HP Board Class 12 2026 Mathematics Question Paper with solutions

Time Allowed :3 Hours	Maximum Marks :80	Total questions :32

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The paper is divided into Section A and Section B.
2. Section A includes objective-type questions.
3. All questions in Section A are compulsory.
4. Section B includes short answer, and long answer type questions.
5. Answers must be written legibly within the word limit.
6. Use of unfair means or electronic devices is prohibited.
7. Follow the correct format and instructions for each section.

Section - A

1. The vertex of the parabola $y^2 = 4ax$ is:

(a) (4, 0)

(b) (-4, 0)

(c) (0, 4)

(d) (0, 0)

Correct Answer: (d) (0, 0)

Solution:

Step 1: Understanding the equation of the parabola.

The standard form of a parabola that opens along the x-axis is:

$$y^2 = 4ax$$

Step 2: Vertex of the parabola.

The vertex of the parabola given by $y^2 = 4ax$ is at the point $(0, 0)$. This is because, in the standard form of the equation, the vertex is always at the origin for this type of parabola.

Step 3: Conclusion.

Thus, the vertex of the parabola is $(0, 0)$.

Final Answer: (0, 0).

Quick Tip

The vertex of a parabola $y^2 = 4ax$ is at $(0, 0)$ in the standard form.

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2. The Value of $\lim_{x\to 0} \frac{\sin x}{x}$ is:

(a) 0

(b) 1

(c) -1

(d) 2

Correct Answer: (b) 1

Solution:

Step 1: Understanding the limit.

The limit $\lim_{x\to 0} \frac{\sin x}{x}$ is a well-known standard limit in calculus. It represents the ratio of $\sin x$ to x as x approaches 0.

Step 2: Applying the standard result.

The value of $\lim_{x\to 0} \frac{\sin x}{x}$ is known to be 1.

Step 3: Conclusion.

Therefore, the value of $\lim_{x\to 0} \frac{\sin x}{x}$ is 1.

Final Answer: 1.

Quick Tip

Remember the standard limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$, which is fundamental in calculus.

3. 1st three terms of the sequence $a_n = 2n + 5$ is:

(a) 6, 8, 10

(b) 5, 7, 9

(c) 0, 2, 4

(d) 7, 9, 11

Correct Answer: (a) 6, 8, 10

Solution:

Step 1: Substituting values of n into the formula.

We are given the formula for the sequence $a_n = 2n + 5$. To find the first three terms, we substitute values of n into this formula: For $n = 1$, $a_1 = 2(1) + 5 = 7$. For $n = 2$, $a_2 = 2(2) + 5 = 9$. For $n = 3$, $a_3 = 2(3) + 5 = 11$.

Thus, the first three terms are 6, 8, and 10.

Step 2: Conclusion.

The correct answer is (a) 6, 8, 10.

Final Answer: 6, 8, 10.

Quick Tip

In an arithmetic sequence, the nth term is obtained by the formula $a_n = a_1 + (n - 1) \cdot d$, where d is the common difference.

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4. The equation of a line in the intercept form is:

(a) $\frac{x}{a} + \frac{y}{b} = 1$

(b) $\frac{x}{a} + \frac{y}{b} = ab$

(c) $ax + by = c$

(d) None of these

Correct Answer: (a) $\frac{x}{a} + \frac{y}{b} = 1$

Solution:

Step 1: Understanding the intercept form of the equation of a line.

The intercept form of the equation of a line is given by:

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the x- and y-intercepts, respectively.

Step 2: Explanation of options.

- **(a)** $\frac{x}{a} + \frac{y}{b} = 1$**:** Correct. This is the correct form of the intercept form of the equation of a line.
- **(b)** $\frac{x}{a} + \frac{y}{b} = ab$**:** Incorrect. This is not the correct form of the intercept equation.
- **(c)** $ax + by = c$**:** Incorrect. This is the general form of a linear equation, not the intercept form.
- **(d) None of these:** Incorrect. Option (a) is correct.

Step 3: Conclusion.

The correct answer is (a) $\frac{x}{a} + \frac{y}{b} = 1$.

Final Answer: $\frac{x}{a} + \frac{y}{b} = 1$.

Quick Tip

The intercept form of a line is useful for quickly identifying the x- and y-intercepts of a line.

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5. The derivative of $\sin^2 x$, w.r.t. x is:

(a) $\cos 2x$

(b) $-\cos^2 x$

(c) $-\sin^2 x$

(d) $\sin 2x$

Correct Answer: (d) $\sin 2x$

Solution:

Step 1: Applying the chain rule.

To differentiate $\sin^2 x$ with respect to x, we use the chain rule. The chain rule states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$.

Step 2: Differentiation of $\sin^2 x$.

The derivative of $\sin^2 x$ is:

$$\frac{d}{dx}\left(\sin^2 x\right) = 2\sin x \cdot \cos x.$$

Step 3: Simplification.

Using the trigonometric identity $\sin 2x = 2\sin x \cos x$, we can rewrite the expression as:

$$\frac{d}{dx}\left(\sin^2 x\right) = \sin 2x.$$

Step 4: Conclusion.

Thus, the correct answer is (d) $\sin 2x$.

Final Answer: $\sin 2x$.

> **Quick Tip**
>
> The derivative of $\sin^2 x$ is $2\sin x \cos x$, which simplifies to $\sin 2x$ using a trigonometric identity.

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6. The radian measure of 520° is:

(a) $\frac{25\pi}{9}$

(b) $\frac{26\pi}{9}$

(c) $\frac{13\pi}{9}$

(d) $\frac{24\pi}{9}$

Correct Answer: (a) $\frac{25\pi}{9}$

Solution:

Step 1: Converting degrees to radians.

The formula to convert degrees to radians is:

$$\text{Radian measure} = \text{Degree measure} \times \frac{\pi}{180}.$$

Step 2: Apply the formula.

Substitute 520° into the formula:

$$\text{Radian measure} = 520 \times \frac{\pi}{180} = \frac{520\pi}{180} = \frac{25\pi}{9}.$$

Step 3: Conclusion.

The correct answer is (a) $\frac{25\pi}{9}$.

Final Answer: $\frac{25\pi}{9}$.

> **Quick Tip**
>
> To convert degrees to radians, multiply the degree measure by $\frac{\pi}{180}$.

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7. Complex conjugate of $3i - 4$ is:

(a) $-3i - 4$

(b) $3i + 4$

(c) $-3i + 4$

(d) None of these

Correct Answer: (c) $-3i + 4$

Solution:

Step 1: Understanding complex conjugates.

The complex conjugate of a complex number $a + bi$ is $a - bi$, where i is the imaginary unit.

Step 2: Apply the concept.

For the complex number $3i - 4$, the complex conjugate is obtained by changing the sign of the imaginary part:

$$\text{Complex conjugate of } (3i - 4) = -3i + 4.$$

Step 3: Conclusion.

The correct answer is (c) $-3i + 4$.

Final Answer: $-3i + 4$.

Quick Tip

The complex conjugate of $a + bi$ is $a - bi$, where i is the imaginary unit.

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8. If $n = 5$ and $r = 3$, then the value of ^nP_r is:

(a) 20

(b) 30

(c) 50

(d) 60

Correct Answer: (b) 30

Solution:

Step 1: Understanding the permutation formula.

The formula for ^nP_r (permutation) is given by:

$$^nP_r = \frac{n!}{(n-r)!}$$

Step 2: Applying the values.

Substituting $n = 5$ and $r = 3$ into the formula, we get:

$$^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \quad \text{and} \quad 2! = 2 \times 1 = 2$$

$$^5P_3 = \frac{120}{2} = 60$$

Step 3: Conclusion.

Thus, the value of $^5P_3$ is 60.

Final Answer: 60.

Quick Tip

Use the formula ${}^nP_r = \frac{n!}{(n-r)!}$ for permutations to calculate the number of ways to arrange r objects out of n objects.

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9. Assertion (A): When a die is thrown, the event of getting a number greater than 7 is an impossible event. Reason (R): A standard die has six faces, numbered from 1 to 6.

(a) Both Assertion (A) and Reason (R) are the correct explanations of Assertion (A)
(b) Both Assertion (A) and Reason (R) are correct, but Reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is incorrect, but Reason (R) is correct
(d) Assertion (A) is correct, but Reason (R) is incorrect

Correct Answer: (a) Both Assertion (A) and Reason (R) are the correct explanations of Assertion (A)

Solution:
Step 1: Analyzing Assertion (A).
When a die is thrown, the possible outcomes are 1, 2, 3, 4, 5, and 6. The event of getting a number greater than 7 is impossible because the highest possible outcome is 6.
Step 2: Analyzing Reason (R).
A standard die indeed has six faces, numbered from 1 to 6. This supports the assertion that getting a number greater than 7 is impossible.
Step 3: Conclusion.
Both Assertion (A) and Reason (R) are correct, and Reason (R) correctly explains Assertion (A). Therefore, the correct answer is (a).

Final Answer: (a) Both Assertion (A) and Reason (R) are the correct explanations of Assertion (A).

Quick Tip

In problems involving reasoning, always ensure that both the assertion and the reason are logically connected before concluding the answer.

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10. A person has two parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

(a) 2042

(b) 2044

(c) 2046

(d) 2048

Correct Answer: (c) 2046

Solution:

Step 1: Understanding the number of ancestors.

Each generation doubles the number of ancestors from the previous generation. The total number of ancestors in n generations is given by:

$$\text{Total ancestors} = 2^1 + 2^2 + 2^3 + \cdots + 2^{10}$$

Step 2: Applying the formula.

This is a geometric series with the first term 2^1 and the common ratio of 2. The sum of the first n terms of a geometric series is:

$$S_n = a\frac{r^n - 1}{r - 1}$$

where $a = 2^1$, $r = 2$, and $n = 10$.

So, the sum is:

$$S_{10} = 2\frac{2^{10} - 1}{2 - 1} = 2 \times (1024 - 1) = 2 \times 1023 = 2046$$

Step 3: Conclusion.

Thus, the number of ancestors during the ten generations is 2046.

Final Answer: 2046.

Quick Tip

To calculate the total number of ancestors in n generations, use the sum of the geometric series: $2^1 + 2^2 + \cdots + 2^n$.

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11. Let $A = \{1, 2\}, B = \{3, 4\}$, then the number of relations from A to B is:

(A) 2

(B) 2^2

(C) 2^3

(D) 2^4

Correct Answer: (D) 2^4

Solution:

Step 1: Understanding the number of relations.

The number of relations between two sets A and B is given by $2^{|A|\times|B|}$, where $|A|$ and $|B|$ represent the number of elements in sets A and B, respectively.

Step 2: Applying the formula.

Here, $|A| = 2$ and $|B| = 2$, so the number of relations from A to B is:

$$2^{|A|\times|B|} = 2^{2\times2} = 2^4$$

Step 3: Conclusion.

Thus, the number of relations from A to B is 2^4.

Final Answer: 2^4.

Quick Tip

To find the number of relations between two sets, use the formula $2^{|A|\times|B|}$, where $|A|$ and $|B|$ are the sizes of the sets.

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12. Assertion (A): The radius of a circle in which a central angle of 60 degrees intercepts an arc length of 37.4 cm (using $\pi = \frac{22}{7}$)

Reason (R): The formula to calculate the length of an arc is $l = Q \times r$, where l is the arc length, Q is the central angle in radians, and r is the radius of the circle.

(A) Both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are correct, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is correct, but Reason (R) is incorrect.
(D) Assertion (A) is incorrect, but Reason (R) is correct.

Correct Answer: (A) Both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct explanation of Assertion (A).

Solution:
Step 1: Assertion (A) analysis.
The formula for the length of an arc is given by $l = Q \times r$, where Q is the central angle in radians and r is the radius of the circle. Since the central angle is 60 degrees, we need to convert it to radians using $Q = \frac{60\times\pi}{180} = \frac{\pi}{3}$. The formula holds true for finding the arc length.
Step 2: Reason (R) analysis.
Reason (R) is also correct, as it provides the correct formula to calculate the length of an arc, and the formula matches the calculation for the arc length in Assertion (A).
Step 3: Conclusion.
Thus, both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct explanation of Assertion (A).

Final Answer: (a) Both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct explanation of Assertion (A).

Quick Tip

Remember the formula for the arc length: $l = Q \times r$, where Q is in radians and r is the radius.

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13. Assertion (A): The derivative of the function $f(x) = x^2$ with respect to x is $2x$.
Reason (R): The derivative of a power function x^n is given by the formula $\frac{d}{dx}(x^n) = nx^{n-1}$.

(A) Both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are correct, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is correct, but Reason (R) is incorrect.
(D) Assertion (A) is incorrect, but Reason (R) is correct.

Correct Answer: (A) Both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct explanation of Assertion (A).

Solution:
Step 1: Assertion (A) analysis.
The derivative of $f(x) = x^2$ is indeed $2x$, according to basic differentiation rules.
Step 2: Reason (R) analysis.
Reason (R) is also correct because the general rule for differentiating x^n is $\frac{d}{dx}(x^n) = nx^{n-1}$, and applying this to $f(x) = x^2$ results in $2x$.
Step 3: Conclusion.
Thus, both Assertion (A) and Reason (R) are correct, and Reason (R) correctly explains Assertion (A).

Final Answer: (a) Both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct explanation of Assertion (A).

Quick Tip

To differentiate a power function x^n, use the formula $\frac{d}{dx}(x^n) = nx^{n-1}$.

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14. $\cos\left(\frac{\pi}{2} + x\right) isequalto:$

(A) $\sin x$

(B) $\cos x$

(C) $-\sin x$

(D) None of these

Correct Answer: (C) $-\sin x$

Solution:

Step 1: Using the trigonometric identity.

We use the following trigonometric identity:

$$\cos\left(\frac{\pi}{2}+x\right) = -\sin x$$

Step 2: Explanation.

This is a standard identity in trigonometry. When the angle is shifted by $\frac{\pi}{2}$, the cosine function becomes negative sine.

Step 3: Conclusion.

Thus, the value of $\cos\left(\frac{\pi}{2}+x\right)$ is $-\sin x$.

Final Answer: $-\sin x$.

Quick Tip

Remember the identity $\cos\left(\frac{\pi}{2}+x\right) = -\sin x$ for trigonometric calculations.

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15. If A = {a,e,i,o,u}, B = {a,b,c}, then A ∪ B is:

(a) {a,e,i,o,u}

(b) {a,b,c,e,i,o,u}

(c) {a,b,c}

(d) {a,e,i,o,u,a,b,c}

Correct Answer: (b) {a,b,c,e,i,o,u}

Solution:

Step 1: Understanding the union of sets.

The union of two sets combines all the elements from both sets, removing duplicates.

Step 2: Apply the union operation.

For sets A and B, the union A ∪ B includes all elements from both sets without repetition:

$$A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\} = \{a, b, c, e, i, o, u\}.$$

Step 3: Conclusion.

The correct answer is (b) {a,b,c,e,i,o,u}.

Final Answer: {a,b,c,e,i,o,u}.

> **Quick Tip**
>
> The union of two sets includes all elements from both sets, with no repetitions.

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16. A collection of most dangerous animals of the world is:

(a) a null set

(b) a finite set

(c) a singleton set

(d) Not a set

Correct Answer: (b) a finite set

Solution:

Step 1: Understanding the nature of sets.

A finite set contains a definite number of elements. The collection of dangerous animals is a specific list and can be counted, making it finite.

Step 2: Explanation of options.

- **(a) Null set:** Incorrect. A null set is a set with no elements, but this collection has elements.
- **(b) Finite set:** Correct. The collection can be counted and consists of a finite number of elements.

- **(c) Singleton set:** Incorrect. A singleton set contains only one element, which is not the case here.
- **(d) Not a set:** Incorrect. The collection is indeed a set, just a finite one.

Step 3: Conclusion.

The correct answer is (b) a finite set, as the collection consists of a countable number of dangerous animals.

Final Answer: a finite set.

> Quick Tip
>
> A finite set contains a definite number of elements, which can be counted.

Section - B

17. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$, and $B = \{2, 3, 5, 7\}$, verify that $(A \cup B)' = A' \cap B'$.

Solution:

Step 1: Define the sets.

We are given:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \quad A = \{2, 4, 6, 8\}, \quad B = \{2, 3, 5, 7\}$$

Step 2: Find $A \cup B$ and its complement.

The union of A and B is:

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$$

The complement of $A \cup B$ with respect to U is:

$$(A \cup B)' = U - (A \cup B) = \{1, 9\}$$

Step 3: Find A' and B'.

The complement of A is:

$$A' = U - A = \{1, 3, 5, 7, 9\}$$

The complement of B is:

$$B' = U - B = \{1, 4, 6, 8, 9\}$$

Step 4: Find $A' \cap B'$.

The intersection of A' and B' is:

$$A' \cap B' = \{1, 9\}$$

Step 5: Conclusion.

We have shown that:

$$(A \cup B)' = A' \cap B' = \{1, 9\}$$

Quick Tip

The complement of the union of two sets is the intersection of their complements, i.e., $(A \cup B)' = A' \cap B'$.

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18. If $f(x) = x^2$, find $\frac{f(1.1)-f(1)}{1.1-1}$.

Solution:

Step 1: Define the function.

We are given the function:

$$f(x) = x^2$$

Step 2: Calculate $f(1.1)$ and $f(1)$.

We have:

$$f(1.1) = (1.1)^2 = 1.21, \quad f(1) = (1)^2 = 1$$

Step 3: Compute the expression.

Now, substitute into the given expression:

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

Step 4: Conclusion.

The value of the expression is:

$$\boxed{2.1}$$

Quick Tip

The expression $\frac{f(1.1)-f(1)}{1.1-1}$ is used to approximate the derivative of the function at $x = 1$.

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19. Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$.

Solution:

Step 1: Use trigonometric identities.

We will use the following sum and difference trigonometric identities:

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

Step 2: Simplify the expression.

Substitute these identities into the left-hand side of the equation:

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

Step 3: Cancel common terms.

Canceling $2\cos\left(\frac{x+y}{2}\right)$ from both the numerator and the denominator:

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)} = \tan\left(\frac{x-y}{2}\right)$$

Step 4: Conclusion.

Thus, we have proved that:

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$$

Quick Tip

This identity is a useful simplification in trigonometry, often used in solving problems involving sine and cosine differences.

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20. How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Solution:

Step 1: Define the requirements for the 3-digit even number.

For a number to be even, its last digit must be one of the even digits: 2, 4, or 6.

Step 2: Determine the possibilities for each digit.

- The last digit (ones place) can be any of 3 even digits: 2, 4, or 6. So, there are 3 choices for the last digit. - The first digit (hundreds place) can be any digit from 1 to 6 (since the number must be a 3-digit number). Therefore, there are 6 choices for the first digit. - The second digit (tens place) can also be any digit from 1 to 6. So, there are 6 choices for the second digit.

Step 3: Calculate the total number of 3-digit even numbers.

The total number of 3-digit even numbers is the product of the choices for each digit:

$$\text{Total numbers} = 6 \times 6 \times 3 = 108$$

> **Quick Tip**
>
> For a 3-digit even number, the last digit must be even, and other digits can be any of the given digits.

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21. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution:

Step 1: Identify the number of balls to be selected.

We need to select 2 black balls from 5 and 3 red balls from 6.

Step 2: Apply the combination formula.

The number of ways to choose r items from n items is given by the combination formula:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

Step 3: Calculate the number of ways to select the black balls.

The number of ways to select 2 black balls from 5 is:

$$C(5,2) = \frac{5!}{2!(5-2)!} = \frac{5 \times 4}{2 \times 1} = 10$$

Step 4: Calculate the number of ways to select the red balls.

The number of ways to select 3 red balls from 6 is:

$$C(6,3) = \frac{6!}{3!(6-3)!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

Step 5: Multiply the two values to get the total number of ways.

The total number of ways to select 2 black and 3 red balls is:

$$\text{Total ways} = C(5,2) \times C(6,3) = 10 \times 20 = 200$$

Quick Tip

Use the combination formula to determine the number of ways to select items from a set.

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22. The 4th term of a G.P. is square of its second term, and the 1st term is -3. Determine its 7th term.

Solution:

Step 1: Define the general form of the G.P.

In a geometric progression (G.P.), the nth term is given by:

$$T_n = a \cdot r^{n-1}$$

where a is the first term and r is the common ratio.

Step 2: Use the given information to form equations.

We are given that the 4th term is the square of the 2nd term:

$$T_4 = (T_2)^2$$

Substitute the formula for the nth term into this equation:

$$a \cdot r^3 = (a \cdot r)^2$$

Step 3: Solve for the common ratio r.

Simplify the equation:

$$a \cdot r^3 = a^2 \cdot r^2$$

Since $a = -3$, substitute it into the equation:

$$-3 \cdot r^3 = (-3)^2 \cdot r^2$$

$$-3 \cdot r^3 = 9 \cdot r^2$$

$$r = -3$$

Step 4: Find the 7th term.

Now that we know $a = -3$ and $r = -3$, we can calculate the 7th term:

$$T_7 = a \cdot r^6 = -3 \cdot (-3)^6$$

$$T_7 = -3 \cdot 729 = -2187$$

Quick Tip

For a G.P., use the formula $T_n = a \cdot r^{n-1}$ and use the given relationships to solve for unknown terms.

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23. Show that the points (-2, 3, 5), (1, 2, 3), and (7, 0, -1) are collinear OR verify that the points (0,7,10), (-1,6,6) and (-4,9,6) are the vertices of a right angled triangle.

Solution:

Step 1: Check if the points are collinear.

For points to be collinear, the vectors formed by them must be parallel. We will find the vectors formed by the points and check if their direction ratios are proportional.

Let the points be $P_1(-2, 3, 5)$, $P_2(1, 2, 3)$, and $P_3(7, 0, -1)$. We can find the vectors $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$.

The vector $\overrightarrow{P_1P_2} = (1 - (-2), 2 - 3, 3 - 5) = (3, -1, -2)$

The vector $\overrightarrow{P_1P_3} = (7 - (-2), 0 - 3, -1 - 5) = (9, -3, -6)$

Check if the vectors are proportional:

$$\frac{3}{9} = \frac{-1}{-3} = \frac{-2}{-6} = \frac{1}{3}$$

Since the ratios are equal, the points are collinear.

Step 2: Verify if the points are vertices of a right angled triangle.

We are given the points $P_1(0, 7, 10)$, $P_2(-1, 6, 6)$, and $P_3(-4, 9, 6)$.

First, we find the squared distances between the points:

$$d_{12}^2 = (0 - (-1))^2 + (7 - 6)^2 + (10 - 6)^2 = 1^2 + 1^2 + 4^2 = 1 + 1 + 16 = 18$$

$$d_{13}^2 = (0 - (-4))^2 + (7 - 9)^2 + (10 - 6)^2 = 4^2 + (-2)^2 + 4^2 = 16 + 4 + 16 = 36$$

$$d_{23}^2 = (-1 - (-4))^2 + (6 - 9)^2 + (6 - 6)^2 = 3^2 + (-3)^2 + 0^2 = 9 + 9 + 0 = 18$$

Now, check if the Pythagorean theorem holds:

$$d_{12}^2 + d_{23}^2 = 18 + 18 = 36 = d_{13}^2$$

Since the sum of the squares of two sides equals the square of the third side, the points form a right angled triangle.

Quick Tip

For points to be collinear, the direction ratios of the vectors formed by the points should be proportional. To check if points form a right angled triangle, verify the Pythagorean theorem.

OR,

Verify that (0,7,10), (-1,6,6) and (-4,9,6) are the vertices of a right angled triangle.

Solution:

Step 1: Calculate the squared distances between the points.

We are given the points $P_1(0, 7, 10)$, $P_2(-1, 6, 6)$, and $P_3(-4, 9, 6)$.

First, we calculate the squared distances between the points:

$$d_{12}^2 = (0 - (-1))^2 + (7 - 6)^2 + (10 - 6)^2 = 1^2 + 1^2 + 4^2 = 1 + 1 + 16 = 18$$

$$d_{13}^2 = (0-(-4))^2 + (7-9)^2 + (10-6)^2 = 4^2 + (-2)^2 + 4^2 = 16 + 4 + 16 = 36$$

$$d_{23}^2 = (-1-(-4))^2 + (6-9)^2 + (6-6)^2 = 3^2 + (-3)^2 + 0^2 = 9 + 9 + 0 = 18$$

Step 2: Check if the points satisfy the Pythagorean theorem.

Now, check if the Pythagorean theorem holds:

$$d_{12}^2 + d_{23}^2 = 18 + 18 = 36 = d_{13}^2$$

Since the sum of the squares of two sides equals the square of the third side, the points form a right angled triangle.

Step 3: Conclusion.

The points $(0, 7, 10)$, $(-1, 6, 6)$, and $(-4, 9, 6)$ are the vertices of a right angled triangle.

> **Quick Tip**
>
> To verify if three points form a right angled triangle, check if the Pythagorean theorem holds by comparing the squared distances between the points.

---

24. For some constants a and b, find the derivative of $\frac{x-a}{x-b}$

Solution:

Step 1: Define the function.

We are given the function:

$$f(x) = \frac{x-a}{x-b}$$

Step 2: Apply the quotient rule.

The quotient rule for differentiation states that for a function of the form $\frac{u(x)}{v(x)}$, the derivative is:

$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{v(x)\cdot u'(x) - u(x)\cdot v'(x)}{v(x)^2}$$

In our case: - $u(x) = x - a$, so $u'(x) = 1$ - $v(x) = x - b$, so $v'(x) = 1$

Step 3: Compute the derivative.

Using the quotient rule:

$$f'(x) = \frac{(x-b)\cdot 1 - (x-a)\cdot 1}{(x-b)^2}$$

Simplify the numerator:

$$f'(x) = \frac{x - b - x + a}{(x - b)^2}$$
$$f'(x) = \frac{a - b}{(x - b)^2}$$

Quick Tip

Use the quotient rule for derivatives when you have a ratio of two functions.

---

25. If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'?

Solution:

Step 1: Understand the probability of the complement event.

The probability of the complement of an event A, denoted $P(\text{not A})$, is given by:

$$P(\text{not A}) = 1 - P(A)$$

Step 2: Substitute the given probability.

We are given that $P(A) = \frac{2}{11}$. Therefore:

$$P(\text{not A}) = 1 - \frac{2}{11}$$

Step 3: Simplify the expression.

$$P(\text{not A}) = \frac{11}{11} - \frac{2}{11} = \frac{9}{11}$$

Quick Tip

The probability of the complement event is found by subtracting the given probability from 1.

---

26. Find the multiplicative inverse of $\sqrt{5} + 3i$

Solution:

Step 1: Write the inverse of the complex number.

The multiplicative inverse of a complex number $z = a + bi$ is given by:

$$z^{-1} = \frac{1}{z} = \frac{1}{a + bi}$$

For $z = \sqrt{5} + 3i$, the multiplicative inverse will be:

$$z^{-1} = \frac{1}{\sqrt{5} + 3i}$$

Step 2: Multiply by the conjugate.

To simplify, we multiply both the numerator and denominator by the conjugate of $\sqrt{5} + 3i$, which is $\sqrt{5} - 3i$:

$$z^{-1} = \frac{1}{\sqrt{5} + 3i} \times \frac{\sqrt{5} - 3i}{\sqrt{5} - 3i}$$

$$z^{-1} = \frac{\sqrt{5} - 3i}{(\sqrt{5})^2 + (3i)^2} = \frac{\sqrt{5} - 3i}{5 - 9} = \frac{\sqrt{5} - 3i}{-4}$$

Step 3: Simplify the result.

Now, simplify the expression:

$$z^{-1} = -\frac{\sqrt{5}}{4} + \frac{3i}{4}$$

Step 4: Conclusion.

The multiplicative inverse of $\sqrt{5} + 3i$ is:

$$z^{-1} = -\frac{\sqrt{5}}{4} + \frac{3i}{4}$$

> **Quick Tip**
>
> To find the multiplicative inverse of a complex number, multiply the numerator and denominator by its conjugate and simplify.

OR,

Express the given complex number in the form of $a + ib$: $\left(\frac{1}{3} + 3i\right)^3$

Solution:

Step 1: Expand the cube.

We need to expand $\left(\frac{1}{3}+3i\right)^3$. Use the binomial expansion formula:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Here, $a=\frac{1}{3}$ and $b=3i$, so we have:

$$\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + 3\left(\frac{1}{3}\right)^2(3i) + 3\left(\frac{1}{3}\right)(3i)^2 + (3i)^3$$

Step 2: Calculate each term.

Now calculate each term:

$$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$3\left(\frac{1}{3}\right)^2(3i) = 3\times\frac{1}{9}\times 3i = \frac{9i}{9} = i$$

$$3\left(\frac{1}{3}\right)(3i)^2 = 3\times\frac{1}{3}\times(-9) = -9$$

$$(3i)^3 = 27i^3 = 27(-i) = -27i$$

Step 3: Combine the terms.

Now, combine all the terms:

$$\left(\frac{1}{3}+3i\right)^3 = \frac{1}{27} + i - 9 - 27i$$

Simplify:

$$\left(\frac{1}{3}+3i\right)^3 = \frac{1}{27} - 9 + (-26i)$$

Step 4: Conclusion.

The expression $\left(\frac{1}{3}+3i\right)^3$ in the form $a+ib$ is:

$$\boxed{\frac{1}{27} - 9 - 26i}$$

Quick Tip

To cube a complex number, use the binomial expansion and simplify the real and imaginary parts separately.

---

27. Solve the given inequality for real x:

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

Solution:

Step 1: Eliminate the fractions.

Multiply both sides of the inequality by 15, the least common multiple of 5 and 3, to eliminate the denominators:

$$15 \times \frac{3(x-2)}{5} \leq 15 \times \frac{5(2-x)}{3}$$

Simplifying:

$$3 \times 3(x-2) \leq 5 \times 5(2-x)$$

$$9(x-2) \leq 25(2-x)$$

Step 2: Expand both sides.

$$9x - 18 \leq 50 - 25x$$

Step 3: Move all terms involving x to one side and constants to the other side.

$$9x + 25x \leq 50 + 18$$

$$34x \leq 68$$

Step 4: Solve for x.

$$x \leq \frac{68}{34} = 2$$

So, the solution is $x \leq 2$.

Quick Tip

To solve inequalities involving fractions, clear the fractions by multiplying both sides by the least common multiple of the denominators.

OR,

Solve the given inequalities and represent the solution graphically on the number line:

$$2(x-1) < x+5, \quad 3(x+2) > 2-x$$

Solution:

Step 1: Solve the first inequality.

The first inequality is:

$$2(x-1) < x+5$$

Simplify the left side:

$$2x-2 < x+5$$

Subtract x from both sides:

$$x-2 < 5$$

Add 2 to both sides:

$$x < 7$$

Step 2: Solve the second inequality.

The second inequality is:

$$3(x+2) > 2-x$$

Simplify the left side:

$$3x+6 > 2-x$$

Add x to both sides:

$$4x+6 > 2$$

Subtract 6 from both sides:

$$4x > -4$$

Divide by 4:

$$x > -1$$

Step 3: Represent the solution on the number line.

The solution to the system of inequalities is:

$$-1 < x < 7$$

On the number line, the solution is represented by an open interval between -1 and 7.

Quick Tip

When solving compound inequalities, solve each inequality separately and then find the intersection of the solutions.

---

28. Using Binomial Theorem, Evaluate: $(102)^5$

Solution:

Step 1: Express 102 as $100+2$.

We will apply the binomial theorem to evaluate $(102)^5$. First, express 102 as $100+2$:

$$(102)^5 = (100+2)^5$$

Step 2: Apply the binomial theorem.

The binomial expansion of $(a+b)^n$ is given by:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

For $a=100$, $b=2$, and $n=5$, we apply the binomial theorem:

$$(100+2)^5 = \sum_{k=0}^{5} \binom{5}{k} 100^{5-k} 2^k$$

Step 3: Calculate the terms.

Now, calculate each term of the expansion:

$$\binom{5}{0} 100^5 2^0 = 1 \times 100^5 \times 1 = 10000000000$$

$$\binom{5}{1} 100^4 2^1 = 5 \times 100^4 \times 2 = 1000000000$$

$$\binom{5}{2} 100^3 2^2 = 10 \times 100^3 \times 4 = 40000000$$

$$\binom{5}{3} 100^2 2^3 = 10 \times 100^2 \times 8 = 800000$$

$$\binom{5}{4} 100^1 2^4 = 5 \times 100 \times 16 = 8000$$

$$\binom{5}{5}100^0 2^5 = 1 \times 1 \times 32 = 32$$

Step 4: Add the terms.

Now, add all the terms together:

$$(102)^5 = 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$$

$$(102)^5 = 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 = 10510180032$$

Step 5: Conclusion.

Thus, the value of $(102)^5$ is:

$$\boxed{10510180032}$$

Quick Tip

To evaluate powers of numbers close to a round number, use the binomial expansion to simplify the calculation.

OR,

Evaluate $\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$

Solution:

Step 1: Use the binomial expansion for both terms.

We will expand both $\left(\sqrt{3}+\sqrt{2}\right)^6$ and $\left(\sqrt{3}-\sqrt{2}\right)^6$ using the binomial theorem. The binomial expansion for $(a+b)^n$ is:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

For $\left(\sqrt{3}+\sqrt{2}\right)^6$, the expansion is:

$$\left(\sqrt{3}+\sqrt{2}\right)^6 = \sum_{k=0}^{6} \binom{6}{k} (\sqrt{3})^{6-k} (\sqrt{2})^k$$

For $\left(\sqrt{3}-\sqrt{2}\right)^6$, the expansion is:

$$\left(\sqrt{3}-\sqrt{2}\right)^6 = \sum_{k=0}^{6} \binom{6}{k} (\sqrt{3})^{6-k} (-\sqrt{2})^k$$

Step 2: Subtract the expansions.

When subtracting the two expansions, we observe that the odd powers of $\sqrt{2}$ will cancel out, leaving only even powers. This simplifies to:

$$\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6 = 2\sum_{k\text{ even}} \binom{6}{k} (\sqrt{3})^{6-k}(\sqrt{2})^k$$

Step 3: Conclusion.

By calculating the above sum, we can find the exact value of the expression.

Quick Tip

When expanding expressions like $(a+b)^n - (a-b)^n$, use symmetry to simplify the calculation by focusing only on the even or odd powers.

---

29. Find the equation of the right bisector of the line segment joining the points $(3, 4)$ and $(1, 2)$.

Solution:

Step 1: Find the midpoint of the line segment.

The midpoint M of the line segment joining two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute the coordinates of the points $(3, 4)$ and $(1, 2)$:

$$M = \left(\frac{3+1}{2}, \frac{4+2}{2}\right) = \left(\frac{4}{2}, \frac{6}{2}\right) = (2, 3)$$

Step 2: Find the slope of the line segment.

The slope m of the line joining two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the coordinates $(3, 4)$ and $(1, 2)$:

$$m = \frac{2-4}{1-3} = \frac{-2}{-2} = 1$$

Step 3: Find the slope of the perpendicular bisector.

The slope of the right bisector (perpendicular to the line) is the negative reciprocal of the slope of the original line. Therefore, the slope of the bisector is:

$$m_{\text{bisector}} = -\frac{1}{1} = -1$$

Step 4: Use the point-slope form of the equation.

The equation of a line with slope m passing through a point (x_1, y_1) is given by:

$$y - y_1 = m(x - x_1)$$

Substitute the slope $m = -1$ and the point $(2, 3)$ into the equation:

$$y - 3 = -1(x - 2)$$

Simplify the equation:

$$y - 3 = -x + 2$$

$$y = -x + 5$$

So, the equation of the right bisector is $y = -x + 5$.

Quick Tip

The right bisector of a line segment passes through its midpoint and has a slope that is the negative reciprocal of the slope of the line segment.

OR,

The vertices of $\triangle PQR$ are $P(2, 1)$, $Q(-2, 3)$, and $R(4, 5)$. Find the equation of the median through the vertex R.

Solution:

Step 1: Find the midpoint of side PQ.

The midpoint M of the line segment joining points $P(2, 1)$ and $Q(-2, 3)$ is:

$$M = \left(\frac{2 + (-2)}{2}, \frac{1 + 3}{2}\right) = (0, 2)$$

Step 2: Find the slope of the median.

The median is the line joining vertex $R(4,5)$ to the midpoint $M(0,2)$. The slope m_{median} is:

$$m_{\text{median}} = \frac{5-2}{4-0} = \frac{3}{4}$$

Step 3: Use the point-slope form to find the equation of the median.

Using the point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

Substitute $m = \frac{3}{4}$ and the point $(0,2)$:

$$y - 2 = \frac{3}{4}(x - 0)$$

Simplify the equation:

$$y - 2 = \frac{3}{4}x$$

$$y = \frac{3}{4}x + 2$$

So, the equation of the median is $y = \frac{3}{4}x + 2$.

Quick Tip

The median of a triangle connects a vertex to the midpoint of the opposite side. To find its equation, first find the midpoint of the opposite side, then use the point-slope formula.

---

30. Find the values of the other five trigonometric functions, if $\cos x = -\frac{1}{2}$, and x lies in the third quadrant.

Solution:

Step 1: Recall the trigonometric identities.

The six trigonometric functions are:

$$\sin x, \cos x, \tan x, \csc x, \sec x, \text{ and } \cot x$$

Step 2: Given information.

We are given that $\cos x = -\frac{1}{2}$ and that x lies in the third quadrant. In the third quadrant, sine and cosine are negative, while tangent is positive.

Step 3: Use the Pythagorean identity.

We can use the identity:

$$\sin^2 x + \cos^2 x = 1$$

Substitute $\cos x = -\frac{1}{2}$ into the identity:

$$\sin^2 x + \left(-\frac{1}{2}\right)^2 = 1$$

$$\sin^2 x + \frac{1}{4} = 1$$

$$\sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\sin x = -\frac{\sqrt{3}}{2} \quad \text{(since } x \text{ is in the third quadrant, where sine is negative)}$$

Step 4: Find the remaining trigonometric functions.

Now, we can find the other functions:

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$

$$\csc x = \frac{1}{\sin x} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{-\frac{1}{2}} = -2$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Step 5: Conclusion.

The values of the other five trigonometric functions are:

$$\sin x = -\frac{\sqrt{3}}{2}, \quad \tan x = \sqrt{3}, \quad \csc x = -\frac{2\sqrt{3}}{3}, \quad \sec x = -2, \quad \cot x = \frac{\sqrt{3}}{3}$$

Quick Tip

In the third quadrant, both sine and cosine are negative, but tangent is positive. Use the Pythagorean identity to find the missing trigonometric functions.

31. The sum of the first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the 1st term, the common ratio, and the sum to n terms of the G.P.

Solution:

Step 1: Use the formula for the sum of the first n terms of a G.P.

The sum of the first n terms of a geometric progression is given by:

$$S_n = a\frac{1-r^n}{1-r} \quad (\text{for } r \neq 1)$$

where a is the first term, and r is the common ratio.

Step 2: Sum of the first three terms.

We are given that the sum of the first three terms is 16:

$$a + ar + ar^2 = 16$$

$$a(1 + r + r^2) = 16$$

Step 3: Sum of the next three terms.

We are also given that the sum of the next three terms is 128:

$$ar^3 + ar^4 + ar^5 = 128$$

$$ar^3(1 + r + r^2) = 128$$

Since $1 + r + r^2$ is common in both equations, we can divide the second equation by the first equation:

$$\frac{ar^3(1 + r + r^2)}{a(1 + r + r^2)} = \frac{128}{16}$$

$$r^3 = 8$$

$$r = 2$$

Step 4: Find the first term.

Substitute $r = 2$ into the first equation:

$$a(1 + 2 + 2^2) = 16$$

$$a(1 + 2 + 4) = 16$$

$$a(7) = 16$$

$$a = \frac{16}{7}$$

Step 5: Find the sum to n terms.

The sum to n terms of a G.P. is given by:

$$S_n = a\frac{1 - r^n}{1 - r}$$

Substitute $a = \frac{16}{7}$ and $r = 2$:

$$S_n = \frac{16}{7}\frac{1 - 2^n}{1 - 2} = \frac{16}{7} \times (2^n - 1)$$

Step 6: Conclusion.

The first term $a = \frac{16}{7}$, the common ratio $r = 2$, and the sum to n terms is:

$$S_n = \frac{16}{7}(2^n - 1)$$

Quick Tip

To find the sum of the terms in a G.P., use the formula $S_n = a\frac{1-r^n}{1-r}$ for $r \neq 1$. When the sum of terms is given, create a system of equations to find the first term and common ratio.

OR,

Find the value of 'n' so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Solution:

Step 1: Define the geometric mean.

The geometric mean between two numbers a and b is given by:

$$\text{Geometric mean} = \sqrt{ab}$$

Step 2: Set up the equation.

We are given that:

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

Now, multiply both sides by $a^n + b^n$:

$$a^{n+1} + b^{n+1} = \sqrt{ab}(a^n + b^n)$$

Step 3: Solve for n.

To find the value of n, we solve the equation for specific values of a and b. The exact solution will depend on the values of a and b, but the general approach involves simplifying the equation and solving for n.

> **Quick Tip**
>
> The geometric mean between two numbers a and b is $\sqrt{ab}$. To solve such equations, manipulate the expression and use algebraic techniques to find the value of n.

---

32. Find the coordinates of the focus, axis of the parabola, the equation of the directrix, and the length of the latus rectum, if $y^2 = 12x$.

Solution:

Step 1: Recognize the standard form of the equation.

The equation $y^2 = 12x$ is in the standard form of a parabola that opens to the right, which is:

$$y^2 = 4ax$$

where a is the distance from the vertex to the focus.

Step 2: Compare with the standard equation.

Comparing $y^2 = 12x$ with $y^2 = 4ax$, we get $4a = 12$, so $a = 3$.

Step 3: Find the coordinates of the focus.

The focus of a parabola $y^2 = 4ax$ is at $(a, 0)$. Therefore, the coordinates of the focus are $(3, 0)$.

Step 4: Find the equation of the directrix.

The directrix of the parabola $y^2 = 4ax$ is given by $x = -a$. Therefore, the equation of the directrix is:

$$x = -3$$

Step 5: Find the axis of the parabola.

The axis of the parabola is the line that passes through the focus and the vertex. In this case, the axis is the x-axis, or $y = 0$.

Step 6: Find the length of the latus rectum.

The length of the latus rectum for a parabola $y^2 = 4ax$ is given by $4a$. Therefore, the length of the latus rectum is:

$$4a = 4 \times 3 = 12$$

So, the answers are: - Focus: $(3, 0)$ - Axis: $y = 0$ - Directrix: $x = -3$ - Length of the latus rectum: 12

Quick Tip

For a parabola of the form $y^2 = 4ax$, the focus is at $(a, 0)$, the axis is the x-axis, and the directrix is the line $x = -a$.

OR,

Find the coordinates of the foci, the vertices, the length of the major axis, the minor axis, the eccentricity, and the length of the latus rectum of the ellipse:

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

Solution:

Step 1: Recognize the standard form of the ellipse equation.

The given equation is in the standard form of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a^2 is the square of the length of the semi-major axis and b^2 is the square of the length of the semi-minor axis.

Step 2: Identify a^2 and b^2.

From the equation $\frac{x^2}{36} + \frac{y^2}{16} = 1$, we see that:

$$a^2 = 36, \quad b^2 = 16$$

So, $a = 6$ and $b = 4$.

Step 3: Find the foci of the ellipse.

The foci of an ellipse are located at $(\pm c, 0)$ along the major axis, where c is given by:

$$c = \sqrt{a^2 - b^2}$$

Substitute the values of a^2 and b^2:

$$c = \sqrt{36 - 16} = \sqrt{20} = 2\sqrt{5}$$

So, the foci are at $(\pm 2\sqrt{5}, 0)$.

Step 4: Find the coordinates of the vertices.

The vertices are located at $(\pm a, 0)$ along the major axis. Therefore, the vertices are at $(6, 0)$ and $(-6, 0)$.

Step 5: Find the length of the major axis.

The length of the major axis is $2a = 2 \times 6 = 12$.

Step 6: Find the length of the minor axis.

The length of the minor axis is $2b = 2 \times 4 = 8$.

Step 7: Find the eccentricity.

The eccentricity e of an ellipse is given by:

$$e = \frac{c}{a}$$

Substitute the values of c and a:

$$e = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

Step 8: Find the length of the latus rectum.

The length of the latus rectum L is given by:

$$L = \frac{2b^2}{a}$$

Substitute the values of b^2 and a:

$$L = \frac{2 \times 16}{6} = \frac{32}{6} = \frac{16}{3}$$

Quick Tip

For an ellipse, the length of the major axis is $2a$, the length of the minor axis is $2b$, the foci are at $(\pm c, 0)$, and $c = \sqrt{a^2 - b^2}$.
