Series–A

Roll No.....

(GRAPH PAPER)

Total No. of Questions–34] [Total No. of Printed Pages–16

A–857–A–XII–2325

MATHEMATICS

Time Allowed—3 Hours Maximum Marks—80

Candidates are required to give their answers in their own words as far as practicable.

Marks allotted to each question are indicated against it.

Special Instructions :

(i) You must write Question Paper Series in the circle at the top left side of title page of your Answer-book.

(ii) While answering your Questions, you must indicate on your Answer-book the same Question No. as appears in your Question Paper.

(iii) Do not leave blank page/pages in your Answer book.

(iv) All questions are compulsory.

(v) This question paper contains four sections A, B, C, & D. Each section is compulsory.

(vi) Section–A has 13 MCQ (Multiple Choice Questions) and 3 Assertion, Reason based questions of 1 mark each.

(vii) Section–B has 12 Very Short answer questions of 3 marks each i.e. Question Nos. 17 to 28.

(viii) Section–C has 2 Short answer questions of 4 marks each i.e. Question Nos. 29 & 30.

(ix) Section–D has 4 Long answer questions of 5 marks each i.e. Question Nos. 31 to 34.

(x) Graph paper must be attached in between the Answer-book.

(xi) All questions given in Section–A (Multiple Choice Questions) are to be attempt on OMR sheet provided with Answer book.

SECTION–A

1. Let $f : R \to R$ be defined as $f(x) = x^4$. Choose the correct answer : 1

(a) f is one-one onto

(b) f is many-one onto

(c) f is one-one but not onto

(d) f is neither one-one nor onto.

2. If $\cot^{-1} x = y$, then : 1

(a) $0 < y < \pi$ (b) $0 \leq y < \frac{\pi}{2}$

(c) $0 \leq y \leq \frac{\pi}{2}$ (d) $0 \leq y \leq \pi$.

3. The value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to: 1

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) 1.

4. Matrices A and B will be inverse of each other only if 1

(a) $AB = BA$

(b) $AB = BA = O$

(c) $AB = O$, $BA = I$

(d) $AB = BA = I$.

5. The matrix $B = \left[a_{ij}\right]_{1\times n}$, when $n > 1$ is : 1

(a) Square matrix

(b) Column matrix

(c) Diagonal matrix

(d) Row matrix.

6. The derivative of e^{-nx} w.r.t. 'x' is : 1

(a) $-ne^{-nx}$ (b) ne^{-nx}

(c) $-ne^{nx}$ (d) ne^{nx}.

7. The function $f(x) = \log x$ is : 1

(a) Strictly increasing on $(0, \infty)$

(b) Strictly decreasing on $(0, \infty)$

(c) Neither increasing nor decreasing on $(0, \infty)$

(d) None of these.

8. The rate of change of area of a circle w.r.t. its radius 'r' when r = 5 cm is : 1

(a) $8\pi \text{ cm}^2/\text{cm}$

(b) $10\pi \text{ cm}^2/\text{cm}$

(c) $9\pi \text{ cm}^2/\text{cm}$

(d) $16\pi \text{ cm}^2/\text{cm}$.

9. $\int \sin^3 x \cdot \cos^2 x \, dx$ is equal to : 1

(a) $-\frac{2}{3}\sin^3 x + \frac{1}{4}\sin x + C$

(b) $\frac{\sin^4 x}{4} + \frac{\cos^3 x}{3} + C$

(c) $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

(d) None of these.

10. The degree of differential equation

$$2x^2\left(\frac{d^2y}{dx^2}\right)-3\frac{dy}{dx}+y=0 \text{ is :}$$ 1

(a) 2 (b) 1

(c) 0 (d) Not defined.

11. The integrating factor of $ydx-\left(x+2y^2\right)dy=0$ is : 1

(a) $\frac{1}{y^2}$ (b) $-\frac{1}{y}$

(c) $\frac{1}{y}$ (d) y.

12. The projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ is : 1

(a) $\frac{10}{\sqrt{6}}$ (b) $\frac{-10}{\sqrt{6}}$

(c) $\frac{5}{\sqrt{6}}$ (d) $\frac{-5}{\sqrt{6}}$.

13. The relation between direction Cosines l, m and n of a line is : 1

(a) $l^2 + m^2 + n^2 = 1$

(b) $l^2 + m^2 + n^2 = -1$

(c) $l^2 + m^2 + n^2 = 0$

(d) $l^2 + m^2 + n^2 = \frac{1}{\sqrt{2}}$.

Assertion Reason based Questions

In the following questions, a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following from (Question No. 14 to 16) :

14. Assertion (A): The line $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular when $\vec{b}_1 - \vec{b}_2 = 0$. 1

Reason (R): The angle 'θ' between the line $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by $\cos\theta = \frac{\vec{b}_1 - \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$.

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

15. Assertion (A): Let A and B be two events such that $P(A)=\frac{1}{5}$ while $P(A \text{ or } B)=\frac{1}{2}$. Let P(B) = p, then for $p=\frac{3}{8}$, A and B are independent. 1

Reason (R): For independent events

$P(A \cap B)=P(A)\cdot P(B)$

then $P(A \cup B)=$

$P(A)+P(B)-P(A)\cdot P(B)$

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

16. Assertion (A): Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely if E and F denotes the events the card drawn is a spade and the card drawn is an ace respectively then

$$P(E/F) = \frac{1}{4}, \quad P(F/E) = \frac{1}{13}.$$ 1

Reason (R): E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other such events are called. Independent events.

Option:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

SECTION–B

17. Check whether the relation R in $\mathbb{R}$ defined by $R = \{(a, b); a \leq b^3\}$ is reflexive symmetric or transitive. 3

18. Express the matrix $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. 3

19. If

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}.$$

Verify $(AB)^{-1} = B^{-1}A^{-1}$. 3

20. Find the value of 'k' so that the function 'f' is continuous at the indicated point

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x} & , \text{ if } x \neq \dfrac{\pi}{2} \\ 3 & , \text{ if } x = \dfrac{\pi}{2} \end{cases} \text{ at } x = \frac{\pi}{2}. \quad 3$$

21. Evaluate :

$$\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\, dx. \quad 3$$

22. By using properties of definite integral evaluate :

$$\int_{-1}^{2} \left| x^3 - x \right| dx.$$

3

23. Find the area of the region bounded by circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ in first quadrant. 3

24. Solve the differential equation :

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0.$$

Or

Find the general solution of differential equation :

$$\frac{x\,dy}{dx} + 2y = x^2 \log x.$$

3

25. Find the area of parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. 3

26. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that

$$|\vec{a}| = 3, \ |\vec{b}| = 4 \text{ and } |\vec{c}| = 5$$

and each of them being perpendicular to sum of other two Find $|\vec{a} + \vec{b} + \vec{c}|$. 3

27. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coin, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold. 3

28. Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is know that the number on the drawn card is more than 3, what is the probability that it is an even number.

Or

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that :

(a) youngest is a girl.

(b) at least one is a girl ? 3

SECTION–C

29. Prove that

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2},\quad x\in\left(0,\frac{\pi}{4}\right).$$

Or

Prove that

$$\tan^{-1}\left(\frac{63}{16}\right)=\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right).$$

4

30. Differentiate w.r.t 'x'; $(\log x)^x + x^{\log x}$.

Or

If $y = \sin^{-1} x$, show that

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0. \qquad 4$$

SECTION–D

31. Solve the system of linear equation using matrix method : 5

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12.$$

32. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

Or

Find the maximum and minimum value of function : $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on [0, 3]. 5

33. Find the shortest distance between lines

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

and $\vec{r} = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$.

Or

Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines : 5

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

34. Solve the following Linear Programming Problem (LPP) graphically :

Maximize $Z = 5x + 3y$

Subject to Constraints

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x, y \geq 0.$$

5