

IBSAT Quantitative Ability Sample Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :100	Total Questions :140
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test paper contains 140 Multiple Choice Questions (MCQs) for a total of 140 marks.
2. **There will be Negative Marking for Multiple Choice Questions (MCQs).** For every wrong answer 0.25 marks will be deducted.
3. Candidates have to indicate the most appropriate answer by darkening one of the four responses provided, with only **BLACK/BLUE BALL POINT PEN** in the OMR Response Sheet.
4. Answering the question by any method other than the method mentioned above shall be considered wrong answer.
5. More than one response to a question shall be counted as wrong answer.
6. The candidate shall not write anything on the OMR Response Sheet other than the details required and, in the spaces, provided for.
7. After the examination is over, the candidate can carry the Question Booklet along with candidate's copy of the OMR Response Sheet. Candidate will hand over the original OMR Response Sheet to the invigilator.
8. Electronic gadgets like mobile phones, digital watch, pagers and calculators etc. are strictly not permitted inside the Test Centre/Hall.
9. The test paper contains 140 Multiple Choice Questions (MCQs) for a total of 140 marks.
10. **There will be Negative Marking for Multiple Choice Questions (MCQs).** For every wrong answer 0.25 marks will be deducted.
11. Candidates have to indicate the most appropriate answer by darkening one of the four responses provided, with only **BLACK/BLUE BALL POINT PEN** in the OMR Response Sheet.
12. Answering the question by any method other than the method mentioned above shall be considered wrong answer.
13. More than one response to a question shall be counted as wrong answer.
14. Electronic gadgets like mobile phones, digital watch, pagers and calculators etc. are strictly not permitted inside the Test Centre/Hall.

1. Five bells begin to toll together and toll respectively at intervals of 6,7,8,9 and 12 seconds. How many times, will they toll together in one hour excluding the one at the start?

- (A) 5
- (B) 6
- (C) 7
- (D) 4
- (E) None of these

Correct Answer: (C) 7

Solution:

Step 1: Understanding the Question:

The problem asks for the number of times five bells, which ring at different intervals, will ring simultaneously within a one-hour period. This is a classic application of the Least Common Multiple (LCM).

Step 2: Key Formula or Approach:

To find when the bells will toll together, we need to find the LCM of their individual tolling intervals (6, 7, 8, 9, and 12 seconds).

Step 3: Detailed Explanation:

First, we find the prime factorization of each interval:

$$6 = 2 \times 3$$

$$7 = 7$$

$$8 = 2^3$$

$$9 = 3^2$$

$$12 = 2^2 \times 3$$

The LCM is the product of the highest powers of all prime factors that appear in any of the numbers.

$$\text{LCM}(6, 7, 8, 9, 12) = 2^3 \times 3^2 \times 7$$

$$\text{LCM} = 8 \times 9 \times 7 = 504 \text{ seconds}$$

This means the bells will toll together every 504 seconds.

Now, we need to find out how many times this happens in one hour.

One hour = 60 minutes = $60 \times 60 = 3600$ seconds.

Number of times they toll together in 3600 seconds is given by the integer part of the division:

$$\text{Number of times} = \left\lfloor \frac{3600}{504} \right\rfloor$$

$$\frac{3600}{504} \approx 7.14$$

So, the bells will toll together 7 times within the one-hour period. The question specifies to exclude the one at the start, and this calculation naturally does that as it counts the occurrences at 504s, 1008s, ..., up to the 7th multiple before 3600s.

Step 4: Final Answer:

The bells will toll together 7 times in one hour, excluding the initial toll.

Quick Tip

For problems involving events that repeat at different intervals and need to occur simultaneously, always find the LCM of the intervals. Be careful to check if the question includes or excludes the starting event.

2. A two digit number is such that the unit digit multiplied by 3 is equal to three more than the sum of the digits. When the digits are reversed the resulting number is 18 less than the original number. Find the units digit of the original number.

- (A) 8
- (B) 5
- (C) 4
- (D) 2
- (E) 1

Correct Answer: (B) 5

Solution:

Step 1: Understanding the Question:

We need to find the units digit of a two-digit number based on two given algebraic conditions relating its digits.

Step 2: Key Formula or Approach:

Represent the two-digit number algebraically and form a system of linear equations from the given conditions. Let the tens digit be 't' and the units digit be 'u'.

The original number can be written as $10t + u$.

The reversed number can be written as $10u + t$.

Step 3: Detailed Explanation:

Condition 1: "The unit digit multiplied by 3 is equal to three more than the sum of the digits."

$$3u = (t + u) + 3$$

$$3u - u = t + 3$$

$$2u = t + 3 \quad \rightarrow \quad (\text{Equation I})$$

Condition 2: "When the digits are reversed the resulting number is 18 less than the original number."

$$10u + t = (10t + u) - 18$$

$$18 = 10t - t + u - 10u$$

$$18 = 9t - 9u$$

Dividing by 9, we get:

$$2 = t - u \rightarrow t = u + 2 \rightarrow \text{(Equation II)}$$

Now, we solve the system of two equations. Substitute the value of 't' from Equation II into Equation I:

$$2u = (u + 2) + 3$$

$$2u = u + 5$$

$$2u - u = 5$$

$$u = 5$$

To find the complete number, substitute $u=5$ back into Equation II:

$$t = 5 + 2 = 7$$

The original number is 75. The question asks for the units digit.

Step 4: Final Answer:

The units digit of the original number is 5.

Quick Tip

For problems involving digits of a number, always express the number in its expanded form (e.g., $10t + u$ for a two-digit number). This makes it easier to set up algebraic equations.

3. When an integer 'n' is divided by 'k', the remainder is one. When another integer 'm' is divided by 'k', the remainder is 2. What is the remainder when 'n x m' is divided by 'k'?

- (A) 1
- (B) 2
- (C) 3
- (D) n-1
- (E) n/2

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Question:

This question is based on the properties of remainders, a concept from modular arithmetic. We are given the remainders of two numbers when divided by 'k' and asked to find the remainder of their product when divided by the same 'k'.

Step 2: Key Formula or Approach:

We can use the properties of modular arithmetic. If $a \equiv r_1 \pmod{k}$ and $b \equiv r_2 \pmod{k}$, then $a \times b \equiv r_1 \times r_2 \pmod{k}$. This means the remainder of the product is the product of the remainders (modulo k).

Step 3: Detailed Explanation:

From the problem statement:

When 'n' is divided by 'k', the remainder is 1. This can be written as:

$$n \equiv 1 \pmod{k}$$

When 'm' is divided by 'k', the remainder is 2. This can be written as:

$$m \equiv 2 \pmod{k}$$

We need to find the remainder when $n \times m$ is divided by 'k'. Using the property of modular arithmetic:

$$n \times m \equiv (1 \times 2) \pmod{k}$$

$$n \times m \equiv 2 \pmod{k}$$

This means that when the product $n \times m$ is divided by 'k', the remainder is 2.

Step 4: Final Answer:

The remainder when 'n x m' is divided by 'k' is 2.

Quick Tip

The remainder of a product is the same as the product of the remainders. For example, Remainder of $(17 \times 18)/5$ is the same as the remainder of $(17 \pmod{5} \times 18 \pmod{5})/5$, which is $(2 \times 3)/5$, giving a remainder of 1.

4. An apple seller sells to the first customer half of the total number of apples that he has and one-half of an apple. To the second customer he sells half of the remaining stock and one-half of an apple. The same thing continues with the third and the fourth customers. He then finds that he is left with 15 apples. How many apples did he have initially?

- (A) 250
- (B) 255
- (C) 260
- (D) 200
- (E) 190

Correct Answer: (B) 255

Solution:

Step 1: Understanding the Question:

An apple seller's stock decreases in a specific pattern over four transactions. We know the final quantity and need to find the initial quantity.

Step 2: Key Formula or Approach:

The most effective method for this type of problem is to work backwards from the final state to the initial state, reversing the operations at each step.

If the seller sells $\frac{x}{2} + 0.5$ apples from a stock of x , the remaining apples are $x - (\frac{x}{2} + 0.5) = \frac{x}{2} - 0.5$. To reverse this, if we have y apples remaining, the quantity before the sale was $(y + 0.5) \times 2$.

Step 3: Detailed Explanation:

Let's work backwards from the 15 apples left at the end.

Before the 4th customer:

The seller was left with 15 apples. So, the number of apples before selling to the 4th customer was:

$$(15 + 0.5) \times 2 = 15.5 \times 2 = 31 \text{ apples}$$

Before the 3rd customer:

He had 31 apples before selling to the 4th customer. So, the number of apples before the 3rd customer was:

$$(31 + 0.5) \times 2 = 31.5 \times 2 = 63 \text{ apples}$$

Before the 2nd customer:

He had 63 apples before selling to the 3rd customer. So, the number of apples before the 2nd customer was:

$$(63 + 0.5) \times 2 = 63.5 \times 2 = 127 \text{ apples}$$

Initially (Before the 1st customer):

He had 127 apples before selling to the 2nd customer. So, the initial number of apples was:

$$(127 + 0.5) \times 2 = 127.5 \times 2 = 255 \text{ apples}$$

Step 4: Final Answer:

The apple seller initially had 255 apples.

Quick Tip

For problems where a sequence of operations leads to a final result, working backward is often the simplest and quickest strategy. Reverse each mathematical operation step-by-step.

5. A reduction of 20% in the price of mangoes enables a man to buy 25 mangoes more for Rs.40. Find the reduced price of the basket that contains 200 mangoes.

- (A) Rs. 100
- (B) Rs. 75
- (C) Rs. 60
- (D) Rs. 64
- (E) Rs. 80

Correct Answer: (D) Rs. 64

Solution:

Step 1: Understanding the Question:

A price decrease allows a person to buy more items for the same amount of money. We need to find the new (reduced) price for a specific quantity (200 mangoes).

Step 2: Key Formula or Approach:

The core concept is that the expenditure is constant (Rs. 40). The saving from the price reduction is used to buy the extra mangoes.

Saving = 20% of Rs. 40.

This saving amount is equal to the new price of the extra 25 mangoes.

Step 3: Detailed Explanation:

The total expenditure is fixed at Rs. 40.

The price of mangoes is reduced by 20%.

The saving on the original purchase due to the price reduction is:

$$\text{Saving} = 20\% \text{ of } 40 = \frac{20}{100} \times 40 = \text{Rs. } 8$$

This saving of Rs. 8 is what allows the man to buy 25 more mangoes.

Therefore, the new (reduced) price of 25 mangoes is Rs. 8.

Now we can find the reduced price per mango:

$$\text{Reduced price per mango} = \frac{\text{Price}}{\text{Quantity}} = \frac{\text{Rs. } 8}{25}$$

The question asks for the reduced price of a basket of 200 mangoes.

$$\text{Reduced price for 200 mangoes} = 200 \times (\text{Reduced price per mango})$$

$$\text{Reduced price for 200 mangoes} = 200 \times \frac{8}{25}$$

$$= 8 \times \frac{200}{25} = 8 \times 8 = \text{Rs. } 64$$

Step 4: Final Answer:

The reduced price of the basket containing 200 mangoes is Rs. 64.

Quick Tip

When expenditure is constant, the percentage saving due to a price drop is equal to the cost of the extra quantity purchased at the new price. This shortcut can save significant time.

6. One vessel contains milk and water in the ratio a: b. While another vessel contains milk and water in the ratio b:a. In what ratio must the contents of the first vessel be mixed with the contents of the second so that in the final sample, milk and water may be as 2:1?

- (A) (2a+b): (a+2b)
- (B) (2a-b): (a-2b)
- (C) (a+b): (a-b)
- (D) (a-b): (a+b)
- (E) (2a+b): (a-2b)

Correct Answer: (B) (2a-b): (a-2b)

Solution:

Step 1: Understanding the Question:

We are mixing two different milk-water solutions to obtain a third solution with a desired concentration. We need to find the ratio in which the first two solutions must be mixed. This is a problem of alligation or weighted averages.

Step 2: Key Formula or Approach:

We can use the rule of alligation on the concentration of one of the components (e.g., milk).

Let's find the fraction of milk in each mixture.

Vessel 1: Milk concentration $C_1 = \frac{a}{a+b}$

Vessel 2: Milk concentration $C_2 = \frac{b}{a+b}$

Final Mixture: Desired milk concentration $C_m = \frac{2}{2+1} = \frac{2}{3}$

The ratio of quantities (Vessel 1 : Vessel 2) is given by $(C_m - C_2) : (C_1 - C_m)$.

Step 3: Detailed Explanation:

Applying the alligation formula:

$$\text{Quantity of Vessel 1} / \text{Quantity of Vessel 2} = \frac{C_m - C_2}{C_1 - C_m}$$

$$\frac{\text{Qty 1}}{\text{Qty 2}} = \frac{\frac{2}{3} - \frac{b}{a+b}}{\frac{a}{a+b} - \frac{2}{3}}$$

Now, let's simplify the numerator and the denominator separately.

Numerator:

$$\frac{2}{3} - \frac{b}{a+b} = \frac{2(a+b) - 3b}{3(a+b)} = \frac{2a + 2b - 3b}{3(a+b)} = \frac{2a - b}{3(a+b)}$$

Denominator:

$$\frac{a}{a+b} - \frac{2}{3} = \frac{3a - 2(a+b)}{3(a+b)} = \frac{3a - 2a - 2b}{3(a+b)} = \frac{a - 2b}{3(a+b)}$$

Now, find the ratio:

$$\frac{\text{Qty 1}}{\text{Qty 2}} = \frac{\frac{2a-b}{3(a+b)}}{\frac{a-2b}{3(a+b)}}$$

The term $3(a+b)$ cancels out.

$$\frac{\text{Qty 1}}{\text{Qty 2}} = \frac{2a - b}{a - 2b}$$

So, the required ratio is $(2a - b) : (a - 2b)$.

Step 4: Final Answer:

The contents must be mixed in the ratio $(2a-b) : (a-2b)$.

Quick Tip

Alligation is a powerful tool for mixture problems. Always pick one component (like milk or water) and work with its concentration (fraction or percentage) across all mixtures.

7. A man is 10 years older than his wife who is now 2.5 times as old as their daughter. The daughter is now 14 years old. What was the man's age when the daughter was born?

- (A) 29 years
- (B) 30 years
- (C) 31 years
- (D) 32 years
- (E) 24 years

Correct Answer: (C) 31 years

Solution:

Step 1: Understanding the Question:

This is a word problem involving ages. We are given relationships between the current ages of a man, his wife, and their daughter. We need to find the man's age at a past event (daughter's

birth).

Step 2: Key Formula or Approach:

Calculate the current ages of all individuals step-by-step based on the given information. Then, calculate the man's age in the past.

Step 3: Detailed Explanation:

Step 1: Find the daughter's current age.

The daughter is now 14 years old.

$$\text{Daughter's current age} = 14$$

Step 2: Find the wife's current age.

The wife is 2.5 times as old as the daughter.

$$\text{Wife's current age} = 2.5 \times 14 = 35 \text{ years}$$

Step 3: Find the man's current age.

The man is 10 years older than his wife.

$$\text{Man's current age} = 35 + 10 = 45 \text{ years}$$

Step 4: Find the man's age when the daughter was born.

The daughter was born 14 years ago. So, we need to find the man's age 14 years ago.

$$\begin{aligned} \text{Man's age at daughter's birth} &= \text{Man's current age} - \text{Daughter's current age} \\ &= 45 - 14 = 31 \text{ years} \end{aligned}$$

Step 4: Final Answer:

The man's age when the daughter was born was 31 years.

Quick Tip

In age problems, it's crucial to establish a baseline time (usually the present) and calculate all ages for that specific time before calculating ages in the past or future.

8. A watch was slow by 5 minutes at 4 p.m. on Wednesday, but it was fast by 10 minutes at 4 p.m on Saturday. At what time did it show the right time?

- (A) 4 p.m. on Thursday
- (B) 4 p.m. on Friday
- (C) 4 a.m. on Friday
- (D) 12 Noon on Friday
- (E) 12 Noon on Thursday

Correct Answer: (A) 4 p.m. on Thursday

Solution:

Step 1: Understanding the Question:

A faulty clock moves from being slow to being fast over a period. This means at some point it must have crossed the correct time. We need to find that exact point in time.

Step 2: Key Formula or Approach:

First, calculate the total time elapsed and the total time gained by the clock. Then, determine the rate at which the clock gains time. Finally, calculate how long it took for the clock to gain the initial time it was lagging by.

Step 3: Detailed Explanation:

1. Calculate the total time interval:

From 4 p.m. Wednesday to 4 p.m. Saturday is exactly 3 days.

Total time = $3 \times 24 = 72$ hours.

2. Calculate the total time gained by the watch:

The watch was 5 minutes slow and became 10 minutes fast. The total gain is the sum of the lag and the lead.

Total gain = 5 min (to cover the lag) + 10 min (to move ahead) = 15 minutes.

3. Determine the rate of gain:

The watch gains 15 minutes in 72 hours.

4. Calculate the time to show the correct time:

The watch shows the correct time when it has exactly compensated for its initial slowness. It was initially 5 minutes slow. We need to find the time it takes to gain these 5 minutes.

We can set up a proportion:

$$\frac{\text{Time taken}}{\text{Time gained}} = \frac{72 \text{ hours}}{15 \text{ minutes}}$$

Time to gain 5 minutes = $\left(\frac{72 \text{ hours}}{15 \text{ minutes}}\right) \times 5 \text{ minutes}$

$$= \frac{72 \times 5}{15} = \frac{72}{3} = 24 \text{ hours}$$

The watch will show the correct time 24 hours after the starting point.

Starting point: 4 p.m. on Wednesday.

Correct time = 4 p.m. Wednesday + 24 hours = 4 p.m. on Thursday.

Step 4: Final Answer:

The watch showed the right time at 4 p.m. on Thursday.

Quick Tip

For faulty clock problems, the total gain is always the sum of the initial lag and the final lead (or vice versa). The clock shows the correct time at the moment it overcomes its initial error.

9. A man sells 10 oranges for a rupee there by gaining 40%. How many oranges did he buy for a rupee?

- (A) 12
- (B) 14
- (C) 13
- (D) 15
- (E) 11

Correct Answer: (B) 14

Solution:

Step 1: Understanding the Question:

We are given the selling price and profit percentage for a transaction and need to find the cost price, expressed in terms of quantity per rupee.

Step 2: Key Formula or Approach:

The relationship between Selling Price (SP), Cost Price (CP), and Gain Percentage is:

$$SP = CP \times \left(1 + \frac{\text{Gain \%}}{100}\right)$$

We can rearrange this to find the CP:

$$CP = \frac{SP}{1 + \frac{\text{Gain \%}}{100}}$$

Step 3: Detailed Explanation:

1. Determine the Selling Price (SP) per orange:

The man sells 10 oranges for Re. 1.

$$\text{SP of 1 orange} = \frac{\text{Re. 1}}{10} = \text{Rs. } 0.10$$

2. Use the formula to find the Cost Price (CP) per orange:

The gain is 40%.

$$\begin{aligned}\text{CP of 1 orange} &= \frac{\text{SP}}{1 + \frac{40}{100}} = \frac{0.10}{1 + 0.40} = \frac{0.10}{1.4} \\ \text{CP} &= \frac{1/10}{14/10} = \frac{1}{14}\end{aligned}$$

So, the cost price of one orange is Rs. $1/14$.

3. Determine how many oranges were bought for a rupee:

If the cost of 1 orange is Rs. $1/14$, then for Re. 1, the number of oranges bought is:

$$\text{Number of oranges} = \frac{\text{Total Amount}}{\text{Price per orange}} = \frac{1}{1/14} = 14$$

Step 4: Final Answer:

He bought 14 oranges for a rupee.

Quick Tip

In questions involving quantity and price, it's often easiest to calculate the price per single item first (both CP and SP) and then answer the question about quantity for a given amount.

10. A committee of five members is to be formed from among six boys and five girls. Find the number of ways of selecting the committee, if it is to consist of at least one boy and at least one girl?

- (A) 455
- (B) 456
- (C) 461
- (D) 477
- (E) None of these

Correct Answer: (A) 455

Solution:

Step 1: Understanding the Question:

This is a combination problem with constraints. We need to find the number of ways to form a 5-member committee from a group of 6 boys and 5 girls, ensuring that the committee is not composed of only boys or only girls.

Step 2: Key Formula or Approach:

The most efficient approach is to use the indirect method:

Total possible committees - Unwanted committees = Desired committees.

The number of ways to choose 'r' items from 'n' is given by the combination formula ${}^n C_r = \frac{n!}{r!(n-r)!}$.

Step 3: Detailed Explanation:

Total number of people = 6 boys + 5 girls = 11 people.

We need to form a committee of 5.

1. Calculate the total number of possible committees of 5 from 11 people (without any restrictions):

$$\text{Total ways} = {}^{11} C_5 = \frac{11!}{5!(11-5)!} = \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} = 11 \times 2 \times 3 \times 7 = 462$$

2. Calculate the unwanted committees:

The condition "at least one boy and at least one girl" means we must exclude committees that have:

Case (a): Only boys (5 boys, 0 girls).

$$\text{Ways for all-boy committee} = {}^6 C_5 \times {}^5 C_0 = 6 \times 1 = 6$$

Case (b): Only girls (0 boys, 5 girls).

$$\text{Ways for all-girl committee} = {}^6 C_0 \times {}^5 C_5 = 1 \times 1 = 1$$

Total unwanted committees = $6 + 1 = 7$.

3. Calculate the desired number of committees:

$$\begin{aligned}\text{Desired ways} &= \text{Total ways} - \text{Unwanted ways} \\ &= 462 - 7 = 455\end{aligned}$$

Step 4: Final Answer:

The number of ways of selecting the committee is 455.

Quick Tip

When you see phrases like "at least one" or "at least two" in combination problems, it's often faster to calculate the total combinations and subtract the cases that do not meet the condition.

11. A right circular cylinder is inscribed in a sphere and the height of the cylinder is equal to the diameter of its base. Find the ratio of the volume of the sphere to that of the cylinder.

- (A) $4: \sqrt{3}$
- (B) $4\sqrt{2}:3$
- (C) $2\sqrt{2}:1$
- (D) $1:2$
- (E) None of these

Correct Answer: (B) $4\sqrt{2}:3$

Solution:

Step 1: Understanding the Question:

This is a 3D geometry problem. We need to find the relationship between the dimensions of a sphere and an inscribed cylinder with a specific constraint (height = diameter) and then compute the ratio of their volumes.

Step 2: Key Formula or Approach:

Let 'r' be the radius of the cylinder and 'h' its height. Let 'R' be the radius of the sphere. We need to establish a relationship between R and r using the Pythagorean theorem on a cross-section. Then, we use the volume formulas.

$$\text{Volume of Sphere: } V_s = \frac{4}{3}\pi R^3$$

$$\text{Volume of Cylinder: } V_c = \pi r^2 h$$

Step 3: Detailed Explanation:**1. Relate cylinder dimensions:**

Given: height of the cylinder = diameter of its base.

$$h = 2r$$

2. Relate sphere and cylinder dimensions:

Imagine a vertical cross-section through the center of the sphere. The sphere appears as a circle, and the cylinder appears as a rectangle inscribed in it. The diagonal of this rectangle is equal to the diameter of the sphere ($2R$).

Using the Pythagorean theorem on the triangle formed by the radius of the sphere, half the height of the cylinder, and the radius of the cylinder:

$$R^2 = r^2 + \left(\frac{h}{2}\right)^2$$

Substitute $h = 2r$ into this equation:

$$\begin{aligned} R^2 &= r^2 + \left(\frac{2r}{2}\right)^2 = r^2 + r^2 = 2r^2 \\ R &= \sqrt{2r^2} = r\sqrt{2} \end{aligned}$$

3. Calculate the volumes in terms of a single variable (e.g., r):

Volume of Cylinder (V_c):

$$V_c = \pi r^2 h = \pi r^2 (2r) = 2\pi r^3$$

Volume of Sphere (V_s):

$$V_s = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (r\sqrt{2})^3 = \frac{4}{3}\pi (r^3 \cdot 2\sqrt{2}) = \frac{8\sqrt{2}}{3}\pi r^3$$

4. Find the ratio $V_s : V_c$:

$$\frac{V_s}{V_c} = \frac{\frac{8\sqrt{2}}{3}\pi r^3}{2\pi r^3}$$

The πr^3 terms cancel out.

$$\frac{V_s}{V_c} = \frac{8\sqrt{2}/3}{2} = \frac{8\sqrt{2}}{3 \times 2} = \frac{4\sqrt{2}}{3}$$

The ratio is $4\sqrt{2} : 3$.

Step 4: Final Answer:

The ratio of the volume of the sphere to that of the cylinder is $4\sqrt{2}:3$.

Quick Tip

For problems involving inscribed 3D shapes, drawing a 2D cross-section is the key. It simplifies the problem to a 2D geometry question, where you can use tools like the Pythagorean theorem to relate the dimensions.

12. A merchant mixes two varieties of rice, one costing Rs.12.75 per kg with another variety costing Rs.12 per kg in the proportion 1: r. He sells the mixture at Rs. 13.50 per kg and there by gains 10%. Find the value of r.

- (A) 2
- (B) 1.75
- (C) 2.5
- (D) 2.75
- (E) 1.5

Correct Answer: (B) 1.75

Solution:

Step 1: Understanding the Question:

This problem combines mixtures and profit/loss. We need to find the ratio of mixing given the costs of the ingredients, the selling price of the mixture, and the profit percentage.

Step 2: Key Formula or Approach:

First, we must find the Cost Price (CP) of the mixture from the given Selling Price (SP) and gain percent. Then, we use the weighted average formula to relate the CP of the mixture to the CPs of its components and their mixing ratio.

$$\text{CP of mixture} = \frac{SP}{1+(\text{Gain\%/100})}$$
$$\text{Weighted Average CP} = \frac{q_1c_1+q_2c_2}{q_1+q_2}$$

Step 3: Detailed Explanation:

1. Calculate the Cost Price (CP) of the mixture:

SP of mixture = Rs. 13.50 per kg

Gain = 10%

$$CP_{mixture} = \frac{13.50}{1 + \frac{10}{100}} = \frac{13.50}{1.1} = \frac{135}{11}$$

The cost price of the mixture is Rs. 135/11 per kg.

2. Apply the weighted average formula:

The two rice varieties are mixed in the ratio 1 : r.

Cost of 1st variety (c_1) = Rs. 12.75

Cost of 2nd variety (c_2) = Rs. 12

$$CP_{mixture} = \frac{(1 \times c_1) + (r \times c_2)}{1 + r}$$
$$\frac{135}{11} = \frac{(1 \times 12.75) + (r \times 12)}{1 + r}$$
$$\frac{135}{11} = \frac{12.75 + 12r}{1 + r}$$

Cross-multiply:

$$\begin{aligned}135(1 + r) &= 11(12.75 + 12r) \\135 + 135r &= 140.25 + 132r\end{aligned}$$

Rearrange the terms to solve for 'r':

$$\begin{aligned}135r - 132r &= 140.25 - 135 \\3r &= 5.25 \\r &= \frac{5.25}{3} = 1.75\end{aligned}$$

Step 4: Final Answer:

The value of r is 1.75.

Quick Tip

Never use the Selling Price in the alligation or weighted average formula for mixtures. Always calculate the Cost Price of the mixture first if a profit or loss percentage is given.

13. A loan of Rs. 6000 is to be paid back in three equal installments. Find the value of each installment to the nearest whole rupee, if the interest is compounded annually at 12.5%.

- (A) Rs.2240
- (B) Rs.2519
- (C) Rs.2521
- (D) Rs. 2915
- (E) None of these

Correct Answer: (B) Rs.2519

Solution:

Step 1: Understanding the Question:

This problem involves calculating the equal annual installment for a loan with compound interest. The principle is that the present value of all installments must equal the loan amount.

Step 2: Key Formula or Approach:

Let P be the loan amount, x be the equal installment, R be the rate of interest, and n be the number of installments. The formula for the present value (P) is:

$$P = \frac{x}{(1 + R)^1} + \frac{x}{(1 + R)^2} + \dots + \frac{x}{(1 + R)^n}$$

Step 3: Detailed Explanation:

Given values:

Loan Amount (P) = Rs. 6000

Number of installments (n) = 3

Rate of Interest (R) = 12.5% = $\frac{12.5}{100} = \frac{1}{8}$

Let the installment be 'x'.

The factor $(1 + R)$ is $1 + \frac{1}{8} = \frac{9}{8}$.

Using the formula:

$$6000 = \frac{x}{\left(\frac{9}{8}\right)^1} + \frac{x}{\left(\frac{9}{8}\right)^2} + \frac{x}{\left(\frac{9}{8}\right)^3}$$
$$6000 = x \left(\frac{8}{9} + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^3 \right)$$

The expression in the bracket is a geometric progression with first term $a = 8/9$, common ratio $r = 8/9$, and $n = 3$.

The sum is $S_n = a \frac{1-r^n}{1-r}$.

$$S_3 = \frac{8}{9} \frac{1 - (8/9)^3}{1 - 8/9} = \frac{8}{9} \frac{1 - 512/729}{1/9} = 8 \left(\frac{729 - 512}{729} \right) = 8 \left(\frac{217}{729} \right) = \frac{1736}{729}$$

Now, substitute this sum back into the main equation:

$$6000 = x \left(\frac{1736}{729} \right)$$
$$x = \frac{6000 \times 729}{1736}$$
$$x = \frac{4374000}{1736} \approx 2519.585\dots$$

To the nearest whole rupee, the value of each installment is Rs. 2520. However, looking at the options, Rs. 2519 is the closest available choice. In competitive exams, sometimes the intended answer is the one closest to the calculation.

Step 4: Final Answer:

The value of each installment is approximately Rs. 2519.59, which is closest to Rs. 2519 among the given options.

Quick Tip

For installment problems, remember that the loan amount represents the sum of the present values of all future payments. Converting percentage rates to fractions (like $12.5\% = 1/8$) can simplify calculations significantly.

14. There are 15 books on a shelf, 5 of which are on fiction. If five books are selected at random, find the probability that 4 of them are books on fiction.

- (A) $5/72$
(B) $5/273$

- (C) 50/3003
- (D) 50/273
- (E) None of these

Correct Answer: (C) 50/3003

Solution:

Step 1: Understanding the Question:

This is a probability problem that involves combinations. We need to find the probability of a specific outcome (selecting 4 fiction and 1 non-fiction book) when selecting a total of 5 books from a larger collection.

Step 2: Key Formula or Approach:

$$\text{Probability} = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}.$$

We will use the combination formula ${}^n C_r$ to calculate the number of outcomes.

Step 3: Detailed Explanation:

1. Identify the groups of books:

Total books = 15

Fiction books = 5

Non-fiction books = 15 - 5 = 10

2. Calculate the total number of possible outcomes:

This is the number of ways to select any 5 books from the 15 available.

$$\begin{aligned} \text{Total ways} &= {}^{15} C_5 = \frac{15!}{5!(15-5)!} = \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} \\ &= 3 \times 7 \times 13 \times 1 \times 11 = 3003 \end{aligned}$$

3. Calculate the number of favorable outcomes:

We want to select exactly 4 fiction books and, consequently, 1 non-fiction book to make a total of 5 books.

Ways to choose 4 fiction books from 5 = ${}^5 C_4 = \frac{5!}{4!1!} = 5$

Ways to choose 1 non-fiction book from 10 = ${}^{10} C_1 = \frac{10!}{1!9!} = 10$

Total favorable outcomes = (Ways to choose fiction) \times (Ways to choose non-fiction)

$$= {}^5 C_4 \times {}^{10} C_1 = 5 \times 10 = 50$$

4. Calculate the probability:

$$P(4 \text{ fiction, } 1 \text{ non-fiction}) = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}} = \frac{50}{3003}$$

Step 4: Final Answer:

The probability that 4 of the selected books are fiction is 50/3003.

Quick Tip

In probability problems involving selections, always break down the "favorable outcome" into the required number of items from each distinct group and multiply their combinations.

15. A swimming pool has 6 inlet pipes, each of which can fill the pool in 6 hrs. At noon, when the pool is empty, tap one is opened and after one hour the second tap is opened. After another hour, the third tap is opened and the remaining taps are opened one after the other with a gap of one hour till the pool is filled. At what time will the pool be filled?

- (A) 3.00p.m.
- (B) 3.30 p.m.
- (C) 4 p.m.
- (D) 4.30 p.m
- (E) 5.00 p.m

Correct Answer: (A) 3.00p.m.

Solution:

Step 1: Understanding the Question:

This is a "Pipes and Cisterns" problem, a variation of Work and Time. Pipes are opened sequentially, so the rate of filling increases every hour. We need to find the total time to fill the pool.

Step 2: Key Formula or Approach:

First, determine the rate of work for a single pipe. Then, calculate the fraction of the pool filled in each one-hour interval as more pipes are opened. Sum these fractions until the total work (filling the pool) equals 1.

Step 3: Detailed Explanation:

1. Determine the rate of a single pipe:

Each of the 6 pipes can fill the pool in 6 hours.

The rate of one pipe = $\frac{1}{6}$ of the pool per hour.

2. Track the work done hour by hour, starting from noon (12:00 p.m.):

Hour 1 (12:00 p.m. to 1:00 p.m.):

Only 1 pipe is open.

Work done = $1 \times \frac{1}{6} = \frac{1}{6}$.

Total work completed = $\frac{1}{6}$.

Hour 2 (1:00 p.m. to 2:00 p.m.):

Now, 2 pipes are open.

Work done in this hour = $2 \times \frac{1}{6} = \frac{2}{6}$.

Total work completed = $\frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$.

Hour 3 (2:00 p.m. to 3:00 p.m.):

Now, 3 pipes are open.

Work done in this hour = $3 \times \frac{1}{6} = \frac{3}{6}$.

Total work completed = (Work from first 2 hours) + (Work in 3rd hour) = $\frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1$.

The pool is completely filled.

3. Determine the final time:

The pool is filled exactly at the end of the third hour. Since the process started at 12:00 p.m., the time is:

12:00 p.m. + 3 hours = 3:00 p.m.

Step 4: Final Answer:

The pool will be filled at 3:00 p.m.

Quick Tip

For work problems where the rate changes at intervals, calculate the work done in each interval and keep a running total. This systematic approach prevents confusion. Defining the total work as the LCM of the times involved (e.g., 6 units of water) can also make calculations with fractions easier.

16. A certain number of men can complete a piece of work in 60 days. If there were 8 more men, the work could be finished in 10 days less. How many men were there originally?

- (A) 40
- (B) 50
- (C) 80
- (D) 100
- (E) 110

Correct Answer: (A) 40

Solution:

Step 1: Understanding the Question:

This is a classic work and time problem where the total work done is constant. The total work can be expressed as the product of the number of men and the number of days they work.

Step 2: Key Formula or Approach:

Let 'M' be the original number of men and 'D' be the original number of days. The total work is constant, so we can use the formula $M_1 \times D_1 = M_2 \times D_2$.

Step 3: Detailed Explanation:

Let the original number of men be M.

The original number of days taken is $D_1 = 60$ days.

The total work can be represented as $W = M \times 60$.

In the second scenario, the number of men is $M_2 = M + 8$.
The work is finished 10 days less, so the number of days taken is $D_2 = 60 - 10 = 50$ days.
The total work in the second scenario is $W = (M + 8) \times 50$.
Since the work is the same in both cases, we can equate the two expressions:

$$\begin{aligned}60 \times M &= 50 \times (M + 8) \\60M &= 50M + 400 \\60M - 50M &= 400 \\10M &= 400 \\M &= \frac{400}{10} = 40\end{aligned}$$

Step 4: Final Answer:

There were originally 40 men.

Quick Tip

In work-related problems, the concept of 'man-days' (number of men \times number of days) represents the total amount of work. If the work is constant, you can always equate the man-days from different scenarios.

17. Find the cost of papering the walls of a room 10m long, 5m broad and 5 m high with paper 75 cm wide at 90 paisa per meter.

- (A) Rs.120
- (B) Rs.150
- (C) Rs.180
- (D) Rs.200
- (E) Rs. 220

Correct Answer: (C) Rs.180

Solution:

Step 1: Understanding the Question:

This is a mensuration problem. We need to find the area of the four walls of a room, then determine the length of wallpaper required to cover that area, and finally calculate the total cost.

Step 2: Key Formula or Approach:

Area of four walls (Lateral Surface Area of a cuboid) = $2 \times (\text{length} + \text{breadth}) \times \text{height}$.

Area of wallpaper = Length of wallpaper \times Width of wallpaper.

Total Cost = Length of wallpaper \times Cost per meter.

Step 3: Detailed Explanation:

1. Calculate the area of the four walls:

Given: Length (L) = 10 m, Breadth (B) = 5 m, Height (H) = 5 m.

$$\begin{aligned}\text{Area of walls} &= 2 \times (L + B) \times H \\ &= 2 \times (10 + 5) \times 5 = 2 \times 15 \times 5 = 150 \text{ m}^2\end{aligned}$$

2. Calculate the required length of the wallpaper:

The area of the paper must be equal to the area of the walls.

Width of paper = 75 cm = 0.75 m.

Let the required length of the paper be 'l'.

$$\begin{aligned}\text{Area of paper} &= l \times \text{width} \\ 150 &= l \times 0.75 \\ l &= \frac{150}{0.75} = \frac{150}{\frac{3}{4}} = 150 \times \frac{4}{3} = 200 \text{ meters}\end{aligned}$$

3. Calculate the total cost:

Cost of paper = 90 paise per meter = Rs. 0.90 per meter.

$$\begin{aligned}\text{Total Cost} &= \text{Length of paper} \times \text{Cost per meter} \\ &= 200 \times 0.90 = \text{Rs. } 180\end{aligned}$$

Step 4: Final Answer:

The cost of papering the walls is Rs. 180.

Quick Tip

Always ensure that all units are consistent before performing calculations. In this case, converting the paper's width from cm to meters is a crucial first step.

18. The base of a triangle is increased in length by 20% and its height reduced by 20%. How does its area change?

- (A) reduced by 4%
- (B) increased by 4%
- (C) does not change
- (D) reduced by 4.166%
- (E) increases by 4.166 %

Correct Answer: (A) reduced by 4%

Solution:

Step 1: Understanding the Question:

We need to determine the net percentage change in the area of a triangle when its base and height undergo given percentage changes.

Step 2: Key Formula or Approach:

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$.

For successive percentage changes of $x\%$ and $y\%$, the net percentage change is given by the formula: $(x + y + \frac{xy}{100})\%$. An increase is positive, and a decrease is negative.

Step 3: Detailed Explanation:

Let the original base be 'b' and the original height be 'h'.

Original Area, $A = \frac{1}{2}bh$.

The base is increased by 20%. So, the new base, $b' = b + 0.20b = 1.2b$.

The height is reduced by 20%. So, the new height, $h' = h - 0.20h = 0.8h$.

New Area, $A' = \frac{1}{2} \times b' \times h' = \frac{1}{2} \times (1.2b) \times (0.8h)$.

$$A' = (1.2 \times 0.8) \times (\frac{1}{2}bh) = 0.96 \times A$$

The new area is 0.96 times the original area, which means it is 96% of the original area.

The change in area is a reduction.

Percentage reduction = $(1 - 0.96) \times 100\% = 0.04 \times 100\% = 4\%$.

Alternative Method (Using formula):

Let $x = +20$ (for 20% increase) and $y = -20$ (for 20% decrease).

Net change = $(20 + (-20) + \frac{20 \times (-20)}{100})\%$

$$= (0 + \frac{-400}{100})\% = -4\%$$

The negative sign indicates a reduction.

Step 4: Final Answer:

The area is reduced by 4%.

Quick Tip

The successive percentage change formula $(x + y + xy/100)\%$ is extremely useful for finding the net effect of two consecutive percentage changes on any quantity that is a product of two factors (like area = length \times width).

19. A boat which can travel at 10km/hr in still water goes 91km down a river and returns to the starting point in 20 hours. Find the speed of the flow of the river.

- (A) 3.5kmph
- (B) 2.5kmph
- (C) 2kmph
- (D) 3kmph
- (E) 4 kmph

Correct Answer: (D) 3kmph

Solution:

Step 1: Understanding the Question:

This problem involves the concept of relative speed in the context of boats and streams. We are given the boat's speed, the distance of a round trip, and the total time taken, and we need to find the speed of the stream.

Step 2: Key Formula or Approach:

Let the speed of the boat in still water be 'u' and the speed of the river be 'v'.

Downstream speed = $u + v$.

Upstream speed = $u - v$.

Total time = Time Downstream + Time Upstream = $\frac{\text{Distance}}{u+v} + \frac{\text{Distance}}{u-v}$.

Step 3: Detailed Explanation:

Given:

Speed of boat in still water (u) = 10 km/hr.

Distance (one way) = 91 km.

Total time for the round trip = 20 hours.

Let the speed of the river be 'v' km/hr.

Downstream speed = $10 + v$.

Upstream speed = $10 - v$.

Using the time formula:

$$\begin{aligned}\text{Total Time} &= \frac{91}{10 + v} + \frac{91}{10 - v} \\ 20 &= 91 \left(\frac{1}{10 + v} + \frac{1}{10 - v} \right) \\ 20 &= 91 \left(\frac{(10 - v) + (10 + v)}{(10 + v)(10 - v)} \right) \\ 20 &= 91 \left(\frac{20}{10^2 - v^2} \right)\end{aligned}$$

Divide both sides by 20:

$$\begin{aligned}1 &= \frac{91}{100 - v^2} \\ 100 - v^2 &= 91 \\ v^2 &= 100 - 91 = 9 \\ v &= \sqrt{9} = 3 \text{ km/hr}\end{aligned}$$

(Speed cannot be negative, so we take the positive root).

Step 4: Final Answer:

The speed of the flow of the river is 3 kmph.

Quick Tip

In boat and stream problems, you can often save time by plugging the options into the equation. For example, trying $v=3$ gives $\text{Time} = 91/13 + 91/7 = 7 + 13 = 20$ hours, which matches the question.

20. A train 110 metres long is running with a speed of 60kmph. In what time will it pass a man who is running at 6 kmph in the direction opposite to that in which the train is going?

- (A) 5sec
- (B) 6sec
- (C) 10sec
- (D) 15sec
- (E) 12sec

Correct Answer: (B) 6sec

Solution:

Step 1: Understanding the Question:

This is a relative speed problem involving a train and a moving person. We need to find the time taken for the train to completely pass the person.

Step 2: Key Formula or Approach:

When two objects move in opposite directions, their relative speed is the sum of their individual speeds.

The distance to be covered for the train to pass a point object (like a man) is the length of the train itself.

$$\text{Time} = \frac{\text{Distance}}{\text{Relative Speed}}$$

Remember to convert units to be consistent (km/hr to m/s).

Step 3: Detailed Explanation:

1. Calculate the relative speed:

Speed of train = 60 kmph.

Speed of man = 6 kmph.

Since they are moving in opposite directions, their relative speed is:

$$\text{Relative Speed} = 60 + 6 = 66 \text{ kmph}$$

2. Convert the speed to m/s:

To convert km/hr to m/s, multiply by $\frac{5}{18}$.

$$\text{Relative Speed in m/s} = 66 \times \frac{5}{18} = 11 \times \frac{5}{3} = \frac{55}{3} \text{ m/s}$$

3. Calculate the time taken:

The distance the train needs to cover to pass the man is its own length.

Distance = 110 m.

$$\text{Time} = \frac{\text{Distance}}{\text{Relative Speed}} = \frac{110}{\frac{55}{3}}$$

$$\text{Time} = 110 \times \frac{3}{55} = 2 \times 3 = 6 \text{ seconds}$$

Step 4: Final Answer:

The train will pass the man in 6 seconds.

Quick Tip

For train problems, remember the conversion factor from km/hr to m/s is 5/18. For opposite directions, add speeds; for the same direction, subtract speeds.

21. A garrison of 750 men has provisions for 20 weeks. If at the end of 4 weeks, the garrison is reinforced by 450 men, for how many more weeks will the provisions last now?

- (A) 9weeks
- (B) 10weeks
- (C) 6weeks
- (D) 15weeks
- (E) None of these

Correct Answer: (B) 10weeks

Solution:

Step 1: Understanding the Question:

This is an indirect proportion problem. The core idea is to find the amount of remaining provisions and then calculate how long it will last for the new, larger group of men.

Step 2: Key Formula or Approach:

The amount of work (or in this case, consumption of provisions) is measured in "man-weeks". We can use the formula $M_1 \times D_1 = M_2 \times D_2$, where M is the number of men and D is the duration.

Step 3: Detailed Explanation:

1. Calculate the remaining provisions:

Initially, 750 men had provisions for 20 weeks.

After 4 weeks, the provisions for 750 men have been consumed for 4 weeks.

The remaining provisions would be sufficient for the original 750 men for $20 - 4 = 16$ more weeks.

So, the total amount of remaining provisions is equivalent to 750 men \times 16 weeks. This is our

$$M_1 \times D_1.$$

2. Calculate the new number of men:

The garrison is reinforced by 450 men.

The new total number of men is $M_2 = 750 + 450 = 1200$ men.

3. Calculate how long the remaining provisions will last for the new number of men:

Let the remaining provisions last for D_2 weeks for the 1200 men.

Using the formula $M_1 \times D_1 = M_2 \times D_2$:

$$\begin{aligned}750 \times 16 &= 1200 \times D_2 \\D_2 &= \frac{750 \times 16}{1200} \\D_2 &= \frac{75 \times 16}{120} = \frac{15 \times 16}{24} = \frac{15 \times 2}{3} = 5 \times 2 = 10 \text{ weeks}\end{aligned}$$

Step 4: Final Answer:

The provisions will last for 10 more weeks.

Quick Tip

For provision problems, always focus on the remaining food and the remaining time for the original group. This "remaining work" is then distributed among the new group of people.

22. A cylindrical bucket 28 cm in diameter and 72 cm height is full of water. The water is emptied into a rectangular vessel 66cm long and 28 cm wide. Find the height of the water level in the tank.

- (A) 20cm
- (B) 30cm
- (C) 24cm
- (D) 35cm
- (E) None of these

Correct Answer: (C) 24cm

Solution:

Step 1: Understanding the Question:

This is a volume conservation problem. The volume of water from the cylindrical bucket is transferred to a rectangular vessel, so the volume remains the same. We need to find the height the water reaches in the rectangular vessel.

Step 2: Key Formula or Approach:

Volume of a cylinder = $\pi r^2 h$.

Volume of a rectangular vessel (cuboid) = $L \times W \times H$.

We will equate the volume of water from the cylinder to the volume of water in the rectangular vessel.

Step 3: Detailed Explanation:

1. Calculate the volume of water in the cylindrical bucket:

Diameter = 28 cm, so radius (r) = 14 cm.

Height (h) = 72 cm.

$$\begin{aligned}\text{Volume}_{cylinder} &= \pi r^2 h = \frac{22}{7} \times (14)^2 \times 72 \\ &= \frac{22}{7} \times 196 \times 72 = 22 \times 28 \times 72 \text{ cm}^3\end{aligned}$$

(It's often better to leave the calculation in factored form to simplify later).

2. Set up the equation for the volume in the rectangular vessel:

Length (L) = 66 cm.

Width (W) = 28 cm.

Let the height of the water level be H_{water} .

$$\text{Volume}_{rectangular} = L \times W \times H_{water} = 66 \times 28 \times H_{water}$$

3. Equate the volumes and solve for H_{water} :

$$\begin{aligned}\text{Volume}_{cylinder} &= \text{Volume}_{rectangular} \\ 22 \times 28 \times 72 &= 66 \times 28 \times H_{water}\end{aligned}$$

We can cancel the '28' from both sides.

$$\begin{aligned}22 \times 72 &= 66 \times H_{water} \\ H_{water} &= \frac{22 \times 72}{66}\end{aligned}$$

Divide 22 and 66 by 22:

$$H_{water} = \frac{1 \times 72}{3} = 24 \text{ cm}$$

Step 4: Final Answer:

The height of the water level in the tank will be 24 cm.

Quick Tip

In volume transfer problems, avoid fully calculating the initial volume if possible. Keeping it in factored form often allows for easy cancellation and simplifies the final calculation.

23. The average mark obtained by 100 students was 40. It was later realised that while calculating the average, a mark '53' was wrongly read as 83. Find the correct

average.

- (A) 39.7
- (B) 38.9
- (C) 39.3
- (D) 39.5
- (E) 37.5

Correct Answer: (A) 39.7

Solution:

Step 1: Understanding the Question:

This is a problem on averages where an error in one of the data points needs to be corrected. We need to find the new, correct average after adjusting the total sum.

Step 2: Key Formula or Approach:

$$\text{Average} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

Therefore, $\text{Sum} = \text{Average} \times \text{Number}$.

$\text{Correct Sum} = \text{Incorrect Sum} - \text{Incorrect Value} + \text{Correct Value}$.

$$\text{Correct Average} = \frac{\text{Correct Sum}}{\text{Number of observations}}$$

Step 3: Detailed Explanation:

1. Calculate the incorrect total sum of marks:

Number of students = 100.

Incorrect average = 40.

$$\text{Incorrect Sum} = 100 \times 40 = 4000$$

2. Find the error and calculate the correct sum:

A mark was wrongly read as 83 instead of the correct value 53.

The error in the sum = Incorrect value - Correct value = $83 - 53 = 30$.

The calculated sum is 30 more than the actual sum.

$$\text{Correct Sum} = \text{Incorrect Sum} - \text{Error} = 4000 - 30 = 3970$$

3. Calculate the correct average:

The number of students remains the same (100).

$$\text{Correct Average} = \frac{\text{Correct Sum}}{\text{Number of students}} = \frac{3970}{100} = 39.7$$

Step 4: Final Answer:

The correct average is 39.7.

Quick Tip

To correct an average, you don't need to recalculate everything. Simply find the total error in the sum and distribute this error over the number of observations. Change in Average = Total Error / Number of Observations. Here, change is $-30/100 = -0.3$. New average = $40 - 0.3 = 39.7$.

24. What values of k will make $9x^2 + 3kx + 4$ a perfect square?

- (A) ± 1
- (B) ± 2
- (C) ± 3
- (D) ± 4
- (E) ± 5

Correct Answer: (D) ± 4

Solution:

Step 1: Understanding the Question:

We are given a quadratic expression and asked to find the value of a constant 'k' that makes the expression a perfect square trinomial.

Step 2: Key Formula or Approach:

A quadratic expression $Ax^2 + Bx + C$ is a perfect square if its discriminant is zero ($B^2 - 4AC = 0$). Alternatively, we can compare it to the standard form of a perfect square, $(ax + b)^2 = a^2x^2 + 2abx + b^2$.

Step 3: Detailed Explanation:

Method 1: Using the Discriminant

The given expression is $9x^2 + 3kx + 4$.

Here, $A = 9$, $B = 3k$, and $C = 4$.

For it to be a perfect square, the discriminant must be zero.

$$\begin{aligned} B^2 - 4AC &= 0 \\ (3k)^2 - 4(9)(4) &= 0 \\ 9k^2 - 144 &= 0 \\ 9k^2 &= 144 \\ k^2 &= \frac{144}{9} = 16 \\ k &= \pm\sqrt{16} = \pm 4 \end{aligned}$$

Method 2: Comparing with Standard Form

The expression is $9x^2 + 3kx + 4$.

We can write $9x^2 = (3x)^2$ and $4 = (2)^2$.

So, the expression should be of the form $(3x \pm 2)^2$.

Let's expand $(3x \pm 2)^2$:

$$(3x \pm 2)^2 = (3x)^2 \pm 2(3x)(2) + (2)^2 = 9x^2 \pm 12x + 4$$

Comparing the middle term of this expansion with the middle term of the given expression:

$$3kx = \pm 12x$$

$$3k = \pm 12$$

$$k = \frac{\pm 12}{3} = \pm 4$$

Step 4: Final Answer:

The values of k are ± 4 .

Quick Tip

For making a quadratic a perfect square, comparing it to the $(a \pm b)^2$ form is often quicker than calculating the discriminant, especially when the first and last terms are perfect squares themselves.

25. If $\log_a x = 7$ and $\log_a y = 3$, find the value of $\log_y x$.

- (A) $5/3$
- (B) $7/3$
- (C) $3/7$
- (D) $3/5$
- (E) None of these

Correct Answer: (B) $7/3$

Solution:

Step 1: Understanding the Question:

This problem involves the properties of logarithms, specifically the change of base rule. We are given the logarithms of two numbers to a common base and asked to find the logarithm of one number to the base of the other. Note: The question in the image had typos, which have been interpreted as standard logarithm notation for this solution.

Step 2: Key Formula or Approach:

The change of base formula for logarithms is:

$$\log_b c = \frac{\log_a c}{\log_a b}$$

where 'a' can be any new valid base.

Step 3: Detailed Explanation:

We are given:

$$\log_a x = 7$$

$$\log_a y = 3$$

We need to find the value of $\log_y x$.

Using the change of base formula, we can express $\log_y x$ in terms of the base 'a':

$$\log_y x = \frac{\log_a x}{\log_a y}$$

Now, substitute the given values into this formula:

$$\log_y x = \frac{7}{3}$$

Step 4: Final Answer:

The value of $\log_y x$ is $7/3$.

Quick Tip

Remember the change of base rule: $\log_b a = 1/\log_a b$. A more general and powerful version is $\log_b a = (\log_c a)/(\log_c b)$. This question is a direct application of the latter.

26. A car dealer lost 6% on a car. Had he sold it for Rs 3600 more, he would have gained 6%. At what price did he buy the car?

- (A) Rs.40, 000
- (B) Rs.24, 000
- (C) Rs.30, 000
- (D) Rs.36,000
- (E) Rs.32,000

Correct Answer: (C) Rs.30, 000

Solution:

Step 1: Understanding the Question:

This is a profit and loss problem. The difference in selling price is directly related to the difference between a loss percentage and a profit percentage. We need to find the original cost price.

Step 2: Key Formula or Approach:

Let the Cost Price (CP) be C.

A loss of x% means Selling Price (SP) = $CP \times (1 - x/100)$.

A gain of y% means Selling Price (SP) = $CP \times (1 + y/100)$.

The difference between the two selling prices corresponds to the percentage swing from loss to

profit.

Step 3: Detailed Explanation:

Let the cost price of the car be C .

Scenario 1: Loss of 6%

Selling Price, $SP_1 = C \times (1 - \frac{6}{100}) = 0.94C$.

Scenario 2: Gain of 6%

This happens if the car is sold for Rs. 3600 more than SP_1 .

Selling Price, $SP_2 = SP_1 + 3600 = 0.94C + 3600$.

We also know that SP_2 corresponds to a 6% gain on the cost price.

$$SP_2 = C \times (1 + \frac{6}{100}) = 1.06C$$

Now, we can equate the two expressions for SP_2 :

$$0.94C + 3600 = 1.06C$$

$$3600 = 1.06C - 0.94C$$

$$3600 = 0.12C$$

$$C = \frac{3600}{0.12} = \frac{3600 \times 100}{12} = 300 \times 100 = 30,000$$

Shortcut Method:

The difference in selling price (Rs. 3600) is the result of swinging from a 6% loss to a 6% gain.

The total percentage difference relative to the cost price is $6\% - (-6\%) = 12\%$.

So, 12% of the Cost Price = Rs. 3600.

$$0.12 \times C = 3600$$

$$C = \frac{3600}{0.12} = 30,000$$

Step 4: Final Answer:

The dealer bought the car for Rs. 30,000.

Quick Tip

When a problem describes a shift from a loss to a profit, the difference in selling price is equal to the sum of the loss and profit percentages multiplied by the cost price.

27. A man deposits Rs.50 in a bank at the beginning of every month. If 10% simple interest is reckoned, how much money is he eligible to get at the end of 24 months?

- (A) Rs.1350
- (B) Rs.1300
- (C) Rs.1250

- (D) Rs.1325
- (E) Rs. 1200

Correct Answer: (D) Rs.1325

Solution:

Step 1: Understanding the Question:

This problem involves calculating the future value of a series of regular deposits (an annuity) with simple interest. We need to calculate the interest earned on each deposit and sum it up with the total principal deposited.

Step 2: Key Formula or Approach:

Total Amount = Total Principal + Total Simple Interest.

Simple Interest (SI) = $\frac{P \times R \times T}{100}$, where P is principal, R is the annual rate, and T is time in years.

Step 3: Detailed Explanation:

1. Calculate the total principal deposited:

Deposit per month = Rs. 50.

Number of months = 24.

$$\text{Total Principal} = 50 \times 24 = \text{Rs. } 1200$$

2. Calculate the total simple interest earned:

The interest rate is 10% per annum.

The first deposit (at the beginning of month 1) earns interest for 24 months.

The second deposit (at the beginning of month 2) earns interest for 23 months.

...

The last deposit (at the beginning of month 24) earns interest for 1 month.

The total interest is the sum of interest on Rs. 50 for (24 + 23 + ... + 1) months.

The sum of the first n natural numbers is $\frac{n(n+1)}{2}$. For n=24, the sum is $\frac{24(25)}{2} = 300$.

So, the total interest is equivalent to the interest on Rs. 50 for 300 months.

Time in years = $\frac{300}{12} = 25$ years.

$$\text{Total SI} = \frac{P_{\text{monthly}} \times R \times T_{\text{equivalent}}}{100} = \frac{50 \times 10 \times 25}{100} = \frac{12500}{100} = \text{Rs. } 125$$

Alternatively, using a monthly interest rate:

Monthly interest on Rs. 50 = $50 \times \frac{10}{100} \times \frac{1}{12}$.

Total interest = $(50 \times \frac{10}{100} \times \frac{1}{12}) \times (24 + 23 + \dots + 1) = (\frac{5}{12}) \times 300 = 5 \times 25 = \text{Rs. } 125$.

3. Calculate the total amount at the end:

$$\begin{aligned} \text{Total Amount} &= \text{Total Principal} + \text{Total Interest} \\ &= 1200 + 125 = \text{Rs. } 1325 \end{aligned}$$

Step 4: Final Answer:

He is eligible to get Rs. 1325 at the end of 24 months.

Quick Tip

For recurring deposit problems with simple interest, you can calculate interest on a single deposit for the sum of all time periods. The sum of the first 'n' integers is a useful shortcut here.

28. A student scores 25% of the total marks of a paper and fails by 30 marks. Another student scores 35% of total marks and fails by 5 marks. What is the pass mark for that paper?

- (A) 85
- (B) 82.5
- (C) 92.5
- (D) 90
- (E) None of these

Correct Answer: (C) 92.5

Solution:

Step 1: Understanding the Question:

This problem involves setting up equations based on percentages and passing marks. The key insight is that the difference in marks obtained corresponds to the difference in their percentages.

Step 2: Key Formula or Approach:

Let T be the total marks and P be the passing marks.

From the given information, we can form two linear equations for P in terms of T and then solve for T and P.

Step 3: Detailed Explanation:

Let the total marks be T.

Student 1:

Score = 25% of T = $0.25T$.

This student fails by 30 marks, which means the passing mark is 30 more than their score.

$$\text{Pass Mark (P)} = 0.25T + 30 \quad \text{---(Equation 1)}$$

Student 2:

Score = 35% of T = $0.35T$.

This student fails by 5 marks.

$$\text{Pass Mark (P)} = 0.35T + 5 \quad \text{---(Equation 2)}$$

Since the passing mark is the same in both cases, we can equate Equation 1 and Equation 2:

$$0.25T + 30 = 0.35T + 5$$

$$30 - 5 = 0.35T - 0.25T$$

$$25 = 0.10T$$
$$T = \frac{25}{0.10} = 250$$

So, the total marks for the paper are 250.

Now, we can find the pass mark by substituting T back into either equation. Using Equation 1:

$$P = 0.25(250) + 30 = 62.5 + 30 = 92.5$$

Using Equation 2 to verify:

$$P = 0.35(250) + 5 = 87.5 + 5 = 92.5$$

Step 4: Final Answer:

The pass mark for the paper is 92.5.

Quick Tip

A quick way to solve such problems is to see that the difference in percentage points ($35\% - 25\% = 10\%$) corresponds to the difference in marks needed to pass ($30 - 5 = 25$ marks). So, 10% of Total Marks = 25, which gives Total Marks = 250.

29. The length of the shadow of a pole is 12m when the angle of elevation of sun is 30° . What would be the length of the shadow when the elevation of sun is 45° ?

- (A) 8m
- (B) $6\sqrt{3}$ m
- (C) 6m
- (D) $4\sqrt{3}$ m
- (E) 5m

Correct Answer: (D) $4\sqrt{3}$ m

Solution:

Step 1: Understanding the Question:

This is a trigonometry problem involving heights and distances. The height of the pole is constant, but the length of its shadow changes with the sun's angle of elevation. We first need to find the pole's height and then use it to find the new shadow length.

Step 2: Key Formula or Approach:

In a right-angled triangle formed by the pole, its shadow, and the sun's ray, the tangent of the angle of elevation (θ) is given by:

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\text{Height of Pole}}{\text{Length of Shadow}}$$

Step 3: Detailed Explanation:

Let H be the height of the pole.

Case 1: Angle of elevation is 30°

Length of shadow, $S_1 = 12$ m.

$$\tan(30^\circ) = \frac{H}{S_1}$$

$$\frac{1}{\sqrt{3}} = \frac{H}{12}$$

$$H = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3} \text{ m}$$

So, the height of the pole is $4\sqrt{3}$ meters.

Case 2: Angle of elevation is 45°

Let the new length of the shadow be S_2 . The height H remains the same.

$$\tan(45^\circ) = \frac{H}{S_2}$$

We know that $\tan(45^\circ) = 1$.

$$1 = \frac{4\sqrt{3}}{S_2}$$

$$S_2 = 4\sqrt{3} \text{ m}$$

Step 4: Final Answer:

The length of the shadow would be $4\sqrt{3}$ m.

Quick Tip

Remember the standard values for trigonometric ratios, especially for 30° , 45° , and 60° . Knowing that $\tan(45^\circ)=1$ immediately tells you that the height and base of the right triangle are equal in that scenario.

30. If $a + b + c = 11$ and $ab + bc + ca = 35$, find the value of $(a-b)^2 + (b-c)^2 + (c-a)^2$.

- (A) 30
- (B) 32
- (C) 35
- (D) 40
- (E) 25

Correct Answer: (B) 32

Solution:

Step 1: Understanding the Question:

This is an algebraic identity problem. We need to manipulate the given expression and use the values of $a + b + c$ and $ab + bc + ca$ to find the final result.

Step 2: Key Formula or Approach:

We will use two key algebraic identities:

- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a - b)^2 + (b - c)^2 + (c - a)^2 = 2(a^2 + b^2 + c^2 - ab - bc - ca)$

Step 3: Detailed Explanation:**1. Find the value of $a^2 + b^2 + c^2$:**

Using the first identity:

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

Substitute the given values: $a + b + c = 11$ and $ab + bc + ca = 35$.

$$(11)^2 = (a^2 + b^2 + c^2) + 2(35)$$

$$121 = (a^2 + b^2 + c^2) + 70$$

$$a^2 + b^2 + c^2 = 121 - 70 = 51$$

2. Evaluate the required expression:

Let's expand the expression we need to find:

$$\begin{aligned} (a - b)^2 + (b - c)^2 + (c - a)^2 &= (a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2) \\ &= 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca \\ &= 2(a^2 + b^2 + c^2) - 2(ab + bc + ca) \end{aligned}$$

Now substitute the values we know: $a^2 + b^2 + c^2 = 51$ and $ab + bc + ca = 35$.

$$= 2(51) - 2(35)$$

$$= 102 - 70 = 32$$

Step 4: Final Answer:

The value of the expression is 32.

Quick Tip

Memorizing the identity $(a - b)^2 + (b - c)^2 + (c - a)^2 = 2(a^2 + b^2 + c^2 - (ab + bc + ca))$ can be very helpful for solving such problems quickly.

31. Find the value of $x^2 + \frac{1}{x^2}$ when it is given that $x - \frac{1}{x} = 4$.

- (A) 16
(B) 18

- (C) 20
- (D) 22
- (E) None of these

Correct Answer: (B) 18

Solution:

Step 1: Understanding the Question:

This algebraic problem requires finding the value of the sum of squares of a number and its reciprocal, given the difference between them.

Step 2: Key Formula or Approach:

The key is to square the given equation. We use the identity $(a - b)^2 = a^2 - 2ab + b^2$.

Step 3: Detailed Explanation:

We are given the equation:

$$x - \frac{1}{x} = 4$$

To get terms like x^2 and $\frac{1}{x^2}$, we square both sides of the equation:

$$\left(x - \frac{1}{x}\right)^2 = (4)^2$$

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$, with $a = x$ and $b = 1/x$:

$$\begin{aligned}x^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 &= 16 \\x^2 - 2 + \frac{1}{x^2} &= 16\end{aligned}$$

Now, isolate the term we want to find by adding 2 to both sides:

$$\begin{aligned}x^2 + \frac{1}{x^2} &= 16 + 2 \\x^2 + \frac{1}{x^2} &= 18\end{aligned}$$

Step 4: Final Answer:

The value of $x^2 + \frac{1}{x^2}$ is 18.

Quick Tip

Remember these two useful relations:

If $x - \frac{1}{x} = k$, then $x^2 + \frac{1}{x^2} = k^2 + 2$.

If $x + \frac{1}{x} = k$, then $x^2 + \frac{1}{x^2} = k^2 - 2$.

32. If $x = 2^{\frac{1}{3}} - 2^{-\frac{1}{3}}$, find the value of $2x^3 + 6x$.

- (A) 8
- (B) 3
- (C) 6
- (D) 4
- (E) None of these

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Question:

This problem involves an expression with cube roots. The structure of the expression to be found ($2x^3 + 6x$) suggests that we should cube the given equation for x .

Step 2: Key Formula or Approach:

We will use the binomial expansion for a cube: $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.

Step 3: Detailed Explanation:

We are given:

$$x = 2^{\frac{1}{3}} - 2^{-\frac{1}{3}}$$

Cube both sides of the equation:

$$x^3 = \left(2^{\frac{1}{3}} - 2^{-\frac{1}{3}}\right)^3$$

Apply the formula $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$, where $a = 2^{1/3}$ and $b = 2^{-1/3}$.

$$x^3 = (2^{\frac{1}{3}})^3 - (2^{-\frac{1}{3}})^3 - 3(2^{\frac{1}{3}})(2^{-\frac{1}{3}}) \left(2^{\frac{1}{3}} - 2^{-\frac{1}{3}}\right)$$

Let's simplify each part:

$$(2^{1/3})^3 = 2^1 = 2.$$

$$(2^{-1/3})^3 = 2^{-1} = \frac{1}{2}.$$

$$3(2^{1/3})(2^{-1/3}) = 3(2^{1/3-1/3}) = 3(2^0) = 3(1) = 3.$$

The term $\left(2^{\frac{1}{3}} - 2^{-\frac{1}{3}}\right)$ is just the original expression for x .

Substitute these back into the equation for x^3 :

$$x^3 = 2 - \frac{1}{2} - 3(x)$$

$$x^3 = \frac{3}{2} - 3x$$

Now, we need to find the value of $2x^3 + 6x$. Let's rearrange our equation to match that form. First, multiply the entire equation by 2 to clear the fraction:

$$2x^3 = 3 - 6x$$

Now, move the $-6x$ term to the left side:

$$2x^3 + 6x = 3$$

Step 4: Final Answer:

The value of $2x^3 + 6x$ is 3.

Quick Tip

When you see an expression for x involving cube roots like $a^{1/3} \pm b^{1/3}$, and you are asked to find the value of an expression involving x^3 and x , the first step should almost always be to cube the expression for x . The cross-product terms will simplify nicely.

33. If p and q are roots of equation $x^2 + x - 7 = 0$, then find the value of $p^4 + q^4$.

- (A) 133
- (B) 129
- (C) 125
- (D) 127
- (E) 137

Correct Answer: (D) 127

Solution:

Step 1: Understanding the Question:

We are given a quadratic equation and need to find the sum of the fourth powers of its roots, p and q . We will use Vieta's formulas and algebraic identities.

Step 2: Key Formula or Approach:

For a quadratic equation $ax^2 + bx + c = 0$, the sum of roots is $p + q = -b/a$ and the product of roots is $pq = c/a$.

We will use the identities:

1. $p^2 + q^2 = (p + q)^2 - 2pq$
2. $p^4 + q^4 = (p^2 + q^2)^2 - 2(pq)^2$

Step 3: Detailed Explanation:

The given equation is $x^2 + x - 7 = 0$.

Here, $a=1$, $b=1$, $c=-7$.

1. Find the sum and product of the roots:

Sum of roots: $p + q = -1/1 = -1$.

Product of roots: $pq = -7/1 = -7$.

2. Calculate $p^2 + q^2$:

$$p^2 + q^2 = (p + q)^2 - 2pq$$

$$= (-1)^2 - 2(-7) = 1 + 14 = 15$$

3. Calculate $p^4 + q^4$:

$$\begin{aligned} p^4 + q^4 &= (p^2 + q^2)^2 - 2(pq)^2 \\ &= (15)^2 - 2(-7)^2 \\ &= 225 - 2(49) = 225 - 98 = 127 \end{aligned}$$

Step 4: Final Answer:

The value of $p^4 + q^4$ is 127.

Quick Tip

For finding sums of higher powers of roots ($S_n = p^n + q^n$), you can establish a recurrence relation from the equation. From $x^2 + x - 7 = 0$, we have $x^2 = -x + 7$. Thus, $p^n = -p^{n-1} + 7p^{n-2}$ and $S_n = -S_{n-1} + 7S_{n-2}$. This is a fast way to compute successive powers.

34. ABC is a triangle. BQ and CR are the bisectors of angles $\angle ABC$ and $\angle BCA$ respectively, Q and R are two points on the sides AC and AB respectively. The bisectors meet at a point O. If AQOR is a cyclic quadrilateral, find the measure of $\angle BAC$.

- (A) 90°
- (B) 45°
- (C) 30°
- (D) 60°
- (E) None of these

Correct Answer: (D) 60°

Solution:

Step 1: Understanding the Question:

We have a triangle where two angle bisectors meet at a point O. A quadrilateral formed by the vertex A and points on the sides is given to be cyclic. We need to find the angle at vertex A using properties of angle bisectors and cyclic quadrilaterals.

Step 2: Key Formula or Approach:

1. The angle formed by two angle bisectors at the incenter O is given by $\angle BOC = 90^\circ + \frac{1}{2}\angle A$.
2. In a cyclic quadrilateral, the sum of opposite angles is 180° . For quadrilateral AQOR, $\angle A + \angle ROQ = 180^\circ$.
3. $\angle ROQ$ and $\angle BOC$ are vertically opposite angles, so $\angle ROQ = \angle BOC$.

Step 3: Detailed Explanation:

From the properties of angle bisectors meeting at the incenter O:

$$\angle BOC = 90^\circ + \frac{1}{2}\angle BAC \quad \text{---(1)}$$

Since AQOR is a cyclic quadrilateral, the sum of opposite angles is 180° .

$$\begin{aligned}\angle QAR + \angle ROQ &= 180^\circ \\ \angle BAC + \angle ROQ &= 180^\circ \quad \text{---(2)}\end{aligned}$$

The angles $\angle ROQ$ and $\angle BOC$ are vertically opposite, so they are equal.

$$\angle ROQ = \angle BOC$$

Substitute this into equation (2):

$$\angle BAC + \angle BOC = 180^\circ \quad \text{---(3)}$$

Now we have two equations relating $\angle BAC$ and $\angle BOC$. Substitute the expression for $\angle BOC$ from equation (1) into equation (3):

$$\begin{aligned}\angle BAC + \left(90^\circ + \frac{1}{2}\angle BAC\right) &= 180^\circ \\ \frac{3}{2}\angle BAC &= 180^\circ - 90^\circ \\ \frac{3}{2}\angle BAC &= 90^\circ \\ \angle BAC &= 90^\circ \times \frac{2}{3} = 60^\circ\end{aligned}$$

Step 4: Final Answer:

The measure of $\angle BAC$ is 60° .

Quick Tip

Remember the formula for the angle at the incenter: $\angle BIC = 90^\circ + A/2$. Combining this with properties of cyclic quadrilaterals is key to solving this type of geometry problem.

35. Find the possible value of $\cos x$, if $3 \sin x = 2(1 - \cos x)$.

- (A) 1
- (B) -1
- (C) $1/\sqrt{2}$
- (D) $\sqrt{3}/2$
- (E) 0

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Question:

We are given a trigonometric equation and asked to find a possible value for $\cos(x)$ from the given options. A common method to solve equations involving both $\sin(x)$ and $\cos(x)$ is to square both sides to get an equation in terms of a single trigonometric function.

Step 2: Key Formula or Approach:

1. Square both sides of the equation.
2. Use the identity $\sin^2 x = 1 - \cos^2 x$ to form a quadratic equation in terms of $\cos(x)$.
3. Solve the quadratic equation and check for extraneous solutions.

Step 3: Detailed Explanation:

The given equation is:

$$3 \sin x = 2(1 - \cos x)$$

Squaring both sides:

$$\begin{aligned}(3 \sin x)^2 &= (2(1 - \cos x))^2 \\ 9 \sin^2 x &= 4(1 - \cos x)^2\end{aligned}$$

Using the identity $\sin^2 x = 1 - \cos^2 x$:

$$\begin{aligned}9(1 - \cos^2 x) &= 4(1 - 2 \cos x + \cos^2 x) \\ 9 - 9 \cos^2 x &= 4 - 8 \cos x + 4 \cos^2 x\end{aligned}$$

Rearrange into a standard quadratic form (let $y = \cos x$):

$$13 \cos^2 x - 8 \cos x - 5 = 0$$

This is a quadratic equation for $\cos x$. We can solve it using the quadratic formula or by factoring.

Factoring: $13y^2 - 13y + 5y - 5 = 0 \implies 13y(y - 1) + 5(y - 1) = 0 \implies (13y + 5)(y - 1) = 0$. This gives two possible solutions for $\cos x$:

$$\cos x = 1 \quad \text{or} \quad \cos x = -\frac{5}{13}$$

We must check these solutions in the original equation, as squaring can introduce extraneous roots.

Check 1: $\cos x = 1$

If $\cos x = 1$, then $x = 2n\pi$, which implies $\sin x = 0$.

Substitute into the original equation: $3(0) = 2(1 - 1) \implies 0 = 0$. This is a valid solution.

Check 2: $\cos x = -5/13$

If $\cos x = -5/13$, then $\sin x = \pm \sqrt{1 - (-5/13)^2} = \pm 12/13$.

From the original equation, $3 \sin x = 2 - 2 \cos x = 2 - 2(-5/13) = 2 + 10/13 = 36/13$.

So, $\sin x = (36/13)/3 = 12/13$. Since $\sin x = 12/13$ is a possible value, $\cos x = -5/13$ is also a valid solution.

However, only $\cos x = 1$ is present in the options.

Step 4: Final Answer:

The possible value of $\cos x$ from the given options is 1.

Quick Tip

When squaring both sides of a trigonometric equation, it's crucial to check your final solutions against the original equation. Squaring can introduce solutions that do not satisfy the initial relationship (e.g., they might satisfy $A = -B$ instead of $A = B$).

36. Find the value of $\sin^2 25^\circ + \sin^2 65^\circ$.

- (A) 0
- (B) 1
- (C) -1
- (D) $1/\sqrt{2}$
- (E) None of these

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Question:

We need to evaluate a trigonometric expression involving the sum of squares of sine functions with complementary angles.

Step 2: Key Formula or Approach:

We will use the co-function identity $\sin(90^\circ - \theta) = \cos(\theta)$ and the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

Step 3: Detailed Explanation:

The given expression is $\sin^2 25^\circ + \sin^2 65^\circ$.

Notice that the angles 25° and 65° are complementary because $25^\circ + 65^\circ = 90^\circ$.

We can rewrite $\sin 65^\circ$ using the co-function identity:

$$\sin 65^\circ = \sin(90^\circ - 25^\circ)$$

According to the identity $\sin(90^\circ - \theta) = \cos(\theta)$, this becomes:

$$\sin 65^\circ = \cos 25^\circ$$

Now substitute this back into the original expression:

$$\sin^2 25^\circ + \sin^2 65^\circ = \sin^2 25^\circ + (\cos 25^\circ)^2 = \sin^2 25^\circ + \cos^2 25^\circ$$

Using the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$:

$$\sin^2 25^\circ + \cos^2 25^\circ = 1$$

Step 4: Final Answer:

The value of the expression is 1.

Quick Tip

Whenever you see a sum of squares of sines or cosines, check if the angles are complementary (add up to 90°). If they are, you can likely use the identity $\sin^2 \theta + \cos^2 \theta = 1$ to simplify the expression to 1.

37. The second term of an A.P is 15 and the fifth term is double the first term. Find the sum of the first 20 terms of the series.

- (A) 750
- (B) 810
- (C) 800
- (D) 850
- (E) None of these

Correct Answer: (B) 810

Solution:

Step 1: Understanding the Question:

We are given information about two terms of an Arithmetic Progression (A.P.) and need to find the sum of the first 20 terms. We must first find the first term (a) and the common difference (d).

Step 2: Key Formula or Approach:

The nth term of an A.P. is given by $a_n = a + (n - 1)d$.

The sum of the first n terms is $S_n = \frac{n}{2}[2a + (n - 1)d]$.

Step 3: Detailed Explanation:

Let 'a' be the first term and 'd' be the common difference of the A.P.

Condition 1: The second term is 15.

$$a_2 = a + (2 - 1)d = a + d = 15 \quad \text{---(1)}$$

Condition 2: The fifth term is double the first term.

$$a_5 = 2 \times a_1$$

$$a + (5 - 1)d = 2a$$

$$a + 4d = 2a$$

$$a = 4d \quad \text{---(2)}$$

Now we solve the system of two linear equations. Substitute equation (2) into equation (1):

$$(4d) + d = 15$$

$$5d = 15$$

$$d = 3$$

Now find 'a' using equation (2):

$$a = 4(3) = 12$$

The A.P. has a first term $a = 12$ and a common difference $d = 3$.

Calculate the sum of the first 20 terms (S_{20}):

$$\begin{aligned} S_{20} &= \frac{20}{2}[2a + (20 - 1)d] \\ &= 10[2(12) + 19(3)] \\ &= 10[24 + 57] \\ &= 10[81] = 810 \end{aligned}$$

Step 4: Final Answer:

The sum of the first 20 terms of the series is 810.

Quick Tip

For A.P. problems, always start by translating the word statements into algebraic equations using the standard formulas for the n th term and sum. This turns the problem into a solvable system of equations.

38. A person owns 150 shares (face value Rs.25) of a company which declares a dividend of 12%. He sells the share at Rs. 40 and invests the proceeds in 7% stock (par value Rs.100) at Rs.80. What is the change in his income?

- (A) Rs.50 less
- (B) Rs.70 more
- (C) Rs.60 less
- (D) Rs.75 more
- (E) Cannot be determined

Correct Answer: (D) Rs.75 more

Solution:

Step 1: Understanding the Question:

This is a stocks and shares problem. We need to calculate the income from the initial investment, then calculate the income from the new investment after selling the old shares, and finally find the difference between the two incomes.

Step 2: Key Formula or Approach:

Annual Income (Dividend) = Number of shares \times Face Value per share \times Dividend Rate.

Sale Proceeds = Number of shares \times Market Value per share.

Number of new shares = Total Investment / Market Value of new share.

Step 3: Detailed Explanation:**1. Calculate the initial income:**

Number of shares = 150.

Face Value (FV) = Rs. 25.

Dividend Rate = 12%.

$$\text{Initial Income} = 150 \times 25 \times \frac{12}{100} = 150 \times 3 = \text{Rs. } 450$$

2. Calculate the proceeds from selling the shares:

Market Value (MV) for selling = Rs. 40.

$$\text{Sale Proceeds} = 150 \times 40 = \text{Rs. } 6000$$

This amount is the investment for the new stock.

3. Calculate the new income:

Investment = Rs. 6000.

Par Value (FV) of new stock = Rs. 100.

Market Value (MV) of new stock = Rs. 80.

Dividend Rate of new stock = 7%.

First, find the number of new shares purchased:

$$\text{Number of new shares} = \frac{\text{Investment}}{\text{MV of new stock}} = \frac{6000}{80} = 75 \text{ shares}$$

Now, calculate the income from these new shares (dividend is on Par/Face Value):

$$\text{New Income} = 75 \times 100 \times \frac{7}{100} = 75 \times 7 = \text{Rs. } 525$$

4. Calculate the change in income:

$$\text{Change} = \text{New Income} - \text{Initial Income} = 525 - 450 = \text{Rs. } 75$$

Since the new income is higher, the change is an increase of Rs. 75.

Step 4: Final Answer:

The change in his income is Rs. 75 more.

Quick Tip

A crucial point in stocks problems: Dividend is always calculated on the Face Value (or Par Value) of a share, not on its Market Value. The Market Value is used for buying and selling.

39. If 25000 copies of the TIMES OF INDIA be issued daily, each copy consisting of 10 sheets and each sheet measuring 75cmx50cm, how many hectares will one edition cover?

- (A) 9.375 hectare
- (B) 9375 hectare
- (C) 93.75 hectares
- (D) 937.5 hectare
- (E) None of these

Correct Answer: (A) 9.375 hectare

Solution:

Step 1: Understanding the Question:

This is a mensuration and unit conversion problem. We need to find the total area of all sheets in one edition and then convert that area into hectares.

Step 2: Key Formula or Approach:

1. Calculate the total number of sheets.
2. Calculate the total area in cm^2 .
3. Convert the area from cm^2 to m^2 .
4. Convert the area from m^2 to hectares.

Conversion factors: $1 \text{ m} = 100 \text{ cm}$, $1 \text{ hectare} = 10,000 \text{ m}^2$.

Step 3: Detailed Explanation:

1. Calculate the total number of sheets:

$$\text{Total sheets} = 25,000 \text{ copies} \times 10 \text{ sheets/copy} = 250,000 \text{ sheets}$$

2. Calculate the area of one sheet:

$$\text{Area per sheet} = 75 \text{ cm} \times 50 \text{ cm} = 3,750 \text{ cm}^2$$

3. Calculate the total area in cm^2 :

$$\text{Total area} = 250,000 \text{ sheets} \times 3,750 \text{ cm}^2/\text{sheet} = 937,500,000 \text{ cm}^2$$

4. Convert the total area to m^2 :

Since $1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10,000 \text{ cm}^2$, we divide by 10,000.

$$\text{Total area in m}^2 = \frac{937,500,000}{10,000} = 93,750 \text{ m}^2$$

5. Convert the total area to hectares:

Since 1 hectare = 10,000 m², we divide by 10,000 again.

$$\text{Total area in hectares} = \frac{93,750}{10,000} = 9.375 \text{ hectares}$$

Step 4: Final Answer:

One edition will cover 9.375 hectares.

Quick Tip

Be systematic with unit conversions to avoid errors. It's often easier to convert the initial measurements (cm to m) before calculating the total area, which can prevent dealing with very large numbers.

40. If 8:x=12:30, find the value of x.

- (A) 18
- (B) 20
- (C) 25
- (D) 15
- (E) None of these

Correct Answer: (B) 20

Solution:

Step 1: Understanding the Question:

This is a problem of ratio and proportion. We are given a proportion and need to find the value of the unknown variable 'x'.

Step 2: Key Formula or Approach:

The fundamental rule of proportion states that the product of the means is equal to the product of the extremes. For a proportion a:b = c:d, this means $b \times c = a \times d$.

Step 3: Detailed Explanation:

The given proportion is $8 : x = 12 : 30$.

This can be written in fractional form as:

$$\frac{8}{x} = \frac{12}{30}$$

To solve for x, we can cross-multiply:

$$12 \times x = 8 \times 30$$

$$12x = 240$$

Now, divide both sides by 12:

$$x = \frac{240}{12}$$
$$x = 20$$

Step 4: Final Answer:

The value of x is 20.

Quick Tip

For proportions, you can often simplify one of the ratios before solving. Here, 12:30 can be simplified by dividing both sides by 6 to get 2:5. The equation becomes $8:x = 2:5$, or $8/x = 2/5$. This gives $2x = 40$, so $x = 20$.

41. Which term of the series 545, 525, 505, 485.... is closest to zero?

- (A) 30
- (B) 29
- (C) 28
- (D) 37
- (E) None of these

Correct Answer: (C) 28

Solution:

Step 1: Understanding the Question:

The given series is an Arithmetic Progression (A.P.) with a negative common difference. We need to find which term number (n) corresponds to a value closest to 0.

Step 2: Key Formula or Approach:

The n th term of an A.P. is given by $a_n = a + (n - 1)d$. We can set a_n to 0 to find the non-integer value of 'n' where the series crosses the zero mark. The integer terms surrounding this value of 'n' will be the candidates for the closest term.

Step 3: Detailed Explanation:

First, identify the parameters of the A.P.:

First term, $a = 545$.

Common difference, $d = 525 - 545 = -20$.

Let's find the value of 'n' for which the term a_n would be zero:

$$a_n = 545 + (n - 1)(-20) = 0$$

$$545 = 20(n - 1)$$

$$n - 1 = \frac{545}{20} = 27.25$$

$$n = 28.25$$

This means the series becomes zero between the 28th and 29th terms. To find which one is closer, we must calculate the value of both terms.

Value of the 28th term (a_{28}):

$$a_{28} = 545 + (28 - 1)(-20) = 545 + 27(-20) = 545 - 540 = 5$$

Value of the 29th term (a_{29}):

$$a_{29} = 545 + (29 - 1)(-20) = 545 + 28(-20) = 545 - 560 = -15$$

Now, we compare the absolute values of these terms to see which is closer to 0.

Distance of a_{28} from zero = $|5| = 5$.

Distance of a_{29} from zero = $|-15| = 15$.

Since 5 is less than 15, the 28th term is closer to zero.

Step 4: Final Answer:

The 28th term is closest to zero.

Quick Tip

To find the term of an A.P. closest to a certain value (like zero), set the formula for the n th term equal to that value and solve for 'n'. If 'n' is not an integer, check the integer terms on either side of your result.

42. A man invested Rs. 2375 when he bought shares of a company at Rs. 125 each, the face value of share was Rs. 100. The company paid 13% dividend. Find the dividend earned by the man at the end of the year.

- (A) 273
- (B) 247
- (C) 221
- (D) 234
- (E) 324

Correct Answer: (B) 247

Solution:

Step 1: Understanding the Question:

We need to calculate the total dividend earned from an investment in shares. This requires first finding the number of shares purchased and then calculating the dividend based on the face value.

Step 2: Key Formula or Approach:

1. Number of shares = Total Investment / Market Value per share.

2. Total Dividend = Number of shares \times Face Value per share \times Dividend Rate.

Step 3: Detailed Explanation:

1. Find the number of shares bought:

Total Investment = Rs. 2375.

Market Value (MV) per share = Rs. 125.

$$\text{Number of shares} = \frac{2375}{125}$$

To simplify the division: $2375/125 = (2500 - 125)/125 = 2500/125 - 125/125 = 20 - 1 = 19$.
So, the man bought 19 shares.

2. Calculate the total dividend earned:

Number of shares = 19.

Face Value (FV) per share = Rs. 100.

Dividend Rate = 13%.

The dividend is always calculated on the face value.

$$\begin{aligned}\text{Total Dividend} &= 19 \times 100 \times \frac{13}{100} \\ &= 19 \times 13 \\ &= 247\end{aligned}$$

The total dividend earned is Rs. 247.

Step 4: Final Answer:

The dividend earned by the man at the end of the year is Rs. 247.

Quick Tip

Always distinguish between Face Value and Market Value. Shares are bought and sold at Market Value, but dividends and returns are calculated on Face Value. This is a common point of confusion.

43. If 8 men or 15 boys can do a work in 60 days. In how many days can 48 men and 10 boys complete the same work?

- (A) 10
- (B) 9
- (C) 12
- (D) 17
- (E) 14

Correct Answer: (B) 9

Solution:

Step 1: Understanding the Question:

This is a work and time problem where we first need to establish the relative work rates of men and boys and then use this relationship to find the time taken by a combined group.

Step 2: Key Formula or Approach:

1. Equate the total work done to find the relationship between a man's and a boy's work rate. (e.g., 1 man = x boys).
2. Convert the combined workforce into a single unit (either all men or all boys).
3. Use the formula $M_1D_1 = M_2D_2$, where M is the number of workers and D is the number of days.

Step 3: Detailed Explanation:**1. Establish the work rate equivalence:**

"8 men or 15 boys can do a work in 60 days" means the work done by 8 men is equal to the work done by 15 boys in the same amount of time.

$$8 \text{ Men's work rate} = 15 \text{ Boys' work rate}$$

$$1 \text{ Man} = \frac{15}{8} \text{ Boys}$$

2. Convert the new workforce into a single unit (boys):

The new workforce is 48 men and 10 boys.

Convert the 48 men to their equivalent in boys:

$$48 \text{ Men} = 48 \times \frac{15}{8} \text{ Boys} = 6 \times 15 \text{ Boys} = 90 \text{ Boys}$$

The total combined workforce is:

$$90 \text{ Boys} + 10 \text{ Boys} = 100 \text{ Boys}$$

3. Calculate the required time:

We know from the problem that 15 boys can complete the work in 60 days. Let's use this as our base case ($M_1 = 15, D_1 = 60$).

Our new workforce is $M_2 = 100$ boys, and we need to find the new number of days, D_2 .

$$M_1D_1 = M_2D_2$$

$$15 \times 60 = 100 \times D_2$$

$$900 = 100 \times D_2$$

$$D_2 = \frac{900}{100} = 9 \text{ days}$$

Step 4: Final Answer:

48 men and 10 boys can complete the work in 9 days.

Quick Tip

The key to "or" type work problems (e.g., X men or Y women) is to find the equivalence ratio between the different types of workers. Once you have that, always convert the mixed group into a single equivalent type of worker before applying the $MD = W$ formula.

44. Find the value of $97+101+105+\dots\dots\dots+221$.

- (A) 3975
- (B) 4929
- (C) 4770
- (D) 5088
- (E) 4292

Correct Answer: (D) 5088

Solution:

Step 1: Understanding the Question:

The given series is an Arithmetic Progression (A.P.). We need to find the sum of all its terms. To do this, we must first find the number of terms in the series.

Step 2: Key Formula or Approach:

For an A.P.:

1. To find the number of terms (n): $l = a + (n - 1)d$, where 'a' is the first term, 'l' is the last term, and 'd' is the common difference.
2. To find the sum: $S_n = \frac{n}{2}(a + l)$.

Step 3: Detailed Explanation:

1. Identify the parameters of the A.P.:

First term, $a = 97$.

Last term, $l = 221$.

Common difference, $d = 101 - 97 = 4$.

2. Find the number of terms (n):

$$221 = 97 + (n - 1)4$$

$$221 - 97 = 4(n - 1)$$

$$124 = 4(n - 1)$$

$$n - 1 = \frac{124}{4} = 31$$

$$n = 32$$

There are 32 terms in the series.

3. Calculate the sum of the series (S_{32}):

$$\begin{aligned} S_{32} &= \frac{32}{2}(97 + 221) \\ &= 16(318) \\ &= 5088 \end{aligned}$$

Step 4: Final Answer:

The value of the series is 5088.

Quick Tip

For calculating the sum of an A.P., using the formula $S_n = \frac{n}{2}(a + l)$ is faster if the first and last terms are known. Remember to correctly calculate the number of terms first. A common mistake is to forget to add 1 after dividing the difference by 'd'.

45. Find the sum of all natural numbers from 48 to 99.

- (A) 3822
- (B) 3922
- (C) 3969
- (D) 3725
- (E) 3229

Correct Answer: (A) 3822

Solution:

Step 1: Understanding the Question:

We need to find the sum of a consecutive sequence of natural numbers, which is an Arithmetic Progression (A.P.).

Step 2: Key Formula or Approach:

For an A.P. with consecutive numbers:

1. Number of terms (n) = (Last term - First term) + 1.
2. Sum $S_n = \frac{n}{2}(\text{First term} + \text{Last term})$.

Step 3: Detailed Explanation:

The series is 48, 49, 50, ..., 99.

1. Identify the parameters of the A.P.:

First term, $a = 48$.

Last term, $l = 99$.

Common difference, $d = 1$.

2. Find the number of terms (n):

$$n = (99 - 48) + 1 = 51 + 1 = 52$$

There are 52 numbers from 48 to 99, inclusive.

3. Calculate the sum (S_{52}):

$$\begin{aligned} S_{52} &= \frac{52}{2}(48 + 99) \\ &= 26(147) \end{aligned}$$

Calculation: $26 \times 147 = 26 \times (100 + 40 + 7) = 2600 + 1040 + 182 = 3822$.

Alternatively: $26 \times 147 = (25 + 1) \times 147 = 25 \times 147 + 147 = 3675 + 147 = 3822$.

Step 4: Final Answer:

The sum of all natural numbers from 48 to 99 is 3822.

Quick Tip

A useful method to find the sum of a range of numbers is to calculate the sum from 1 to the last number and subtract the sum from 1 to the number just before the first number. Sum of first n numbers = $n(n + 1)/2$. Sum(48 to 99) = Sum(1 to 99) - Sum(1 to 47).

46. A trader purchased 20 apples. Half of the stock was sold at a profit of 10% and with that money he purchased five mangoes. After that he sold his entire stock at a profit of 20% thereby gaining Rs.21. Find the cost of each mango.

- (A) Rs.11
- (B) Rs.10
- (C) Rs.5
- (D) Rs.9
- (E) Rs. 7

Correct Answer: (A) Rs.11

Solution:

Step 1: Understanding the Question:

This is a multi-step profit and loss problem. The wording "sold his entire stock" is ambiguous. A plausible interpretation is that the final profit calculation is on the new stock composition (remaining apples + newly bought mangoes).

Step 2: Key Formula or Approach:

We will set up equations based on the transactions. Let 'c' be the cost price of one apple. We will track the cost and profit through the stages.

Step 3: Detailed Explanation:

Let the Cost Price (CP) of one apple be 'c'.

Total initial CP of 20 apples = $20c$.

Transaction 1: Selling half the apples

He sells 10 apples. CP of these 10 apples = $10c$.

They are sold at a 10% profit.

Selling Price (SP) of these 10 apples = $10c \times (1 + 10/100) = 10c \times 1.1 = 11c$.

Transaction 2: Buying mangoes

He uses the proceeds ($11c$) to buy 5 mangoes.

Cost of 5 mangoes = $11c$.

Cost of 1 mango = $11c/5$.

Transaction 3: Selling the "entire stock"

The "entire stock" at this point consists of the remaining 10 apples and the 5 newly purchased mangoes.

CP of this remaining stock = (CP of 10 apples) + (CP of 5 mangoes) = $10c + 11c = 21c$.

He sells this stock at a 20% profit.

Profit = 20% of the CP of this stock = $0.20 \times 21c = 4.2c$.

The problem states that this gain is Rs. 21.

So, $4.2c = 21$.

$$c = \frac{21}{4.2} = \frac{210}{42} = 5.$$

The cost price of one apple is Rs. 5.

Find the cost of each mango:

$$\text{Cost of 1 mango} = \frac{11c}{5} = \frac{11 \times 5}{5} = \text{Rs. } 11.$$

Step 4: Final Answer:

The cost of each mango is Rs. 11.

Quick Tip

In complex profit and loss problems, carefully define the "stock" and its "cost price" at each stage of the transaction. The final profit or loss is always calculated on the relevant cost price for that specific transaction.

47. A can do a job in 24 days and B can do the same job in 26 days. A and B do the work in alternative days. If 'B' starts the work, find in how many days the job got completed.

- (A) 25
- (B) 27
- (C) $24\frac{12}{13}$
- (D) 35
- (E) None of these

Correct Answer: (A) 25

Solution:

Step 1: Understanding the Question:

This is an alternating days work problem. We need to calculate the work done in a 2-day cycle and see how many cycles are needed to complete the job.

Step 2: Key Formula or Approach:

1. Find the rate of work for each person (work/day).
2. Calculate the work done in one cycle (2 days).
3. Find the number of full cycles needed to get close to completing the work.
4. Calculate the remaining work and find the time needed by the next person to finish it.

Step 3: Detailed Explanation:

A's rate of work = $\frac{1}{24}$ of the job per day.

B's rate of work = $\frac{1}{26}$ of the job per day.

B starts the work. The work pattern is B, A, B, A, ...

Work done in one 2-day cycle (Day 1 by B, Day 2 by A):

$$\text{Work per cycle} = \frac{1}{26} + \frac{1}{24} = \frac{12 + 13}{312} = \frac{25}{312}$$

Let's find how many full cycles can be completed. Total work is 1.

$$\text{Number of cycles} \approx \frac{1}{25/312} = \frac{312}{25} = 12.48$$

So, we can complete 12 full cycles.

Work done after 12 cycles:

Time taken = $12 \times 2 = 24$ days.

Work completed = $12 \times \frac{25}{312} = \frac{300}{312}$.

Remaining work:

$$\text{Remaining work} = 1 - \frac{300}{312} = \frac{12}{312} = \frac{1}{26}$$

After 24 days (12 cycles), the 25th day begins, and it is B's turn to work.

B's rate is $\frac{1}{26}$ of the job per day.

The remaining work is exactly $\frac{1}{26}$.

Time for B to finish the remaining work = $\frac{\text{Remaining Work}}{\text{B's Rate}} = \frac{1/26}{1/26} = 1$ day.

Total time to complete the job:

$$\text{Total time} = 24 \text{ days (for cycles)} + 1 \text{ day (for remaining work)} = 25 \text{ days}$$

Step 4: Final Answer:

The job got completed in 25 days.

Quick Tip

For alternating work problems, the "cycle" method is very effective. Calculate work done in one full cycle, find the number of cycles to get close to the total work, and then handle the remaining fraction of work manually for the subsequent days.

48. Eight years ago, Rajesh was half as old as Shiva. If the ratio of their ages after 4 years becomes 3: 4, find the present age of Rajesh.

- (A) 14 years
- (B) 18 years
- (C) 24 years
- (D) 20 years
- (E) 12 years

Correct Answer: (A) 14 years

Solution:

Step 1: Understanding the Question:

This is a word problem on ages involving two individuals and two different points in time. We need to set up a system of linear equations to find their present ages.

Step 2: Key Formula or Approach:

Let the present ages of Rajesh and Shiva be R and S, respectively. Translate the given statements into algebraic equations.

Step 3: Detailed Explanation:

Let R be the present age of Rajesh and S be the present age of Shiva.

Condition 1: Eight years ago...

Rajesh's age was $R - 8$.

Shiva's age was $S - 8$.

The condition is: $R - 8 = \frac{1}{2}(S - 8)$.

$$2(R - 8) = S - 8$$

$$2R - 16 = S - 8$$

$$S = 2R - 8 \quad \text{---(1)}$$

Condition 2: After 4 years...

Rajesh's age will be $R + 4$.

Shiva's age will be $S + 4$.

The ratio of their ages will be 3:4.

$$\frac{R + 4}{S + 4} = \frac{3}{4}$$

$$4(R + 4) = 3(S + 4)$$

$$4R + 16 = 3S + 12 \quad \text{---(2)}$$

Now, substitute the expression for S from equation (1) into equation (2):

$$4R + 16 = 3(2R - 8) + 12$$

$$4R + 16 = 6R - 24 + 12$$

$$4R + 16 = 6R - 12$$

$$16 + 12 = 6R - 4R$$

$$28 = 2R$$

$$R = 14$$

The present age of Rajesh is 14 years.

Step 4: Final Answer:

The present age of Rajesh is 14 years.

Quick Tip

Always define your variables based on the present ages. Then, express past and future ages relative to the present age (e.g., age 't' years ago is Present - t, age 't' years hence is Present + t).

49. Two trains start at the same time from two stations A and B proceeding towards each other at 36 kmph and 42 kmph respectively. When they meet it was noticed that one train has moved 48 km more than the other. Find the distance between A and B.

- (A) 468 km
- (B) 672 km
- (C) 624 km
- (D) 684 km
- (E) 486 km

Correct Answer: (C) 624 km

Solution:

Step 1: Understanding the Question:

This is a relative speed problem. Two trains travel towards each other, and we are given the difference in the distance they cover when they meet. We need to find the total initial distance between them.

Step 2: Key Formula or Approach:

1. The time taken for the trains to meet is the same for both. Let this time be 't'.
2. Distance = Speed \times Time.
3. The difference in distance covered is due to the difference in their speeds over the same time 't'.
4. The total distance between A and B is the sum of the distances covered by each train.

Step 3: Detailed Explanation:

Let the speed of the first train from A be $S_A = 36$ kmph.

Let the speed of the second train from B be $S_B = 42$ kmph.

Let 't' be the time in hours after which they meet.

Distance covered by the first train, $D_A = S_A \times t = 36t$.

Distance covered by the second train, $D_B = S_B \times t = 42t$.

The second train is faster, so it covers more distance. The difference is given as 48 km.

$$D_B - D_A = 48$$

$$42t - 36t = 48$$

$$6t = 48$$

$$t = \frac{48}{6} = 8 \text{ hours}$$

So, the trains meet after 8 hours.

The total distance between stations A and B is the sum of the distances traveled by both trains until they meet.

$$\text{Total Distance} = D_A + D_B = 36t + 42t = 78t$$

Substitute the value of t:

$$\text{Total Distance} = 78 \times 8 = 624 \text{ km}$$

Step 4: Final Answer:

The distance between A and B is 624 km.

Quick Tip

A shortcut for this type of problem: The ratio of distances covered is the same as the ratio of their speeds, since time is constant ($D_A/D_B = S_A/S_B$). The difference in distance (48 km) corresponds to the difference in speed (6 kmph). Time to meet = Difference in Distance / Difference in Speed = $48/6 = 8$ hours.

50. A plot of land 45m * 65m is divided into four equal rectangular plots by 2 roads that are perpendicular to each other. If the width of the roads is 5 m, find the area of the crossroads.

- (A) 625 sq.m.
- (B) 525 sq.m.
- (C) 550 sq.m.
- (D) 600 sq.m.
- (E) 652 sq.m

Correct Answer: (B) 525 sq.m.

Solution:**Step 1: Understanding the Question:**

We have a rectangular plot with two perpendicular roads running through it. We need to find the total area covered by these roads. It's implied the roads run parallel to the sides of the plot.

Step 2: Key Formula or Approach:

The total area of the crossroads is the sum of the areas of the two rectangular roads minus the area of the central part where they overlap (to avoid counting it twice).

Area of crossroads = (Area of road 1) + (Area of road 2) - (Area of overlap).

Step 3: Detailed Explanation:

Dimensions of the plot = 45 m × 65 m.

Width of the roads = 5 m.

Area of the road parallel to the length (65m side):

This road will have dimensions 65 m × 5 m.

$$\text{Area}_1 = 65 \times 5 = 325 \text{ m}^2$$

Area of the road parallel to the width (45m side):

This road will have dimensions 45 m × 5 m.

$$\text{Area}_2 = 45 \times 5 = 225 \text{ m}^2$$

Area of the central overlapping square:

The two roads intersect in a square region. The dimensions of this square will be the width of the roads.

$$\text{Area}_{\text{overlap}} = 5 \text{ m} \times 5 \text{ m} = 25 \text{ m}^2$$

Total area of the crossroads:

$$\begin{aligned} \text{Total Area} &= \text{Area}_1 + \text{Area}_2 - \text{Area}_{\text{overlap}} \\ &= 325 + 225 - 25 \\ &= 550 - 25 = 525 \text{ m}^2 \end{aligned}$$

Step 4: Final Answer:

The area of the crossroads is 525 sq.m.

Quick Tip

For calculating the area of intersecting paths inside a rectangle of length L and breadth B , where path width is ' w ', use the formula: $\text{Area} = w(L + B - w)$. Here, $5(65 + 45 - 5) = 5(110 - 5) = 5(105) = 525$. This is a quick and reliable shortcut.

51. One bell rings at intervals of 40 minutes and another at intervals of 30 minutes. If they ring at 10 am together, when will they ring together again for the next time?

- (A) 12.30 am
- (B) 11.30 am
- (C) 12 Noon
- (D) 1.30 pm
- (E) 2.00 pm

Correct Answer: (C) 12 Noon

Solution:

Step 1: Understanding the Question:

The problem asks for the next time two bells, with different ringing intervals, will ring simultaneously. This is a classic problem that can be solved by finding the Least Common Multiple (LCM) of their intervals.

Step 2: Key Formula or Approach:

To find when the bells will ring together, we need to find the LCM of their ringing intervals.

$$\text{Next ringing time} = \text{Initial time} + \text{LCM of intervals}$$

Step 3: Detailed Explanation:

The intervals are 40 minutes and 30 minutes.

First, we find the prime factorization of each number:

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$$

$$30 = 2 \times 3 \times 5$$

Now, we find the LCM by taking the highest power of each prime factor present in the numbers:

$$\text{LCM}(40, 30) = 2^3 \times 3^1 \times 5^1 = 8 \times 3 \times 5 = 120 \text{ minutes}$$

120 minutes is equal to 2 hours (since 60 minutes = 1 hour).

The bells ring together at 10:00 am. The next time they will ring together is after 120 minutes (2 hours).

$$\text{Next ringing time} = 10 : 00 \text{ am} + 2 \text{ hours} = 12 : 00 \text{ pm (12 Noon)}$$

Step 4: Final Answer:

The bells will ring together again at 12 Noon.

Quick Tip

Whenever a question involves events repeating at different intervals and asks when they will occur together (like bells ringing, lights flashing, or runners meeting at a starting point), the key is always to find the LCM of the intervals.

52. What is the least number of students in a class if they can be made to stand in rows of 8, 12 or 14 each?

- (A) 248
- (B) 224
- (C) 196
- (D) 168
- (E) 242

Correct Answer: (D) 168

Solution:

Step 1: Understanding the Question:

The question asks for the smallest number of students that can be perfectly divided into groups of 8, 12, and 14. This means we need to find the least number which is a multiple of 8, 12, and 14. This is the Least Common Multiple (LCM).

Step 2: Key Formula or Approach:

The required number of students is the LCM of the given row sizes.

$$\text{Least number of students} = \text{LCM}(8, 12, 14)$$

Step 3: Detailed Explanation:

We will find the LCM of 8, 12, and 14 using prime factorization. First, find the prime factors of each number:

$$8 = 2 \times 2 \times 2 = 2^3$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$14 = 2 \times 7$$

To find the LCM, we take the highest power of each prime factor that appears in any of the numbers.

The prime factors are 2, 3, and 7. The highest powers are 2^3 , 3^1 , and 7^1 .

$$\text{LCM}(8, 12, 14) = 2^3 \times 3^1 \times 7^1 = 8 \times 3 \times 7 = 168$$

Step 4: Final Answer:

The least number of students required is 168.

Quick Tip

The keyword "least", "smallest", or "minimum" number that is divisible by a set of numbers is a strong indicator that you need to calculate the LCM. Conversely, if the question asks for the "greatest" or "largest" number that divides a set of numbers, you should calculate the HCF (Highest Common Factor).

53. A dealer buys an article listed as Rs.3000 at successive discounts of 15% and 20%. Find the price at which he should sell the article so as to make a profit of 25%.

- (A) Rs. 2448
- (B) Rs. 2500
- (C) Rs. 2550
- (D) Rs. 2750
- (E) Rs. 2844

Correct Answer: (C) Rs. 2550

Solution:

Step 1: Understanding the Question:

First, we need to calculate the actual cost price (CP) for the dealer after two successive discounts on the listed price. Then, we need to calculate the selling price (SP) that will give him a 25% profit on his cost price.

Step 2: Key Formula or Approach:

1. Calculate the Cost Price (CP) after successive discounts:

$$\text{CP} = \text{Listed Price} \times \left(1 - \frac{\text{Discount}_1}{100}\right) \times \left(1 - \frac{\text{Discount}_2}{100}\right)$$

2. Calculate the Selling Price (SP) for a given profit:

$$SP = CP \times \left(1 + \frac{\text{Profit}\%}{100}\right)$$

Step 3: Detailed Explanation:

Part 1: Calculate the Cost Price (CP) for the dealer

Listed Price = Rs. 3000.

First discount = 15%.

Price after first discount = $3000 \times \left(1 - \frac{15}{100}\right) = 3000 \times \frac{85}{100} = \text{Rs. } 2550$.

Second discount = 20% (this is applied on the discounted price, Rs. 2550).

Final CP for the dealer = $2550 \times \left(1 - \frac{20}{100}\right) = 2550 \times \frac{80}{100} = \text{Rs. } 2040$.

Part 2: Calculate the required Selling Price (SP)

The dealer's Cost Price (CP) is Rs. 2040.

He wants to make a profit of 25%.

Required SP = $CP \times \left(1 + \frac{\text{Profit}\%}{100}\right)$.

$$SP = 2040 \times \left(1 + \frac{25}{100}\right) = 2040 \times \frac{125}{100} = 2040 \times 1.25$$

$$SP = 2040 \times \frac{5}{4} = 510 \times 5 = \text{Rs. } 2550$$

Step 4: Final Answer:

The dealer should sell the article for Rs. 2550 to make a 25% profit.

Quick Tip

For successive discounts of $d_1\%$ and $d_2\%$, the single equivalent discount is not $(d_1 + d_2)\%$. It is given by $\left(d_1 + d_2 - \frac{d_1 d_2}{100}\right)\%$. Always apply discounts sequentially on the reduced price.

54. A shopkeeper proposes to sell his goods at cost price but uses a weight of 850gms instead of a kilogram weight. What is his profit percentage?

- (A) $17\frac{14}{17}\%$
- (B) $17\frac{11}{17}\%$
- (C) $17\frac{16}{17}\%$
- (D) $17\frac{12}{17}\%$
- (E) None of these

Correct Answer: (B) $17\frac{11}{17}\%$

Solution:**Step 1: Understanding the Question:**

The shopkeeper cheats by giving the customer less quantity than what he charges for. He claims to sell at cost price, but his profit comes from the weight manipulation. We need to calculate the profit percentage based on this deception.

Step 2: Key Formula or Approach:

The profit percentage in cases of false weight is calculated as:

$$\text{Profit \%} = \left(\frac{\text{True Weight} - \text{False Weight}}{\text{False Weight}} \right) \times 100$$

Alternatively, we can use the concept of cost price (CP) and selling price (SP). The shopkeeper's cost is for the weight he actually gives (False Weight), and his revenue is for the weight he claims to give (True Weight).

Step 3: Detailed Explanation:

Let the cost price of 1 gram of the goods be Re. 1.

The shopkeeper claims to sell 1 kilogram (1000 grams) at cost price.

So, the Selling Price (SP) he charges the customer is for 1000 grams, which is Rs. 1000.

However, he only gives 850 grams of goods.

The actual Cost Price (CP) for the shopkeeper for the goods sold is for 850 grams, which is Rs. 850.

Now we can calculate the profit and profit percentage.

$$\text{Profit} = \text{SP} - \text{CP} = 1000 - 850 = \text{Rs. } 150$$

$$\text{Profit \%} = \left(\frac{\text{Profit}}{\text{CP}} \right) \times 100 = \left(\frac{150}{850} \right) \times 100$$

$$\text{Profit \%} = \frac{15}{85} \times 100 = \frac{3}{17} \times 100 = \frac{300}{17} \%$$

To convert this into a mixed fraction:

$$300 \div 17 = 17 \text{ with a remainder of } 11$$

So, the profit percentage is $17\frac{11}{17}\%$.

Step 4: Final Answer:

The shopkeeper's profit percentage is $17\frac{11}{17}\%$.

Quick Tip

In false weight problems, the profit percentage is always calculated on the quantity the shopkeeper actually parts with (the false weight), as this represents his true cost. A common mistake is to calculate it on the true weight.

55. If the profit on the selling price is 20%, find the actual profit percentage.

- (A) 15%
- (B) 25%
- (C) 30%
- (D) 20%
- (E) None of these

Correct Answer: (B) 25%

Solution:

Step 1: Understanding the Question:

The problem gives the profit percentage calculated on the selling price (SP), but standard or "actual" profit percentage is always calculated on the cost price (CP). We need to convert the profit percentage from being based on SP to being based on CP.

Step 2: Key Formula or Approach:

We know the relationships:

1. Profit = Selling Price (SP) - Cost Price (CP)

2. Actual Profit % = $\frac{\text{Profit}}{\text{CP}} \times 100$

We are given Profit = 20% of SP.

Step 3: Detailed Explanation:

Let's assume the Selling Price (SP) is Rs. 100 for easy calculation.

Given that the profit on the selling price is 20%.

$$\text{Profit} = 20\% \text{ of SP} = \frac{20}{100} \times 100 = \text{Rs. } 20$$

Now, we can find the Cost Price (CP) using the formula $\text{SP} = \text{CP} + \text{Profit}$.

$$\text{CP} = \text{SP} - \text{Profit}$$

$$\text{CP} = 100 - 20 = \text{Rs. } 80$$

Now we can calculate the actual profit percentage, which is based on the Cost Price (CP).

$$\text{Actual Profit \%} = \left(\frac{\text{Profit}}{\text{CP}} \right) \times 100 = \left(\frac{20}{80} \right) \times 100$$

$$\text{Actual Profit \%} = \frac{1}{4} \times 100 = 25\%$$

Step 4: Final Answer:

The actual profit percentage is 25%.

Quick Tip

Profit percentage on SP is always numerically less than the profit percentage on CP for the same transaction. If profit is $x\%$ on SP, the actual profit on CP is $\left(\frac{x}{100-x} \right) \times 100\%$. Here, $x = 20$, so profit is $\left(\frac{20}{100-20} \right) \times 100 = \frac{20}{80} \times 100 = 25\%$.

56. The total age of a group of 20 children is 160 years. If the average age of 8 of the children of the group is 12 years, find the average age of the remaining group.

- (A) 5 years
- (B) 5 years and 3 months
- (C) 5 years and 4 months
- (D) 5 years and 6 months
- (E) None of these

Correct Answer: (C) 5 years and 4 months

Solution:

Step 1: Understanding the Question:

We are given information about a whole group and a subgroup. We need to find the average age of the remaining members of the group.

Step 2: Key Formula or Approach:

The core formula for averages is:

$$\text{Average} = \frac{\text{Sum of Observations}}{\text{Number of Observations}}$$

Therefore, Sum of Observations = Average \times Number of Observations.

Step 3: Detailed Explanation:

Total number of children = 20.

Total age of these 20 children = 160 years.

Now, consider the subgroup of 8 children.

Number of children in the subgroup = 8.

Average age of this subgroup = 12 years.

Total age of these 8 children = Average \times Number = $12 \times 8 = 96$ years.

Now, we find the details for the remaining group.

Number of remaining children = Total children - Subgroup children = $20 - 8 = 12$.

Total age of remaining children = Total age of group - Total age of subgroup = $160 - 96 = 64$ years.

Finally, calculate the average age of the remaining 12 children.

$$\text{Average age of remaining group} = \frac{\text{Total age of remaining children}}{\text{Number of remaining children}} = \frac{64}{12} \text{ years}$$

To simplify the fraction: $\frac{64}{12} = \frac{16}{3}$ years.

To convert this into years and months:

$$\frac{16}{3} = 5\frac{1}{3} \text{ years.}$$

This is 5 full years and $\frac{1}{3}$ of a year.

To convert $\frac{1}{3}$ of a year into months, we multiply by 12:

$$\frac{1}{3} \times 12 = 4 \text{ months.}$$

Step 4: Final Answer:

The average age of the remaining group is 5 years and 4 months.

Quick Tip

In problems involving averages of combined groups, always work with the sums (total age, total marks, etc.). Averages themselves cannot be directly added or subtracted. Find the total sum, subtract the sum of the known part, and then calculate the average of the remaining part.

57. A train crossed a platform of 1200 m long in 15 seconds and a bridge 3 km long in 35 seconds. Find the length of the train.

- (A) 250m
- (B) 150m
- (C) 200m
- (D) 190m
- (E) None of these

Correct Answer: (B) 150m

Solution:**Step 1: Understanding the Question:**

A train of a certain length is moving at a constant speed. It crosses two objects of different lengths (a platform and a bridge) in different times. We need to find the length of the train.

Step 2: Key Formula or Approach:

When a train crosses an object with length (like a platform or bridge), the total distance covered by the train is the sum of its own length and the length of the object.

$$\text{Distance} = \text{Length of Train} + \text{Length of Object}$$

The speed of the train is constant:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Step 3: Detailed Explanation:

Let the length of the train be L meters and its speed be S m/s.

Ensure all units are consistent. The bridge length is 3 km, which is $3 \times 1000 = 3000$ meters.

Case 1: Crossing the platform

Length of platform = 1200 m.

Time taken = 15 s.

Total distance covered = $L + 1200$ m.

Speed of the train, $S = \frac{L+1200}{15}$ — (Equation 1)

Case 2: Crossing the bridge

Length of bridge = 3000 m.

Time taken = 35 s.

Total distance covered = $L + 3000$ m.

Speed of the train, $S = \frac{L+3000}{35}$ — (Equation 2)

Since the speed S is the same in both cases, we can equate the two expressions for speed:

$$\frac{L + 1200}{15} = \frac{L + 3000}{35}$$

To solve for L , we can cross-multiply:

$$35(L + 1200) = 15(L + 3000)$$

Divide both sides by 5 to simplify the calculation:

$$7(L + 1200) = 3(L + 3000)$$

$$7L + 8400 = 3L + 9000$$

$$7L - 3L = 9000 - 8400$$

$$4L = 600$$

$$L = \frac{600}{4} = 150$$

Step 4: Final Answer:

The length of the train is 150 meters.

Quick Tip

A faster way to solve this is to see that the train travels an extra distance of $(3000 - 1200) = 1800$ m in an extra time of $(35 - 15) = 20$ s. This extra distance is covered at the train's speed. So, Speed = $1800\text{m} / 20\text{s} = 90$ m/s. Now use this speed in either case to find L. For case 1: $90 = \frac{L+1200}{15} \implies 1350 = L + 1200 \implies L = 150$ m.

58. If the lines $7x+6y+9=0$ and $ax+14y+8=0$ are perpendicular to each other, find the value of a.

- (A) 12
- (B) -12
- (C) 6
- (D) -6
- (E) None of these

Correct Answer: (B) -12

Solution:

Step 1: Understanding the Question:

We are given the equations of two straight lines and told they are perpendicular. We need to find the value of an unknown coefficient 'a' in the second equation.

Step 2: Key Formula or Approach:

For two lines given in the general form $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$, the

condition for them to be perpendicular is:

$$A_1A_2 + B_1B_2 = 0$$

Alternatively, if the slopes are m_1 and m_2 , the condition is $m_1 \times m_2 = -1$. The slope of a line $Ax + By + C = 0$ is $m = -\frac{A}{B}$.

Step 3: Detailed Explanation:

Let's use the condition $A_1A_2 + B_1B_2 = 0$.

First line: $7x + 6y + 9 = 0$.

Here, $A_1 = 7$ and $B_1 = 6$.

Second line: $ax + 14y + 8 = 0$.

Here, $A_2 = a$ and $B_2 = 14$.

Now, apply the perpendicularity condition:

$$(7)(a) + (6)(14) = 0$$

$$7a + 84 = 0$$

$$7a = -84$$

$$a = -\frac{84}{7}$$

$$a = -12$$

Step 4: Final Answer:

The value of 'a' is -12.

Quick Tip

For perpendicular lines in the form $Ax + By + C = 0$, remember the condition $A_1A_2 + B_1B_2 = 0$. For parallel lines, the condition is $\frac{A_1}{A_2} = \frac{B_1}{B_2}$. These are quicker than finding slopes and using $m_1m_2 = -1$ or $m_1 = m_2$.

59. A and B started a business with Rs.1200 and Rs.1500. After some time C joined with Rs.2400. C and A got equal amount as their share of profit at the end of the year. After how many months did C join the firm?

- (A) 9 months
- (B) 4 months
- (C) 6 months
- (D) 10 months
- (E) None of these

Correct Answer: (C) 6 months

Solution:

Step 1: Understanding the Question:

This is a partnership problem where partners invest different amounts for different periods. The profit sharing is proportional to the product of their investment and the time period for which they invested. We are given a condition about the profits of A and C and need to find when C joined the business.

Step 2: Key Formula or Approach:

The ratio of profits is equal to the ratio of the product of capital and time.

$$\text{Profit}_A : \text{Profit}_B : \text{Profit}_C = (\text{Capital}_A \times \text{Time}_A) : (\text{Capital}_B \times \text{Time}_B) : (\text{Capital}_C \times \text{Time}_C)$$

Step 3: Detailed Explanation:

The total duration of the business is one year (12 months).

Investment of A = Rs. 1200. Time period for A = 12 months.

Investment of B = Rs. 1500. Time period for B = 12 months.

Investment of C = Rs. 2400.

Let's assume C was in the business for t months. This means C joined after $(12 - t)$ months.

Now, let's find the ratio of their effective investments (Capital \times Time).

A's share is proportional to $1200 \times 12 = 14400$.

B's share is proportional to $1500 \times 12 = 18000$.

C's share is proportional to $2400 \times t = 2400t$.

The profit sharing ratio is $P_A : P_B : P_C = 14400 : 18000 : 2400t$.

We can simplify this ratio by dividing all parts by a common factor. Let's divide by 100:

$$144 : 180 : 24t$$

Let's divide by 12:

$$12 : 15 : 2t$$

The problem states that C and A got an equal amount of profit. This means their shares in the ratio are equal.

Share of A = Share of C

$$12 = 2t$$

$$t = \frac{12}{2} = 6$$

So, C invested his money for a period of 6 months.

The question asks, "After how many months did C join the firm?"

Time after which C joined = Total duration - C's investment period.

$$\text{Time after joining} = 12 - 6 = 6 \text{ months}$$

Step 4: Final Answer:

C joined the firm after 6 months.

Quick Tip

In partnership problems, be very careful about what the variable 't' represents. If you define 't' as the time for which a person invested, the question might ask for the time after which they joined (Total Time - t). Reading the question carefully is crucial.

60. The salaries of Sudhakar, Sekhar and Subhash are in the ratio 2:3:4 and they got increments in the ratio 1:2:1 and finally their salaries were in the ratio of 5:8:9. If Sudhakar got Rs.1500 as increment then, find the ratio of percentage of increments with respect to their salaries.

- (A) 5:7:3
- (B) 6:8:3
- (C) 7:8:5
- (D) 5:8:3
- (E) None of these

Correct Answer: (B) 6:8:3

Solution:

Step 1: Understanding the Question:

We are given initial salary ratios, increment ratios, and final salary ratios for three individuals. We are also given the actual increment amount for one person. The goal is to find the ratio of

their percentage increments relative to their initial salaries.

Step 2: Key Formula or Approach:

1. Represent the initial salaries and increments using variables based on their ratios.
2. Use the given increment amount to find the value of the variable for increments.
3. Use the final salary ratio to establish an equation and find the value of the variable for initial salaries.
4. Calculate the initial salaries and increments for all three individuals.
5. Calculate the percentage increment for each: $(\text{Increment}/\text{Initial Salary}) \times 100$.
6. Find the ratio of these percentages.

Step 3: Detailed Explanation:

Let the initial salaries of Sudhakar, Sekhar, and Subhash be $2x$, $3x$, and $4x$ respectively. Let their increments be y , $2y$, and y respectively.

We are given that Sudhakar's increment is Rs. 1500.

So, $y = 1500$.

This means the increments are:

Sudhakar's increment = $y = 1500$.

Sekhar's increment = $2y = 2 \times 1500 = 3000$.

Subhash's increment = $y = 1500$.

The final salaries are the sum of the initial salary and the increment.

Final salary of Sudhakar = $2x + 1500$.

Final salary of Sekhar = $3x + 3000$.

Final salary of Subhash = $4x + 1500$.

The ratio of their final salaries is given as 5:8:9. We can use any pair to set up an equation. Let's use Sudhakar and Sekhar.

$$\frac{\text{Final Salary of Sudhakar}}{\text{Final Salary of Sekhar}} = \frac{2x + 1500}{3x + 3000} = \frac{5}{8}$$

Cross-multiplying gives:

$$8(2x + 1500) = 5(3x + 3000)$$

$$16x + 12000 = 15x + 15000$$

$$16x - 15x = 15000 - 12000$$

$$x = 3000$$

Now we can find their initial salaries:

Initial salary of Sudhakar = $2x = 2 \times 3000 = \text{Rs. } 6000$.

Initial salary of Sekhar = $3x = 3 \times 3000 = \text{Rs. } 9000$.

Initial salary of Subhash = $4x = 4 \times 3000 = \text{Rs. } 12000$.

The question asks for the ratio of the percentage of increments. The ratio of percentage increments is the same as the ratio of the fractions $\frac{\text{Increment}}{\text{Initial Salary}}$.

$$\text{Ratio} = \left(\frac{1500}{6000}\right) : \left(\frac{3000}{9000}\right) : \left(\frac{1500}{12000}\right)$$

Simplify the fractions:

$$\text{Ratio} = \frac{1}{4} : \frac{1}{3} : \frac{1}{8}$$

To get a ratio of integers, we multiply by the LCM of the denominators (4, 3, 8), which is 24.

$$\text{Ratio} = \left(\frac{1}{4} \times 24\right) : \left(\frac{1}{3} \times 24\right) : \left(\frac{1}{8} \times 24\right)$$

$$\text{Ratio} = 6 : 8 : 3$$

Step 4: Final Answer:

The ratio of percentage of increments is 6:8:3.

Quick Tip

When asked for the ratio of percentages (like % increment), you can often work directly with the ratio of the fractions $\frac{\text{part}}{\text{whole}}$. There is no need to multiply by 100, as the factor of 100 will cancel out from all terms in the ratio.

61. 1, 27, 125, ?, 729, 1331

- (A) 241
- (B) 343
- (C) 289
- (D) 641
- (E) 214

Correct Answer: (B) 343

Solution:

Step 1: Understanding the Question:

This is a number series problem where we need to identify the pattern and find the missing term.

Step 2: Detailed Explanation:

Let's analyze the terms in the series:

The first term is 1, which can be written as 1^3 .

The second term is 27, which is $3 \times 3 \times 3 = 3^3$.

The third term is 125, which is $5 \times 5 \times 5 = 5^3$.

The fifth term is 729, which is $9 \times 9 \times 9 = 9^3$.

The sixth term is 1331, which is $11 \times 11 \times 11 = 11^3$.

The pattern is the cube of consecutive odd numbers: $1^3, 3^3, 5^3, \dots, 9^3, 11^3$.

The missing term must be the cube of the next odd number in the sequence, which is 7.

Missing Term = 7^3 .

$$7^3 = 7 \times 7 \times 7 = 49 \times 7 = 343$$

Step 3: Final Answer:

The missing term in the series is 343.

Quick Tip

When you see numbers like 8, 27, 64, 125, 216, 343, 512, 729, 1000 in a series, immediately think of cubes. Being familiar with squares up to 30 and cubes up to 15 can save a lot of time in number series questions.

62. 81, 9, 64, 8, 49, ?

- (A) 10
- (B) 7
- (C) 5
- (D) 4
- (E) 36

Correct Answer: (B) 7

Solution:

Step 1: Understanding the Question:

This is a number series problem. We need to find the pattern to determine the missing number.

Step 2: Detailed Explanation:

This series can be viewed as a set of pairs. Let's group the terms:

Pair 1: (81, 9)

Pair 2: (64, 8)

Pair 3: (49, ?)

Let's analyze the relationship within each pair.

In Pair 1, 81 is the square of 9. $9^2 = 81$.

In Pair 2, 64 is the square of 8. $8^2 = 64$.

Following this pattern, the first number in each pair is the square of the second number. For Pair 3, we have 49. We need to find the number whose square is 49.

$$?^2 = 49$$

$$? = \sqrt{49} = 7$$

The missing term is 7. The series follows the pattern $9^2, 9, 8^2, 8, 7^2, 7$.

Step 3: Final Answer:

The missing term is 7.

Quick Tip

For series with an even number of terms or where the pattern seems to jump, consider it as two or more interleaved series or a series of pairs. This often reveals a simple underlying pattern.

63. 1, 3, 4, 8, 15, 27, ?

- (A) 37
- (B) 44
- (C) 50
- (D) 55
- (E) 22

Correct Answer: (C) 50

Solution:

Step 1: Understanding the Question:

We need to find the next term in the given number series by identifying the rule that governs it.

Step 2: Detailed Explanation:

Let's examine the relationship between the terms. Simple differences don't reveal an obvious pattern.

Let's try a summation pattern.

The first few terms are 1, 3, 4. Let's see if the next term, 8, can be formed from these.

$1 + 3 + 4 = 8$. This works.

Let's test this "sum of previous three terms" rule for the subsequent terms.

To get 15: Previous three terms are 3, 4, 8.

$3 + 4 + 8 = 15$. This is correct.

To get 27: Previous three terms are 4, 8, 15.

$4 + 8 + 15 = 27$. This is also correct.

The pattern is that each term (starting from the fourth term) is the sum of the three preceding terms.

To find the missing term, we need to sum the last three known terms: 8, 15, and 27.

Missing Term = $8 + 15 + 27$.

$$8 + 15 = 23$$

$$23 + 27 = 50$$

Step 3: Final Answer:

The next term in the series is 50.

Quick Tip

Series like this, where a term is the sum of a fixed number of previous terms, are known as recurrence relations. The Fibonacci series (sum of previous two) is the most famous example. If simple differences or multiplications don't work, check for summation patterns.

64. 2, 5, 11, 23, 47, ?

- (A) 49
- (B) 52
- (C) 95
- (D) 106
- (E) None of these

Correct Answer: (C) 95

Solution:

Step 1: Understanding the Question:

We need to find the missing term in the sequence by identifying the underlying logical pattern.

Step 2: Detailed Explanation:

There are two common ways to identify the pattern in this series.

Method 1: Analyzing the differences

Let's find the difference between consecutive terms.

$$5 - 2 = 3$$

$$11 - 5 = 6$$

$$23 - 11 = 12$$

$$47 - 23 = 24$$

The differences are 3, 6, 12, 24. This is a geometric progression where each term is double the previous one. The next difference should be $24 \times 2 = 48$.

So, the missing term is the previous term (47) plus the next difference (48).

$$\text{Missing Term} = 47 + 48 = 95.$$

Method 2: Analyzing the relationship between terms

Let's see how we can get from one term to the next.

$$\text{To get from 2 to 5: } (2 \times 2) + 1 = 4 + 1 = 5.$$

$$\text{To get from 5 to 11: } (5 \times 2) + 1 = 10 + 1 = 11.$$

$$\text{To get from 11 to 23: } (11 \times 2) + 1 = 22 + 1 = 23.$$

$$\text{To get from 23 to 47: } (23 \times 2) + 1 = 46 + 1 = 47.$$

The pattern is: Next Term = (Previous Term \times 2) + 1.

Applying this rule to find the missing term:

$$\text{Missing Term} = (47 \times 2) + 1 = 94 + 1 = 95.$$

Step 3: Final Answer:

The missing term is 95.

Quick Tip

For number series, always try two basic approaches first: 1) Find the differences between terms (and if needed, the differences of the differences). 2) Look for a multiplication/division pattern, often combined with addition/subtraction. One of these two methods solves most common series problems.

65. What is the length of the longest rod that can be put on the floor of a rectangular room measuring 45 m in length and 28 m in breadth?

- (A) 45 m
- (B) 50 m
- (C) 53 m
- (D) 35 m
- (E) None of these

Correct Answer: (C) 53 m

Solution:

Step 1: Understanding the Question:

The longest rod that can be put on the floor of a rectangular room corresponds to the length of the diagonal of the floor.

Step 2: Key Formula or Approach:

The problem requires us to find the diagonal of a rectangle. According to the Pythagorean theorem, for a rectangle with length L and breadth B , the diagonal D is given by:

$$D = \sqrt{L^2 + B^2}$$

Step 3: Detailed Explanation:

Given dimensions of the rectangular floor:

Length (L) = 45 m.

Breadth (B) = 28 m.

Using the formula for the diagonal:

$$D = \sqrt{(45)^2 + (28)^2}$$

First, calculate the squares of the length and breadth:

$$45^2 = 2025$$

$$28^2 = 784$$

Now, add the squares:

$$D^2 = 2025 + 784 = 2809$$

Finally, find the square root of the sum:

$$D = \sqrt{2809}$$

To find the square root of 2809, we can estimate. We know $50^2 = 2500$ and $60^2 = 3600$. So the root is between 50 and 60. The number ends in 9, so its square root must end in 3 or 7. Let's try 53.

$$53 \times 53 = 2809$$

So, the diagonal is 53 m.

Step 4: Final Answer:

The length of the longest rod that can be put on the floor is 53 m.

Quick Tip

Be careful to distinguish between the longest rod on the *floor* (diagonal of the rectangle) and the longest rod in the *room* (space diagonal of the cuboid, $\sqrt{L^2 + B^2 + H^2}$). Always read the question carefully to understand what is being asked. Recognizing Pythagorean triplets (like 3-4-5, 5-12-13, 8-15-17, etc.) can also speed up calculations. Here (28, 45, 53) is a less common triplet.

66. The sides of a rectangular field are in the ratio of 3:5 and its area is 2535 m². Calculate the cost of fencing it at the rate of Rs.2.50/m.

- (A) Rs. 250
- (B) Rs. 260
- (C) Rs. 324
- (D) Rs. 423
- (E) None of these

Correct Answer: (E) None of these

Solution:

Step 1: Understanding the Question:

First, we need to find the actual dimensions (length and breadth) of the rectangular field using the given ratio and area. Second, we need to calculate the perimeter of the field, as fencing is done along the boundary. Finally, we calculate the total cost of fencing using the given rate.

Step 2: Key Formula or Approach:

1. Area of a rectangle = Length \times Breadth.
2. Perimeter of a rectangle = $2 \times$ (Length + Breadth).
3. Total Cost = Perimeter \times Rate per meter.

Step 3: Detailed Explanation:

Let the sides of the rectangular field be $3x$ and $5x$. Conventionally, Length is the longer side.

Length (L) = $5x$.

Breadth (B) = $3x$.

The area is given as 2535 m².

$$\text{Area} = L \times B = (5x)(3x) = 15x^2$$

$$15x^2 = 2535$$

$$x^2 = \frac{2535}{15}$$

To divide 2535 by 15: $2535 \div 15 = 169$.

$$x^2 = 169$$

$$x = \sqrt{169} = 13$$

Now we can find the actual dimensions:

$$\text{Length } (L) = 5x = 5 \times 13 = 65 \text{ m.}$$

$$\text{Breadth } (B) = 3x = 3 \times 13 = 39 \text{ m.}$$

Next, calculate the perimeter of the field for fencing.

$$\text{Perimeter} = 2 \times (L + B) = 2 \times (65 + 39)$$

$$\text{Perimeter} = 2 \times (104) = 208 \text{ m}$$

Finally, calculate the total cost of fencing.

Rate of fencing = Rs. 2.50 per meter.

$$\text{Total Cost} = \text{Perimeter} \times \text{Rate} = 208 \times 2.50$$

$$\text{Total Cost} = 208 \times \frac{5}{2} = 104 \times 5 = \text{Rs. } 520$$

The calculated cost is Rs. 520. Since this value is not among options (A), (B), (C), or (D), the correct choice is (E).

Step 4: Final Answer:

The cost of fencing the field is Rs. 520, so the correct option is (E) None of these.

Quick Tip

When dealing with ratios for dimensions, always introduce a variable (like 'x'). Be careful not to use the ratio numbers (3 and 5) directly as the lengths. Remember that fencing refers to the perimeter, while carpeting or ploughing refers to the area.

Directions for the questions 67 to 75:

Each of the following problems has a question and two statements which are labeled I and II. Use the data given in I and II together with other available information to decide whether the statements are sufficient to answer the question. Then choose:

- (a) If statement I by itself is sufficient to answer the question, but statement II by itself is not sufficient.
 - (b) If statement II by itself is sufficient to answer the question, but statement I by itself is not sufficient.
 - (c) If statements I and II taken together are sufficient to answer the question, even though neither statement by itself is sufficient.
 - (d) If either statement by itself is sufficient to answer the question.
 - (e) If statements I and II together are not sufficient to answer the question, meaning that further information would be needed to answer the question.
-

67. Is Narayan a relative of Ashok?

I. None, but the relatives of Ashok help him

II. Narayan helps Ashok.

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (C) Both statements I and II together are sufficient.

Solution:

Step 1: Understanding the Question:

The question asks for a definitive "Yes" or "No" answer regarding whether Narayan is a relative of Ashok.

Step 2: Detailed Explanation:

Analyzing Statement I: "None, but the relatives of Ashok help him". This is phrased ambiguously. A logical interpretation is "Only the relatives of Ashok help him". This statement establishes a rule, but it does not mention Narayan. Therefore, we cannot determine if Narayan is a relative. So, Statement I alone is not sufficient.

Analyzing Statement II: "Narayan helps Ashok." This statement tells us an action performed by Narayan. However, people other than relatives can also help. So, this statement alone does not establish a relationship. So, Statement II alone is not sufficient.

Analyzing Statements I and II Together:

From Statement I, we establish the rule: "If someone helps Ashok, then that person is a relative of Ashok".

From Statement II, we are given a fact: "Narayan helps Ashok".

By combining the rule from Statement I and the fact from Statement II, we can logically conclude that Narayan must be a relative of Ashok. This provides a definite "Yes" to the question.

Step 3: Final Answer:

Since both statements together are required to answer the question, the correct option is (C).

Quick Tip

In data sufficiency, the goal is not to find the answer but to determine if you *can* find the answer. A definite "Yes" or a definite "No" both count as sufficient information.

68. Is Vinayak a relative of Shekhar?

I. None of the relatives of Shekhar helps him.

II. Vinayak does not help him.

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (E) Both statements I and II together are not sufficient.

Solution:

Step 1: Understanding the Question:

We need to determine with certainty whether Vinayak is a relative of Shekhar.

Step 2: Detailed Explanation:

Analyzing Statement I: "None of the relatives of Shekhar helps him." This can be written as a conditional statement: If a person is a relative of Shekhar, then that person does not help him. This gives a property of relatives, but it doesn't mention Vinayak. So, Statement I alone is not sufficient.

Analyzing Statement II: "Vinayak does not help him." This tells us about Vinayak's action (or inaction). It doesn't provide information about his relationship with Shekhar. So, Statement II alone is not sufficient.

Analyzing Statements I and II Together:

From Statement I: All relatives of Shekhar do not help him.

From Statement II: Vinayak does not help him.

This means Vinayak behaves in a way that is consistent with him being a relative. However, it's also possible that Vinayak is a non-relative who also does not help Shekhar. The statements do not exclude this possibility. Since we cannot definitively conclude whether Vinayak is a relative or not, the information is insufficient. This is an example of the logical fallacy of affirming the

consequent.

Step 3: Final Answer:

Even with both statements, we cannot answer the question with certainty. The correct option is (E).

Quick Tip

Be careful with conditional statements (If P, then Q). Given Q is true, you cannot conclude P is true. However, if you are given that Q is false, you can conclude that P is false (this is called the contrapositive).

69. Does Prasad a student of our college come to the college by bus?

I. None of the students of our college comes on foot or by car.

II. All the students of our college come by cycle.

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (B) Statement II alone is sufficient.

Solution:

Step 1: Understanding the Question:

The question asks for a "Yes" or "No" answer to whether Prasad, a student, travels to college by bus.

Step 2: Detailed Explanation:

Analyzing Statement I: "None of the students of our college comes on foot or by car." This statement eliminates two modes of transport for all students, including Prasad. However, it doesn't rule out other modes like bus, train, or cycle. We cannot be certain if Prasad uses a bus. So, Statement I alone is not sufficient.

Analyzing Statement II: "All the students of our college come by cycle." Since Prasad is a student of the college, this statement implies that Prasad must come by cycle. If Prasad comes by cycle, he does not come by bus. This gives a definitive "No" answer to the question. So, Statement II alone is sufficient.

Step 3: Final Answer:

Since Statement II alone is sufficient to answer the question, but Statement I alone is not, the correct option is (B).

Quick Tip

In data sufficiency, a statement is sufficient if it leads to a single, unambiguous answer. "No" is as valid an answer as "Yes".

70. Is Lakshman a relative of Babu?

I. Babu helps only his relatives.

II. Lakshman helps Babu.

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (E) Both statements I and II together are not sufficient.

Solution:

Step 1: Understanding the Question:

We need to determine if there is a family relationship between Lakshman and Babu.

Step 2: Detailed Explanation:

Analyzing Statement I: "Babu helps only his relatives." This statement sets a condition on whom **Babu helps**. It doesn't provide any information about Lakshman or who helps Babu. So, Statement I alone is not sufficient.

Analyzing Statement II: "Lakshman helps Babu." This statement describes an action from Lakshman to Babu. It does not provide any context about the reason for the help or their relationship. So, Statement II alone is not sufficient.

Analyzing Statements I and II Together:

Statement I is about the people that **Babu helps**.

Statement II is about **Lakshman helping Babu**.

There is no connection between the two statements. Statement I's condition applies when Babu is the one giving help. Statement II describes a situation where Babu is receiving help. The information is unrelated and cannot be combined to reach a conclusion.

Step 3: Final Answer:

Even when combined, the statements do not provide enough information to answer the question. The correct option is (E).

Quick Tip

Pay close attention to the subject and object of the action in each statement. A statement like "A helps B" is very different from "B helps A", and rules applying to one case cannot be applied to the other.

71. One of the three candidates (A, B and C) won the election with a clear majority. Was it A?

I. B got 5000 votes more than C

II. A and C got the same number of votes.

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (C) Both statements I and II together are sufficient.

Solution:

Step 1: Understanding the Question:

We are told that one candidate (A, B, or C) won with a "clear majority" (more than 50% of the total votes). The question is specifically "Was it A?"

Step 2: Detailed Explanation:

Analyzing Statement I: "B got 5000 votes more than C." Let the votes be V_A, V_B, V_C . This gives us $V_B = V_C + 5000$. This tells us $V_B > V_C$, but we know nothing about V_A . A could have received the most or fewest votes. So, Statement I alone is not sufficient.

Analyzing Statement II: "A and C got the same number of votes." This means $V_A = V_C$. This doesn't tell us how B's votes compare. B could have more or fewer votes than A and C. So, Statement II alone is not sufficient.

Analyzing Statements I and II Together:

From statement II, we have $V_A = V_C$.

From statement I, we have $V_B > V_C$.

Combining these, we get $V_B > V_C = V_A$. This means B received the most votes.

The initial problem states that one candidate won with a clear majority. Since B received more votes than A and C, B must be the candidate who won with the majority.

Therefore, A did not win the election. We can answer the question "Was it A?" with a definite "No".

Step 3: Final Answer:

Since we need both statements to get a definitive answer, the correct option is (C).

Quick Tip

In data sufficiency, if you can determine the unique winner of a contest, you can answer the question "Did person X win?". Even if X did not win, that is a sufficient answer.

72. Is $x - y > 1$?

I. $x = 4y$

II. $x + y = 5$

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (C) Both statements I and II together are sufficient.

Solution:

Step 1: Understanding the Question:

We need to determine if the expression $x - y$ is definitively greater than 1.

Step 2: Key Formula or Approach:

We will analyze each statement to see if it constrains the values of x and y enough to answer the question. If not, we will solve the system of equations formed by both statements.

Step 3: Detailed Explanation:

Analyzing Statement I: $x = 4y$.

Substitute this into the inequality: Is $4y - y > 1$? This simplifies to "Is $3y > 1$?", or "Is $y > 1/3$?"

Since we don't know the value of y , we cannot answer. If $y = 1$, then $3 > 1$ (Yes). If $y = 0$, then $0 > 1$ (No). So, Statement I alone is not sufficient.

Analyzing Statement II: $x + y = 5$.

From this, $x = 5 - y$. Substitute into the inequality: Is $(5 - y) - y > 1$? This simplifies to "Is $5 - 2y > 1$ ", or "Is $4 > 2y$ ", or "Is $2 > y$?"

Again, we don't know the value of y . If $y = 1$, then $2 > 1$ (Yes). If $y = 3$, then $2 > 3$ (No). So, Statement II alone is not sufficient.

Analyzing Statements I and II Together:

We have a system of two linear equations:

$$x = 4y \quad \text{---(1)}$$

$$x + y = 5 \quad \text{---(2)}$$

Substitute equation (1) into equation (2):

$$(4y) + y = 5$$

$$5y = 5$$

$$y = 1$$

Now find x using equation (1):

$$x = 4(1) = 4$$

We have found unique values for x and y . Now we can check the condition: Is $x - y > 1$? Is $4 - 1 > 1$? Is $3 > 1$? Yes. This gives a definite "Yes" answer.

Step 4: Final Answer:

Both statements together are required to find a unique solution and answer the question. The correct option is (C).

Quick Tip

For algebraic data sufficiency questions, you don't always need to solve completely. The goal is to see if you *can* find a unique answer. If you have two distinct linear equations for two variables, you can generally find a unique solution.

73. What is the relation between Uma and Tilak?

I. Tilak is the brother of Sudhakar and he is doing his graduation.

II. Uma is the sister of Sudhakar.

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (C) Both statements I and II together are sufficient.

Solution:

Step 1: Understanding the Question:

The question asks to establish the specific family relationship between Uma and Tilak.

Step 2: Detailed Explanation:

Analyzing Statement I: "Tilak is the brother of Sudhakar...". This statement establishes the relationship between Tilak and Sudhakar. It gives no information about Uma. The part about graduation is irrelevant. So, Statement I alone is not sufficient.

Analyzing Statement II: "Uma is the sister of Sudhakar." This statement establishes the relationship between Uma and Sudhakar. It gives no information about Tilak. So, Statement II alone is not sufficient.

Analyzing Statements I and II Together:

From Statement I: Tilak is the brother of Sudhakar. (Tilak and Sudhakar are brothers).

From Statement II: Uma is the sister of Sudhakar. (Uma and Sudhakar are siblings).

Since Tilak is a brother to Sudhakar, and Uma is a sister to Sudhakar, they share a common sibling. This means Tilak and Uma are also siblings. Specifically, Tilak is Uma's brother, and Uma is Tilak's sister. This answers the question completely.

Step 3: Final Answer:

Both statements are necessary to connect Uma and Tilak through their common relative, Sudhakar. The correct option is (C).

Quick Tip

In blood relation problems, look for a common link between the individuals mentioned in the statements. Combining statements often allows you to build a complete family tree or chain of relationships.

74. Did Satya go to the temple yesterday?

I. If Satya goes to temple, Ramesh also go along with him.

II. Ramesh went to the market yesterday.

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (E) Both statements I and II together are not sufficient.

Solution:

Step 1: Understanding the Question:

We need a definite "Yes" or "No" answer to the question of whether Satya visited the temple yesterday.

Step 2: Detailed Explanation:

Analyzing Statement I: "If Satya goes to temple, Ramesh also go along with him." This is a conditional rule. Let S = "Satya goes to temple" and R = "Ramesh goes to temple". The rule is $S \implies R$. This rule alone does not tell us whether S happened. So, Statement I alone is not sufficient.

Analyzing Statement II: "Ramesh went to the market yesterday." This statement provides information about Ramesh's whereabouts, but it is about the market, not the temple. It also doesn't mention Satya. So, Statement II alone is not sufficient.

Analyzing Statements I and II Together:

We have the rule $S \implies R$. Statement II tells us that Ramesh went to the market. This does not confirm or deny whether Ramesh went to the temple. He could have gone to both the market and the temple, or only to the market. Since we cannot be sure if R (Ramesh goes to temple) is true or false, we cannot use the rule $S \implies R$ to deduce anything about S (Satya goes to temple). The information is insufficient.

Step 3: Final Answer:

The statements together do not provide enough information to answer the question. The correct option is (E).

Quick Tip

Information that seems related might be irrelevant. Knowing Ramesh went to the market doesn't impact whether he went to the temple unless a statement explicitly links the two activities (e.g., "he only went to one place").

75. Who is the tallest among A, B, C and D?**I. A is taller than C and D.****II. B is taller than C and shorter than D.**

- (A) Statement I alone is sufficient.
- (B) Statement II alone is sufficient.
- (C) Both statements I and II together are sufficient.
- (D) Either statement I or II is sufficient.
- (E) Both statements I and II together are not sufficient.

Correct Answer: (C) Both statements I and II together are sufficient.

Solution:**Step 1: Understanding the Question:**

We need to identify the single tallest person from a group of four (A, B, C, D).

Step 2: Detailed Explanation:**Analyzing Statement I:** "A is taller than C and D."

This gives us the relationships: $A > C$ and $A > D$.

This tells us that A is taller than C and D, but it does not provide any information about B. B could be taller than A, making B the tallest. We cannot be certain who is the tallest. So, Statement I alone is not sufficient.

Analyzing Statement II: "B is taller than C and shorter than D."

This gives us the relationship: $D > B > C$.

This provides an order for B, C, and D, but it does not include A. A could be taller than D, making A the tallest. We cannot be certain who is the tallest. So, Statement II alone is not

sufficient.

Analyzing Statements I and II Together:

From Statement I, we know: $A > D$.

From Statement II, we know: $D > B > C$.

We can combine these two inequalities:

Since $A > D$ and $D > B$, it follows that $A > B$.

Since $A > D$ and $D > C$, it follows that $A > C$.

We have established that A is taller than D, B, and C. Therefore, A is the tallest among the four.

Step 3: Final Answer:

Both statements together are needed to establish the complete order and identify the tallest person. The correct option is (C).

Quick Tip

For ordering and ranking problems, try to combine the inequalities from the statements to form a single chain of comparison. If you can establish a unique first or last position, the data is sufficient to answer questions about the "tallest", "shortest", "highest", etc.