

Haryana Board Class 12 Physics Question Paper with Solutions(Memory Based)

Time Allowed :3 Hour	Maximum Marks :60	Total Questions :24
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General Instructions

Read the following instructions very carefully and strictly follow them:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes
- This time is to be spent in reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers,
- The paper has four Sections.
- Section A is compulsory - All questions in Section A must be answered.
- You must attempt one question from each of the Sections B, C and D and one other question from any Section of your choice.

1. State Gauss's Theorem and use it to find the electric field due to an infinitely long charged wire or a thin spherical shell.

Correct Answer: Electric field derived using Gauss's law for cylindrical and spherical symmetry.

Solution:

Concept: Gauss's Theorem (Gauss's Law) relates the electric flux through a closed surface to the charge enclosed by that surface. It is especially useful for systems with high symmetry such as spherical, cylindrical, or planar distributions.

Mathematically,

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

where:

- \vec{E} = electric field
- $d\vec{A}$ = area vector
- Q_{enc} = charge enclosed
- ϵ_0 = permittivity of free space

Gauss's law simplifies calculations when symmetry ensures that the electric field is constant over the Gaussian surface.

Part 1: Electric field due to an infinitely long charged wire

Let the wire have linear charge density λ (charge per unit length).

Step 1: Choosing Gaussian surface

Because of cylindrical symmetry, choose a cylindrical Gaussian surface of:

- Radius r
- Length L

The electric field is:

- Radially outward
- Same magnitude everywhere on curved surface

Step 2: Electric flux

Flux passes only through the curved surface (not the flat ends):

$$\text{Flux} = E \times (2\pi rL)$$

Step 3: Charge enclosed

$$Q_{\text{enc}} = \lambda L$$

Step 4: Apply Gauss's law

$$E(2\pi rL) = \frac{\lambda L}{\epsilon_0}$$

Cancel L :

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Result:

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

The electric field decreases inversely with distance from the wire.

Part 2: Electric field due to a thin spherical shell

Let total charge on the shell be Q and radius be R .

Case 1: Outside the shell ($r > R$)

Step 1: Gaussian surface Choose a concentric sphere of radius r .

Step 2: Flux

$$\text{Flux} = E \times 4\pi r^2$$

Step 3: Enclosed charge

$$Q_{\text{enc}} = Q$$

Applying Gauss's law:

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Thus, the shell behaves like a point charge at the center.

Case 2: Inside the shell ($r < R$)

Step 1: Enclosed charge No charge is enclosed:

$$Q_{\text{enc}} = 0$$

From Gauss's law:

$$E = 0$$

Result:

$$E = 0 \quad \text{inside a charged spherical shell}$$

Final Results:

- Infinitely long wire: $E = \frac{\lambda}{2\pi\epsilon_0 r}$
- Outside spherical shell: $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
- Inside spherical shell: $E = 0$

Quick Tip

Use Gauss's law when symmetry exists: - Cylindrical symmetry \rightarrow Use cylindrical Gaussian surface - Spherical symmetry \rightarrow Use spherical Gaussian surface Also remember: electric field inside a charged spherical shell is always zero.

2. Derive the expression for Electric Field Intensity on the axial and equatorial line of a dipole.

Correct Answer: Electric field expressions derived using superposition of fields of charges forming an electric dipole.

Solution:

Concept: An electric dipole consists of two equal and opposite charges $+q$ and $-q$ separated by a small distance $2a$.

Dipole moment:

$$\vec{p} = q \cdot (2a) \quad \text{directed from } -q \text{ to } +q$$

The electric field due to a dipole is obtained using the principle of superposition by adding fields due to both charges.

Part 1: Electric Field on the Axial Line of a Dipole

The axial line is the line passing through both charges.

Let:

- Point P be at distance r from the center of dipole
- Charges located at $-a$ and $+a$

Step 1: Electric field due to each chargeField due to $+q$:

$$E_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2}$$

Field due to $-q$:

$$E_- = \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2}$$

Both fields act along the same line but in opposite directions.

Step 2: Net electric field

$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{(r-a)^2} - \frac{q}{(r+a)^2} \right]$$

Taking common denominator and simplifying:

$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{4qar}{(r^2 - a^2)^2}$$

Using dipole moment $p = 2aq$:

$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2pr}{(r^2 - a^2)^2}$$

For a short dipole ($r \gg a$)Neglect a^2 :

$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

Direction: Along dipole moment.

Part 2: Electric Field on the Equatorial Line of a Dipole

The equatorial line is perpendicular to the dipole axis and passes through the center.

Let point P be at distance r from the center.**Step 1: Distance from charges**

Distance from each charge:

$$\sqrt{r^2 + a^2}$$

Field due to each charge:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + a^2}$$

Step 2: Resolve components

Horizontal components cancel due to symmetry. Vertical components add.

Vertical component of each field:

$$E \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + a^2} \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

Step 3: Net electric field

$$E_{\text{equatorial}} = 2 \times \frac{1}{4\pi\epsilon_0} \frac{qa}{(r^2 + a^2)^{3/2}}$$

Using $p = 2aq$:

$$E_{\text{equatorial}} = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}$$

For a short dipole ($r \gg a$)

$$E_{\text{equatorial}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

Direction: Opposite to dipole moment.

Comparison: Axial vs Equatorial Field

- Axial field is twice the equatorial field at same distance.
- Both vary inversely as r^3 .

$$E_{\text{axial}} = 2E_{\text{equatorial}}$$

Quick Tip

Key dipole results to remember: - Axial field: $E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$ - Equatorial field: $E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$ - Both decrease as $1/r^3$, faster than a point charge field.

3. Derive the expression for the capacitance of a parallel plate capacitor (with and without dielectric slab).

Correct Answer: Capacitance derived using electric field between plates and potential difference, with modification due to dielectric constant.

Solution:

Concept: Capacitance is defined as the charge stored per unit potential difference:

$$C = \frac{Q}{V}$$

For a parallel plate capacitor:

- Plate area = A
- Separation between plates = d
- Medium between plates affects capacitance

The derivation is based on electric field and potential difference between plates.

Part 1: Capacitance of Parallel Plate Capacitor (Without Dielectric)

Step 1: Electric field between plates

Surface charge density:

$$\sigma = \frac{Q}{A}$$

Electric field due to one plate:

$$E = \frac{\sigma}{2\epsilon_0}$$

Between two oppositely charged plates:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

Step 2: Potential difference

$$V = Ed = \frac{Qd}{A\epsilon_0}$$

Step 3: Capacitance

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\epsilon_0}}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

This is the capacitance in vacuum or air.

Part 2: Capacitance with Dielectric Slab Filling Entire Space

Let dielectric constant be K .

When dielectric is inserted:

- Electric field reduces by factor K
- Permittivity becomes $\epsilon = K\epsilon_0$

New electric field:

$$E = \frac{E_0}{K}$$

Potential difference:

$$V = \frac{Ed}{K}$$

Capacitance:

$$C = \frac{Q}{V} = \frac{K\epsilon_0 A}{d}$$

$$\boxed{C = \frac{K\epsilon_0 A}{d}}$$

Thus, dielectric increases capacitance by factor K .

Part 3: Dielectric Slab Partially Filling the Capacitor

Let:

- Thickness of dielectric slab = t
- Remaining air gap = $d - t$

This behaves like two capacitors in series:

- One with dielectric

- One with air

Capacitances of parts:

$$C_1 = \frac{K\varepsilon_0 A}{t}, \quad C_2 = \frac{\varepsilon_0 A}{d-t}$$

For series combination:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{t}{K\varepsilon_0 A} + \frac{d-t}{\varepsilon_0 A}$$

Taking common factor:

$$\frac{1}{C} = \frac{1}{\varepsilon_0 A} \left(\frac{t}{K} + d - t \right)$$

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

Special Cases:

- If $t = 0$: $C = \frac{\varepsilon_0 A}{d}$ (air)
- If $t = d$: $C = \frac{K\varepsilon_0 A}{d}$ (fully filled)

Final Results:

- Without dielectric: $C = \frac{\varepsilon_0 A}{d}$
- Fully filled dielectric: $C = \frac{K\varepsilon_0 A}{d}$
- Partially filled: $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

Quick Tip

Remember: - Capacitance increases with area and dielectric constant. - Capacitance decreases with plate separation. - Dielectric reduces electric field and increases charge storing ability.

4. Derive the condition for a balanced Wheatstone Bridge.

Correct Answer: A Wheatstone bridge is balanced when the ratio of resistances in one arm equals the ratio in the other arm, i.e., $\frac{P}{Q} = \frac{R}{S}$.

Solution:

Concept: A Wheatstone bridge is used to determine an unknown resistance by comparing it with known resistances. It consists of four resistors arranged in a diamond shape with a galvanometer connecting the middle junctions.

Let:

- P, Q be resistances in one branch
- R, S be resistances in the other branch
- Galvanometer connects junctions B and D

The bridge is said to be balanced when no current flows through the galvanometer.

Step 1: Balanced Condition

If no current flows through the galvanometer:

$$I_g = 0$$

This means the potentials at points B and D are equal:

$$V_B = V_D$$

Step 2: Apply Kirchhoff's Law

Let current I_1 flow through branch $P \rightarrow Q$, and current I_2 flow through branch $R \rightarrow S$.

Potential drop from A to B :

$$V_{AB} = I_1 P$$

Potential drop from A to D :

$$V_{AD} = I_2 R$$

Since $V_B = V_D$:

$$I_1 P = I_2 R \quad \dots (1)$$

Step 3: Consider lower arms

Potential drop from B to C :

$$V_{BC} = I_1 Q$$

Potential drop from D to C :

$$V_{DC} = I_2 S$$

Again using equal potentials:

$$I_1 Q = I_2 S \quad \dots (2)$$

Step 4: Divide equations (1) and (2)

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}$$

Cancel currents:

$$\frac{P}{Q} = \frac{R}{S}$$

$$\frac{P}{Q} = \frac{R}{S}$$

This is the condition for a balanced Wheatstone bridge.

Physical Meaning:

- No current flows through the galvanometer.
- Potential difference between its terminals is zero.
- The bridge behaves like two independent voltage dividers.

Application: If one resistance is unknown (say S):

$$S = \frac{RQ}{P}$$

Thus Wheatstone bridge is used for precise resistance measurement.

Quick Tip

In a balanced Wheatstone bridge: - No current flows through the galvanometer. - Ratio of resistances in opposite arms are equal. - Used in meter bridge and strain gauge measurements.

5. Define Drift Velocity and establish its relation with electric current.

Correct Answer: Electric current is related to drift velocity by $I = nqAv_d$, where n is charge carrier density and v_d is drift velocity.

Solution:

Concept: In a conductor, free electrons move randomly due to thermal energy. When an electric field is applied, these electrons acquire a small net velocity in a particular direction. This average velocity is called **drift velocity**.

Definition: Drift velocity is the average velocity acquired by charge carriers in a conductor under the influence of an external electric field.

It is denoted by v_d .

Step 1: Motion of Charge Carriers

Consider:

- A conductor of cross-sectional area A
- Number of charge carriers per unit volume = n
- Charge of each carrier = q
- Drift velocity = v_d

In time t , charge carriers move a distance:

$$\ell = v_d t$$

Step 2: Volume of Charge Flowing

Volume of conductor through which charges move:

$$\text{Volume} = A \times \ell = Av_d t$$

Number of charge carriers in this volume:

$$N = nAv_d t$$

Step 3: Total Charge Flow

Total charge crossing the section in time t :

$$Q = nqAv_d t$$

Step 4: Relation with Electric Current

By definition of current:

$$I = \frac{Q}{t}$$

Substituting Q :

$$I = \frac{nqAv_d t}{t}$$

$$\boxed{I = nqAv_d}$$

This is the required relation between current and drift velocity.

Special Case: Electrons as Charge Carriers

If electrons are charge carriers:

$$q = e$$

$$I = neAv_d$$

Direction:

- Electron drift is opposite to electric field
- Current direction is opposite to electron motion

Drift Velocity in terms of Electric Field

From microscopic theory:

$$v_d = \mu E$$

where μ is mobility.

Thus:

$$I = nqA\mu E$$

This leads to Ohm's law at microscopic level.

Key Observations:

- Current depends on number density of charge carriers.
- Larger cross-section gives larger current.
- Drift velocity is very small compared to speed of light.

Quick Tip

Important formulas: - Drift velocity: $v_d = \mu E$ - Current relation: $I = nqAv_d$ - Even though drift velocity is small, current appears instantly due to electric field propagation.

6. Explain the principle, construction, and derive the current sensitivity of a moving coil galvanometer.

Correct Answer: Current sensitivity of a moving coil galvanometer is $\frac{\theta}{I} = \frac{NBA}{k}$, where N is number of turns, B magnetic field, A area of coil, and k torsional constant.

Solution:

Concept: A moving coil galvanometer is a sensitive instrument used to detect and measure small electric currents. It works on the principle that a current-carrying conductor placed in a magnetic field experiences a torque.

Principle: When a current-carrying coil is placed in a uniform magnetic field, it experiences a torque given by:

$$\tau = NBIA \sin \theta$$

In a galvanometer, the magnetic field is radial, so $\sin \theta = 1$:

$$\tau = NBAI$$

This torque causes rotation of the coil, producing deflection proportional to current.

Construction: A moving coil galvanometer consists of the following parts:

- **Coil:** Rectangular coil of fine insulated copper wire with many turns.
- **Magnet:** Strong horseshoe magnet providing a radial magnetic field.
- **Soft Iron Core:** Cylindrical core placed inside the coil to make field radial and increase sensitivity.
- **Suspension System:** Coil is suspended using phosphor-bronze strip or springs.
- **Mirror/Pointer:** Attached to coil to measure angular deflection.
- **Scale:** Calibrated to read deflection.

The radial field ensures linear scale (deflection proportional to current).

Working: When current flows through the coil:

- Magnetic torque rotates the coil.

- Suspension fiber provides restoring torque.
- Coil stops when both torques balance.

Derivation of Current Sensitivity

Let:

- N = number of turns
- B = magnetic field
- A = area of coil
- I = current
- k = torsional constant of suspension wire
- θ = angular deflection

Step 1: Magnetic Torque

$$\tau_m = NBAI$$

Step 2: Restoring Torque

When the coil rotates, suspension wire twists and produces restoring torque:

$$\tau_r = k\theta$$

Step 3: Equilibrium Condition

At steady deflection:

$$\tau_m = \tau_r$$

$$NBAI = k\theta$$

Step 4: Current Sensitivity

Current sensitivity is defined as deflection per unit current:

$$\text{Current Sensitivity} = \frac{\theta}{I}$$

From above equation:

$$\theta = \frac{NBA}{k}I$$

$$\boxed{\frac{\theta}{I} = \frac{NBA}{k}}$$

Factors Affecting Sensitivity:

Sensitivity increases when:

- Number of turns N increases

- Magnetic field B is strong
- Coil area A is large
- Torsional constant k is small

Voltage Sensitivity: Sometimes defined as:

$$\frac{\theta}{V} = \frac{NBA}{kR}$$

where R is resistance of galvanometer.

Applications:

- Detect small currents
- Used in ammeters and voltmeters (after modification)
- Null detection in bridge circuits

Quick Tip

Key results: - Torque on coil: $\tau = NBAI$ - Current sensitivity: $\frac{\theta}{I} = \frac{NBA}{k}$ - Radial magnetic field ensures linear scale and high sensitivity.

7. Explain the principle and working of an AC Generator or a Transformer (including energy losses).

Correct Answer: Based on electromagnetic induction. AC generator converts mechanical energy to electrical energy, while transformer transfers electrical energy between coils using mutual induction (with energy losses accounted).

Solution:

Concept: Both AC generators and transformers work on the principle of electromagnetic induction discovered by Faraday. A changing magnetic flux induces an emf in a circuit.

Option 1: AC Generator

Principle: Based on Faraday's law of electromagnetic induction:

$$e = -\frac{d\Phi}{dt}$$

An emf is induced when a coil rotates in a magnetic field, causing change in magnetic flux.

Construction:

- Rectangular coil of wire (armature)
- Strong magnetic field (permanent magnet or electromagnet)
- Slip rings

- Carbon brushes
- Axle for rotation

Working:

- Coil rotates in magnetic field.
- Magnetic flux linked with coil changes continuously.
- Induced emf alternates in direction every half rotation.
- Slip rings collect alternating current.

If the coil rotates with angular velocity ω :

$$\Phi = BA \cos \omega t$$

Induced emf:

$$e = -\frac{d\Phi}{dt} = BA\omega \sin \omega t$$

$$e = e_0 \sin \omega t$$

Thus, output is alternating current.

Applications:

- Power generation in power plants
- Bicycle dynamos

Option 2: Transformer

Principle: Based on mutual induction — a changing current in one coil induces emf in another nearby coil.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

where:

- V_p, V_s = primary and secondary voltages
- N_p, N_s = number of turns

Construction:

- Two insulated coils (primary and secondary)
- Soft iron laminated core
- Core provides low reluctance path for magnetic flux

Working:

- AC applied to primary coil.

- Alternating current produces changing magnetic flux in core.
- This changing flux links secondary coil.
- Induced emf appears in secondary coil.

If $N_s > N_p \rightarrow$ Step-up transformer If $N_s < N_p \rightarrow$ Step-down transformer

Energy Losses in Transformer:

1. **Copper Loss (I^2R Loss):** Due to resistance of windings.
2. **Iron Losses:**
 - Hysteresis loss (magnetization cycles)
 - Eddy current loss (circulating currents in core)
3. **Flux Leakage:** Not all magnetic flux links both coils.
4. **Mechanical Losses:** Vibrations and heating.

Minimizing Losses:

- Laminated iron core (reduces eddy currents)
- Soft iron (reduces hysteresis loss)
- Thick copper windings (reduces resistance)

Efficiency of Transformer:

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100\%$$

Practical transformers have efficiency up to 95–99%.

Quick Tip

- AC Generator: Converts mechanical energy \rightarrow electrical energy (Faraday's law).
- Transformer: Transfers electrical energy using mutual induction.
- Laminated cores reduce eddy current losses.

8. Derive the Lens Maker's Formula for a thin convex lens.

Correct Answer: The lens maker's formula for a thin lens is $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$, where μ is refractive index and R_1, R_2 are radii of curvature.

Solution:

Concept: A convex lens has two spherical surfaces. The focal length depends on:

- Refractive index of lens material (μ)

- Radii of curvature of its surfaces (R_1, R_2)

Lens maker's formula gives relation between focal length and lens geometry. We derive it using refraction at spherical surfaces.

Refraction Formula at a Spherical Surface:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Step 1: Refraction at First Surface

Let:

- Object at infinity $\rightarrow u = \infty$
- Refractive index of air = 1
- Refractive index of lens = μ
- Radius of first surface = R_1

Using refraction formula:

$$\frac{\mu}{v_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1}$$

$$\frac{\mu}{v_1} = \frac{\mu - 1}{R_1}$$

$$v_1 = \frac{\mu R_1}{\mu - 1}$$

This image acts as virtual object for second surface.

Step 2: Refraction at Second Surface

Now light goes from lens to air:

- Object distance = $u_2 = -v_1$ (sign convention)
- Final image at focal point $\rightarrow v_2 = f$
- Radius of second surface = R_2

Applying refraction formula:

$$\frac{1}{f} - \frac{\mu}{-v_1} = \frac{1 - \mu}{R_2}$$

$$\frac{1}{f} + \frac{\mu}{v_1} = \frac{1 - \mu}{R_2}$$

Step 3: Substitute $\frac{\mu}{v_1}$

From first surface:

$$\frac{\mu}{v_1} = \frac{\mu - 1}{R_1}$$

Substitute:

$$\frac{1}{f} + \frac{\mu - 1}{R_1} = \frac{1 - \mu}{R_2}$$

Rearranging:

$$\frac{1}{f} = \frac{1 - \mu}{R_2} - \frac{\mu - 1}{R_1}$$

Factor out $(\mu - 1)$:

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\boxed{\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}$$

This is the Lens Maker's Formula.

Special Cases:

- For symmetric biconvex lens: $R_1 = R, R_2 = -R$

$$\frac{1}{f} = (\mu - 1) \left(\frac{2}{R} \right)$$

- If lens in medium of refractive index μ_m :

$$\frac{1}{f} = \left(\frac{\mu}{\mu_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Sign Convention:

- $R_1 > 0$ for convex surface
- $R_2 < 0$ for convex lens second surface

Importance:

- Used in lens design
- Helps control focal length
- Basis for optical instruments

Quick Tip

Lens Maker's Formula:

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Focal length increases if curvature decreases or refractive index decreases.

9. State Bohr's postulates and derive the expression for the radius of the n^{th} orbit.

Correct Answer: Radius of the n^{th} Bohr orbit is $r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$ or $r_n = n^2 a_0$, where a_0 is Bohr radius.

Solution:

Concept: Bohr proposed a model for hydrogen atom to explain atomic stability and discrete spectral lines. He introduced quantized orbits where electrons revolve around the nucleus without radiating energy.

Bohr's Postulates:

1. **Stationary Orbits:** Electrons revolve in certain stable circular orbits without emitting radiation.
2. **Quantization of Angular Momentum:** Angular momentum of electron is quantized:

$$mvr = \frac{nh}{2\pi}, \quad n = 1, 2, 3, \dots$$

3. **Emission/Absorption of Radiation:** Radiation is emitted or absorbed only when an electron jumps between orbits:

$$h\nu = E_2 - E_1$$

Derivation of Radius of n^{th} Orbit

Consider hydrogen atom:

- Electron mass = m
- Electron charge = e
- Orbit radius = r

Step 1: Electrostatic Force = Centripetal Force

Attraction between nucleus and electron provides centripetal force.

Coulomb force:

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

Centripetal force:

$$F = \frac{mv^2}{r}$$

Equating:

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

$$mv^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad \dots (1)$$

Step 2: Quantization of Angular Momentum

From Bohr's second postulate:

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \quad \dots (2)$$

Step 3: Substitute velocity in equation (1)

Substitute v from (2) into (1):

$$m \left(\frac{nh}{2\pi mr} \right)^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$
$$\frac{n^2 h^2}{4\pi^2 m r^2} = \frac{e^2}{4\pi\epsilon_0 r}$$

Step 4: Solve for r

Multiply both sides by $4\pi^2 m r^2$:

$$n^2 h^2 = \frac{e^2}{4\pi\epsilon_0 r} \cdot 4\pi^2 m r^2$$

$$n^2 h^2 = \frac{\pi m e^2 r}{\epsilon_0}$$

$$r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

Bohr Radius:

For $n = 1$:

$$a_0 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

Numerically:

$$a_0 = 0.529 \text{ \AA}$$

Thus:

$$r_n = n^2 a_0$$

Key Results:

- Radius increases as n^2
- Higher orbits are farther from nucleus
- Explains hydrogen spectrum

Limitations of Bohr Model:

- Works only for hydrogen-like atoms
- Cannot explain fine spectral lines
- Ignores wave nature of electron

Quick Tip

Important formulas: - Angular momentum: $mvr = \frac{nh}{2\pi}$ - Orbit radius: $r_n = n^2 a_0$ - Bohr radius: $a_0 = 0.529 \text{ \AA}$ Radius increases rapidly with quantum number n .
