Series–C

Roll No.....

(GRAPH PAPER)

Total No. of Questions–34] [Total No. of Printed Pages–16

A–857–C–XII–2325

MATHEMATICS

Time Allowed—3 Hours Maximum Marks—80

Candidates are required to give their answers in their own words as far as practicable.

Marks allotted to each question are indicated against it.

Special Instructions :

(i) You must write Question Paper Series in the circle at the top left side of title page of your Answer-book.

(ii) While answering your Questions, you must indicate on your Answer-book the same Question No. as appears in your Question Paper.

(iii) Do not leave blank page/pages in your Answer book.

(iv) All questions are compulsory.

(v) This question paper contains four sections A, B, C, & D. Each section is compulsory.

(vi) Section–A has 13 MCQ (Multiple Choice Questions) and 3 Assertion, Reason based questions of 1 mark each.

(vii) Section–B has 12 Very Short answer questions of 3 marks each i.e. Question Nos. 17 to 28.

(viii) Section–C has 2 Short answer questions of 4 marks each i.e. Question Nos. 29 & 30.

(ix) Section–D has 4 Long answer questions of 5 marks each i.e. Question Nos. 31 to 34.

(x) Graph paper must be attached in between the Answer-book.

(xi) All questions given in Section–A (Multiple Choice Questions) are to be attempt on OMR sheet provided with Answer book.

SECTION–A

1. Let $f : R \to R$ be defined as $f(x) = x^3$. Choose the correct answer : 1

(a) f is one-one onto

(b) f is many-one onto

(c) f is one-one but not onto

(d) f is neither one-one nor onto.

2. If $\cos^{-1} x = y$, then : 1

(a) $0 \le y < \pi$ (b) $0 \le y \le \pi$

(c) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

3. The value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to: 1

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$.

4. $A = \left[a_{ij}\right]_{m \times n}$ is a square matrix if : 1

(a) $m < n$

(b) $m > n$

(c) $m = n$

(d) None of these.

5. The element a_{33} of a 3×3 matrix $A = \left[a_{ij}\right]$ where $a_{ij} = 2i - j$ is : 1

(a) 0 (b) 1

(c) 3 (d) 4.

6. The derivative of $\cos^{-1}\left(e^x\right)$ w.r.t. 'x' is: 1

(a) $\sin^{-1}\left(e^x\right) \cdot e^x$

(b) $\dfrac{-e^x}{\sqrt{1-e^{2x}}}$

(c) $\dfrac{e^x}{\sqrt{1-e^{2x}}}$

(d) None of these.

7. The rate of change of the area of the circle with respect to its radius 'r' where r = 6 cm is : 1

(a) $10\pi \text{ cm}^2/\text{cm}$ (b) $12\pi \text{ cm}^2/\text{cm}$

(c) $8\pi \text{ cm}^2/\text{cm}$ (d) None of these.

8. The interval in which the function $f(x) = x^2 - 4x + 6$ is increasing : 1

(a) $(0, \infty)$ (b) $(-\infty, \infty)$

(c) $(2, \infty)$ (d) $(-\infty, 0)$.

9. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to : 1

(a) $\tan x + \cot x + C$

(b) $\tan x + \text{cosec}\, x + C$

(c) $-\tan x + \cot x + C$

(d) $\tan x + \sec x + C$.

10. The degree of differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$ is : 1

(a) 4 (b) 3

(c) 0 (d) Not defined.

11. The integrating factor of

$\left(1+x^2\right)dy + 2xy\,dx = \cot x\,dx \quad (x \neq 0)$ is : 1

(a) $\log\left(1+y^2\right)$ (b) $1 + y^2$

(c) $\log\left(1+x^2\right)$ (d) $1 + x^2$.

12. If $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$, then '$\theta$' = 1

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $0°$ (d) π.

13. If a line has direction ratio (2, –1, –2), then its Direction Cosines are : 1

(a) $\left(\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$

(b) $\left(\frac{-2}{3}, \frac{-1}{3}, \frac{2}{3}\right)$

(c) $\left(\frac{-2}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$

(d) None of these.

Assertion Reason based Question

In the following questions, a statement of Assertion(A) is followed by a Reason(R). Choose the correct answer out of the following from (Question No. 14 to 16) :

14. Assertion (A): The acute angle between the line $\vec{r} = \hat{i} + \hat{j} + 2\hat{k} + \lambda\left(\hat{i} - \hat{j}\right)$ and the x-axis is $\frac{\pi}{4}$. 1

Reason (R): The acute angle 'θ' between the lines $\vec{r} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda\left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right)$ & $\vec{r} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} + \mu\left(a_2\hat{i} + b_2\hat{j} + c_2\hat{k}\right)$ is given by

$$\cos\theta = \frac{\left|a_1a_2 + b_1b_2 + c_1c_2\right|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

15. Assertion (A): The probability of drawing either an ace or a king from a pack of 52 playing card is $\frac{2}{13}$. 1

Reason (R): For two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

16. Assertion (A): An urn contain 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random Then the probability that the second ball is red is $\frac{1}{2}$. 1

Reason (R): A bag contains 4 red and 4 black balls; another bag contains 2 red and 6 black balls. One of the bag is selected at random and a ball is drawn from the bag which is found to be red.

Then, the probability that the ball is drawn from the first bag is $\frac{2}{3}$.

Options:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Both Assertion (A) and Reason (R) are false.

SECTION–B

17. Show that the relation R in $\mathbb{R}$ defined by $R = \{(a, b); a \leq b\}$ is reflexive and transitive, but not symmetric. 3

18. Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}_{3\times3}$. 3

19. Find the area of triangle by using determinants whose vertices are (1, 0) (6, 0) (4, 3). 3

20. Find the value of 'k' so that the function 'f' is continuous at the indicated point

$$f(x) = \begin{cases} kx + 1 & , \text{ if } x \leq \pi \\ \cos x & , \text{ if } x > \pi \end{cases} \text{ at } x = \pi. \quad 3$$

21. Evaluate :

$$\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx. \quad 3$$

22. By using properties of definite integral evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx. \quad 3$$

23. Find the area of the region bounded by ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1. \quad 3$$

24. Solve the differential equation :

$$\cos^2 x \frac{dy}{dx} + y = \tan x, \quad 0 \le x \le \frac{\pi}{2}.$$

Or

Solve the differential equation :

$$x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0. \qquad 3$$

25. Find 'λ' and 'μ' if

$$\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \vec{0}. \qquad 3$$

26. Find the area of triangle using vectors with vertices

A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5). 3

27. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively

$\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late ? What is the probability that he comes by train ? 3

28. Two balls are drawn at random with replacement from a box. Containing 10 black and 8 red balls, find the probability that :

(a) both balls are red

(b) First ball is black and second is red

(c) One of them black and other is red.

Or

Given that the two numbers appearing on throwing two dice are different. Find the probability of the event "the sum of numbers on the dice is 4". 3

SECTION–C

29. Express :

$$\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right], -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

in the simplest form.

Or

Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right).$$ 4

30. If $e^y(x+1) = 1$, show that

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2.$$

Or

Differentiate w.r.t. 'x':

$$(\sin x)^x + x^{\sin x}.$$ 4

SECTION–D

31. Solve the system of linear equation by using matrix method : 5

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2.$$

32. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Or

Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$. 5

33. Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

Or

Find the angle between the lines l_1 and l_2 whose Cartesian equations are 5

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and}$$

$$\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}.$$

34. Solve the Linear Programming Problem (LPP) graphically : 5

Minimize $Z = -3x + 4y$

Subject to :

$$x + 2y \leq 8$$

$$3x + 2y \leq 12$$

where $x \geq 0$

$y \geq 0.$