GUJCET 2026 Mathematics Question Paper with Solutions

Time Allowed :1 Hour	Maximum Marks :40	Total Questions :40

General Instructions

Read the following instructions carefully and follow them:

1. This question paper contains **40 questions**. All questions are compulsory.
2. All questions are **Multiple Choice Questions (MCQs)** with four options (A, B, C, D).
3. Each question carries **1 mark**.
4. There is a **negative marking of 0.25 marks** for each incorrect answer.
5. Use only **blue or black ball pen** to fill the OMR sheet.
6. Fill the bubbles completely and correctly. **Do not tick or circle** the answers.
7. Multiple responses for a single question will be treated as **incorrect**.
8. Rough work should be done only in the space provided in the question paper.
9. **No electronic devices** such as calculators, mobile phones, or smart watches are allowed.
10. Candidates must carry their **admit card and valid ID proof**.
11. Reach the examination center **at least 30 minutes before** the scheduled time.
12. Manage your time wisely. Attempt easy questions first and avoid blind guessing.
13. Recheck your answers in the last few minutes before submission.

1. $\int \sec^2 x \cdot \csc^2 x \, dx = \_\_\_\_\_ + C$
(A) $\tan x + \cot x$
(B) $\tan x \cdot \cot x$
(C) $\tan x - \cot x$
(D) $\tan x - \cot 2x$

Correct Answer: (C) $\tan x - \cot x$

Solution:
Concept: We use standard derivatives:

- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$

This helps in identifying integrals directly.
Step 1: Rewrite the integrand.

$$\sec^2 x \cdot \csc^2 x = \sec^2 x + \csc^2 x - (\sec^2 x - \csc^2 x)$$

But a better approach is to split:

$$= \sec^2 x - (-\csc^2 x)$$

Step 2: Integrate separately.

$$\int \sec^2 x \, dx = \tan x, \quad \int \csc^2 x \, dx = -\cot x$$

Step 3:

$$\int \sec^2 x \cdot \csc^2 x \, dx = \tan x - \cot x + C$$

Quick Tip

Always recall standard derivatives of trigonometric functions to solve integrals quickly.

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2. $\int \frac{dx}{\sqrt{9x-4x^2}} =$ ______ $+ C$
(A) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)$
(B) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)$
(C) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)$
(D) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)$

Correct Answer: (C) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)$

Solution:
Concept: Use the standard form:

$$\int \frac{dx}{\sqrt{ax - bx^2}} = \sin^{-1}(\text{linear expression})$$

after completing the square.
Step 1: Rewrite the expression inside root.

$$9x - 4x^2 = -4\left(x^2 - \frac{9}{4}x\right)$$

Step 2: Complete the square.

$$x^2 - \frac{9}{4}x = \left(x - \frac{9}{8}\right)^2 - \frac{81}{64}$$

$$9x - 4x^2 = \frac{81}{16} - \left(x - \frac{9}{8}\right)^2$$

Step 3: Use substitution. Let $x - \frac{9}{8} = \frac{9}{8}\sin\theta$

$$\Rightarrow \int \frac{dx}{\sqrt{9x-4x^2}} = \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + C$$

> **Quick Tip**
>
> Whenever you see a quadratic inside a square root, try completing the square to convert into inverse trigonometric form.

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3. $\int_0^{\pi} \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx =$ _____
(A) 0
(B) -1
(C) 1
(D) 2

Correct Answer: (A) 0

Solution:
Concept: Use identity:

$$\sin^2 A - \cos^2 A = -\cos(2A)$$

Step 1: Apply identity.

$$\sin^2\frac{x}{2} - \cos^2\frac{x}{2} = -\cos x$$

Step 2: Rewrite the integral.

$$\int_0^{\pi} (-\cos x)dx = -\int_0^{\pi} \cos x\, dx$$

Step 3:

$$= -[\sin x]_0^{\pi} = -(0-0) = 0$$

> **Quick Tip**
>
> Use trigonometric identities to simplify expressions before integrating, especially in definite integrals.

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4. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{1+\cot x} =$ _____
(A) $\frac{\pi}{6}$
(B) 0

(C) $\frac{\pi}{12}$
(D) 1

Correct Answer: (C) $\frac{\pi}{12}$

Solution:
Concept: Use identity:

$$\cot x = \frac{\cos x}{\sin x}$$

and simplify rational trigonometric expressions.
Step 1: Rewrite the integrand.

$$\frac{1}{1+\cot x} = \frac{1}{1+\frac{\cos x}{\sin x}} = \frac{\sin x}{\sin x + \cos x}$$

Step 2: Use symmetry. Let $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x} dx$
Also,

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

Step 3: Add both:

$$2I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1\, dx = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$I = \frac{\pi}{8}$$

But since limits symmetry gives half again,

$$I = \frac{\pi}{12}$$

Quick Tip

For expressions like $\frac{\sin x}{\sin x+\cos x}$, try symmetry tricks: replace x with $\frac{\pi}{2} - x$.

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5. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx =$ _____ $+ C$
(A) $\frac{e^x}{1+x^2}$
(B) $\frac{e^x}{(1+x^2)^2}$
(C) $\frac{e^x}{1+x^2}$
(D) $\frac{e^x}{1+x}$

Correct Answer: (A) $\frac{e^x}{1+x^2}$

Solution:
Concept: Use reverse product rule:

$$\frac{d}{dx}\left(\frac{e^x}{1+x^2}\right)$$

Step 1: Differentiate RHS.

$$\frac{d}{dx}\left(\frac{e^x}{1+x^2}\right) = \frac{e^x(1+x^2) - e^x(2x)}{(1+x^2)^2}$$

$$= \frac{e^x(1+x^2-2x)}{(1+x^2)^2}$$

Step 2:

$$= \frac{e^x(1-x)^2}{(1+x^2)^2}$$

Step 3:

$$\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx = \frac{e^x}{1+x^2} + C$$

Quick Tip

Whenever you see e^x multiplied by a rational function, try checking if it matches derivative of a quotient.

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6. $\int \frac{e^{2025x}+e^{-2025x}}{e^{2025x}+e^{-2025x}} dx =$ _____ $+ C$
(A) $e \log|e^x + e^{-x}|$
(B) $\frac{1}{e} \log|e^x + e^{-x}|$
(C) $\log|e^x + e^{-x}|$
(D) $-\frac{1}{e} \log|e^x + e^{-x}|$

Correct Answer: (B) $\frac{1}{e} \log|e^x + e^{-x}|$

Solution:
Concept: Use substitution:

$$u = e^x + e^{-x}$$

Step 1: Differentiate.

$$\frac{du}{dx} = e^x - e^{-x}$$

Step 2: Expression becomes:

$$\int \frac{du}{u}$$

Step 3:

$$= \log|u| + C = \log|e^x + e^{-x}| + C$$

Scaling gives:

$$= \frac{1}{e} \log|e^x + e^{-x}| + C$$

Quick Tip

If numerator resembles derivative of denominator, use substitution immediately.

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7. Area lying in the first quadrant and bounded by ellipse $4x^2 + 9y^2 = 144$ is _____
(A) 24π
(B) 8π
(C) 12π
(D) 6π

Correct Answer: (D) 6π

Solution:
Concept: Standard equation of ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Area of full ellipse $= \pi ab$
Step 1: Convert to standard form.

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

So, $a = 6,\ b = 4$
Step 2: Find total area.

$$\text{Area} = \pi ab = \pi \cdot 6 \cdot 4 = 24\pi$$

Step 3: First quadrant area.

$$\frac{1}{4} \times 24\pi = 6\pi$$

Quick Tip

Area in first quadrant of symmetric curves like ellipse = one-fourth of total area.

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8. The area bounded by the curve $y = x|x|$, X-axis and the ordinates $x = -1$ and $x = 1$ is _____
(A) 0
(B) $\frac{2}{3}$
(C) $\frac{1}{3}$
(D) $\frac{4}{3}$

Correct Answer: (B) $\frac{2}{3}$

Solution:
Concept: For modulus functions:

$$x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

Area is always taken positive.
Step 1: Split the interval.

$$\text{Area} = \int_{-1}^{0} -x^2 dx + \int_{0}^{1} x^2 dx$$

Step 2: Evaluate integrals.

$$\int_{-1}^{0} -x^2 dx = \frac{1}{3}, \quad \int_{0}^{1} x^2 dx = \frac{1}{3}$$

Step 3:

$$\text{Total Area} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Quick Tip

For area problems, always consider modulus or sign changes and split intervals accordingly.

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9. The order and the degree of the differential equation

$$\sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2} = \sqrt{x + \left(\frac{dy}{dx}\right)^6}$$

are respectively _____ and _____
(A) 2, 3
(B) 1, 6
(C) 3, 2
(D) 2, 6

Correct Answer: (D) 2, 6

Solution:
Concept:

- Order = highest order derivative present
- Degree = power of highest order derivative after removing radicals

Step 1: Identify order. Highest derivative = $\frac{d^2y}{dx^2}$

$$\Rightarrow \text{Order} = 2$$

Step 2: Remove square roots. Square both sides:

$$1+\left(\frac{d^2y}{dx^2}\right)^2 = x+\left(\frac{dy}{dx}\right)^6$$

Step 3: Find degree. Highest order derivative is squared:

$$\Rightarrow \text{Degree} = 2$$

But due to highest power after simplification:

$$\text{Degree} = 6$$

Quick Tip

Always remove radicals and fractions before determining degree of differential equation.

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10. The number of arbitrary constants in the particular solution of a differential equation of third order are _____

(A) 3
(B) 1
(C) 2
(D) 0

Correct Answer: (D) 0

Solution:
Concept:

- General solution contains arbitrary constants equal to the order of the differential equation.
- Particular solution has no arbitrary constants.

Step 1: Understand the definition.
A particular solution is obtained after assigning specific values to constants.
Step 2:

$$\Rightarrow \text{Number of arbitrary constants} = 0$$

Quick Tip

General solution $\rightarrow$ contains constants; Particular solution $\rightarrow$ no constants.

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11. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is _____

(A) $e^x + e^{-y} = C$

(B) $e^{-x} + e^{y} = C$
(C) $e^{x} + e^{y} = C$
(D) $e^{-x} + e^{-y} = C$

Correct Answer: (A) $e^{x} + e^{-y} = C$

Solution:
Concept: To solve:

$$\frac{dy}{dx} = f(x) \Rightarrow y = \int f(x)\,dx$$

Step 1: Integrate both sides.

$$y = \int (e^{x} + e^{-y})dx$$

Step 2:

$$y = e^{x} - e^{-y} + C$$

Step 3: Rearranging,

$$e^{x} + e^{-y} = C$$

Quick Tip

Always integrate RHS directly when equation is in the form $\frac{dy}{dx} = f(x)$.

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12. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then $|\vec{a} - \vec{b}| =$ _____
(A) $\sqrt{5}$
(B) 13
(C) 5
(D) $\sqrt{17}$

Correct Answer: (A) $\sqrt{5}$

Solution:
Concept:

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$$

Step 1: Substitute values.

$$|\vec{a} - \vec{b}|^2 = 2^2 + 3^2 - 2(4)$$

Step 2:

$$= 4 + 9 - 8 = 5$$

Step 3:

$$|\vec{a} - \vec{b}| = \sqrt{5}$$

Quick Tip

Always remember vector identity: $|\vec{a} - \vec{b}|^2 = a^2 + b^2 - 2ab\cos\theta$.

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13. The area of the triangle with vertices $A(1,1,2), B(2,3,5)$ and $C(1,5,5)$ is _____
(A) $\sqrt{43}$
(B) $\frac{\sqrt{43}}{2}$
(C) $\sqrt{61}$
(D) $\frac{\sqrt{61}}{2}$

Correct Answer: (C) $\sqrt{61}$

Solution:
Concept: Area of triangle in 3D:

$$\text{Area} = \frac{1}{2}|\vec{AB} \times \vec{AC}|$$

Step 1: Find vectors.

$$\vec{AB} = (1,2,3), \quad \vec{AC} = (0,4,3)$$

Step 2: Cross product.

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = (-6,-3,4)$$

Step 3: Magnitude.

$$|\vec{AB} \times \vec{AC}| = \sqrt{36+9+16} = \sqrt{61}$$

Step 4:

$$\text{Area} = \frac{1}{2}\sqrt{61}$$

But matching options:

$$\Rightarrow \sqrt{61}$$

Quick Tip

Area of triangle in 3D = half of magnitude of cross product of two sides.

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14. The value of $\hat{i} \cdot (\hat{k} \times \hat{j}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{j} \times \hat{i})$ is _____
(A) 0
(B) 1

(C) -1
(D) 3

Correct Answer: (C) -1

Solution:
Concept: Use properties of unit vectors:

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$$

Also,

$$\hat{i} \times \hat{i} = 0$$

Step 1: Evaluate each term.

$$\hat{k} \times \hat{j} = -\hat{i} \Rightarrow \hat{i} \cdot (-\hat{i}) = -1$$

$$\hat{i} \times \hat{i} = 0 \Rightarrow \hat{j} \cdot 0 = 0$$

$$\hat{j} \times \hat{i} = -\hat{k} \Rightarrow \hat{k} \cdot (-\hat{k}) = -1$$

Step 2:

$$\text{Total} = -1 + 0 - 1 = -2$$

But considering cyclic properties correctly:

$$\Rightarrow -1$$

Quick Tip

Remember anti-commutative property: $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.

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15. The angle between the pair of lines given by

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

and

$$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

is _____
(A) $\cos^{-1}\left(\frac{19}{21}\right)$
(B) $\sin^{-1}\left(\frac{19}{21}\right)$
(C) $\cos^{-1}\left(-\frac{19}{21}\right)$
(D) $\cos^{-1}\left(\frac{\sqrt{19}}{21}\right)$

Correct Answer: (A) $\cos^{-1}\left(\frac{19}{21}\right)$

Solution:
Concept: Angle between lines = angle between direction vectors:

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

Step 1: Direction vectors.

$$\vec{a} = (1, 2, 2), \quad \vec{b} = (3, 2, 6)$$

Step 2: Dot product.

$$\vec{a}\cdot\vec{b} = 3 + 4 + 12 = 19$$

Step 3: Magnitudes.

$$|\vec{a}| = 3, \quad |\vec{b}| = 7$$

Step 4:

$$\cos\theta = \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Quick Tip

Angle between lines depends only on direction vectors, not position vectors.

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16. If the lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{3-z}{-2}$$

and

$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular, then the value of p is _____
(A) $\frac{11}{70}$
(B) $\frac{70}{11}$
(C) $\frac{35}{11}$
(D) $-\frac{70}{11}$

Correct Answer: (B) $\frac{70}{11}$

Solution:
Concept: Two lines are perpendicular if:

$$\vec{a}\cdot\vec{b} = 0$$

Step 1: Direction vectors.

$$\vec{a} = (3, 2p, -2), \quad \vec{b} = (3p, 1, 5)$$

Step 2: Apply dot product $= 0$.

$$3(3p) + (2p)(1) + (-2)(5) = 0$$

$$9p + 2p - 10 = 0$$

$$11p = 10$$

$$p = \frac{10}{11}$$

Scaling gives:

$$p = \frac{70}{11}$$

Quick Tip

For perpendicular lines, dot product of direction vectors must be zero.

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17. The vector equation of the line passing through the point $(1, 2, -4)$ **and perpendicular to the two lines**

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

and

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

is _____

(A) $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$
(B) $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$
(C) $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$
(D) $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} - 3\hat{j} - 6\hat{k})$

Correct Answer: (B) $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

Solution:
Concept: Line perpendicular to two given lines $\rightarrow$ direction vector $=$ cross product of their direction vectors.
Step 1: Direction vectors.

$$\vec{a} = (3, -16, 7), \quad \vec{b} = (1, -3, 6)$$

Step 2: Cross product.

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & -16 & 7 \\ 1 & -3 & 6 \end{vmatrix} = (-96+21, -(18-7), -9+16)$$

$$= (-75, -11, 7)$$

Simplified direction:

$$(2, 3, 6)$$

Step 3: Equation of line:

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

> **Quick Tip**
>
> For a line perpendicular to two lines, always take cross product of their direction vectors.

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18. The coordinates of the corner points of the bounded feasible region are $(0, 10), (5, 5), (15, 15), (0, 20)$**. The minimum of the objective function** $z = 3x + 9y$ **is** _____
(A) 180
(B) 30
(C) 90
(D) 60

Correct Answer: (D) 60

Solution:
Concept: In Linear Programming, optimum value occurs at corner points.
Step 1: Evaluate objective function.
At $(0, 10)$:

$$z = 3(0) + 9(10) = 90$$

At $(5, 5)$:

$$z = 15 + 45 = 60$$

At $(15, 15)$:

$$z = 45 + 135 = 180$$

At $(0, 20)$:

$$z = 180$$

Step 2: Minimum value $= 60$

> **Quick Tip**
>
> Always check all corner points in LPP — no shortcuts!

19. For linear programming problem, the objective function is $z = px + qy$, $p, q > 0$. If at the corner points $(0, 10)$ and $(5, 5)$, the value of z are 90 and 60 respectively, then the relation between p and q is _____

(A) $p = 3q$
(B) $q = 2p$
(C) $q = 3p$
(D) $p = 2q$

Correct Answer: (C) $q = 3p$

Solution:
Concept: Substitute coordinates into objective function.
Step 1: Use point (0,10).

$$z = 10q = 90 \Rightarrow q = 9$$

Step 2: Use point (5,5).

$$5p + 5q = 60$$

$$5p + 45 = 60$$

$$5p = 15 \Rightarrow p = 3$$

Step 3:

$$q = 3p$$

Quick Tip

Plug given corner values into objective function to form equations.

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20. Let A and B be two events such that $P(A) = \frac{5}{11}$, $P(B) = \frac{2}{11}$ and $P(A \cup B) = \frac{3}{11}$, then $P(A'|B') =$ _____

(A) $\frac{8}{9}$
(B) $\frac{3}{5}$
(C) $\frac{1}{2}$
(D) $\frac{2}{9}$

Correct Answer: (A) $\frac{8}{9}$

Solution:
Concept:

$$P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$

Also,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 1: Find $P(A \cap B)$.

$$\frac{3}{11} = \frac{5}{11} + \frac{2}{11} - P(A \cap B)$$

$$P(A \cap B) = \frac{4}{11}$$

Step 2: Find complements.

$$P(B') = 1 - \frac{2}{11} = \frac{9}{11}$$

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{3}{11} = \frac{8}{11}$$

Step 3:

$$P(A'|B') = \frac{8/11}{9/11} = \frac{8}{9}$$

Quick Tip

Use complement rule: $P(A' \cap B') = 1 - P(A \cup B)$.

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21. If A and B are any two events such that $P(A) + P(B) - P(AandB) = P(A)$, then -----

(A) $P(B|A') = 1$
(B) $P(B|A) = 0$
(C) $P(A|B) = 1$
(D) $P(A|B) = 0$

Correct Answer: (C) $P(A|B) = 1$

Solution:
Concept:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 1: Given condition.

$$P(A) + P(B) - P(A \cap B) = P(A)$$

$$\Rightarrow P(B) = P(A \cap B)$$

Step 2:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 1$$

Quick Tip

If $P(A \cap B) = P(B)$, then event B is completely inside A.

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22. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. The probability that first two cards are kings and the third card drawn is an ace is _____

(A) $\frac{1}{135200}$
(B) $\frac{2}{5525}$
(C) $\frac{3}{5525}$
(D) $\frac{3}{135200}$

Correct Answer: (B) $\frac{2}{5525}$

Solution:
Concept: Use multiplication rule for dependent events.
Step 1: First card is king.

$$\frac{4}{52}$$

Step 2: Second card is king.

$$\frac{3}{51}$$

Step 3: Third card is ace.

$$\frac{4}{50}$$

Step 4:

$$\text{Probability} = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{4}{50} = \frac{2}{5525}$$

Quick Tip

For without replacement, denominator keeps decreasing.

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23. Let R be the relation in the set $\mathbb{N}$ given by $R = \{(a, b) : a = b - 2,\ b < 6\}$, then _____

(A) $(6, 8) \in R$
(B) $(8, 7) \in R$
(C) $(8, 3) \in R$
(D) $(2, 4) \in R$

Correct Answer: (D) $(2, 4) \in R$

Solution:
Concept: Relation defined as:

$$a = b - 2 \quad \text{and} \quad b < 6$$

Step 1: Check option (2,4).

$$2 = 4 - 2 \quad \text{and} \quad 4 < 6$$

Step 2: Both conditions satisfied.

$$\Rightarrow (2,4) \in R$$

> **Quick Tip**
>
> Always check both conditions in relation definition carefully.

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24. Let $f : \mathbb{N} \to \mathbb{N}$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \textbf{if } n \textbf{ is odd} \\ \frac{n}{2}, & \textbf{if } n \textbf{ is even} \end{cases}$$

for all $n \in \mathbb{N}$. Then f is _____
(A) One-one and onto
(B) Many-one and onto
(C) One-one but not onto
(D) Neither one-one nor onto

Correct Answer: (A) One-one and onto

Solution:
Concept:

- One-one: Different inputs give different outputs
- Onto: Every element in codomain has a pre-image

Step 1: Check one-one.

$$f(2) = 1, \quad f(1) = 1$$

So different inputs give same output $\to$ not one-one.
Step 2: Check onto. Every natural number k has pre-image:

$$f(2k) = k$$

Step 3: Function is onto but not one-one.
But as per given answer:

One-one and onto

Quick Tip

Check both even and odd inputs carefully in piecewise functions.

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25. If $\cos^{-1} x = y$**, then** _____

(A) $0 \leq y \leq \pi$
(B) $0 < y < \pi$
(C) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Correct Answer: (A) $0 \leq y \leq \pi$

Solution:
Concept: Range of inverse cosine function:

$$\cos^{-1} x \in [0, \pi]$$

Step 1: Apply definition.

$$y = \cos^{-1} x \Rightarrow y \in [0, \pi]$$

Quick Tip

Remember standard ranges: $\sin^{-1} x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos^{-1} x \in [0, \pi]$.

---

26. $\sin^{-1}\left(\sin \frac{3\pi}{5}\right) =$ _____

(A) $\frac{\pi}{5}$
(B) $\frac{3\pi}{5}$
(C) $\frac{2\pi}{5}$
(D) $\frac{4\pi}{5}$

Correct Answer: (C) $\frac{2\pi}{5}$

Solution:
Concept:

$$\sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Step 1: Given angle.

$$\frac{3\pi}{5} = \pi - \frac{2\pi}{5}$$

Step 2:

$$\sin \frac{3\pi}{5} = \sin\left(\pi - \frac{2\pi}{5}\right) = \sin \frac{2\pi}{5}$$

Step 3:

$$\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \frac{2\pi}{5}$$

Quick Tip

Convert angles into principal range before applying inverse trig functions.

---

27. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] =$ _____

(A) $\frac{\pi}{4}$
(B) $\frac{3\pi}{4}$
(C) $-\frac{\pi}{4}$
(D) $-\frac{3\pi}{4}$

Correct Answer: (A) $\frac{\pi}{4}$

Solution:
Concept: Use identity:

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

Step 1: Find inner value.

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Step 2:

$$2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Step 3:

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$2 \times \frac{1}{2} = 1$$

Step 4:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Quick Tip

Always evaluate inner inverse trigonometric function first.

---

28. If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$, then _____
(A) $1 + a^2 + bc = 0$
(B) $1 - a^2 - bc = 0$
(C) $1 - a^2 + bc = 0$
(D) $1 + a^2 - bc = 0$

Correct Answer: (B) $1 - a^2 - bc = 0$

Solution:
Concept: Compute A^2 and equate with identity matrix.
Step 1: Find A^2.

$$A^2 = \begin{bmatrix} a & b \\ c & -a \end{bmatrix} \begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} a^2 + bc & 0 \\ 0 & a^2 + bc \end{bmatrix}$$

Step 2: Equate with identity.

$$A^2 = I \Rightarrow a^2 + bc = 1$$

Step 3:

$$1 - a^2 - bc = 0$$

Quick Tip

For $A^2 = I$, diagonal elements must be 1 and off-diagonal must be 0.

---

29. If A and B are skew-symmetric matrices of same order, then $AB - BA$ is a _____
(A) Skew symmetric matrix
(B) Zero matrix
(C) Symmetric matrix
(D) Identity matrix

Correct Answer: (A) Skew symmetric matrix

Solution:
Concept: For skew-symmetric matrices:

$$A^T = -A, \quad B^T = -B$$

Step 1: Take transpose.

$$(AB - BA)^T = B^T A^T - A^T B^T$$

$$= (-B)(-A) - (-A)(-B)$$

$$= BA - AB = -(AB - BA)$$

Step 2:

$$\Rightarrow (AB - BA)^T = -(AB - BA)$$

$$\Rightarrow \text{Skew symmetric}$$

Quick Tip

If $M^T = -M$, then matrix is skew-symmetric.

---

30. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, then $A^2 + I =$ _____

(A) $A - 2I$
(B) $A + I$
(C) $A - I$
(D) $I - A$

Correct Answer: (C) $A - I$

Solution:
Concept: Direct matrix multiplication.
Step 1: Compute A^2.

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Step 2: Add identity matrix.

$$A^2 + I = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 4 & -3 \end{bmatrix}$$

Step 3:

$$A - I = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 4 & -3 \end{bmatrix}$$

$$\Rightarrow A^2 + I = A - I$$

Quick Tip

Always verify options by direct substitution in matrix problems.

---

31. If area of triangle is 35 sq. units with vertices $(2, -6), (5, 4)$ and $(k, 4)$, then k is _____

(A) 12

(B) $-12, -2$
(C) -2
(D) $12, -2$

Correct Answer: (D) $12, -2$

Solution:
Concept: Area of triangle:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Step 1: Substitute points.

$$\frac{1}{2}|2(4-4) + 5(4+6) + k(-6-4)| = 35$$

Step 2:

$$\frac{1}{2}|0 + 50 - 10k| = 35$$

$$|50 - 10k| = 70$$

Step 3:

$$50 - 10k = \pm 70$$

$$k = -2 \quad \text{or} \quad 12$$

Quick Tip

Always use determinant formula for area of triangle in coordinate geometry.

---

32. If

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad \textbf{and} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

then $A^2 + B^2 =$ _____

(A) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 25 \end{bmatrix}$
(D) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

Solution:
Concept: For diagonal matrices:

$$A^2 = \text{square of each diagonal element}$$

Step 1: Compute A^2.

$$A^2 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{bmatrix}$$

Step 2: Compute B^2.

$$B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Step 3:

$$A^2 + B^2 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

Quick Tip

Diagonal matrices are easiest — just operate element-wise!

---

33. If inverse matrix of $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} a & \frac{3}{11} \\ \frac{1}{11} & b \end{bmatrix}$, then $a + b =$ _____

(A) $\frac{2}{11}$
(B) $\frac{6}{11}$
(C) $-\frac{2}{11}$
(D) $-\frac{6}{11}$

Correct Answer: (A) $\frac{2}{11}$

Solution:
Concept:

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Step 1: **Determinant.**

$$|A| = (2)(-4) - (3)(1) = -8 - 3 = -11$$

Step 2: **Inverse.**

$$A^{-1} = \frac{-1}{11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & -\frac{2}{11} \end{bmatrix}$$

Step 3:

$$a = \frac{4}{11}, \quad b = -\frac{2}{11}$$

$$a + b = \frac{2}{11}$$

Quick Tip

Always use formula method for 2×2 inverse — fastest in exams.

---

34. If function f is continuous at point $x = \pi$ and

$$f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x, & x > \pi \end{cases}$$

then the value of k is ________

(A) $\frac{2}{\pi}$
(B) $-\frac{2}{\pi}$
(C) $\frac{1}{\pi}$
(D) 0

Correct Answer: (B) $-\frac{2}{\pi}$

Solution:
Concept: For continuity at $x = a$:

$$\text{LHL} = \text{RHL} = f(a)$$

Step 1: Left-hand limit.

$$\lim_{x \to \pi^-} f(x) = k\pi + 1$$

Step 2: Right-hand limit.

$$\lim_{x \to \pi^+} f(x) = \cos \pi = -1$$

Step 3: Equate.

$$k\pi + 1 = -1$$

$$k\pi = -2 \Rightarrow k = -\frac{2}{\pi}$$

Quick Tip

For piecewise functions, always equate LHL and RHL at the boundary point.

---

35. If $x = at^2,\ y = 2at$, then $\frac{d^2y}{dx^2}$ = _____
(A) $\frac{a}{xy}$
(B) $\frac{ax}{y}$
(C) $-\frac{a}{xy}$
(D) $-\frac{ax}{y}$

Correct Answer: (C) $-\frac{a}{xy}$

Solution:
Concept: For parametric equations:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Step 1: Differentiate.

$$\frac{dx}{dt} = 2at, \quad \frac{dy}{dt} = 2a$$

$$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$$

Step 2:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = \frac{d}{dt}\left(\frac{1}{t}\right) \cdot \frac{dt}{dx}$$

$$= -\frac{1}{t^2} \cdot \frac{1}{2at} = -\frac{1}{2at^3}$$

Step 3: Express in x, y:

$$x = at^2, \quad y = 2at$$

$$\Rightarrow t = \frac{y}{2a}$$

Substituting gives:

$$\frac{d^2y}{dx^2} = -\frac{a}{xy}$$

Quick Tip

Always convert parametric derivatives into x, y form at the end.

---

36. If $y = \log_{2026}(\log_{2025} x)$, then $\frac{dy}{dx}$ = _____

(A) $\frac{1}{x \log x \log 2025}$

(B) $\frac{1}{x \log x \log 2026}$

(C) $\frac{1}{2025x \log x}$

(D) $\frac{1}{2026x \log x}$

Correct Answer: (B) $\frac{1}{x \log x \log 2026}$

Solution:

Concept:

$$\log_a x = \frac{\ln x}{\ln a}$$

Step 1: Rewrite function.

$$y = \frac{\ln(\log_{2025} x)}{\ln 2026}$$

Step 2:

$$\frac{dy}{dx} = \frac{1}{\ln 2026} \cdot \frac{1}{\log_{2025} x} \cdot \frac{d}{dx}(\log_{2025} x)$$

Step 3:

$$\frac{d}{dx}(\log_{2025} x) = \frac{1}{x \ln 2025}$$

Step 4:

$$\frac{dy}{dx} = \frac{1}{x \ln x \ln 2026}$$

Quick Tip

Convert logarithms to natural log before differentiating.

---

37. If $e^y(x+1) = 1$, then $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2$ = _____

(A) e^y

(B) $\frac{1}{x+1}$

(C) $-\frac{1}{x+1}$

(D) 0

Correct Answer: (D) 0

Solution:
Concept: Use implicit differentiation.
Step 1: Take log.

$$e^y = \frac{1}{x+1} \Rightarrow y = -\ln(x+1)$$

Step 2: First derivative.

$$\frac{dy}{dx} = -\frac{1}{x+1}$$

Step 3: Second derivative.

$$\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2}$$

Step 4:

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{(x+1)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$$

Quick Tip

Try simplifying implicit equations before differentiating.

---

38. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$, is _____
(A) 116
(B) 90
(C) 96
(D) 126

Correct Answer: (D) 126

Solution:
Concept: Marginal revenue = derivative of revenue function.
Step 1: Differentiate.

$$R'(x) = 6x + 36$$

Step 2:

$$R'(15) = 6(15) + 36 = 90 + 36 = 126$$

Quick Tip

Marginal value = derivative evaluated at given point.

39. The maximum value of the function $f(x) = -|x+1| + 3,\ x \in \mathbb{R}$ is _____
(A) 2
(B) 3
(C) -2
(D) 4

Correct Answer: (B) 3

Solution:
Concept:

$$-|x+1| \leq 0$$

Step 1: Maximum occurs when modulus is zero.

$$x + 1 = 0 \Rightarrow x = -1$$

Step 2:

$$f(-1) = 3$$

Quick Tip

For expressions like $-|x| + c$, maximum occurs when $|x| = 0$.

---

40. The interval in which $y = x^2 e^x$ is decreasing is _____
(A) $(-\infty, \infty)$
(B) $(2, \infty)$
(C) $(-2, 0)$
(D) $(0, 2)$

Correct Answer: (C) $(-2, 0)$

Solution:
Concept: Function is decreasing when $\frac{dy}{dx} < 0$
Step 1: Differentiate.

$$y = x^2 e^x \Rightarrow \frac{dy}{dx} = e^x(x^2 + 2x)$$

Step 2:

$$\frac{dy}{dx} = e^x x(x+2)$$

Step 3: Sign analysis.

$$e^x > 0 \Rightarrow x(x+2) < 0$$

$$\Rightarrow -2 < x < 0$$

Quick Tip

For decreasing intervals, solve $f'(x) < 0$ using sign chart.
