GRE Sample Paper Set 1 with Solutions

Time Allowed : 1 Hour 58 Minutes	**Maximum Marks :** 340

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GRE General Test is 1 hour and 58 minutes long (with one optional 10-minute break) and consists of 54 questions in total.
2. The GRE exam is comprised of three sections:
   - Quantitative Reasoning: 27 questions, 47 minutes
   - Verbal Reasoning: 27 questions, 41 minutes
3. You can answer the two sections in any order.
4. As you move through a section, you can skip questions, flag them for review, and return to them later within the same section.
5. When you have answered all questions in a section, you can review your responses before time expires.
6. If there is no time remaining in the section, you will automatically be moved to your optional break screen or the next section (if you have already taken your optional break).
7. Each review screen includes a numbered list of the questions in that section and indicates the questions you flagged.
8. Clicking a question number will take you to that specific question.
9. You may change any answer within the time allowed for that section.

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1. Compare Quantity A and Quantity B.
Quantity A: The least prime number greater than 24
Quantity B: The greatest prime number less than 28

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (A) Quantity A is greater.

Solution:

Step 1: Understanding the Concept:
The question requires us to identify two specific prime numbers and then compare them.
A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Step 2: Detailed Explanation:
Analysis of Quantity A:
We need to find the least prime number that is greater than 24.
Let's check the integers greater than 24 in order:
- 25 is not a prime number because it is divisible by 5 ($5 \times 5 = 25$).
- 26 is not a prime number because it is an even number and divisible by 2 ($2 \times 13 = 26$).
- 27 is not a prime number because it is divisible by 3 and 9 ($3 \times 9 = 27$).
- 28 is not a prime number because it is an even number and divisible by 2 ($2 \times 14 = 28$).
- 29 is a prime number because its only divisors are 1 and 29.
Thus, the least prime number greater than 24 is 29.
So, Quantity A = 29.

Analysis of Quantity B:
We need to find the greatest prime number that is less than 28.
Let's check the integers less than 28 in descending order:
- 27 is not prime (divisible by 3).
- 26 is not prime (divisible by 2).
- 25 is not prime (divisible by 5).
- 24 is not prime (divisible by 2).
- 23 is a prime number because its only divisors are 1 and 23.
Thus, the greatest prime number less than 28 is 23.
So, Quantity B = 23.

Step 3: Comparison:
Now we compare Quantity A and Quantity B.
Quantity A = 29
Quantity B = 23
Since $29 > 23$, Quantity A is greater than Quantity B.

Step 4: Final Answer:
The correct option is (A) because Quantity A is greater.

Quick Tip

For questions involving prime numbers, quickly check for divisibility by small primes like 2, 3, 5, 7. Remember that any even number greater than 2 is not prime. For odd numbers, check divisibility by 3 (sum of digits is divisible by 3) and 5 (ends in 5).

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2. Lionel is younger than Maria.

Quantity A: Twice Lionel's age
Quantity B: Maria's age

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (D) The relationship cannot be determined from the information given.

Solution:

Step 1: Understanding the Concept:
We are given a condition (Lionel is younger than Maria) and asked to compare two quantities related to their ages. Let L be Lionel's age and M be Maria's age. The given condition is $L < M$. We need to compare Quantity A (2L) with Quantity B (M).

Step 2: Key Approach:
The best approach for this type of problem is to test different numerical examples (plugging in numbers) that satisfy the given condition ($L < M$) and see if the relationship between Quantity A and Quantity B remains consistent. If the relationship changes, the answer is (D).

Step 3: Detailed Explanation:
Let's test two different scenarios that satisfy the condition $L < M$.

Scenario 1:
Let's assume Lionel's age (L) is 6 years and Maria's age (M) is 10 years.
The condition $L < M$ is satisfied since $6 < 10$.
- Quantity A = Twice Lionel's age = $2 \times L = 2 \times 6 = 12$.
- Quantity B = Maria's age = $M = 10$.
In this scenario, Quantity A (12) is greater than Quantity B (10). So, **A >B**.

Scenario 2:
Now, let's assume Lionel's age (L) is 4 years and Maria's age (M) is 10 years.
The condition $L < M$ is still satisfied since $4 < 10$.
- Quantity A = Twice Lionel's age = $2 \times L = 2 \times 4 = 8$.
- Quantity B = Maria's age = $M = 10$.
In this scenario, Quantity B (10) is greater than Quantity A (8). So, **B ¿ A**.

Step 4: Final Answer:
Since we found one case where Quantity A is greater and another case where Quantity B is greater, the relationship between the two quantities is not fixed. It depends on the specific ages of Lionel and Maria. Therefore, the relationship cannot be determined from the information given.

Quick Tip

When a quantitative comparison question involves variables and inequalities, always try to "break" the problem by testing different types of numbers that fit the criteria: small integers, large integers, fractions, etc. If you can find two cases that produce different outcomes (A >B and B >A), the answer is always (D).

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3. Quantity A: 54% of 360
Quantity B: 150

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (A) Quantity A is greater.

Solution:

Step 1: Understanding the Concept:
The problem requires comparing a calculated percentage of a number with a fixed integer value. We need to determine if 54% of 360 is greater than, less than, or equal to 150.

Step 2: Key Approach:
There are two main ways to solve this: by estimation or by exact calculation. Estimation is often faster in quantitative comparison questions.

Step 3: Detailed Explanation (Method 1: Estimation):
We can use a known, easy-to-calculate percentage as a benchmark. 50% is a good choice.
- First, calculate 50% of 360. 50% is equivalent to $\frac{1}{2}$.

$$50\% \text{ of } 360 = \frac{1}{2} \times 360 = 180$$

- We know that 54% is greater than 50%. Therefore, 54% of 360 must be greater than 50% of 360.

$$54\% \text{ of } 360 > 180$$

- Now, compare this result with Quantity B.
- Quantity A is greater than 180.
- Quantity B is 150.
Since $180 > 150$, and Quantity A is even larger than 180, it is certain that Quantity A is greater than Quantity B.

Step 3: Detailed Explanation (Method 2: Exact Calculation):
To be completely sure, we can calculate the exact value of Quantity A.

- "Percent" means "per hundred," so 54% can be written as $\frac{54}{100}$ or 0.54.

$$\text{Quantity A} = 54\% \text{ of } 360 = \frac{54}{100} \times 360$$

$$\text{Quantity A} = 0.54 \times 360$$

$$\text{Quantity A} = \frac{54 \times 360}{100} = \frac{54 \times 36}{10} = \frac{1944}{10} = 194.4$$

- Now, compare Quantity A (194.4) with Quantity B (150).

$$194.4 > 150$$

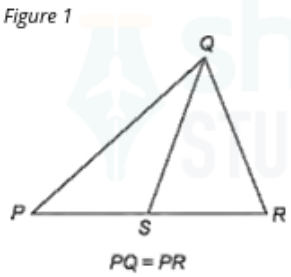
Both methods confirm that Quantity A is greater than Quantity B.

Step 4: Final Answer:
The calculated value of Quantity A is 194.4, which is greater than Quantity B's value of 150. Therefore, the correct answer is (A).

Quick Tip

For percentage problems in quantitative comparison, look for opportunities to estimate using benchmarks like 10%, 25%, or 50%. This can save valuable time compared to performing a full calculation, especially when the quantities are far apart.

4.



Quantity A: PS
Quantity B: SR

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (D) The relationship cannot be determined from the information given.

Solution:

Step 1: Understanding the Concept:
We are given an isosceles triangle PQR, where sides PQ and PR are equal. This implies that the angles opposite these sides are equal ($\angle PRQ = \angle PQR$). Point S is located somewhere on the base PR. We need to compare the lengths of the two segments created by point S, which are PS and SR.

Step 2: Key Approach:
The crucial part of the problem is that the position of point S on the segment PR is not specified. We are not told that S is the midpoint, or that QS is an altitude or an angle bisector. When information is missing, we should test different possible positions for S to see if the relationship between PS and SR changes.

Step 3: Detailed Explanation:
Let's consider the possible locations for point S on the line segment PR.

Case 1: S is the midpoint of PR.
If S is exactly in the middle of PR, then by definition of a midpoint:

$$PS = SR$$

In this case, the two quantities would be equal.

Case 2: S is closer to point P.
Imagine placing S on the segment PR but very close to P. For example, if the length of PR is 10, we could place S such that PS = 2. Then, the remaining length would be:

$$SR = PR - PS = 10 - 2 = 8$$

In this case, $SR > PS$, meaning Quantity B is greater.

Case 3: S is closer to point R.
Imagine placing S on the segment PR but very close to R. Using the same example where PR has a length of 10, we could place S such that SR = 2. Then, the other segment would be:

$$PS = PR - SR = 10 - 2 = 8$$

In this case, $PS > SR$, meaning Quantity A is greater.

Step 4: Final Answer:
We have found three possible scenarios based on the given information:
1. Quantity A = Quantity B (if S is the midpoint)
2. Quantity B ¿ Quantity A (if S is closer to P)
3. Quantity A ¿ Quantity B (if S is closer to R)
Since the relationship between Quantity A and Quantity B can change depending on the position of S, we cannot determine a single, consistent relationship. Therefore, the correct choice is (D).

Quick Tip

In geometry questions, do not make assumptions based on how a figure is drawn. Figures are often "not drawn to scale" to test whether you rely on visual appearance or on the given information. The information that PQ = PR is extra information designed to potentially distract you; the core of the problem is the unspecified location of point S.

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5. $y = 2x^2 + 7x - 3$
Quantity A: x
Quantity B: y

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (D) The relationship cannot be determined from the information given.

Solution:

Step 1: Understanding the Concept:
We are given a functional relationship between two variables, x and y, in the form of a quadratic equation. We need to compare the value of the input, x, with the value of the output, y. The relationship is not fixed and will change depending on the value of x.

Step 2: Key Approach:
The most effective method is to test different values for x and calculate the corresponding value of y. If we can find one value of x where x ¿ y and another value where y ¿ x, then the relationship cannot be determined. It's good practice to test simple integers like 0, 1, and -1.

Step 3: Detailed Explanation:
Let's test a few different values for x.

Case 1: Let x = 0.
Substitute x = 0 into the given equation:

$$y = 2(0)^2 + 7(0) - 3$$
$$y = 0 + 0 - 3$$
$$y = -3$$

In this case:
- Quantity A = x = 0
- Quantity B = y = -3
Comparing them, we find that $0 > -3$. So, for x=0, **Quantity A is greater**.

Case 2: Let x = 1.
Substitute x = 1 into the given equation:

$$y = 2(1)^2 + 7(1) - 3$$
$$y = 2(1) + 7 - 3$$
$$y = 2 + 7 - 3 = 6$$

In this case:
- Quantity A = x = 1
- Quantity B = y = 6
Comparing them, we find that $6 > 1$. So, for x=1, **Quantity B is greater**.

Step 4: Final Answer:
In our first test case (x=0), we found that Quantity A was greater than Quantity B. In our second test case (x=1), we found that Quantity B was greater than Quantity A. Since the relationship between x and y changes depending on the value of x, a consistent comparison is not possible. Therefore, the relationship cannot be determined from the information given.

Quick Tip

For algebraic comparison problems, testing a few strategic numbers is a powerful technique. Always consider testing zero, a positive integer (like 1 or 2), and a negative integer (like -1). These three values often reveal whether the relationship is constant or variable.

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6. $y > 4$
Quantity A: $\frac{3y+2}{5}$
Quantity B: y

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (B) Quantity B is greater.

Solution:

Step 1: Understanding the Concept:
We need to compare two algebraic expressions, Quantity A and Quantity B, given a condition on the variable y (that y is greater than 4). The best approach is to simplify the comparison algebraically.

Step 2: Key Approach:
We can set up a comparison between the two quantities and use standard algebraic manipulations (like multiplying or subtracting terms on both sides) to simplify it. The goal is to isolate the variable y and see what the relationship implies.

Step 3: Detailed Explanation:
Let's compare Quantity A and Quantity B. We can place a placeholder symbol (☐) between them that stands for the relationship we want to find (e.g., $<, >, =$).

$$\frac{3y+2}{5} \quad \square \quad y$$

To eliminate the fraction, we can multiply both sides of the comparison by 5. Since 5 is a positive number, the direction of the inequality will not change.

$$3y+2 \quad \square \quad 5y$$

Next, to gather the terms with y on one side, we can subtract 3y from both sides.

$$2 \quad \square \quad 5y-3y$$

$$2 \quad \square \quad 2y$$

Finally, we can divide both sides by 2 to isolate y. Since 2 is a positive number, the direction of the inequality remains unchanged.

$$1 \quad \square \quad y$$

The comparison has been simplified to comparing 1 and y. Now, we use the given information from the question, which is $y > 4$.
Since y is greater than 4, it must also be greater than 1. So, we can replace the box with a "less than" symbol:

$$1 < y$$

Since all our simplification steps are reversible and did not change the direction of the inequality, the original relationship must be the same.
Therefore, Quantity A is less than Quantity B.

Step 4: Final Answer:
Quantity A is less than Quantity B, which means Quantity B is greater. The correct answer is (B).

Quick Tip

In quantitative comparison questions with variables, try to simplify the comparison algebraically as if it were an inequality. This often reveals a simpler comparison that can be resolved using the given conditions. Always check if multiplying or dividing by a term could change the inequality's direction (it only happens with negative numbers).

7. Quantity A: $\frac{2^{30}-2^{29}}{2}$
Quantity B: 2^{28}

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (C) The two quantities are equal.

Solution:

Step 1: Understanding the Concept:
The question asks us to compare two quantities involving exponents. To do this, we need to simplify Quantity A using the rules of exponents.

Step 2: Key Formula or Approach:
The key exponent rules needed here are:
1. Factoring: $a^m - a^n$ can often be simplified by factoring out the smaller power.
2. The law of exponents for division: $\frac{a^m}{a^n} = a^{m-n}$.
3. The law of exponents for multiplication: $a^m \times a^n = a^{m+n}$.

Step 3: Detailed Explanation:
Let's focus on simplifying the expression for Quantity A.

$$\text{Quantity A} = \frac{2^{30} - 2^{29}}{2}$$

First, simplify the numerator, $2^{30} - 2^{29}$. We can factor out the term with the smaller exponent, which is 2^{29}.
Note that 2^{30} can be rewritten as $2^1 \times 2^{29}$.

$$2^{30} - 2^{29} = (2^1 \times 2^{29}) - (1 \times 2^{29})$$

Now, factor out 2^{29}:

$$= 2^{29} \times (2 - 1)$$
$$= 2^{29} \times 1 = 2^{29}$$

So, the numerator simplifies to 2^{29}. Now, substitute this back into the expression for Quantity A:

$$\text{Quantity A} = \frac{2^{29}}{2}$$

Since 2 is the same as 2^1, we can use the division rule for exponents:

$$\text{Quantity A} = \frac{2^{29}}{2^1} = 2^{29-1} = 2^{28}$$

Now we compare the simplified Quantity A with Quantity B.
- Quantity A = 2^{28}
- Quantity B = 2^{28}
The two quantities are identical.

Step 4: Final Answer:
After simplification, Quantity A is equal to 2^{28}, which is the same as Quantity B. Therefore, the two quantities are equal. The correct choice is (C).

Quick Tip

When you see a sum or difference of terms with exponents (like $a^x \pm a^y$), your first instinct should be to factor out the term with the smaller exponent. This is a very common technique for simplifying such expressions.

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8. Quantity A: $x^2 + 1$
Quantity B: $2x - 1$

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (A) Quantity A is greater.

Solution:

Step 1: Understanding the Concept:
We are asked to compare two algebraic expressions, one quadratic and one linear. Since no conditions are placed on x, the relationship must hold true for all possible real values of x. A good strategy is to analyze the difference between the two quantities.

Step 2: Key Approach:
Let's analyze the difference: Quantity A - Quantity B.
- If Quantity A - Quantity B ¿ 0, then Quantity A is greater.
- If Quantity A - Quantity B ¡ 0, then Quantity B is greater.
- If Quantity A - Quantity B = 0, then they are equal.
- If the sign of the difference can change, the relationship cannot be determined.

Step 3: Detailed Explanation:
Let's calculate the difference between Quantity A and Quantity B:

$$\text{Difference} = (\text{Quantity A}) - (\text{Quantity B})$$

$$\text{Difference} = (x^2 + 1) - (2x - 1)$$
$$\text{Difference} = x^2 + 1 - 2x + 1$$
$$\text{Difference} = x^2 - 2x + 2$$

Now, we need to determine if this expression is always positive, always negative, or if its sign can change. We can analyze this quadratic by completing the square.
To complete the square for $x^2 - 2x$, we take half of the coefficient of x (-2), which is -1, and square it to get 1.
We can rewrite our expression as:

$$\text{Difference} = (x^2 - 2x + 1) + 1$$

The expression in the parenthesis is a perfect square:

$$\text{Difference} = (x - 1)^2 + 1$$

Now let's analyze this simplified form.
- The term $(x - 1)^2$ is the square of a real number. The square of any real number is always greater than or equal to 0. So, $(x - 1)^2 \geq 0$.
- This means the smallest possible value of $(x - 1)^2$ is 0 (which occurs when x = 1).
- Therefore, the smallest possible value of the entire expression $(x - 1)^2 + 1$ is $0 + 1 = 1$.
So, the difference between Quantity A and Quantity B is always greater than or equal to 1. Since the difference is always positive, Quantity A must always be greater than Quantity B.

Step 4: Final Answer:
The expression $(x - 1)^2 + 1$ is always positive (in fact, it's always ≥ 1). This means that $x^2 + 1 > 2x - 1$ for all values of x. Therefore, Quantity A is greater. The correct choice is (A).

Quick Tip

When comparing two algebraic expressions, subtracting one from the other is a powerful technique. If the resulting expression can be shown to be always positive or always negative (e.g., by completing the square to show it's a squared term plus a positive constant), you can determine the relationship definitively.

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9. w > 1
Quantity A: $7w - 4$
Quantity B: $2w + 5$

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Correct Answer: (D) The relationship cannot be determined from the information given.

Solution:

Step 1: Understanding the Concept:
We need to compare two linear expressions of a variable 'w', given the condition that w ¿ 1. We can either simplify the comparison algebraically or test values that satisfy the condition.

Step 2: Key Approach (Algebraic Simplification):
Let's set up a comparison between the two quantities and simplify it to isolate 'w'.

$$7w - 4 \quad \square \quad 2w + 5$$

Subtract $2w$ from both sides:

$$5w - 4 \quad \square \quad 5$$

Add 4 to both sides:

$$5w \quad \square \quad 9$$

Divide both sides by 5 (a positive number, so the relationship does not change):

$$w \quad \square \quad \frac{9}{5}$$

Since $\frac{9}{5} = 1.8$, the original comparison is equivalent to comparing w with 1.8.

Step 3: Detailed Explanation (Testing Values):
The simplified comparison depends on whether w is greater than, less than, or equal to 1.8. The only information we are given is that $w > 1$. This condition allows for w to be on either side of 1.8. Let's test two cases.

Case 1: Choose a value for w such that $1 < w < 1.8$. Let's pick $w = 1.5$.
- Quantity A $= 7(1.5) - 4 = 10.5 - 4 = 6.5$.
- Quantity B $= 2(1.5) + 5 = 3 + 5 = 8$.
In this case, Quantity B (8) is greater than Quantity A (6.5).

Case 2: Choose a value for w such that $w > 1.8$. Let's pick $w = 2$.
- Quantity A $= 7(2) - 4 = 14 - 4 = 10$.
- Quantity B $= 2(2) + 5 = 4 + 5 = 9$.
In this case, Quantity A (10) is greater than Quantity B (9).

Step 4: Final Answer:
Since we found one case where Quantity B is greater and another case where Quantity A is greater, the relationship between the two quantities is not constant and depends on the specific value of w. Therefore, the relationship cannot be determined from the information given. The correct answer is (D).

Quick Tip

For quantitative comparisons, if you simplify the comparison and find it depends on how the variable relates to a specific number (in this case, 1.8), check if the given condition (w >1) is sufficient to fix that relationship. If not, the answer is likely (D). Testing values on either side of that critical number is a great way to confirm this.

Multiple Choice Questions

1. If 5x + 32 = 4 - 2x, what is the value of x?

(A) -4
(B) -3
(C) 4
(D) 7
(E) 12

Correct Answer: (A) -4

Solution:

Step 1: Understanding the Concept:
This question asks to solve a linear equation for the variable x. A linear equation is an equation in which the highest power of the variable is one. The goal is to isolate x on one side of the equation.

Step 2: Key Approach:
We will use algebraic manipulation to solve for x. The standard approach is to gather all terms involving x on one side of the equation and all constant terms on the other side.

Step 3: Detailed Explanation:
We are given the equation:

$$5x + 32 = 4 - 2x$$

First, add 2x to both sides of the equation to move all x terms to the left side.

$$(5x + 2x) + 32 = 4 - 2x + 2x$$

$$7x + 32 = 4$$

Next, subtract 32 from both sides of the equation to move all constant terms to the right side.

$$7x + 32 - 32 = 4 - 32$$

$$7x = -28$$

Finally, divide both sides by 7 to solve for x.

$$\frac{7x}{7} = \frac{-28}{7}$$
$$x = -4$$

Step 4: Final Answer:
The value of x that satisfies the equation is -4. Therefore, the correct answer is (A).

Quick Tip

When solving linear equations, perform inverse operations to isolate the variable. For example, to undo addition, you subtract; to undo multiplication, you divide. Aim to perform these steps systematically to avoid calculation errors.

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2. Which of the following numbers is farthest from the number 1 on the number line?

(A) -10
(B) -5
(C) 0
(D) 5
(E) 10

Correct Answer: (A) -10

Solution:

Step 1: Understanding the Concept:
The question asks to find which of the given numbers has the greatest distance from the number 1 on the number line. The distance between two numbers on a number line is the absolute value of their difference.

Step 2: Key Formula or Approach:
The distance between two numbers, a and b, is given by the formula:

$$\text{Distance} = |a - b|$$

We will calculate the distance of each option from 1 and find the largest value.

Step 3: Detailed Explanation:
Let's calculate the distance for each option from the number 1.
(A) Distance between -10 and 1:

$$|-10 - 1| = |-11| = 11$$

(B) Distance between -5 and 1:

$$|-5-1| = |-6| = 6$$

(C) Distance between 0 and 1:

$$|0-1| = |-1| = 1$$

(D) Distance between 5 and 1:

$$|5-1| = |4| = 4$$

(E) Distance between 10 and 1:

$$|10-1| = |9| = 9$$

Now, we compare the calculated distances: 11, 6, 1, 4, and 9. The largest value is 11.

Step 4: Final Answer:
The number with the greatest distance from 1 is -10, with a distance of 11 units. Therefore, the correct answer is (A).

Quick Tip

To find the distance between two numbers on a number line, simply subtract one from the other and take the absolute value. This method works regardless of whether the numbers are positive or negative. Visually, you can also count the "jumps" from one number to the other.

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3. The figure above shows the graph of the function f, defined by f(x) = $|2x|$ + 4 for all numbers x. For which of the following functions g, defined for all numbers x, does the graph of g intersect the graph of f?

(A) g(x) = x - 2
(B) g(x) = x + 3
(C) g(x) = 2x - 2
(D) g(x) = 2x + 3
(E) g(x) = 3x - 2

Correct Answer: (E) g(x) = 3x - 2

Solution:

Step 1: Understanding the Concept:
We are given the graph of an absolute value function, f(x) = —2x— + 4, and five linear functions. We need to determine which linear function's graph will intersect the graph of f(x). An

intersection occurs if there is at least one value of x for which f(x) = g(x).

Step 2: Key Approach:
First, let's define f(x) as a piecewise function. The absolute value function —2x— can be written as:
- $2x$ if $x \geq 0$
- $-2x$ if $x < 0$
So, the function f(x) is:

$$f(x) = \begin{cases} 2x + 4 & \text{if } x \geq 0 \\ -2x + 4 & \text{if } x < 0 \end{cases}$$

This means that for positive x, the graph of f(x) is a line with slope 2 and y-intercept 4. For negative x, it is a line with slope -2 and y-intercept 4. The vertex of this "V" shape is at (0, 4).
We can test each option g(x) by setting g(x) = f(x) for each piece of f(x) and solving for x. An intersection exists if we find a valid solution for x.

Step 3: Detailed Explanation:
Let's analyze each option.
(A) g(x) = x - 2.
- For $x \geq 0$: $x - 2 = 2x + 4 \implies -6 = x$. This is not valid since we assumed $x \geq 0$.
- For $x < 0$: $x - 2 = -2x + 4 \implies 3x = 6 \implies x = 2$. This is not valid since we assumed $x < 0$. No intersection.

(B) g(x) = x + 3.
- For $x \geq 0$: $x + 3 = 2x + 4 \implies -1 = x$. Not valid.
- For $x < 0$: $x + 3 = -2x + 4 \implies 3x = 1 \implies x = 1/3$. Not valid. No intersection.

(C) g(x) = 2x - 2.
- For $x \geq 0$: $2x - 2 = 2x + 4 \implies -2 = 4$. This is impossible, meaning the lines are parallel. No intersection.
- For $x < 0$: $2x - 2 = -2x + 4 \implies 4x = 6 \implies x = 1.5$. Not valid. No intersection.

(D) g(x) = 2x + 3.
- For $x \geq 0$: $2x + 3 = 2x + 4 \implies 3 = 4$. Impossible, lines are parallel. No intersection.
- For $x < 0$: $2x + 3 = -2x + 4 \implies 4x = 1 \implies x = 1/4$. Not valid. No intersection.

(E) g(x) = 3x - 2.
- For $x \geq 0$: $3x - 2 = 2x + 4 \implies x = 6$. This is a valid solution since $6 \geq 0$. An intersection occurs at x = 6.
- (We don't need to check the other piece since we already found an intersection, but for completeness: For $x < 0$: $3x - 2 = -2x + 4 \implies 5x = 6 \implies x = 1.2$. Not valid.)

Step 4: Final Answer:
The graph of g(x) = 3x - 2 intersects the graph of f(x) at x = 6. Therefore, the correct answer is (E).

Quick Tip

You can often solve graph intersection problems visually or by comparing slopes. The vertex of f(x) is at (0, 4). To the right of the y-axis, its slope is 2. Any line with a slope greater than 2 and a y-intercept less than 4 (like g(x)=3x-2) must eventually intersect it. Any line with a slope of 2 and a lower y-intercept will be parallel and never intersect.

---

4. A car got 33 miles per gallon using gasoline that cost $2.95 per gallon. Approximately what was the cost, in dollars, of the gasoline used in driving the car 350 miles?

(A) $10
(B) $20
(C) $30
(D) $40
(E) $50

Correct Answer: (C) $30

Solution:

Step 1: Understanding the Concept:
This is a multi-step problem that requires calculating the total amount of fuel consumed and then finding the total cost based on the price per unit of fuel. The word "approximately" indicates that we can use estimation to find the answer.

Step 2: Key Approach:
1. Calculate the total gallons of gasoline needed for the trip.
2. Calculate the total cost by multiplying the total gallons by the cost per gallon.
3. Use approximation to simplify the calculations.

Step 3: Detailed Explanation:
Part 1: Calculate Gallons Used
The car drives 350 miles and gets 33 miles per gallon.

$$\text{Gallons Used} = \frac{\text{Total Miles}}{\text{Miles Per Gallon}} = \frac{350}{33}$$

Part 2: Calculate Total Cost
The cost of gasoline is $2.95 per gallon.

$$\text{Total Cost} = (\text{Gallons Used}) \times (\text{Cost Per Gallon}) = \left(\frac{350}{33}\right) \times 2.95$$

Part 3: Estimation
The answer choices are spread out, so estimation is a good strategy. Let's round the numbers

to make the calculation easier.
- Round the fuel efficiency from 33 mpg to a number that divides 350 easily. 35 mpg is a close and convenient number, but let's stick with the PDF's approach.
- Round 33 mpg down to 30 or up to 35. Let's use the numbers given. 33 is close to 35. Or we can see that 350/33 is a bit more than 10 because 33 * 10 = 330.
- Round the cost from $2.95 per gallon up to $3.00 per gallon.
Now let's recalculate the approximate cost:

$$\text{Approximate Gallons} \approx \frac{350}{33} \approx 10.6 \text{ gallons}$$

$$\text{Approximate Cost} \approx 10.6 \times \$3 = \$31.80$$

Alternatively, using the PDF's direct approximation:

$$\text{Approximate Cost} \approx \left(\frac{350}{33}\right) \times 3 \approx (10) \times 3 = \$30$$

This result, $31.80 or $30, is very close to $30.

Step 4: Final Answer:
The estimated cost is approximately $30. This matches option (C).

Quick Tip

When a question uses the word "approximately" and the answer choices are far apart, rounding the numbers in the problem to "compatible numbers" (numbers that are easy to compute with) is a very effective and time-saving strategy.

---

5. A certain jar contains 60 jelly beans — 22 white, 18 green, 11 yellow, 5 red, and 4 purple. If a jelly bean is to be chosen at random, what is the probability that the jelly bean will be neither red nor purple?

(A) 0.09
(B) 0.15
(C) 0.54
(D) 0.85
(E) 0.91

Correct Answer: (D) 0.85

Solution:

Step 1: Understanding the Concept:
This question is about calculating probability. Probability is the ratio of the number of favorable outcomes to the total number of possible outcomes. The event we are interested in is

selecting a jelly bean that is "neither red nor purple".

Step 2: Key Formula or Approach:
The probability of an event E is given by:

$$P(E) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

We can solve this in two ways:
1. Directly count the number of jelly beans that are not red or purple.
2. Use the complement rule: find the probability of the opposite event (picking a red or purple jelly bean) and subtract it from 1.

Step 3: Detailed Explanation (Method 1: Direct Counting):
- Total number of jelly beans = 60. This is the total number of possible outcomes.
- We want a jelly bean that is "neither red nor purple". The number of favorable outcomes is the count of all other colors.
- Number of favorable jelly beans = (white) + (green) + (yellow) = 22 + 18 + 11 = 51.
- Alternatively, number of unfavorable jelly beans (red or purple) = 5 (red) + 4 (purple) = 9.
- Number of favorable jelly beans = Total - Unfavorable = 60 - 9 = 51.
- Now, calculate the probability:

$$P(\text{neither red nor purple}) = \frac{51}{60}$$

- The answer choices are in decimal form, so we need to convert this fraction:

$$\frac{51}{60} = \frac{17 \times 3}{20 \times 3} = \frac{17}{20} = \frac{17 \times 5}{20 \times 5} = \frac{85}{100} = 0.85$$

Step 3: Detailed Explanation (Method 2: Complement Rule):
- First, find the probability of the opposite event: picking a red or purple jelly bean.
- Number of red or purple jelly beans = 5 + 4 = 9.

$$P(\text{red or purple}) = \frac{9}{60} = \frac{3}{20} = 0.15$$

- The probability of an event not happening is 1 minus the probability that it does happen.

$$P(\text{neither red nor purple}) = 1 - P(\text{red or purple})$$
$$= 1 - 0.15 = 0.85$$

Step 4: Final Answer:
Both methods yield a probability of 0.85. Therefore, the correct answer is (D).

Quick Tip

For probability questions with "not" or "neither/nor", using the complement rule (P(not A) = 1 - P(A)) is often faster. Calculating the probability of the event you *don't* want and subtracting from 1 can save you from adding up many different cases.

Multiple Choice Questions

1. Which two of the following numbers have a product that is between -1 and 0? Indicate *both* of the numbers.

(A) -20
(B) -10
(C) 2^{-4}
(D) 3^{-2}

Correct Answer: (B) -10 and (C) 2^{-4}

Solution:

Step 1: Understanding the Concept:
We need to find a pair of numbers from the given options whose product is a negative number greater than -1. The condition "between -1 and 0" can be written as $-1 < \text{product} < 0$.

Step 2: Key Approach:
1. For the product to be negative, one number must be positive and the other must be negative.
2. Evaluate the positive numbers (those with negative exponents).
3. Test the possible pairs to see which product falls within the required range.

Step 3: Detailed Explanation:
First, identify the signs of the numbers.
- Options (A) -20 and (B) -10 are negative.
- Options (C) and (D) involve negative exponents, which represent reciprocals, so they are positive.
- (C) $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$
- (D) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Now, we must form pairs with one negative and one positive number. There are four possible pairs:
- Pair 1: (A) and (C) $\implies (-20) \times \frac{1}{16} = -\frac{20}{16} = -\frac{5}{4} = -1.25$. This is less than -1, so it is not in the range.
- Pair 2: (A) and (D) $\implies (-20) \times \frac{1}{9} = -\frac{20}{9} \approx -2.22$. This is less than -1, so it is not in the range.
- Pair 3: (B) and (C) $\implies (-10) \times \frac{1}{16} = -\frac{10}{16} = -\frac{5}{8} = -0.625$. This is between -1 and 0. This is a correct pair.
- Pair 4: (B) and (D) $\implies (-10) \times \frac{1}{9} = -\frac{10}{9} \approx -1.11$. This is less than -1, so it is not in the range.

Step 4: Final Answer:
The only pair whose product, -0.625, is between -1 and 0 is -10 and 2^{-4}. The correct choices

are (B) and (C).

Quick Tip

Remember that a negative exponent does not make a number negative. For any non-zero number a and positive integer n, $a^{-n} = \frac{1}{a^n}$. This is a common trap in number property questions.

---

2. Which of the following integers are multiples of both 2 and 3? Indicate *all* such integers.

(A) 8
(B) 9
(C) 12
(D) 18
(E) 21
(F) 36

Correct Answer: (C) 12, (D) 18, and (F) 36

Solution:

Step 1: Understanding the Concept:
The question asks us to identify all numbers from the list that are multiples of both 2 and 3. An integer that is a multiple of two or more numbers is a multiple of their least common multiple (LCM).

Step 2: Key Approach:
The least common multiple of 2 and 3 is $2 \times 3 = 6$. Therefore, any integer that is a multiple of both 2 and 3 must be a multiple of 6. The task simplifies to checking which of the given numbers are divisible by 6.

Step 3: Detailed Explanation:
We will test each option for divisibility by 6.
- (A) 8: $8 \div 6$ does not yield an integer. Not a multiple of 6.
- (B) 9: 9 is a multiple of 3 but not 2. Not a multiple of 6.
- (C) 12: $12 \div 6 = 2$. 12 is a multiple of 6. This is a correct answer.
- (D) 18: $18 \div 6 = 3$. 18 is a multiple of 6. This is a correct answer.
- (E) 21: 21 is a multiple of 3 but not 2. Not a multiple of 6.
- (F) 36: $36 \div 6 = 6$. 36 is a multiple of 6. This is a correct answer.

Step 4: Final Answer:
The integers from the list that are multiples of both 2 and 3 (and therefore of 6) are 12, 18,

and 36. The correct options are (C), (D), and (F).

Quick Tip

To quickly check if a number is divisible by 6, use the divisibility rules for 2 and 3. The number must be even (divisible by 2) AND the sum of its digits must be divisible by 3. For example, for 36: it's even, and 3+6=9, which is divisible by 3.

---

3. Each employee of a certain company is in either Department X or Department Y, and there are more than twice as many employees in Department X as in Department Y. The average (arithmetic mean) salary is \$25,000 for the employees in Department X and \$35,000 for the employees in Department Y. Which of the following amounts could be the average salary for all of the employees of the company?
Indicate *all* such amounts.

(A) \$26,000
(B) \$28,000
(C) \$29,000
(D) \$30,000
(E) \$31,000
(F) \$32,000
(G) \$34,000

Correct Answer: (A) \$26,000 and (B) \$28,000

Solution:

Step 1: Understanding the Concept:
This is a weighted average problem. The overall average salary is influenced more by the department with more employees. We are given the average salaries of two groups and an inequality describing the relative sizes of the groups.

Step 2: Key Formula or Approach:
Let N_X and N_Y be the number of employees in departments X and Y, respectively. Let $S_X = \$25,000$ and $S_Y = \$35,000$.
The overall average salary, S_{avg}, is calculated as:

$$S_{avg} = \frac{N_X \cdot S_X + N_Y \cdot S_Y}{N_X + N_Y}$$

We are given the condition $N_X > 2N_Y$. We can find the boundary for the average salary by considering the case where $N_X = 2N_Y$.

Step 3: Detailed Explanation:
The overall average salary must be between the two individual averages, so $\$25,000 < S_{avg} < \$35,000$.
Let's calculate the average salary for the boundary case where there are exactly twice as many employees in X as in Y, i.e., $N_X = 2N_Y$. We can substitute $N_X = 2$ and $N_Y = 1$ into the formula.

$$S_{boundary} = \frac{2 \cdot (\$25,000) + 1 \cdot (\$35,000)}{2+1}$$

$$S_{boundary} = \frac{\$50,000 + \$35,000}{3} = \frac{\$85,000}{3} \approx \$28,333.33$$

This value, \$28,333.33, is the average salary when the ratio of employees in X to Y is exactly 2 to 1. The problem states that there are *more than* twice as many employees in Department X ($N_X > 2N_Y$).
Since Department X has the lower salary (\$25,000), increasing its proportion of employees beyond the 2:1 ratio will pull the overall average *closer* to \$25,000. Therefore, the actual average salary must be *less than* the boundary value we calculated.
So, the possible range for the average salary is $\$25,000 < S_{avg} < \$28,333.33$.
Now let's check the options against this range:
- (A) \$26,000: This is in the range. Correct.
- (B) \$28,000: This is in the range. Correct.
- (C) \$29,000: This is greater than \$28,333.33. Incorrect.
- (D) \$30,000: Incorrect.
- (E) \$31,000: Incorrect.
- (F) \$32,000: Incorrect.
- (G) \$34,000: Incorrect.

Step 4: Final Answer:
The amounts that could be the average salary for all employees are those between \$25,000 and \$28,333.33. From the list, these are \$26,000 and \$28,000. The correct answers are (A) and (B).

Quick Tip

In weighted average problems with inequalities, first find the value at the boundary of the inequality (e.g., using equality). Then, reason about which direction the average will shift. If the group with the lower value becomes larger, the average moves down; if the group with the higher value becomes larger, the average moves up.

---

4. Which of the following could be the units digit of 57^n, where n is a positive integer?
Indicate *all* such digits.

(A) 0
(B) 1

(C) 2
(D) 3
(E) 4
(F) 5
(G) 6
(H) 7
(I) 8
(J) 9

Correct Answer: (B) 1, (D) 3, (H) 7, and (J) 9

Solution:

Step 1: Understanding the Concept:
The units digit of a product of integers is determined solely by the units digits of the integers being multiplied. To find the possible units digits of 57^n, we only need to look at the pattern of the units digits of powers of 7. This pattern is cyclical.

Step 2: Key Approach:
We will calculate the first few powers of 7 and observe the pattern of their units digits. The pattern will repeat, and the set of digits in one cycle will be the set of all possible units digits.

Step 3: Detailed Explanation:
Let's find the units digits for the first few powers of 7.
- For n = 1: 57^1 ends in the same digit as 7^1, which is **7**.
- For n = 2: 57^2 ends in the same digit as $7^2 = 49$, which is **9**.
- For n = 3: 57^3 ends in the same digit as $7^3 = 7 \times 49$. The units digit is $7 \times 9 = 63$, which is **3**.
- For n = 4: 57^4 ends in the same digit as $7^4 = 7 \times 343$. The units digit is $7 \times 3 = 21$, which is **1**.
- For n = 5: 57^5 ends in the same digit as $7^5 = 7 \times 2401$. The units digit is $7 \times 1 = 7$, which is **7**.

The pattern of the units digits is 7, 9, 3, 1. This four-digit pattern repeats indefinitely. Therefore, the only possible units digits for any positive integer power of 57 are 1, 3, 7, and 9.

Step 4: Final Answer:
We check the given options to see which of them are in our set of possible digits {1, 3, 7, 9}. The correct options are (B) 1, (D) 3, (H) 7, and (J) 9.

Quick Tip

To find the cyclicity of units digits, you only need to multiply the previous units digit by the base's units digit. For powers of 7: Start with 7. Then $7 \times 7 = 49 \to 9$. Then $9 \times 7 = 63 \to 3$. Then $3 \times 7 = 21 \to 1$. Then $1 \times 7 = 7$, and the pattern repeats.

Numeric Entry Sample Questions

1. One pen costs $0.25 and one marker costs $0.35. At those prices, what is the total cost of 18 pens and 100 markers?

Correct Answer: 39.50

Solution:

Step 1: Understanding the Concept:
The problem asks for the total cost of two different sets of items. We need to calculate the cost for each set of items separately and then add them together.

Step 2: Key Approach:
1. Calculate the total cost of the 18 pens.
2. Calculate the total cost of the 100 markers.
3. Add the two costs together to find the overall total cost.

Step 3: Detailed Explanation:
Cost of Pens:
The cost of one pen is $0.25. The total cost for 18 pens is:

$$\text{Cost of Pens} = 18 \times \$0.25 = \$4.50$$

Cost of Markers:
The cost of one marker is $0.35. The total cost for 100 markers is:

$$\text{Cost of Markers} = 100 \times \$0.35 = \$35.00$$

Total Cost:
The total cost is the sum of the cost of the pens and the markers.

$$\text{Total Cost} = \$4.50 + \$35.00 = \$39.50$$

Step 4: Final Answer:
The total cost of 18 pens and 100 markers is $39.50.

Quick Tip

For multi-step calculation problems, break the problem down into smaller, manageable parts. Calculate each part separately and then combine them at the end. This helps to reduce errors and makes the process clearer.

2. Rectangle R has length 30 and width 10, and square S has length 5. The perimeter of S is what fraction of the perimeter of R?

Correct Answer: $\frac{1}{4}$

Solution:

Step 1: Understanding the Concept:
This problem requires calculating the perimeters of a rectangle and a square and then expressing the ratio of these two perimeters as a fraction.

Step 2: Key Formula or Approach:
- The formula for the perimeter of a rectangle is $P_R = 2(l + w)$, where l is the length and w is the width.
- The formula for the perimeter of a square is $P_S = 4s$, where s is the side length.
- The required fraction is $\frac{\text{Perimeter of S}}{\text{Perimeter of R}}$.

Step 3: Detailed Explanation:
Perimeter of Rectangle R:
Given length $l = 30$ and width $w = 10$.

$$P_R = 2(30 + 10) = 2(40) = 80$$

Perimeter of Square S:
Given side length $s = 5$.

$$P_S = 4 \times 5 = 20$$

Fraction:
Now, we find the fraction of the perimeter of S to the perimeter of R.

$$\text{Fraction} = \frac{P_S}{P_R} = \frac{20}{80}$$

Simplify the fraction by dividing the numerator and the denominator by their greatest common divisor, which is 20.

$$\frac{20 \div 20}{80 \div 20} = \frac{1}{4}$$

Step 4: Final Answer:
The perimeter of S is $\frac{1}{4}$ of the perimeter of R.

Quick Tip

Always ensure you simplify fractions to their lowest terms unless the question specifies otherwise. In this case, $\frac{20}{80}$ simplifies to $\frac{1}{4}$.

3. For the large cars sold at an auction that is summarized in the table above, what was the average sale price per car?

Correct Answer: 6000

Solution:

Step 1: Understanding the Concept:
This question asks for the average (arithmetic mean) sale price. The average is calculated by dividing the total amount of sales by the number of items sold.

Step 2: Key Formula or Approach:

$$\text{Average Price} = \frac{\text{Total Sales Value}}{\text{Number of Units Sold}}$$

We need to extract the correct values from the "Large Cars" column in the provided table (Figure 7).

Step 3: Detailed Explanation:
From the table, for "Large Cars":
- The "Number of cars sold" is 20.
- The "Actual sales total (in thousands)" is \$120.

It is crucial to note that the sales total is given "in thousands". So, the actual sales total is:

$$\text{Total Sales} = \$120 \times 1,000 = \$120,000$$

Now, we can calculate the average sale price per large car:

$$\text{Average Sale Price} = \frac{\$120,000}{20}$$

To simplify the division:

$$\text{Average Sale Price} = \frac{\$12,000}{2} = \$6,000$$

Step 4: Final Answer:
The average sale price per large car was \$6,000.

Quick Tip

Pay close attention to units given in tables and charts, such as "(in thousands)" or "(in millions)". Forgetting to convert these units to their full value is a common source of error.

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4. A merchant made a profit of \$5 on the sale of a sweater that cost the merchant \$15. What is the profit expressed as a percent of the merchant's cost?

Give your answer to the nearest whole percent.

Correct Answer: 33

Solution:

Step 1: Understanding the Concept:
This problem asks to calculate the percent profit. Percent profit is the ratio of the profit to the original cost, expressed as a percentage.

Step 2: Key Formula or Approach:
The formula for percent profit relative to cost is:

$$\text{Percent Profit} = \left(\frac{\text{Profit}}{\text{Cost}}\right) \times 100\%$$

Step 3: Detailed Explanation:
We are given:
- Profit = \$5
- Cost = \$15

Substitute these values into the formula:

$$\text{Percent Profit} = \left(\frac{5}{15}\right) \times 100\%$$

First, simplify the fraction:

$$\frac{5}{15} = \frac{1}{3}$$

Now, calculate the percentage:

$$\text{Percent Profit} = \frac{1}{3} \times 100\% = 33.333...\%$$

The question asks for the answer to be rounded to the nearest whole percent.

$$33.333...\% \approx 33\%$$

Step 4: Final Answer:
The profit expressed as a percent of the merchant's cost is 33%.

Quick Tip

Be careful to identify what the percentage is based on. In this case, it's "percent of the merchant's cost," so the cost goes in the denominator. If it had asked for percent of the selling price, the selling price (\$15 cost + \$5 profit = \$20) would have been the denominator.

5. Working alone at its constant rate, machine A produces k liters of a chemical in 10 minutes. Working alone at its constant rate, machine B produces k liters of the chemical in 15 minutes. How many minutes does it take machines A and B, working simultaneously at their respective constant rates, to produce k liters of the chemical?

Correct Answer: 6

Solution:

Step 1: Understanding the Concept:
This is a classic "combined work rate" problem. To find the time it takes for two machines to complete a job together, we first need to determine their individual rates of work, then add those rates to get a combined rate.

Step 2: Key Formula or Approach:
- Rate of work = $\frac{\text{Work Done}}{\text{Time Taken}}$
- Combined Rate = Rate of Machine A + Rate of Machine B
- Time Taken = $\frac{\text{Work Done}}{\text{Combined Rate}}$

Step 3: Detailed Explanation:
Let the amount of work be producing k liters of the chemical.
Rate of Machine A:
Machine A produces k liters in 10 minutes.

$$\text{Rate}_A = \frac{k \text{ liters}}{10 \text{ minutes}}$$

Rate of Machine B:
Machine B produces k liters in 15 minutes.

$$\text{Rate}_B = \frac{k \text{ liters}}{15 \text{ minutes}}$$

Combined Rate:
When working together, their rates add up.

$$\text{Combined Rate} = \text{Rate}_A + \text{Rate}_B = \frac{k}{10} + \frac{k}{15}$$

To add these fractions, find a common denominator, which is 30.

$$\text{Combined Rate} = \frac{3k}{30} + \frac{2k}{30} = \frac{5k}{30} = \frac{k}{6} \text{ liters per minute}$$

Time to Produce k Liters Together:
The work to be done is to produce k liters. We use the combined rate to find the time.

$$\text{Time} = \frac{\text{Work}}{\text{Rate}} = \frac{k \text{ liters}}{\frac{k}{6} \text{ liters/minute}}$$

$$\text{Time} = k \times \frac{6}{k} = 6 \text{ minutes}$$

Step 4: Final Answer:
It takes the two machines 6 minutes to produce k liters of the chemical when working together.

Quick Tip

For work-rate problems, it's often easiest to think in terms of "job per unit of time". If Machine A does the job in 10 mins, its rate is 1/10 of the job per minute. Machine B's rate is 1/15 of the job per minute. Their combined rate is $1/10 + 1/15 = 1/6$ of the job per minute. The time to do 1 job is the reciprocal of the rate, which is 6 minutes.

Data Interpretation and Sample Questions

Questions 1 to 3 are based on the following data.
Annual Percent Change in Dollar Amount of Sales at Five Retail Stores from 2006 to 2008

Store	Percent Change from 2006 to 2007	Percent Change from 2007 to 2008
P	10	-10
Q	-20	9
R	5	12
S	-7	-15
T	17	-8

---

1. If the dollar amount of sales at Store P was $800,000 for 2006, what was the dollar amount of sales at that store for 2008?

(A) $727,200
(B) $792,000
(C) $800,000
(D) $880,000
(E) $968,000

Correct Answer: (B) $792,000

Solution:

Step 1: Understanding the Concept:
This problem requires calculating the result of two consecutive percentage changes. The base for the second percentage change is the result after the first change, not the original amount.

Step 2: Key Approach:
1. Calculate the sales amount for 2007 based on the 2006 amount and the first percentage

change.
2. Calculate the sales amount for 2008 based on the 2007 amount and the second percentage change.

Step 3: Detailed Explanation:
Calculate 2007 Sales:
The sales in 2006 for Store P were $800,000.
From the table, the percent change from 2006 to 2007 was +10%.
An increase of 10% is equivalent to multiplying by (1 + 0.10) = 1.10.

$$\text{Sales in } 2007 = \$800,000 \times 1.10 = \$880,000$$

Calculate 2008 Sales:
The base for the next calculation is the 2007 sales amount, which is $880,000.
From the table, the percent change from 2007 to 2008 was -10%.
A decrease of 10% is equivalent to multiplying by (1 - 0.10) = 0.90.

$$\text{Sales in } 2008 = \$880,000 \times 0.90 = \$792,000$$

Step 4: Final Answer:
The dollar amount of sales at Store P for 2008 was $792,000. Therefore, the correct answer is (B).

Quick Tip

A common mistake is to simply add the percentages (+10% and -10% = 0% change). Successive percentage changes are multiplicative, not additive. The base changes after the first percent change, leading to a different final result. A 10% increase followed by a 10% decrease will always result in a net decrease from the original amount.

---

2. At Store T, the dollar amount of sales for 2007 was what percent of the dollar amount of sales for 2008?
Give your answer to the nearest 0.1%.
Enter your answer in the box: [ ] %

Correct Answer: 108.7

Solution:

Step 1: Understanding the Concept:
The question asks us to express the 2007 sales as a percentage of the 2008 sales for Store T. This is a "reverse percentage" problem.

Step 2: Key Formula or Approach:
Let S_{2007} be the sales in 2007 and S_{2008} be the sales in 2008.

The table gives the relationship: S_{2008} is a certain percentage change from S_{2007}.
We need to calculate the ratio $\left(\frac{S_{2007}}{S_{2008}}\right) \times 100\%$.

Step 3: Detailed Explanation:
From the table for Store T, the percent change from 2007 to 2008 was -8%.
This means the sales in 2008 were 8% less than the sales in 2007.
We can write this relationship as an equation:

$$S_{2008} = S_{2007} \times (1 - 0.08)$$
$$S_{2008} = 0.92 \times S_{2007}$$

The question asks for S_{2007} as a percent of S_{2008}. We need to find the value of $\frac{S_{2007}}{S_{2008}}$. We can rearrange the equation above:

$$\frac{S_{2007}}{S_{2008}} = \frac{1}{0.92}$$

Now, we calculate the value of this fraction and convert it to a percentage:

$$\frac{1}{0.92} \approx 1.086956...$$

To express this as a percentage, we multiply by 100:

$$1.086956... \times 100\% \approx 108.6956...\%$$

The question asks to round to the nearest 0.1%.

$$108.6956...\% \approx 108.7\%$$

Step 4: Final Answer:
The dollar amount of sales for 2007 was 108.7% of the dollar amount of sales for 2008.

Quick Tip

When asked for "A is what percent of B?", the formula is always $(A/B) \times 100$. In this problem, A is the 2007 sales and B is the 2008 sales. Set up the relationship between B and A first ($S_{2008} = 0.92 \times S_{2007}$), then solve for the required ratio.

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3. Based on the information given, which of the following statements must be true? Indicate *all* such statements.

(A) For 2008 the dollar amount of sales at Store R was greater than that at each of the other four stores.
(B) The dollar amount of sales at Store S for 2008 was 22% less than that for 2006.
(C) The dollar amount of sales at Store R for 2008 was more than 17% greater than that for 2006.

Correct Answer: (C)

Solution:

Step 1: Understanding the Concept:
This question asks us to evaluate three statements based only on the percent change data. Since we are not given any initial sales figures, we can only make conclusions about the relative percentage changes, not about the absolute dollar amounts.

Step 2: Key Approach:
We will analyze each statement individually to determine if it must be true. For statements involving a combined percent change from 2006 to 2008, we will calculate the effective multiplier.
Multiplier $= (1 + \frac{\%\text{change}_1}{100}) \times (1 + \frac{\%\text{change}_2}{100})$

Step 3: Detailed Explanation:
Analysis of Statement A:
"For 2008 the dollar amount of sales at Store R was greater than that at each of the other four stores."
This statement compares the absolute dollar amounts of sales. The table only gives us percentage changes. A store could have a high percentage growth but start from a very low initial sales value, resulting in a lower final sales amount than a store with low growth but a high initial value. Since we don't know the initial sales for 2006 for any store, we cannot compare the final dollar amounts. Therefore, this statement does not have to be true.

Analysis of Statement B:
"The dollar amount of sales at Store S for 2008 was 22% less than that for 2006."
Let S_{2006} be the sales in 2006.
- Change from 2006 to 2007 is -7%. Multiplier $= 1 - 0.07 = 0.93$.
- Change from 2007 to 2008 is -15%. Multiplier $= 1 - 0.15 = 0.85$.
The combined multiplier from 2006 to 2008 is:

$$\text{Multiplier} = 0.93 \times 0.85 = 0.7905$$

This means $S_{2008} = 0.7905 \times S_{2006}$. The sales are 79.05% of the 2006 level.
The percent decrease is $1 - 0.7905 = 0.2095$, which is a 20.95% decrease.
A 22% decrease is not the same as a 20.95% decrease. Therefore, this statement is false.

Analysis of Statement C:
"The dollar amount of sales at Store R for 2008 was more than 17% greater than that for 2006."
Let R_{2006} be the sales in 2006.
- Change from 2006 to 2007 is +5%. Multiplier $= 1 + 0.05 = 1.05$.
- Change from 2007 to 2008 is +12%. Multiplier $= 1 + 0.12 = 1.12$.
The combined multiplier from 2006 to 2008 is:

$$\text{Multiplier} = 1.05 \times 1.12 = 1.176$$

This means $R_{2008} = 1.176 \times R_{2006}$. This represents a 17.6% increase over the 2006 sales.
The statement says the increase was "more than 17%". Since 17.6% is indeed greater than

17%, this statement is true.

Step 4: Final Answer:
Only statement (C) must be true based on the given information.

Quick Tip

To calculate the total percent change from two successive changes, p_1 and p_2, do not add them. Instead, use the formula: Total Change $= (1 + p_1/100)(1 + p_2/100) - 1$, and then multiply by 100. Remember to use negative values for percentage decreases.
