Gravitation JEE Main PYQ - 1

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

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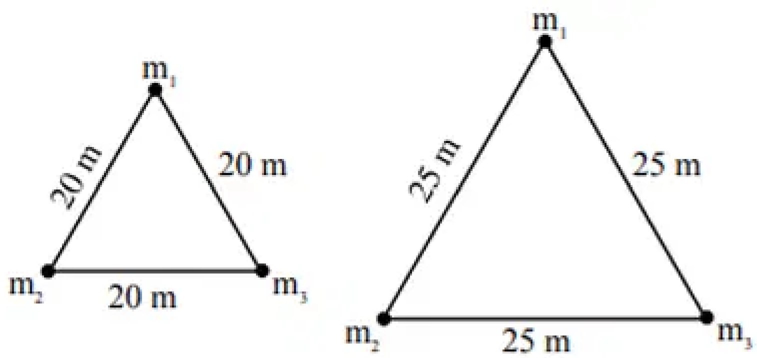
1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Gravitation

1. Three masses m_1 = 200 kg, m_2 = 300 kg and m_3 = 400 kg are kept at the vertices of on equilateral triangle of side 20 m. If the masses are shifted to new configuration such that they are at the vertices of an equilateral triangle of 25 m now. Find the work done in this process : (+4, -1)



a. 1.735×10^{-7}J

b. 17.35×10^{-7}J

c. 173.5×10^{-7}J

d. 1735×10^{-7}J

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2. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon = 27 days and gravitational attraction between the satellite and the Moon is neglected. (+4, -1)

a. 27 days

b. 1 day

c. 81 days

d. 3 days

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3. A satellite is revolving around a planet in orbit radius of 1.5 R. Additional minimum energy required to transfer the satellite to the new orbit radius of 3R is (G and M are mass of satellite \& planet) (+4, -1)

a. $\frac{GMm}{6R}$

b. $\frac{GMm}{2R}$

c. $\frac{G}{6RMm}$

d. $\frac{GMm}{3R}$

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4. For an object revolving around a planet of mass M and radius R_0 at a distance r from the center of the planet. If area velocity of the object is 10 km/sec. Now if the density of the planet increases by +10% and radius of the planet increases by +10%, then find the new area velocity at the same orbital radius. **(+4, -1)**

a. 12.1 km/sec

b. 10 km/sec

c. 15.5 km/sec

d. 8.5 km/sec

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5. A planet 'A' having density ρ and radius R has escape velocity $= 10\,\text{km}/\text{sec}$. Find the escape velocity of a planet B having density and radius both 10% that of planet A. **(+4, -1)**

a. $\frac{1}{\sqrt{10}}$

b. $\frac{1}{\sqrt{20}}$

c. $\frac{1}{\sqrt{30}}$

d. $\frac{1}{\sqrt{50}}$

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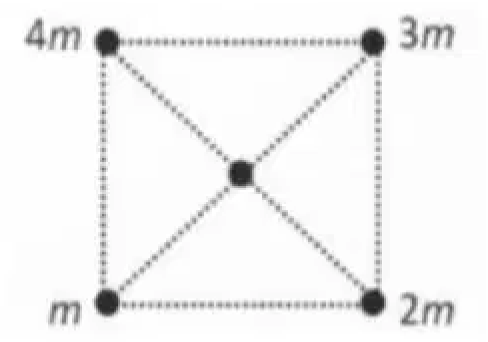
6. In the given situation, the force at the center on 1 kg mass is F_1. Now if $4m$ and $3m$ are interchanged, the force is F_2. Given $\frac{F_1}{F_2} = \frac{2}{\sqrt{\alpha}}$, find α. **(+4, -1)**



a. $\alpha = 5$

b. $\alpha = 3$

c. $\alpha = 7$

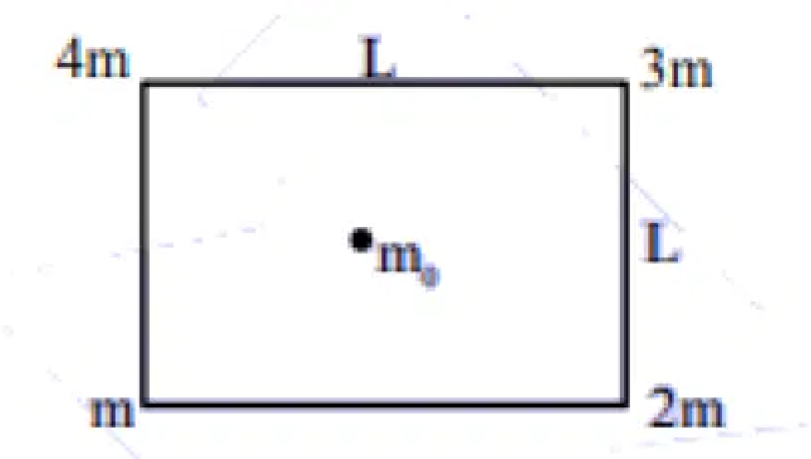
d. $\alpha = 1$

---

7. If initially the force on m_0 is F_0. When the positions of $4m$ and $3m$ are interchanged, the force becomes F'. If (+4, -1)

$$\frac{F_0}{F'} = \frac{\alpha}{\sqrt{5}},$$

find α.



a. 1

b. 2

c. 3

d. 4

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8. Escape velocity from a planet of radius R and density ρ is given as $10\,\mathrm{km\,s^{-1}}$. Find the escape velocity from a planet of radius $\frac{R}{10}$ and density $\frac{\rho}{10}$. (+4, -1)

a. $10\sqrt{100}\,\mathrm{m\,s^{-1}}$

b. $110\sqrt{10}\,\mathrm{m\,s^{-1}}$

c. $100\sqrt{10}\,\mathrm{m\,s^{-1}}$

d. $90\sqrt{10}\,\mathrm{m\,s^{-1}}$

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9. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator? [Use $g = \frac{GM}{R^2} = 9.8\,\mathrm{ms^{-2}}$ and radius of earth, $R = 6400\,\mathrm{km}$.] (+4, -1)

a. 49.17 N

b. 49 N

c. 48.83 N

d. 49.83 N

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10. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is : (+4, -1)

a. 3 T

b. 9 T

c. 27 T

d. 12 T

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11. The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be : (+4, -1)

a. $\frac{v_0 x_2^2}{x_1^2}$

b. $\frac{v_0 x_1}{x_2}$

c. $\frac{v_0 x_2}{x_1}$

d. $\frac{v_0 x_1^2}{x_2^2}$

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12. Two identical particles of mass 1 kg each go round a circle of radius R under the action of their mutual gravitational attraction. The angular speed of each particle is : (+4, -1)

a. $\sqrt{\frac{G}{2R^3}}$

b. $\frac{1}{2}\sqrt{\frac{G}{R^3}}$

c. $\frac{1}{2R}\sqrt{\frac{1}{G}}$

d. $\sqrt{\frac{2G}{R^3}}$

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13. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3 km. Find the mass of Mars. (+4, -1)
$\left\{\text{Given } \frac{4\pi^2}{G} = 6 \times 10^{11}\ \text{N}^{-1}\ \text{m}^{-2}\ \text{kg}^2\right\}$

a. 5.96×10^{19} kg

b. 3.25×10^{21} kg

c. 6.00×10^{23} kg

d. 7.02×10^{25} kg

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14. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is: (+4, -1)

a. $\frac{L}{M}$

b. $\frac{L}{2M}$

c. $\frac{2L}{M}$

d. $\frac{4L}{M}$

---

15. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately: [Take $g = 10$ ms^{-2}, $R = 6400 \times 10^3$ m, $\pi = 3.14$] **(+4, -1)**

a. does not change

b. 1200 minutes

c. 60 minutes

d. 84 minutes

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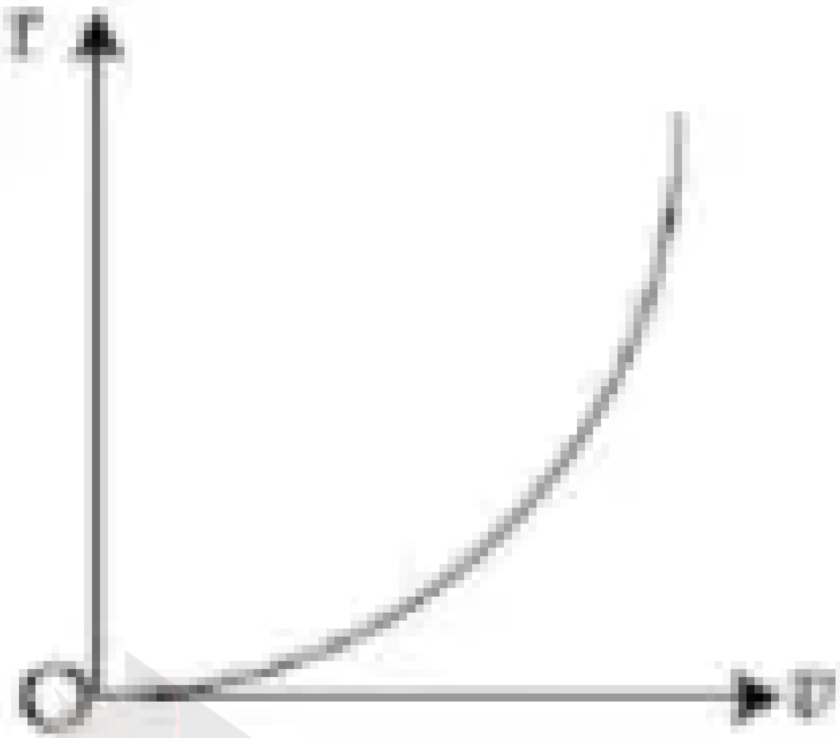
16. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = -\frac{C}{r}$, where C is a positive constant. The correct radius – velocity graph of the particle's motion is: **(+4, -1)**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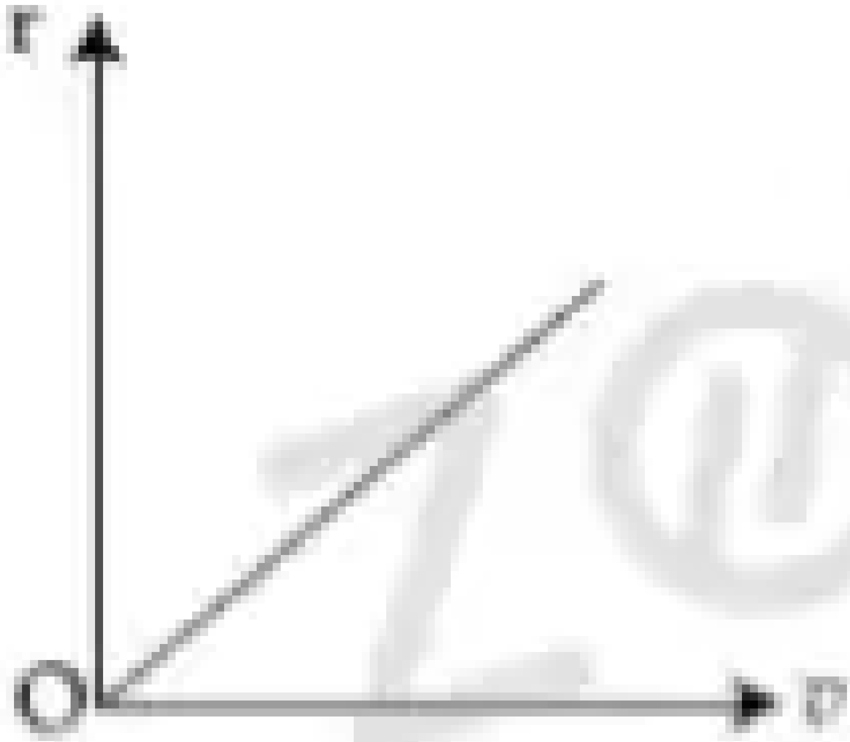
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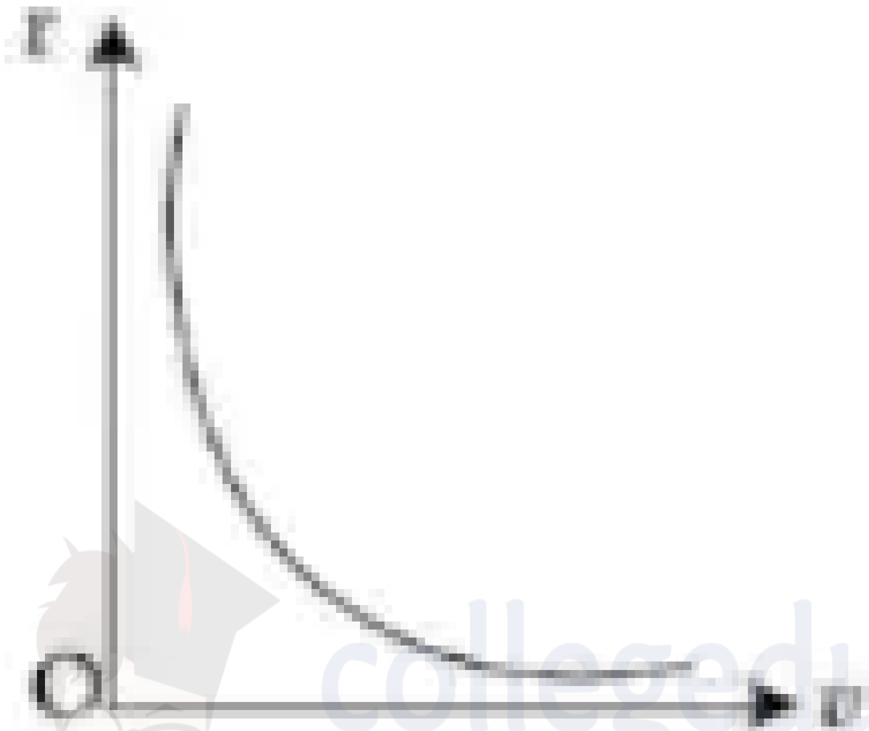
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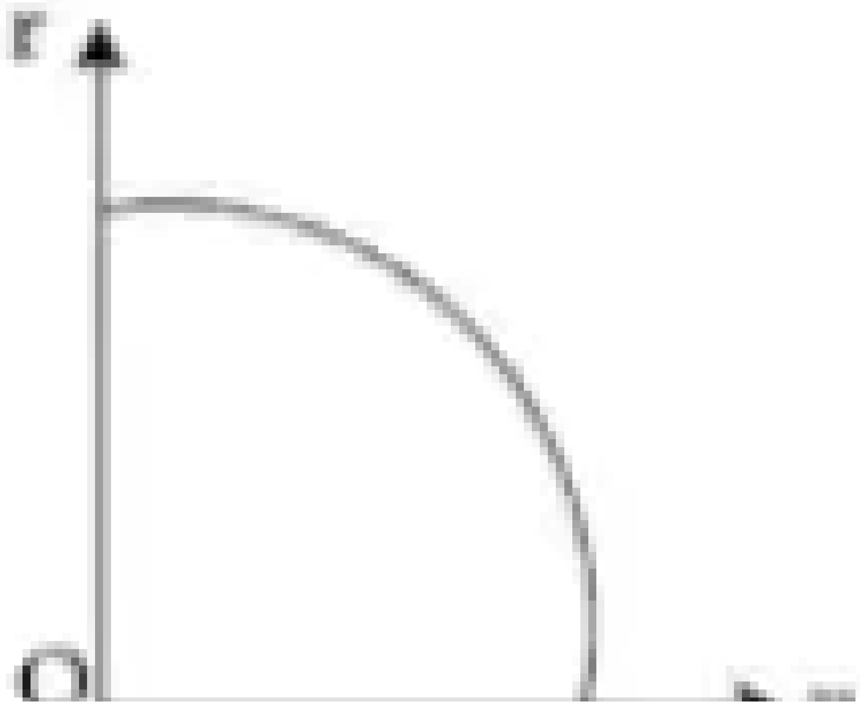
E
O

a. A

b. B

c. C

d. D

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17. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is : (+4, -1)

a. 8 : 1

b. 1 : 8

c. 2 : 1

d. 1 : 4

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18. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be : (+4, -1)

a. $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$

b. $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

c. $\sqrt{G(1+2\sqrt{2})}$

d. $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

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19. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is : (+4, -1)

a. $\frac{1}{2\pi}\sqrt{\frac{d^3}{2Gm}}$

b. $2\pi\sqrt{\frac{d^3}{Gm}}$

c. $2\pi\sqrt{\frac{d^3}{3Gm}}$

d. $\frac{1}{2\pi}\sqrt{\frac{2d^3}{Gm}}$

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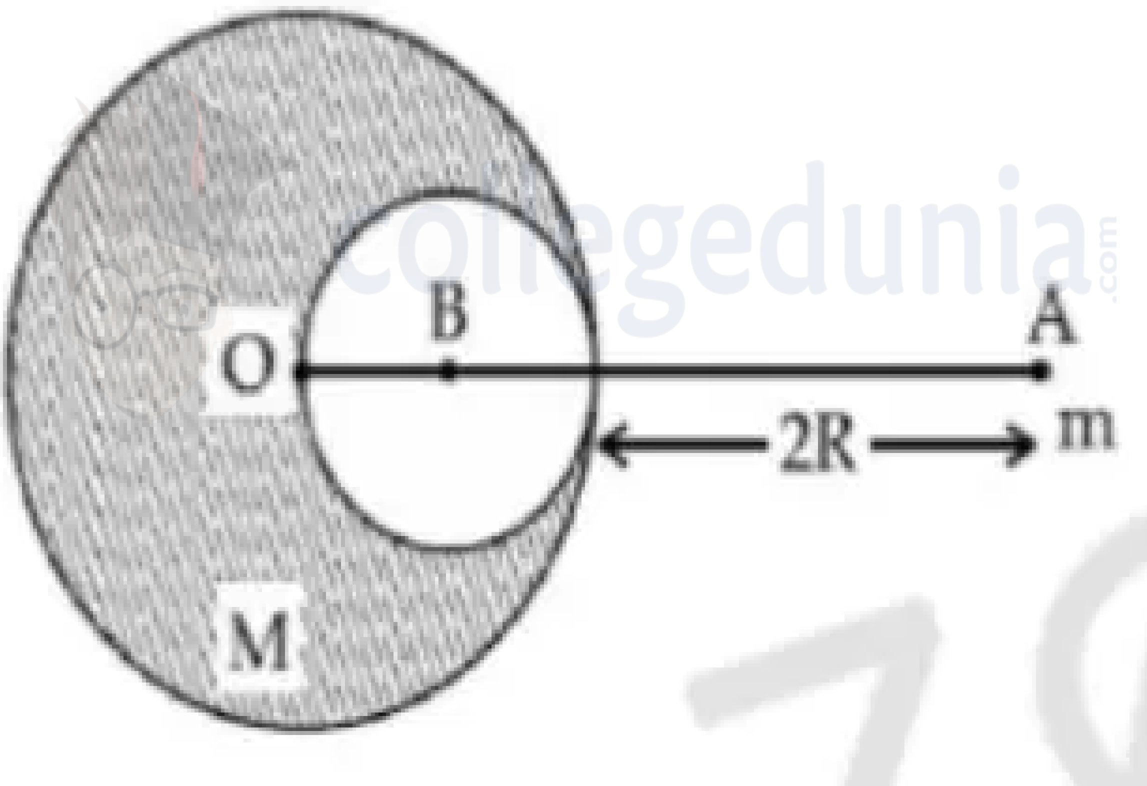
20. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius (R/2) is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is : (+4, -1)



a. 41 : 50

b. 50 : 41

c. 36 : 25

d. 25 : 36

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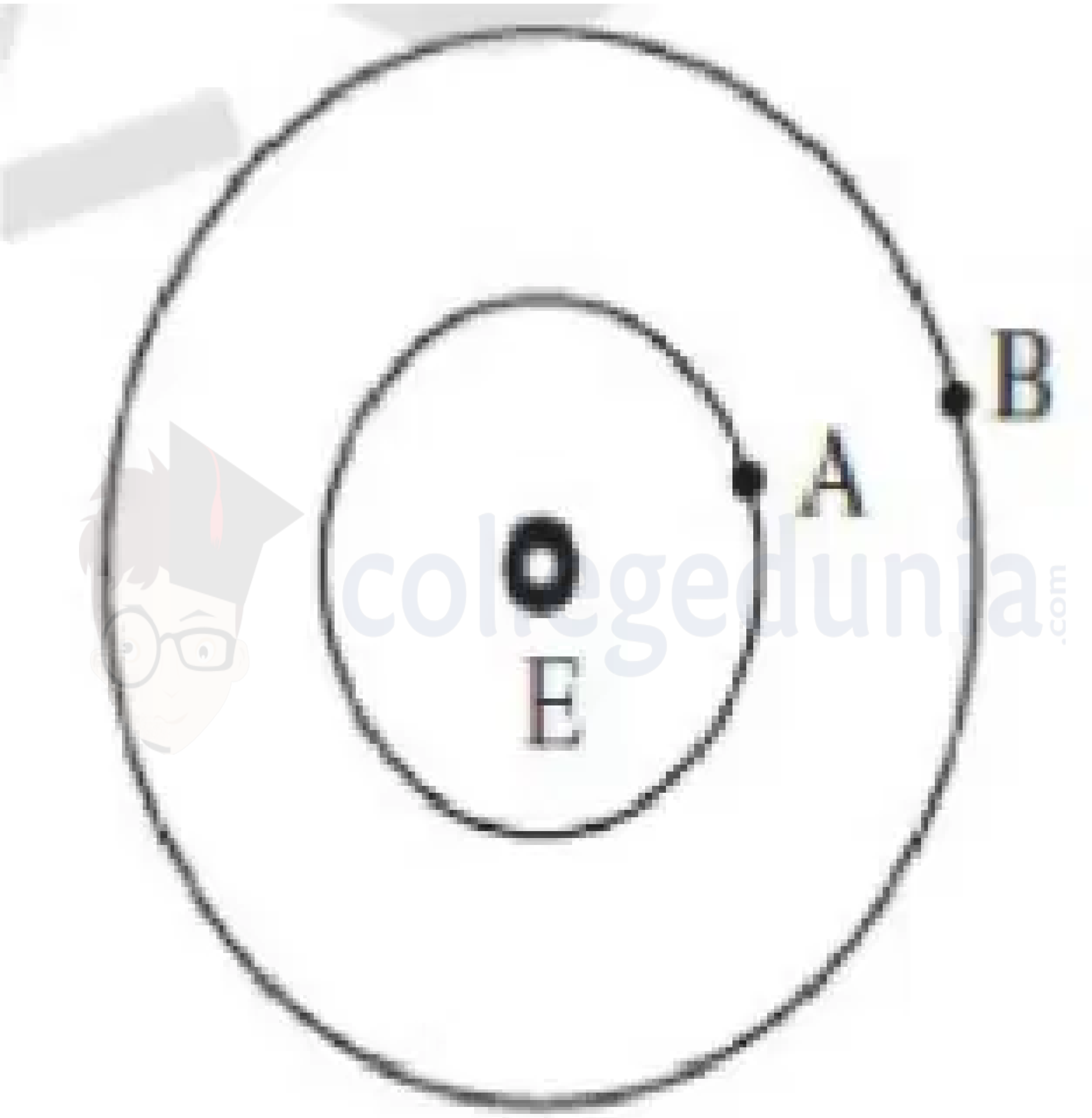
21. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$ is : [$R_e = 6400$ km, $M_e = 6 \times 10^{24}$ kg] (+4, -1)



a. 1.33×10^3 s

b. 4.24×10^3 s

c. 4.24×10^2 s

d. 3.33×10^2 s

22. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R. (+4, -1)

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$ In the light of the above statements, choose the most appropriate answer from the options given below :

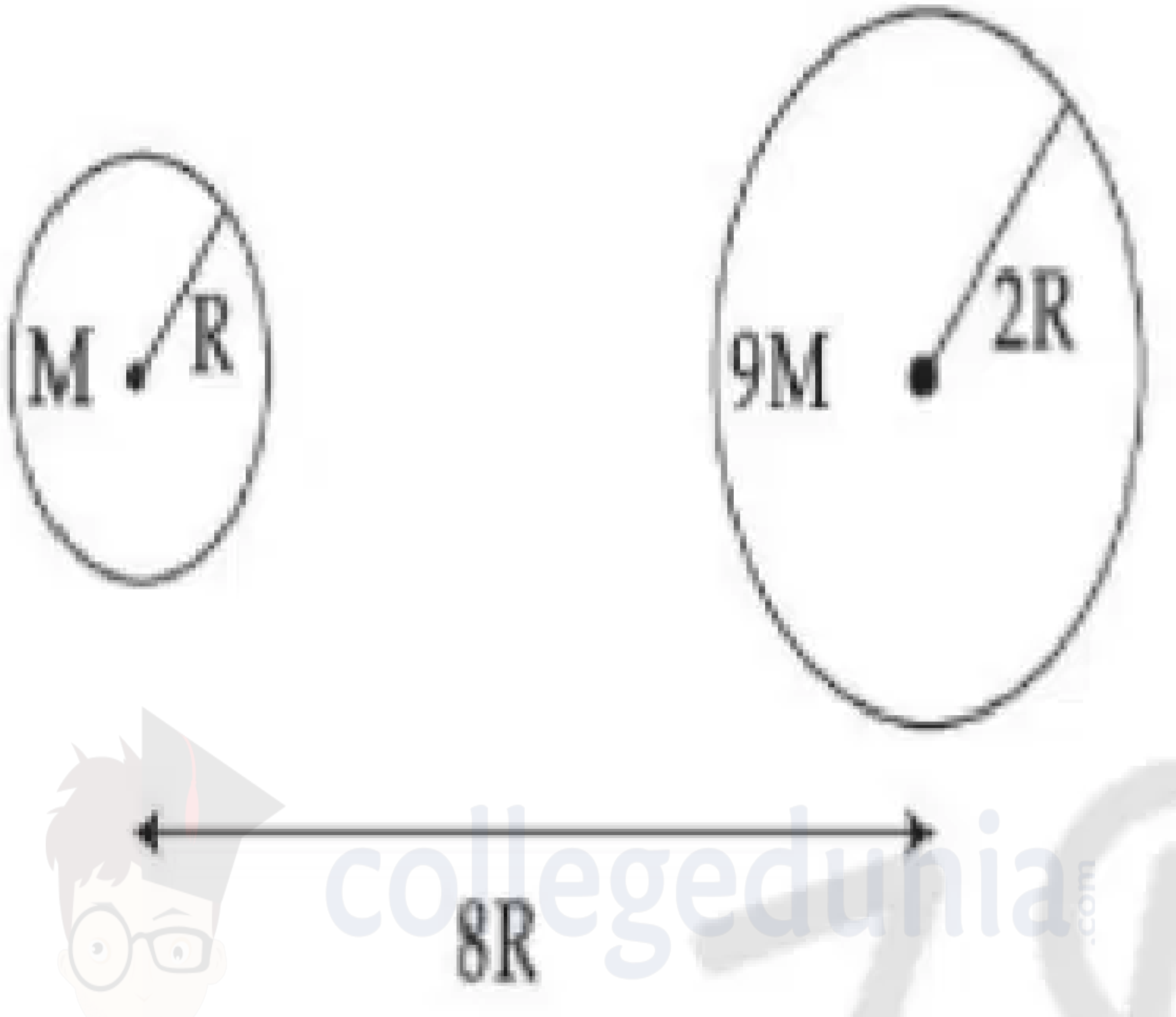
a. Both A and R are correct and R is the correct explanation of A

b. Both A and R are correct but R is NOT the correct explanation of A

c. A is correct but R is not correct

d. A is not correct but R is correct

23. Suppose two planets (spherical in shape) of radii R and 2 R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{aGM}{7R}}$ then the value of 'a' is _________. [Given : The two planets are fixed in their position] (+4, -1)



24. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{11}} v_e$. The value of x will be ________ . (+4, -1)

25. If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is : (Given : $r < R_E$) (+4, -1)

a. $1 + \frac{r}{R_E} + \frac{r^2}{R_E^2} + \frac{r^3}{R_E^3}$

b. $1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$

c. $1 - \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$

d. $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$

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26. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses : (+4, -1)

a. $V=\frac{\sqrt{2G(M_1+M_2)}}{r}$

b. $V=\frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$

c. $V=\sqrt{\frac{4G(M_1+M_2)}{r}}$

d. $V=\frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$

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27. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V. The value of V is: (+4, -1)

a. -60 G

b. -20 G

c. -4 G

d. +2 G

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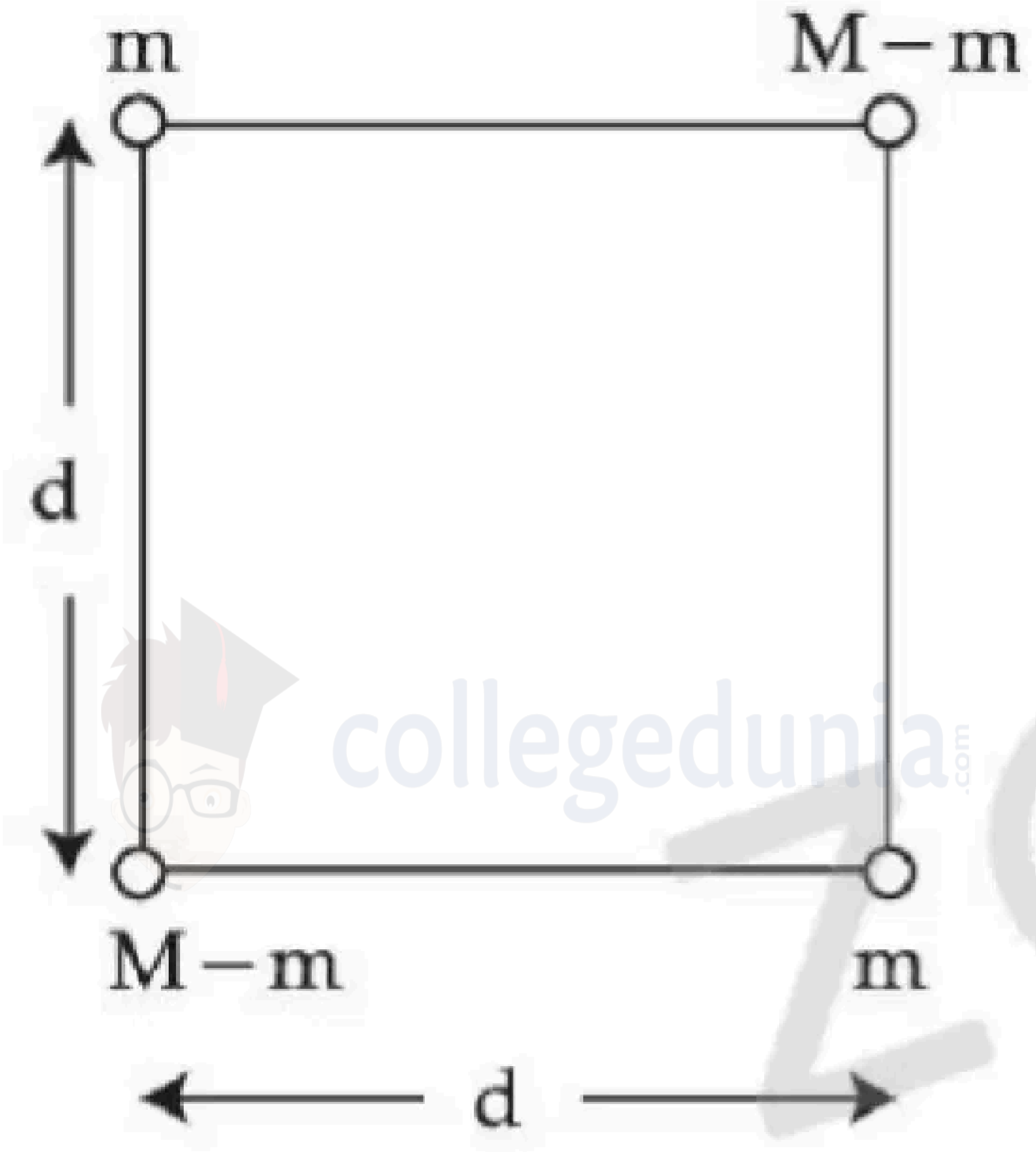
28. A body of mass (2M) splits into four masses m, M-m, m, M-m, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x : 1. The value of x is ________. (+4, -1)



29. Inside a uniform spherical shell : (+4, -1)
(a) the gravitational field is zero.
(b) the gravitational potential is zero.
(c) the gravitational field is same everywhere.
(d) the gravitation potential is same everywhere.

(e) all of the above

Choose the most appropriate answer from the options given below :

a. (b), (c) and (d) only

b. (a), (b) and (c) only

c. (a), (c) and (d) only

d. (e) only

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30. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R). **(+4, -1)**

Assertion (A) : The radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time and thus areal velocity of planet is constant.

Reason (R) : For a central force field the angular momentum is a constant. In the light of the above statements, choose the most appropriate answer from the options given below :

a. Both (A) and (R) are correct and (R) is the correct explanation of (A)

b. Both (A) and (R) are correct but (R) is not the correct explanation of (A)

c. (A) is correct but (R) is not correct

d. (A) is not correct but (R) is correct

Answers

1. Answer: a

Explanation:

Step 1: Understanding the Question:
We are asked to find the work done by an external agent to change the configuration of a system of three masses from one equilateral triangle arrangement to another with a different side length.
Step 2: Key Formula or Approach:
The work done by an external agent in changing the configuration of a system is equal to the change in its potential energy.

$$W_{ext} = \Delta U = U_{final} - U_{initial}$$

The gravitational potential energy of a system of three masses is:

$$U = -G\left(\frac{m_1m_2}{r_{12}} + \frac{m_2m_3}{r_{23}} + \frac{m_3m_1}{r_{31}}\right)$$

For an equilateral triangle, $r_{12} = r_{23} = r_{31} = r$, where r is the side length.
Step 3: Detailed Explanation:
Let's first calculate the sum of the products of masses:

$$\Sigma m_im_j = m_1m_2 + m_2m_3 + m_3m_1$$

$$\Sigma m_im_j = (200)(300) + (300)(400) + (400)(200)$$

$$\Sigma m_im_j = 60000 + 120000 + 80000 = 260000 = 26 \times 10^4 \text{ kg}^2$$

The gravitational constant is $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.
Initial Potential Energy (U_i):
The initial side length is $r_i = 20$ m.

$$U_i = -\frac{G}{r_i}(\Sigma m_im_j) = -\frac{6.67 \times 10^{-11}}{20}(26 \times 10^4)$$

$$U_i = -(0.3335 \times 10^{-11})(26 \times 10^4) = -8.671 \times 10^{-7} \text{ J}$$

Final Potential Energy (U_f):
The final side length is $r_f = 25$ m.

$$U_f = -\frac{G}{r_f}(\Sigma m_i m_j) = -\frac{6.67 \times 10^{-11}}{25}(26 \times 10^4)$$

$$U_f = -(0.2668 \times 10^{-11})(26 \times 10^4) = -6.9368 \times 10^{-7} \text{ J}$$

Work Done:

$$W_{ext} = U_f - U_i = (-6.9368 \times 10^{-7}) - (-8.671 \times 10^{-7})$$

$$W_{ext} = (8.671 - 6.9368) \times 10^{-7} = 1.7342 \times 10^{-7} \text{ J}$$

This value is approximately 1.735×10^{-7} J.
Step 4: Final Answer:
The work done in this process is 1.735×10^{-7} J.

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2. Answer: b

Explanation:

Concept:
For a body revolving around the same central mass, Kepler's third law applies:

$$T^2 \propto r^3$$

or

$$T \propto r^{3/2}$$

where T is the time period and r is the radius of orbit.
Step 1: Let the distance of the Moon from the Earth be r_m and the distance of the satellite be r_s. Given:

$$r_s = \frac{r_m}{9}$$

Step 2: Using Kepler's third law:

$$\frac{T_s}{T_m} = \left(\frac{r_s}{r_m}\right)^{3/2}$$

Step 3: Substituting the values:

$$\frac{T_s}{27} = \left(\frac{1}{9}\right)^{3/2}$$

$$\frac{T_s}{27} = \frac{1}{27}$$

Step 4: Solving for T_s:

$$T_s = 1\,\text{day}$$

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3. Answer: a

Explanation:

Step 1: Use the formula for mechanical energy of satellite.
The mechanical energy $M.E$ of the satellite is given by:

$$M.E = -\frac{GMm}{2a}$$

Where a is the semi-major axis of the orbit.
Step 2: Calculate the work done.
The work required to move the satellite from radius $1.5R$ to $3R$ is given by:

$$W = \Delta M.E = M_f - M_i$$

$$W = -\frac{GMm}{2(3R)} + \frac{GMm}{2(1.5R)}$$

Simplifying the expression:

$$W = \frac{GMm}{R}\left(\frac{1}{6} + \frac{1}{3}\right)$$

$$W = \frac{GMm}{6R}$$

Step 3: Conclusion.
The additional minimum energy required is $\frac{GMm}{6R}$.

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4. Answer: a

Explanation:

Step 1: Use the relationship for area velocity.
The area velocity v_a for an object revolving around a planet is given by:

$$v_a = \frac{dA}{dt} = \text{constant} \times \frac{r^2}{T}$$

where r is the orbital radius and T is the orbital period. **Step 2: Apply the changes in density and radius.**
If the density of the planet increases by 10%, then the mass M increases by 10%, and if the radius of the planet increases by 10%, the area velocity will change. Using the proportional relationship, the new area velocity is:

$$v_a' = 1.1 \times 1.1 \times v_a$$

Step 3: Conclusion.
Substituting the given values and calculating, the new area velocity is $12.1\,\text{km/sec}$.
Final Answer:

$$\boxed{12.1\,\text{km/sec}}$$

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5. **Answer: a**

Explanation:

Step 1: Formula for escape velocity.
The escape velocity v_e is given by:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant, M is the mass, and R is the radius of the planet. **Step 2: Apply the given conditions.**
The mass of a planet is given by $M = \rho R^3$. Hence, the escape velocity can be written as:

$$v_e = \sqrt{\frac{2G\rho R^3}{R}} = \sqrt{2G\rho R^2}$$

For planet B, both the radius and density are 10% that of planet A. Therefore, the escape velocity for planet B is:

$$v_{eB} = \sqrt{2G(0.1\rho)(0.1R)^2} = \frac{1}{\sqrt{10}} v_{eA}$$

Step 3: Conclusion.
The escape velocity of planet B is $\frac{1}{\sqrt{10}}$ of the escape velocity of planet A. **Final Answer:**

$$\boxed{\frac{1}{\sqrt{10}}}$$

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6. **Answer: a**

Explanation:

Step 1: Use the formula for forces.
The force between two masses $F = G\frac{m_1 m_2}{r^2}$, where r is the distance between the masses. In this case, the change in distance will affect the force. **Step 2: Apply the given relation.**
The relationship $\frac{F_1}{F_2} = \frac{2}{\sqrt{\alpha}}$ is used to find α, considering the new distance after interchanging $4m$ and $3m$. **Step 3: Conclusion.**
Solving the equation, we find that $\alpha = 5$. **Final Answer:**

$$\boxed{\alpha = 5}$$

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7. **Answer: b**

Explanation:

Concept:
The gravitational force on a mass due to several surrounding masses is the **vector sum** of individual forces. Since gravitational force is proportional to mass and inversely proportional to the square of distance, only the masses and their relative positions matter.

$$F = G\frac{m_0 m}{r^2}$$

Step 1: Identify forces acting on m_0. From the figure, four masses are placed at the corners of a square of side L:

$$4m,\ 3m,\ m,\ 2m$$

The distance from the center m_0 to each corner is:

$$r = \frac{L}{\sqrt{2}}$$

Hence force due to a mass km is:

$$F_k = G\frac{m_0(km)}{(L/\sqrt{2})^2} = \frac{2Gm_0m}{L^2}k$$

Step 2: Initial force F_0. Resolve forces along x and y axes. Initially, opposite corners contain $4m$ and $2m$ (vertical pair), and $3m$ and m (horizontal pair). Net components:

$$F_{x0} \propto (3m - m) = 2m$$

$$F_{y0} \propto (4m - 2m) = 2m$$

Thus:

$$F_0 \propto \sqrt{(2m)^2 + (2m)^2} = 2m\sqrt{2}$$

Step 3: Final force F' after interchange of $4m$ and $3m$. Now vertical pair is $3m$ and $2m$, horizontal pair is $4m$ and m. Net components:

$$F_{x'} \propto (4m - m) = 3m$$

$$F_{y'} \propto (3m - 2m) = m$$

Thus:

$$F' \propto \sqrt{(3m)^2 + (m)^2} = m\sqrt{10}$$

Step 4: Compute the ratio.

$$\frac{F_0}{F'} = \frac{2m\sqrt{2}}{m\sqrt{10}} = \frac{2\sqrt{2}}{\sqrt{10}} = \frac{2}{\sqrt{5}}$$

Comparing with:

$$\frac{F_0}{F'} = \frac{\alpha}{\sqrt{5}}$$

$$\alpha = 2$$

$$\boxed{\alpha = 2}$$

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8. **Answer: c**

Explanation:

Concept:
Escape velocity from a planet is:

$$v_e = \sqrt{\frac{2GM}{R}}$$

For a planet of uniform density ρ,

$$M = \frac{4}{3}\pi R^3 \rho$$

Hence,

$$v_e \propto R\sqrt{\rho}$$

Step 1: Write the proportionality relation.

$$\frac{v_{e2}}{v_{e1}} = \frac{R_2\sqrt{\rho_2}}{R_1\sqrt{\rho_1}}$$

Step 2: Substitute the given values.

$$R_2 = \frac{R}{10}, \quad \rho_2 = \frac{\rho}{10}$$

$$\frac{v_{e2}}{10\,\text{km s}^{-1}} = \frac{\frac{R}{10}\sqrt{\frac{\rho}{10}}}{R\sqrt{\rho}} = \frac{1}{10\sqrt{10}}$$

Step 3: Find the new escape velocity.

$$v_{e2} = \frac{10}{10\sqrt{10}}\,\text{km s}^{-1} = \frac{1}{\sqrt{10}}\,\text{km s}^{-1}$$

Convert to m/s:

$$v_{e2} = \frac{1000}{\sqrt{10}}\,\text{m s}^{-1} = 100\sqrt{10}\,\text{m s}^{-1}$$

$$\boxed{v_{e2} = 100\sqrt{10}\,\text{m s}^{-1}}$$

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9. **Answer: c**

Explanation:

Step 1: Weight at pole $W_p = mg = 49\text{ N}$. Thus, mass $m = \frac{49}{9.8} = 5\text{ kg}$.
Step 2: Effective gravity at equator $g' = g - R\omega^2$.
Step 3: $R\omega^2$ for Earth is approx 0.0338 m/s^2.
Step 4: $W_e = m(g - R\omega^2) = 5(9.8 - 0.0338) = 49 - 0.169 = 48.831\text{ N}$.

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10. **Answer: c**

Explanation:

Step 1: By Kepler's Third Law, $T^2 \propto R^3$.
Step 2: $\frac{T_2}{T_1} = (\frac{R_2}{R_1})^{3/2} = (\frac{9R}{R})^{3/2}$.
Step 3: $T_2 = T \times (9)^{3/2} = T \times (3^2)^{3/2} = T \times 3^3 = 27T$.

---

11. **Answer: c**

Explanation:

Step 1: Understanding the Concept:
Angular momentum of a planet revolving around the Sun is conserved because the gravitational force is a central force.
The planet moves in an elliptical orbit. The minimum speed occurs at the furthest point (aphelion) and the maximum speed occurs at the nearest point (perihelion).

Step 2: Key Formula or Approach:
Conservation of Angular Momentum: $L = mvr\sin\phi$.
At perihelion and aphelion, the velocity is perpendicular to the position vector ($\phi = 90^\circ$), so:

$$mv_{max}r_{min} = mv_{min}r_{max}$$

Step 3: Detailed Explanation:
Given:
Minimum distance (Perihelion) = x_1.
Maximum distance (Aphelion) = x_2.
Minimum speed (at x_2) = v_0.
Let the maximum speed (at x_1) be v.
By Conservation of Angular Momentum:

$$mvx_1 = mv_0x_2$$

$$v = \frac{v_0x_2}{x_1}$$

Step 4: Final Answer:
The maximum speed will be $\frac{v_0x_2}{x_1}$.

---

12. Answer: b

Explanation:

Let the two identical particles have mass $m = 1$ kg.
They move in a circle of radius R, so they are always diametrically opposite each other.
The center of the circle is the center of mass of the two-particle system.
The distance between the two particles is constant and equal to the diameter of the circle, $d = 2R$.
Consider one of the particles. The only force acting on it is the gravitational attraction from the other particle. This force is directed towards the center of the circle and provides the necessary centripetal force.

Gravitational force: $F_g = \frac{Gm_1m_2}{d^2} = \frac{Gm^2}{(2R)^2} = \frac{Gm^2}{4R^2}$.
Centripetal force: $F_c = ma_c = m\omega^2 R$, where ω is the angular speed.
Equating the two forces:
$m\omega^2 R = \frac{Gm^2}{4R^2}$.
We can cancel one factor of 'm' from both sides.
$\omega^2 R = \frac{Gm}{4R^2}$.
Now, solve for ω^2:
$\omega^2 = \frac{Gm}{4R^3}$.
Take the square root to find the angular speed ω:
$\omega = \sqrt{\frac{Gm}{4R^3}}$.
Given that the mass $m = 1$ kg:
$\omega = \sqrt{\frac{G(1)}{4R^3}} = \sqrt{\frac{G}{4R^3}} = \frac{\sqrt{G}}{2\sqrt{R^3}} = \frac{1}{2}\sqrt{\frac{G}{R^3}}$.

---

13. Answer: c

Explanation:

For a moon orbiting a planet, the gravitational force provides the necessary centripetal force.
$F_g = F_c$
$\frac{GMm}{r^2} = m\omega^2 r$
Where M is the mass of Mars, m is the mass of the moon, r is the orbital radius, and ω is the angular velocity.
$GM = \omega^2 r^3$.
We know that angular velocity $\omega = \frac{2\pi}{T}$, where T is the time period.
$GM = \left(\frac{2\pi}{T}\right)^2 r^3 = \frac{4\pi^2 r^3}{T^2}$.
Rearranging the formula to solve for the mass of Mars (M):
$M = \frac{4\pi^2 r^3}{GT^2} = \left(\frac{4\pi^2}{G}\right)\frac{r^3}{T^2}$.
First, convert the given quantities to SI units.
Orbital radius $r = 9.0 \times 10^3 \text{ km} = 9.0 \times 10^6 \text{ m}$.
Time period T = 7 hours 30 minutes.
$T = (7 \times 3600) + (30 \times 60) = 25200 + 1800 = 27000 \text{ s} = 2.7 \times 10^4 \text{ s}$.
Now, substitute the values into the equation for M.
$M = (6 \times 10^{11}) \times \frac{(9.0\times10^6)^3}{(2.7\times10^4)^2}$.
$M = (6 \times 10^{11}) \times \frac{729\times10^{18}}{7.29\times10^8}$.
$M = (6 \times 10^{11}) \times \left(\frac{729}{7.29}\right) \times \frac{10^{18}}{10^8}$.

$M = (6 \times 10^{11}) \times 100 \times 10^{10}$.
$M = 6 \times 10^{11} \times 10^{2} \times 10^{10} = 6 \times 10^{23}$ kg.

---

14. Answer: b

Explanation:

Step 1: Recall Kepler's Second Law. Areal velocity is the rate at which area is swept: $\frac{dA}{dt}$
.
Step 2: The area swept in small time dt is $dA = \frac{1}{2}|\vec{r} \times \vec{v}|dt$.
Step 3: We know angular momentum is $\vec{L} = M(\vec{r} \times \vec{v})$. Thus, $|\vec{r} \times \vec{v}| = \frac{L}{M}$.
Step 4: Substitute back:

$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{L}{M}\right) = \frac{L}{2M}$$

---

15. Answer: d

Explanation:

Step 1: For bodies to float at the equator, the effective gravity g' must be zero:

$$g' = g - R\omega^2 = 0 \implies \omega = \sqrt{\frac{g}{R}}$$

Step 2: The duration of the day is the time period T:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

Step 3: Substitute values:

$$T = 2 \times 3.14 \times \sqrt{\frac{6400 \times 10^3}{10}} = 6.28 \times \sqrt{640000} = 6.28 \times 800 = 5024 \text{ seconds}$$

Step 4: Convert to minutes:

$$T_{min} = \frac{5024}{60} \approx 83.73 \text{ minutes} \approx 84 \text{ minutes}$$

---

16. Answer: c

Explanation:

Step 1: Find the force F from the potential $U(r)$:

$$F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{C}{r}\right) = -\frac{C}{r^2}$$

The magnitude of the force is $\frac{C}{r^2}$.
Step 2: For circular motion, centripetal force is provided by this field:

$$\frac{mv^2}{r} = \frac{C}{r^2} \implies v^2 = \frac{C}{mr} \implies r = \frac{C}{mv^2}$$

Step 3: Since $r \propto \frac{1}{v^2}$, the graph of r vs v is a hyperbola-like curve where r decreases as v increases.

---

17. Answer: a

Explanation:

Step 1: Angular velocity ω is related to time period T by $\omega = \frac{2\pi}{T}$.
Step 2: Therefore, $\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$.
Step 3: Given $T_1 = 1$ hr and $T_2 = 8$ hr.
Step 4: $\frac{\omega_1}{\omega_2} = \frac{8}{1}$.

---

18. Answer: a

Explanation:

Step 1: Net force on one particle $F_{net} = \frac{mv^2}{R}$. Force from adjacent particles $F_1 = \frac{Gm^2}{(\sqrt{2}R)^2}$ at $45°$. Force from opposite particle $F_2 = \frac{Gm^2}{(2R)^2}$.

Step 2: $F_{net} = 2F_1 \cos 45^\circ + F_2 = 2(\frac{Gm^2}{2R^2})\frac{1}{\sqrt{2}} + \frac{Gm^2}{4R^2} = \frac{Gm^2}{R^2}(\frac{1}{\sqrt{2}} + \frac{1}{4})$.
Step 3: $\frac{mv^2}{R} = \frac{Gm^2}{R^2}(\frac{2\sqrt{2}+1}{4}) \Rightarrow v^2 = \frac{Gm}{R}(\frac{2\sqrt{2}+1}{4})$.
Step 4: With $m = 1, R = 1$, $v = \frac{\sqrt{G(1+2\sqrt{2})}}{2}$.

---

19. Answer: c

Explanation:

Step 1: The centripetal force for mass m is provided by gravitational attraction: $\frac{G(m)(2m)}{d^2} = m\omega^2 r_1$.
Step 2: Distance of mass m from center of mass $r_1 = \frac{2m(d)}{m+2m} = \frac{2d}{3}$.
Step 3: $\frac{2Gm^2}{d^2} = m\omega^2(\frac{2d}{3}) \Rightarrow \omega^2 = \frac{3Gm}{d^3}$.
Step 4: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$.

---

20. Answer: b

Explanation:

Step 1: $F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2}$.
Step 2: Mass of removed part $m' = M(r_{cav}/R)^3 = M(1/2)^3 = M/8$.
Step 3: Distance of cavity center from test particle $= 3R - R/2 = 2.5R = 5R/2$.
Step 4: Force from removed part $F' = \frac{G(M/8)m}{(5R/2)^2} = \frac{GMm}{8\times\frac{25R^2}{4}} = \frac{GMm}{50R^2}$.
Step 5: $F_2 = F_1 - F' = \frac{GMm}{R^2}[1/9 - 1/50] = \frac{GMm}{R^2}[\frac{50-9}{450}] = \frac{41GMm}{450R^2}$.
Step 6: Ratio $F_1/F_2 = (1/9)/(41/450) = \frac{1}{9} \times \frac{450}{41} = 50/41$.

---

21. Answer: a

Explanation:

Step 1: Orbital radii: $r_A = 6400 + 600 = 7000$ km; $r_B = 6400 + 1600 = 8000$ km.
Step 2: Time period formula $T = 2\pi\sqrt{\frac{r^3}{GM}}$.
Step 3: $GM = (6.67 \times 10^{-11}) \times (6 \times 10^{24}) \approx 4 \times 10^{14}$ m^3/s^2.
Step 4: $T_A = 2\pi\sqrt{\frac{(7\times10^6)^3}{4\times10^{14}}} \approx 5840$ s.
Step 5: $T_B = 2\pi\sqrt{\frac{(8\times10^6)^3}{4\times10^{14}}} \approx 7140$ s.
Step 6: $T_B - T_A \approx 1300$ s $= 1.3 \times 10^3$ s. Closest option is (A).

22. **Answer: c**

Explanation:

Step 1: The formula for escape velocity is given by $v_e = \sqrt{\frac{2GM}{R}}$.
Step 2: For the escape velocities of two planets to be equal $(v_{e1} = v_{e2})$, we must have $\frac{M_1}{R_1} = \frac{M_2}{R_2}$.
Step 3: This implies the ratio of mass to radius must be the same, not the product. Thus, Assertion A is correct because unequal masses can have equal escape velocities if their radii are also different in the same proportion.
Step 4: Reason R states $M_1R_1 = M_2R_2$, which is mathematically inconsistent with the escape velocity formula.

---

23. **Answer: 4 - 4**

Explanation:

To find the minimum speed, the satellite must just be able to reach the null point (where the gravitational forces of the two planets cancel out) with zero kinetic energy. After this point, the gravity of the second planet will pull it in.
Let the null point be at a distance x from the center of the first planet (mass M). The distance from the second planet (mass 9M) will be (8R - x).
At the null point, the net gravitational force is zero:
$\frac{GMm}{x^2} = \frac{G(9M)m}{(8R-x)^2}$
$\frac{1}{x^2} = \frac{9}{(8R-x)^2} \implies \frac{1}{x} = \frac{3}{8R-x}$
$8R - x = 3x \implies 4x = 8R \implies x = 2R.$
Now, we apply the principle of conservation of energy. The initial energy at the surface of the first planet must equal the final energy at the null point.
Initial Energy (E_i) at the surface of the first planet:
$E_i = K.E._{initial} + P.E._{initial} = \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{G(9M)m}{(8R-R)} = \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{9GMm}{7R}$
Final Energy (E_f) at the null point (with minimum speed, so $v_{final} = 0$):
$E_f = K.E._{final} + P.E._{final} = 0 - \frac{GMm}{x} - \frac{G(9M)m}{(8R-x)} = -\frac{GMm}{2R} - \frac{9GMm}{6R} = -\frac{GMm}{2R} - \frac{3GMm}{2R} = -\frac{4GMm}{2R} = -\frac{2GMm}{R}$
By conservation of energy, $E_i = E_f$:
$\frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{9GMm}{7R} = -\frac{2GMm}{R}$
Divide by m and rearrange:

$\frac{1}{2}v^2 = \frac{GM}{R} + \frac{9GM}{7R} - \frac{2GM}{R} = GM\left(\frac{1}{R} + \frac{9}{7R} - \frac{2}{R}\right)$
$\frac{1}{2}v^2 = \frac{GM}{R}\left(\frac{7+9-14}{7}\right) = \frac{GM}{R}\left(\frac{2}{7}\right)$
$v^2 = \frac{4GM}{7R} \implies v = \sqrt{\frac{4GM}{7R}}$
Comparing this with the given expression $v = \sqrt{\frac{aGM}{7R}}$, we get $a = 4$.

---

24. Answer: 10 - 10

Explanation:

We will use the principle of conservation of mechanical energy.
The initial energy of the body on the Earth's surface is $E_i = KE_i + PE_i = \frac{1}{2}mv_i^2 - \frac{GMm}{R}$.
The final energy of the body at a height of 10R is $E_f = KE_f + PE_f = 0 - \frac{GMm}{R+10R} = -\frac{GMm}{11R}$. (At maximum height, final velocity is 0).
By conservation of energy, $E_i = E_f$:
$\frac{1}{2}mv_i^2 - \frac{GMm}{R} = -\frac{GMm}{11R}$.
$\frac{1}{2}mv_i^2 = \frac{GMm}{R} - \frac{GMm}{11R} = \frac{GMm}{R}\left(1 - \frac{1}{11}\right) = \frac{10}{11}\frac{GMm}{R}$.
$v_i^2 = \frac{20}{11}\frac{GM}{R}$.
The escape velocity from the Earth's surface is given by $v_e = \sqrt{\frac{2GM}{R}}$, so $v_e^2 = \frac{2GM}{R}$.
Substitute v_e^2 into the equation for v_i^2:
$v_i^2 = \frac{10}{11}\left(\frac{2GM}{R}\right) = \frac{10}{11}v_e^2$.
Taking the square root of both sides: $v_i = \sqrt{\frac{10}{11}}v_e$.
Comparing this with the given expression $v_i = \sqrt{\frac{x}{11}}v_e$, we find that x = 10.

---

25. Answer: b

Explanation:

Step 1: Understanding the Concept:
Acceleration due to gravity varies with height and depth. We need to find the ratio of g_{depth} to g_{height} for a displacement r.
Step 2: Key Formula or Approach:
1. At depth r: $g_d = g\left(1 - \frac{r}{R_E}\right)$
2. At height r: $g_h = \frac{g}{(1+r/R_E)^2} = g\left(1 + \frac{r}{R_E}\right)^{-2}$
Step 3: Detailed Explanation:
We need to find the ratio $\frac{g_d}{g_h}$:

$$\text{Ratio} = \frac{g(1-\frac{r}{R_E})}{g(1+\frac{r}{R_E})^{-2}} = \left(1-\frac{r}{R_E}\right)\left(1+\frac{r}{R_E}\right)^2$$

Expand the squared term:

$$\text{Ratio} = \left(1-\frac{r}{R_E}\right)\left(1+\frac{2r}{R_E}+\frac{r^2}{R_E^2}\right)$$

Multiply the terms:

$$\text{Ratio} = 1\left(1+\frac{2r}{R_E}+\frac{r^2}{R_E^2}\right) - \frac{r}{R_E}\left(1+\frac{2r}{R_E}+\frac{r^2}{R_E^2}\right)$$

$$\text{Ratio} = 1+\frac{2r}{R_E}+\frac{r^2}{R_E^2}-\frac{r}{R_E}-\frac{2r^2}{R_E^2}-\frac{r^3}{R_E^3}$$

Group like terms:

$$\text{Ratio} = 1+\left(\frac{2r}{R_E}-\frac{r}{R_E}\right)+\left(\frac{r^2}{R_E^2}-\frac{2r^2}{R_E^2}\right)-\frac{r^3}{R_E^3}$$

$$\text{Ratio} = 1+\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$$

Step 4: Final Answer:

The ratio is $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$.

---

26. Answer: c

Explanation:

Step 1: Understanding the Concept:

Escape velocity is the minimum velocity required for an object to escape the gravitational field of a system, meaning its total mechanical energy at infinity becomes zero.

Step 2: Key Formula or Approach:
Total Energy = Kinetic Energy + Potential Energy = 0.
Potential Energy at midpoint (distance $r/2$ from both): $U = -\frac{GM_1m}{r/2} - \frac{GM_2m}{r/2}$.
Step 3: Detailed Explanation:

$$U = -\frac{2Gm}{r}(M_1 + M_2)$$

By conservation of energy:

$$\frac{1}{2}mV^2 + U = 0$$

$$\frac{1}{2}mV^2 - \frac{2Gm}{r}(M_1 + M_2) = 0$$

$$V^2 = \frac{4G(M_1 + M_2)}{r}$$

$$V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$$

Step 4: Final Answer:
The minimum escape velocity is $\sqrt{\frac{4G(M_1+M_2)}{r}}$.

---

27. Answer: c

Explanation:

Step 1: Understanding the Question:
We need to find the total gravitational potential at a point inside a spherical shell. The total potential is the sum of the potential due to the shell itself and the potential due to a point mass placed at its center.
Step 2: Key Formula or Approach:
The gravitational potential V at a point is calculated using the principle of superposition.
1. **Potential due to a point mass M at distance r:** $V_p = -\frac{GM}{r}$
2. **Potential due to a uniform spherical shell of mass M_s and radius R:**
- For a point inside the shell ($r < R$), the potential is constant and equal to the potential at the surface: $V_{\text{shell, in}} = -\frac{GM_s}{R}$.
- For a point outside the shell ($r > R$), $V_{\text{shell, out}} = -\frac{GM_s}{r}$.
Step 3: Detailed Explanation:

We are given:
Mass at the center, $M_p = 50\,\text{kg}$.
Mass of the spherical shell, $M_s = 100\,\text{kg}$.
Radius of the shell, $R = 50\,\text{m}$.
We need to find the potential at a distance $r = 25\,\text{m}$ from the center.
The total potential V_{total} at this point is the sum of the potential due to the point mass (V_p) and the potential due to the spherical shell (V_s).

$$V_{\text{total}} = V_p + V_s$$

Calculating Potential due to the Point Mass (V_p):
The point is at $r = 25\,\text{m}$ from the mass $M_p = 50\,\text{kg}$.

$$V_p = -\frac{GM_p}{r} = -\frac{G \times 50}{25} = -2G$$

Calculating Potential due to the Spherical Shell (V_s):
The point $r = 25\,\text{m}$ is inside the shell, since $r < R$ $(25\,\text{m} < 50\,\text{m})$.
For any point inside a uniform spherical shell, the potential is constant and equal to the potential on its surface.

$$V_s = -\frac{GM_s}{R} = -\frac{G \times 100}{50} = -2G$$

Calculating Total Potential (V_{total}):

$$V_{\text{total}} = V_p + V_s = (-2G) + (-2G) = -4G$$

Step 4: Final Answer:
The value of the gravitational potential V at the given point is -4G.

---

28. Answer: 2 - 2

Explanation:

Step 1: Understanding the Question:
We have four masses arranged at the corners of a square. The total gravitational potential energy (GPE) of this system depends on the value of 'm'. We need to find the ratio M/m for which this GPE is maximum.
Step 2: Key Formula or Approach:
The gravitational potential energy between two masses m_1 and m_2 separated by a distance r is $U = -G\frac{m_1 m_2}{r}$.

The total GPE of the system is the sum of the potential energies of all possible pairs of masses. In a square with four masses, there are 6 pairs (4 sides and 2 diagonals). To find the maximum GPE, we will differentiate the total potential energy expression with respect to 'm' and set the derivative to zero.

Step 3: Detailed Explanation:

The arrangement has masses 'm' and 'M-m' at adjacent corners. The four masses at the vertices are $m_1 = m$, $m_2 = M - m$, $m_3 = m$, $m_4 = M - m$. The side length of the square is 'd', and the diagonal length is $d\sqrt{2}$.

Let's calculate the total GPE (U_{total}) by summing the energy of all 6 pairs:

4 pairs along the sides (distance d): $U_{sides} = -G\frac{m(M-m)}{d} - G\frac{(M-m)m}{d} - G\frac{m(M-m)}{d} - G\frac{(M-m)m}{d} = -4G\frac{m(M-m)}{d}$

2 pairs along the diagonals (distance $d\sqrt{2}$): $U_{diag} = -G\frac{m\cdot m}{d\sqrt{2}} - G\frac{(M-m)(M-m)}{d\sqrt{2}} = -\frac{G}{d\sqrt{2}}(m^2 + (M-m)^2)$

Total GPE:

$$U_{total} = -\frac{G}{d}\left[4m(M-m) + \frac{1}{\sqrt{2}}(m^2 + (M-m)^2)\right]$$

To maximize U_{total}, which is a negative quantity, we must minimize its magnitude. Let the term in the brackets be $f(m)$.

$$f(m) = 4mM - 4m^2 + \frac{1}{\sqrt{2}}(m^2 + M^2 - 2mM + m^2) = 4mM - 4m^2 + \frac{1}{\sqrt{2}}(2m^2 - 2mM + M^2)$$

For maximum U, we need $\frac{dU_{total}}{dm} = 0$, which means $\frac{df(m)}{dm} = 0$.

$$\frac{df}{dm} = 4M - 8m + \frac{1}{\sqrt{2}}(4m - 2M) = 0$$

$$4M - 8m + \frac{4}{\sqrt{2}}m - \frac{2}{\sqrt{2}}M = 0$$

$$4M - 8m + 2\sqrt{2}m - \sqrt{2}M = 0$$

Group terms with M and m:

$$M(4-\sqrt{2}) = m(8-2\sqrt{2})$$

$$M(4-\sqrt{2}) = m \cdot 2(4-\sqrt{2})$$

Cancel the $(4-\sqrt{2})$ term from both sides:

$$M = 2m$$

$$\frac{M}{m} = 2$$

The ratio is given as x:1.

$$\frac{M}{m} = \frac{x}{1} \implies x = 2$$

Step 4: Final Answer:
The value of x is 2.

---

29. Answer: c

Explanation:

Step 1: Understanding the Concept:
The shell theorem in gravitation states that the gravitational influence inside a uniform spherical shell follows specific rules based on the inverse square law.
Step 2: Detailed Explanation:
1. **Statement (a) and (c):** The gravitational field (g) inside a uniform shell is zero at all points. Since zero is the same value at every point, it is correct to say the field is zero and that it is the same everywhere.
2. **Statement (b) and (d):** The gravitational potential (V) inside the shell is given by $V = -GM/R$, where R is the shell radius. This is a constant value but is NOT zero. Since it is constant, it is the same everywhere inside the shell.
Conclusion: (a), (c), and (d) are true. (b) is false.
Step 3: Final Answer:
The correct option is (C), which includes (a), (c), and (d).

---

30. Answer: a

Explanation:

To solve the given assertion and reason problem, we first need to analyze the statements individually and then collectively:

1. **Understanding the Assertion (A):** "The radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time and thus areal velocity of planet is constant."

- This statement is based on Kepler's Second Law of Planetary Motion, which states that a line joining a planet to the sun sweeps out equal areas in equal times. This implies that the areal velocity, the area swept per unit time, is constant for a planet in orbit. Thus, Assertion (A) is correct.

2. **Understanding the Reason (R):** "For a central force field the angular momentum is a constant."
   - A central force field is a force field in which the force acts along the line joining the bodies and depends only on the distance between them, such as the gravitational force between a planet and the Sun.
   - For a central force, the angular momentum is conserved because there is no external torque acting on the system, which supports the reason (R) being correct.
3. **Connecting Assertion and Reason:**
   - The constancy of areal velocity as mentioned in Assertion (A) is due to the conservation of angular momentum, as stated in Reason (R). Hence, (R) is indeed the correct explanation of (A).

Therefore, the correct answer is: *Both (A) and (R) are correct and (R) is the correct explanation of (A).*