rightarrow 3x = 12 \Rightarrow x = 4$. This gives a valid integer solution.

- If $y = 3, z = 4$. In Eq 1: $3x - 4(3) = 4 \Rightarrow 3x - 12 = 4 \Rightarrow 3x = 16$. No integer x .
- If $y = 4, z = 3$. In Eq 1: $3x - 4(4) = 4 \Rightarrow 3x - 16 = 4 \Rightarrow 3x = 20$. No integer x .

The only combination of integer factors of 12 that yields an integer solution for x is $y = 2$, which gives $x = 4$ and $z = 6$.

Now we evaluate the expression $3x - z$ using these values:

$$\begin{aligned} 3x - z &= 3(4) - 6 \\ &= 12 - 6 \\ &= 6 \end{aligned}$$

Step 4: Final Answer:

Assuming the typo, the value of the expression is 6.

Quick Tip

If a system of equations appears to be unsolvable (more variables than equations), check for typos or a hidden path to manipulate the equations. If that fails, look for a unique integer solution, as these problems are often designed to have one.

23. If $x^3 = 8$, then $x^2(4/(3-x))(2/(4-x)) - (4/x^2) = ?$

- (A) 16
- (B) 35
- (C) 0
- (D) 15
- (E) 22

Correct Answer: (D) 15

Solution:

Step 1: Understanding the Concept:

This problem requires first solving a simple cubic equation for a variable and then substituting that value into a more complex algebraic expression to evaluate it.

Step 2: Key Formula or Approach:

1. Solve for x from the equation $x^3 = 8$. 2. Substitute the value of x into the expression $x^2(\frac{4}{3-x})(\frac{2}{4-x}) - \frac{4}{x^2}$. 3. Evaluate the expression using the order of operations.

Step 3: Detailed Explanation:

First, find the value of x :

$$x^3 = 8$$

$$x = \sqrt[3]{8}$$

$$x = 2$$

Now, substitute $x = 2$ into the given expression:

$$2^2 \left(\frac{4}{3-2} \right) \left(\frac{2}{4-2} \right) - \frac{4}{2^2}$$

Evaluate each part of the expression:

$$\begin{aligned} &= 4 \left(\frac{4}{1} \right) \left(\frac{2}{2} \right) - \frac{4}{4} \\ &= 4(4)(1) - 1 \end{aligned}$$

Perform the multiplication:

$$\begin{aligned} &= 16 - 1 \\ &= 15 \end{aligned}$$

Step 4: Final Answer:

The value of the expression is 15.

Quick Tip

Break down complex expressions into smaller, manageable parts. Calculate the value of each parenthesis or fraction separately before combining them. This reduces the chance of arithmetic errors.

24. Sarah's current age is three times Ron's age two years ago. Sarah is currently 14 years older than Ron. What is the sum of Sarah and Ron's current age?

- (A) 24
- (B) 36
- (C) 34
- (D) 32

Correct Answer: (C) 34

Solution:

Step 1: Understanding the Concept:

This is a word problem about ages that can be solved by setting up a system of two linear equations with two variables.

Step 2: Key Formula or Approach:

Let S be Sarah's current age and R be Ron's current age. Translate the given information into equations: 1. "Sarah's current age is three times Ron's age two years ago": $S = 3(R - 2)$ 2.

"Sarah is currently 14 years older than Ron": $S = R + 14$ We need to solve for S and R, and then find their sum.

Step 3: Detailed Explanation:

We have a system of two equations for S:

$$S = 3R - 6$$

$$S = R + 14$$

Since both expressions are equal to S, we can set them equal to each other:

$$3R - 6 = R + 14$$

Now, solve for R. Subtract R from both sides:

$$2R - 6 = 14$$

Add 6 to both sides:

$$2R = 20$$

$$R = 10$$

So, Ron's current age is 10. Now find Sarah's age using the simpler equation:

$$S = R + 14 = 10 + 14 = 24$$

Sarah's current age is 24. The question asks for the sum of their current ages:

$$S + R = 24 + 10 = 34$$

Step 4: Final Answer:

The sum of Sarah and Ron's current age is 34.

Quick Tip

For age problems, always define your variables clearly (e.g., "let R be Ron's *current* age"). Pay close attention to phrases like "two years ago" or "in five years" and adjust the variable accordingly (R-2 or R+5).

25. Jack has 14 coins consisting of nickels and dimes that total \$0.90. How many nickels does Jack have?

- (A) 10
- (B) 8
- (C) 4
- (D) 6
- (E) 12

Correct Answer: (A) 10

Solution:

Step 1: Understanding the Concept:

This is a classic coin problem that can be solved with a system of linear equations. One equation represents the total number of coins, and the other represents their total value.

Step 2: Key Formula or Approach:

Let n be the number of nickels and d be the number of dimes. Equation for the number of coins:

$$n + d = 14$$

Equation for the value of the coins (in cents):

$$5n + 10d = 90$$

We need to solve this system for n .

Step 3: Detailed Explanation:

We can use the substitution method. From the first equation, express d in terms of n :

$$d = 14 - n$$

Now, substitute this into the second equation:

$$5n + 10(14 - n) = 90$$

Distribute the 10:

$$5n + 140 - 10n = 90$$

Combine the terms with n :

$$-5n + 140 = 90$$

Subtract 140 from both sides:

$$-5n = 90 - 140$$

$$-5n = -50$$

Divide by -5:

$$n = 10$$

Step 4: Final Answer:

Jack has 10 nickels.

Quick Tip

To avoid decimals, it's often easier to work with cents instead of dollars when solving coin problems. Just remember to convert the total value to cents as well ($\$0.90 = 90$ cents).

26. Abby works at a car dealership and receives a commission "c" which is a percent of the profit the dealership makes from the sale, which is the difference between the price "p" of the car and the value "v" of the car. How much, in dollars, does the dealership earn per transaction?

- (A) $(p - v)(0.01c)$
- (B) $(p - v)(1 - 0.01c)$
- (C) $p(v - 0.01c)$
- (D) $(p - v)(1 - c)$
- (E) $pv(0.01c)$

Correct Answer: (B) $(p - v)(1 - 0.01c)$

Solution:

Step 1: Understanding the Concept:

This is a problem about calculating profit and commission. We need to translate the verbal description into a mathematical expression. The dealership's earning is the total profit from the sale minus the commission paid to the salesperson.

Step 2: Key Formula or Approach:

1. Define the profit. 2. Define the commission amount paid to Abby. 3. Define the dealership's earnings as Profit - Commission. 4. Simplify the resulting expression.

Step 3: Detailed Explanation:

The profit from the sale is the difference between the price (p) and the value (v):

$$\text{Profit} = p - v$$

Abby's commission "c" is a percent of the profit. To use it in a calculation, we must convert the percentage to a decimal by dividing by 100.

$$\text{Commission Amount} = \frac{c}{100} \times \text{Profit} = 0.01c \times (p - v)$$

The dealership's earning is the total profit minus the commission they pay to Abby:

$$\text{Dealership Earns} = \text{Profit} - \text{Commission Amount}$$

$$\text{Dealership Earns} = (p - v) - 0.01c(p - v)$$

We can factor out the common term $(p - v)$:

$$\text{Dealership Earns} = (p - v)(1 - 0.01c)$$

Step 4: Final Answer:

The expression for the dealership's earnings is $(p - v)(1 - 0.01c)$.

Quick Tip

Be very careful with how percentages are defined. If a variable 'c' is "a percent," it means the number itself (e.g., $c=10$ for 10%). To use it in multiplication, you must convert it to a decimal ($c/100$ or $0.01c$). This is a common trap in percentage problems.

27. If $8s - 6k = 4s - 2k$, then, in terms of s , $k = ?$

- (A) Cannot be determined
- (B) $3s$
- (C) $5s$
- (D) $2s$
- (E) s

Correct Answer: (E) s

Solution:

Step 1: Understanding the Concept:

This problem requires rearranging a linear equation to solve for one variable (k) in terms of another variable (s).

Step 2: Key Formula or Approach:

Use basic algebraic operations to isolate all terms with k on one side of the equation and all terms with s on the other side.

Step 3: Detailed Explanation:

The given equation is:

$$8s - 6k = 4s - 2k$$

To solve for k , we want to gather all k -terms on one side. Let's add $6k$ to both sides:

$$8s = 4s - 2k + 6k$$

$$8s = 4s + 4k$$

Now, gather all s -terms on the other side. Subtract $4s$ from both sides:

$$8s - 4s = 4k$$

$$4s = 4k$$

Finally, divide both sides by 4 to solve for k :

$$s = k$$

So, $k = s$.

Step 4: Final Answer:

In terms of s , k is equal to s .

Quick Tip

When isolating a variable, you can move terms to either the left or the right side of the equation. It's often helpful to choose the side that will result in a positive coefficient for the variable you are solving for, which can help prevent sign errors.

28. Kim is twice as old as Claire. Nick is 3 years older than Claire. Kim is 6 years older than Emily. Their ages combined equal 81. How old is Nick?

- (A) 22
- (B) 27
- (C) 17
- (D) 13

Correct Answer: (C) 17

Solution:

Step 1: Understanding the Concept:

This word problem involves multiple age relationships and can be solved by setting up a system of equations. The goal is to express everyone's age in terms of a single base variable and then use the total age to solve for that variable.

Step 2: Key Formula or Approach:

Let K, C, N, and E be the ages of Kim, Claire, Nick, and Emily, respectively. 1. $K = 2C$ 2. $N = C + 3$ 3. $K = E + 6$, which implies $E = K - 6$ 4. $K + C + N + E = 81$ We will express all ages in terms of C.

Step 3: Detailed Explanation:

We have the ages in terms of C:

- Claire's age = C
- Kim's age = $K = 2C$
- Nick's age = $N = C + 3$
- Emily's age = $E = K - 6 = (2C) - 6 = 2C - 6$

Now substitute these into the sum equation:

$$(2C) + C + (C + 3) + (2C - 6) = 81$$

Combine all the terms with C:

$$2C + C + C + 2C = 6C$$

Combine the constant terms:

$$3 - 6 = -3$$

The simplified equation is:

$$6C - 3 = 81$$

Add 3 to both sides:

$$6C = 84$$

Divide by 6:

$$C = \frac{84}{6} = 14$$

Claire's age is 14. The question asks for Nick's age:

$$N = C + 3 = 14 + 3 = 17$$

Step 4: Final Answer:

Nick is 17 years old.

Quick Tip

In multi-person age problems, identify the person whose age is used as a reference for others. In this case, both Kim's and Nick's ages were given relative to Claire's, making Claire's age (C) the best choice for the primary variable.

29. The sum of two consecutive odd integers is 32. What is the value of the next consecutive odd integer?

- (A) 33
- (B) 21
- (C) 17
- (D) Cannot be determined
- (E) 19

Correct Answer: (E) 19

Solution:

Step 1: Understanding the Concept:

Consecutive odd integers are odd numbers that follow each other in sequence, such as 3, 5, 7. They always have a difference of 2. We can set up an algebraic equation to find the integers.

Step 2: Key Formula or Approach:

Let the first odd integer be n . Then the next consecutive odd integer is $n + 2$. Their sum is 32:

$$n + (n + 2) = 32$$

We need to find the integers and then identify the *next* one in the sequence.

Step 3: Detailed Explanation:

Solve the equation for n :

$$2n + 2 = 32$$

Subtract 2 from both sides:

$$2n = 30$$

Divide by 2:

$$n = 15$$

So, the first odd integer is 15. The second consecutive odd integer is $n + 2 = 15 + 2 = 17$. Check: $15 + 17 = 32$. This is correct. The problem asks for the *next* consecutive odd integer after these two. The sequence is 15, 17, ... The next odd integer after 17 is $17 + 2 = 19$.

Step 4: Final Answer:

The value of the next consecutive odd integer is 19.

Quick Tip

Read the question carefully. A common mistake is to solve for one of the two integers (like 17) and select that as the answer. The question here asks for the *next* integer in the sequence, not one of the integers in the sum.

30. A store sells potatoes for \$0.24 and tomatoes for \$0.76. Fred bought 12 individual vegetables. If he paid \$6.52 total, how many potatoes did Fred buy?

- (A) 5
- (B) 8
- (C) 7
- (D) 2

Correct Answer: (A) 5

Solution:

Step 1: Understanding the Concept:

This problem can be solved by setting up a system of two linear equations. One equation represents the total quantity of items, and the other represents the total cost.

Step 2: Key Formula or Approach:

Let p be the number of potatoes and t be the number of tomatoes. Equation 1 (Quantity): $p + t = 12$ Equation 2 (Cost): $0.24p + 0.76t = 6.52$ We need to solve this system for the variable p .

Step 3: Detailed Explanation:

We will use the substitution method. From Equation 1, express t in terms of p :

$$t = 12 - p$$

Substitute this expression for t into Equation 2:

$$0.24p + 0.76(12 - p) = 6.52$$

Distribute the 0.76:

$$0.24p + 9.12 - 0.76p = 6.52$$

Combine the p-terms:

$$-0.52p + 9.12 = 6.52$$

Subtract 9.12 from both sides:

$$-0.52p = 6.52 - 9.12$$

$$-0.52p = -2.60$$

Divide both sides by -0.52:

$$p = \frac{-2.60}{-0.52} = \frac{260}{52}$$

To simplify the fraction, divide the numerator and denominator by 52:

$$p = 5$$

Step 4: Final Answer:

Fred bought 5 potatoes.

Quick Tip

When working with decimals in equations, you can sometimes simplify the calculation by multiplying the entire equation by a power of 10 to eliminate the decimals. In this case, multiplying the cost equation by 100 would give $24p + 76t = 652$, which might be easier to work with for some people.

31. Jon invested part of \$16,000 at 3% and the rest at 5% for a total return of \$680.

Quantity A: The amount Jon invested at 5% interest

Quantity B: The amount Jon invested at 3% interest

- (A) The two quantities are equal
- (B) Quantity A is greater
- (C) The relationship cannot be determined from the information given
- (D) Quantity B is greater

Correct Answer: (B) Quantity A is greater

Solution:

Step 1: Understanding the Concept:

This is an investment problem that can be modeled using a system of two linear equations. One equation will represent the total principal amount invested, and the second will represent the total interest earned.

Step 2: Key Formula or Approach:

Let x be the amount invested at 3% and y be the amount invested at 5%.

The interest earned is given by the formula: Interest = Principal $\times$ Rate.

Equation 1 (Total Principal): $x + y = 16,000$

Equation 2 (Total Interest): $0.03x + 0.05y = 680$

We need to solve for x (Quantity B) and y (Quantity A) and then compare them.

Step 3: Detailed Explanation:

From Equation 1, we can express x in terms of y :

$$x = 16,000 - y$$

Now, substitute this expression into Equation 2:

$$0.03(16,000 - y) + 0.05y = 680$$

Distribute the 0.03:

$$480 - 0.03y + 0.05y = 680$$

Combine the y -terms:

$$480 + 0.02y = 680$$

Subtract 480 from both sides:

$$0.02y = 200$$

Solve for y :

$$y = \frac{200}{0.02} = 10,000$$

So, the amount invested at 5% is \$10,000. This is Quantity A.

Now find the amount invested at 3% (x) using Equation 1:

$$x = 16,000 - y = 16,000 - 10,000 = 6,000$$

So, the amount invested at 3% is \$6,000. This is Quantity B.

Comparison:

Quantity A = \$10,000

Quantity B = \$6,000

Quantity A is greater than Quantity B.

Step 4: Final Answer:

Quantity A is greater.

Quick Tip

In mixed investment problems, setting up a system of equations is the standard approach. To avoid decimals, you can multiply the interest equation by 100: $3x + 5y = 68,000$. This can make the arithmetic easier to handle.

32. Given the system of equations:

$$3x + 4y = 5$$

$$x - y = 6$$

Quantity A: x

Quantity B: y

- (A) The two quantities are equal.
- (B) The relationship cannot be determined from the information given.
- (C) Quantity B is greater.
- (D) Quantity A is greater.

Correct Answer: (D) Quantity A is greater.

Solution:

Step 1: Understanding the Concept:

We are given a system of two linear equations and asked to compare the values of the variables x and y . We need to solve the system to find the unique values for x and y .

Step 2: Key Formula or Approach:

We can use either the substitution or elimination method. The substitution method seems straightforward here.

Equation 1: $3x + 4y = 5$

Equation 2: $x - y = 6$

Step 3: Detailed Explanation:

From Equation 2, we can easily isolate x :

$$x = y + 6$$

Now, substitute this expression for x into Equation 1:

$$3(y + 6) + 4y = 5$$

Distribute the 3:

$$3y + 18 + 4y = 5$$

Combine the y -terms:

$$7y + 18 = 5$$

Subtract 18 from both sides:

$$7y = 5 - 18$$

$$7y = -13$$

$$y = -\frac{13}{7}$$

Now that we have the value of y , substitute it back into the expression for x :

$$x = y + 6 = -\frac{13}{7} + 6 = -\frac{13}{7} + \frac{42}{7} = \frac{29}{7}$$

Comparison:

Quantity A: $x = \frac{29}{7}$ (a positive number)

Quantity B: $y = -\frac{13}{7}$ (a negative number)

Any positive number is greater than any negative number.

Step 4: Final Answer:

Quantity A is greater.

Quick Tip

When comparing a positive and a negative number, the positive number is always greater. Once you found that x was positive and y was negative, you didn't need to worry about their exact fractional values to make the comparison.

33. Bill and Bob are working to build toys. Bill can build k toys in 6 hours. Bob can build k toys in 3 hours. How long would it take Bob and Bill to build $4k$ toys working together?

- (A) 8 hours
- (B) 12 hours
- (C) 9 hours
- (D) 2 hours
- (E) 4 hours

Correct Answer: (A) 8 hours

Solution:

Step 1: Understanding the Concept:

This is a combined work rate problem. The key principle is that when people work together, their individual rates of work add up to a combined rate. The formula relating work, rate, and time is $\text{Work} = \text{Rate} \times \text{Time}$.

Step 2: Key Formula or Approach:

1. Calculate the individual work rate for Bill and Bob. Rate is defined as the amount of work done per unit of time. 2. Add their individual rates to find their combined work rate. 3. Use the combined rate and the total amount of work to be done ($4k$ toys) to calculate the total time required. $\text{Time} = \text{Work} / \text{Rate}$.

Step 3: Detailed Explanation:

Individual Rates:

Bill's rate: $\text{Rate}_{\text{Bill}} = \frac{\text{Work}}{\text{Time}} = \frac{k \text{ toys}}{6 \text{ hours}} = \frac{k}{6} \text{ toys/hour}$.

Bob's rate: $\text{Rate}_{\text{Bob}} = \frac{\text{Work}}{\text{Time}} = \frac{k \text{ toys}}{3 \text{ hours}} = \frac{k}{3} \text{ toys/hour}$.

Combined Rate:

When they work together, their rates add up:

$$\text{Rate}_{\text{Combined}} = \text{Rate}_{\text{Bill}} + \text{Rate}_{\text{Bob}} = \frac{k}{6} + \frac{k}{3}$$

To add these fractions, find a common denominator (6):

$$\text{Rate}_{\text{Combined}} = \frac{k}{6} + \frac{2k}{6} = \frac{3k}{6} = \frac{k}{2} \text{ toys/hour}$$

Time to Complete the Job:

The total work to be done is building $4k$ toys.

$$\text{Time} = \frac{\text{Total Work}}{\text{Combined Rate}} = \frac{4k \text{ toys}}{\frac{k}{2} \text{ toys/hour}}$$

$$\text{Time} = 4k \times \frac{2}{k} = 8 \text{ hours}$$

Step 4: Final Answer:

It would take them 8 hours to build $4k$ toys together.

Quick Tip

In work rate problems, the variable for the amount of work (like 'k' here) often cancels out in the final calculation. Don't be intimidated by it; focus on correctly setting up the rates.

34. If $x = 4$, and $y = 3x + 5$, then $2y - 1$ equals

- (A) 47
- (B) 22
- (C) 15
- (D) 33

Correct Answer: (D) 33

Solution:

Step 1: Understanding the Concept:

This is a two-step algebraic evaluation problem. First, we need to find the value of y by substituting the given value of x . Second, we use that value of y to evaluate the final expression.

Step 2: Key Formula or Approach:

1. Substitute $x = 4$ into the equation $y = 3x + 5$ to find the value of y . 2. Substitute the calculated value of y into the expression $2y - 1$.

Step 3: Detailed Explanation:**Step 1: Find y**

We are given $x = 4$. Substitute this into the equation for y:

$$y = 3(4) + 5$$

$$y = 12 + 5$$

$$y = 17$$

Step 2: Evaluate $2y - 1$

Now substitute the value $y = 17$ into the final expression:

$$2y - 1 = 2(17) - 1$$

$$= 34 - 1$$

$$= 33$$

Step 4: Final Answer:

The value of $2y - 1$ is 33.

Quick Tip

This type of problem tests your ability to follow steps methodically. Don't try to combine the steps in your head. Write down the value of the intermediate variable (y) before moving on to the final calculation to avoid simple errors.

35. A hybrid car gets 40 miles per gallon. Gasoline costs \$3.52 per gallon. What is the cost of the gasoline needed for the car to travel 120 miles?

- (A) \$9.54
- (B) \$10.56
- (C) \$10.36
- (D) \$14.08
- (E) \$12.53

Correct Answer: (B) \$10.56

Solution:

Step 1: Understanding the Concept:

This is a multi-step problem involving rates. We need to first determine the amount of fuel required for the trip and then calculate the total cost of that fuel.

Step 2: Key Formula or Approach:

1. Calculate the number of gallons needed using the formula: Gallons = Total Distance / Miles

per Gallon. 2. Calculate the total cost using the formula: Total Cost = Gallons Needed $\times$ Cost per Gallon.

Step 3: Detailed Explanation:

Step 1: Calculate Gallons Needed

The car travels 120 miles and has a fuel efficiency of 40 miles per gallon.

$$\text{Gallons} = \frac{120 \text{ miles}}{40 \text{ miles/gallon}} = 3 \text{ gallons}$$

Step 2: Calculate Total Cost

The car needs 3 gallons of gasoline, and the cost is \$3.52 per gallon.

$$\text{Total Cost} = 3 \text{ gallons} \times \$3.52/\text{gallon}$$

$$\text{Total Cost} = \$10.56$$

Step 4: Final Answer:

The cost of the gasoline needed is \$10.56.

Quick Tip

Unit analysis is a great way to ensure you're setting up the problem correctly. Notice how in Step 1, "miles" cancels out, leaving "gallons", and in Step 2, "gallons" cancels out, leaving "dollars". This confirms the operations are correct.

36. Two palm trees grow next to each other in Luke's backyard. One of the trees gets sick, so Luke cuts off the top 3 feet. The other tree, however, is healthy and grows 2 feet. How tall are the two trees if the healthy tree is now 4 feet taller than the sick tree, and together they are 28 feet tall?

- (A) 8 and 20 feet
- (B) 11 and 17 feet
- (C) 12 and 16 feet
- (D) cannot be determined
- (E) 14 and 14 feet

Correct Answer: (C) 12 and 16 feet

Solution:

Step 1: Understanding the Concept:

This word problem describes the current state of two trees and can be solved by setting up a system of linear equations based on the information provided. The information about the past changes (cutting and growing) is background context but not needed for the equations about

the current heights.

Step 2: Key Formula or Approach:

Let h be the current height of the healthy tree and s be the current height of the sick tree. Translate the sentences about the *current* state into equations: 1. "the healthy tree is now 4 feet taller than the sick tree": $h = s + 4$ 2. "together they are 28 feet tall": $h + s = 28$ We need to solve this system for h and s .

Step 3: Detailed Explanation:

We have the system:

$$h = s + 4$$

$$h + s = 28$$

We can use the substitution method. Substitute the first equation into the second equation:

$$(s + 4) + s = 28$$

Combine the s -terms:

$$2s + 4 = 28$$

Subtract 4 from both sides:

$$2s = 24$$

Solve for s :

$$s = 12$$

So, the current height of the sick tree is 12 feet.

Now find the current height of the healthy tree using the first equation:

$$h = s + 4 = 12 + 4 = 16$$

The current height of the healthy tree is 16 feet.

Step 4: Final Answer:

The heights of the two trees are 12 and 16 feet.

Quick Tip

Read word problems carefully to distinguish between past information and information describing the current situation. Here, the equations should only model the "now" state of the trees. The past changes explain how they got to this state but are not directly used in the final system of equations.

37. Given the system of equations:

$$x + y = 12$$

$$2x - y = 6$$

Quantity A: x

Quantity B: y

- (A) The relationship cannot be determined from the information given.
- (B) Quantity A is greater.
- (C) Quantity B is greater.
- (D) The two quantities are equal.

Correct Answer: (D) The two quantities are equal.

Solution:

Step 1: Understanding the Concept:

We need to solve a system of two linear equations to find the values of x and y , and then compare them.

Step 2: Key Formula or Approach:

The elimination method is very efficient here because the y -coefficients are opposites ($+1$ and -1). We can add the two equations together to eliminate y .

Equation 1: $x + y = 12$

Equation 2: $2x - y = 6$

Step 3: Detailed Explanation:

Add Equation 1 and Equation 2 directly:

$$(x + y) + (2x - y) = 12 + 6$$

The y -terms cancel out:

$$3x = 18$$

Solve for x by dividing by 3:

$$x = 6$$

Now substitute the value of x back into either of the original equations to find y . Using Equation 1 is simpler:

$$6 + y = 12$$

Subtract 6 from both sides:

$$y = 6$$

Comparison:

Quantity A: $x = 6$

Quantity B: $y = 6$

The two quantities are equal.

Step 4: Final Answer:

The two quantities are equal.

Quick Tip

Always look at the structure of the system of equations before starting. If you see variables with coefficients that are the same or opposites, the elimination method will almost always be the fastest way to solve the system.
