IIITH UGEE 2026 SUPR

Question Paper with Solutions

Conducted by IIIT, Hyderabad

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

General Instructions

(i) Duration and Timing:

- The duration of the SUPR section is **60 minutes**.
- This is a Computer Based Test (CBT). A digital timer will be displayed on the top right corner of the screen.

(ii) Question Structure:

- This section consists of **50 Multiple Choice Questions (MCQs)**.
- Each question has four options, out of which only **one** is correct.

(iii) Marking Scheme:

- Each correct answer will be awarded **+1 mark**.
- For each incorrect answer, **0.25 marks** will be deducted (Negative Marking).
- No marks will be awarded or deducted for unattempted questions.

(iv) Navigation:

- Use the "Save & Next" button to lock your answer and move to the next question.
- Use the "Mark for Review" button if you wish to revisit a question later.
- Questions marked for review will be considered for evaluation only if an option is selected.

(v) Subject Coverage:

- The questions are based on the CBSE 11th and 12th grade syllabus for **Physics, Chemistry, and Mathematics**.

1. If $\frac{d^2y}{dx^2} = \cos\left(\frac{dy}{dx}\right)$, find the order and the degree of the resulting differential equation.

(A) Order 2, Degree 4

(B) Cannot be determined

(C) Order 3, Degree 1

(D) Data Insufficient

(E) None of these

Correct Answer: (E) None of these

Solution:

Concept:

- **Order** of a differential equation is the highest order derivative present in the equation.
- **Degree** is defined only when the differential equation can be expressed as a polynomial in derivatives.
- If derivatives appear inside transcendental functions such as $\sin, \cos, e^x, \log$, the degree is **not defined**.

Step 1: Identify the highest order derivative.

The given equation is

$$\frac{d^2y}{dx^2} = \cos\left(\frac{dy}{dx}\right)$$

The highest derivative present is

$$\frac{d^2y}{dx^2}$$

Thus, the **order** is

$$2$$

Step 2: Determine the degree.

The first derivative $\frac{dy}{dx}$ appears inside the cosine function, which is a transcendental function.

Hence, the differential equation **cannot be expressed as a polynomial in derivatives**.

Therefore, the **degree is not defined**.

Step 3: Select the correct option.

Order $= 2$ but degree **not defined**. Since none of the options match this exactly, the correct choice is

$$\text{None of these}$$

Quick Tip: Degree of a differential equation exists only when the equation can be written as a polynomial in derivatives. If derivatives appear inside functions like sin, cos, or log, the degree is not defined.

---

2. Find the value of $(125)^{\log_{625} 5}$**.**

(A) 15

(B) 25

(C) $5\sqrt{5}$

(D) 3

(E) None

Correct Answer: (E) None

Solution:

Concept: To evaluate expressions involving logarithms and exponents, rewrite the numbers in terms of the same base.

$$125 = 5^3, \qquad 625 = 5^4$$

Also, using the property:

$$\log_{a^m}(b^n) = \frac{n}{m}\log_a b$$

Step 1: Rewrite the given expression using base 5.

$$(125)^{\log_{625} 5} = (5^3)^{\log_{5^4} 5}$$

Step 2: Evaluate the logarithm.

$$\log_{5^4} 5 = \frac{1}{4}\log_5 5 = \frac{1}{4}$$

Step 3: Substitute the value.

$$(5^3)^{1/4} = 5^{3/4}$$

Step 4: Evaluate the final expression.

The value $5^{3/4} = \sqrt[4]{125}$ is approximately 3.34. Since this value does not match any of the options given, the correct choice is

$$\text{None}$$

Quick Tip: When logarithms and exponents appear together, first express all numbers using the same base. This often simplifies the logarithmic expression significantly.

---

3. If $f(x) = \int e^x \left(\frac{x^2+x+1}{\sqrt{x^2+1}}\right) dx$ such that the value of the function is 1 when x vanishes, find the value of $f(1)$.

(A) $\sqrt{3}\,e$

(B) $\sqrt{5}\,e$

(C) $\sqrt{2}\,e$

(D) e

Correct Answer: (C) $\sqrt{2}\,e$

Solution:

Concept: To evaluate an integral containing e^x multiplied by another expression, we often check if the integrand is the derivative of a product involving e^x. This helps simplify the integration directly.

Step 1: Observe the integrand structure.

$$f(x) = \int e^x \left(\frac{x^2+x+1}{\sqrt{x^2+1}}\right) dx$$

Notice that the expression resembles the derivative of

$$e^x\sqrt{x^2+1}$$

Step 2: Differentiate $e^x\sqrt{x^2+1}$.

Using the product rule,

$$\frac{d}{dx}\left(e^x\sqrt{x^2+1}\right)=e^x\sqrt{x^2+1}+e^x\frac{x}{\sqrt{x^2+1}}$$

$$=e^x\left(\frac{x^2+1+x}{\sqrt{x^2+1}}\right)$$

$$=e^x\left(\frac{x^2+x+1}{\sqrt{x^2+1}}\right)$$

Thus,

$$f(x)=e^x\sqrt{x^2+1}+C$$

Step 3: Use the given condition $f(0)=1$.

$$f(0)=e^0\sqrt{0^2+1}+C$$

$$1=1+C$$

$$C=0$$

Thus,

$$f(x)=e^x\sqrt{x^2+1}$$

Step 4: Find $f(1)$.

$$f(1)=e^1\sqrt{1^2+1}$$

$$f(1)=e\sqrt{2}$$

Hence,

$$f(1) = \sqrt{2}\,e$$

Quick Tip: When an integral contains e^x multiplied by another expression, check if the integrand matches the derivative of a product like $e^x g(x)$. This often simplifies the integration immediately.

---

4. Let P be any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Then, what would be the length of the segment of the tangent between the coordinate axes?

(A) a

(B) $2a$

(C) $3a$

(D) $4a$

(E) $5a$

Correct Answer: (A) a

Solution:

Concept: The curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

represents an **astroid**. A convenient parametric representation of this curve is

$$x = a\cos^3\theta, \qquad y = a\sin^3\theta$$

The equation of the tangent at any point θ is given by:

$$\frac{x}{a\cos\theta} + \frac{y}{a\sin\theta} = 1$$

Step 1: Find the intercepts of the tangent.

For the x-intercept (point A), set $y = 0$:

$$\frac{x}{a\cos\theta} = 1 \implies x = a\cos\theta$$

For the y-intercept (point B), set $x = 0$:

$$\frac{y}{a\sin\theta} = 1 \implies y = a\sin\theta$$

Step 2: Find the length of the intercept between the axes.

The length of the segment AB is calculated using the distance formula:

$$L = \sqrt{(a\cos\theta - 0)^2 + (0 - a\sin\theta)^2}$$

$$L = \sqrt{a^2\cos^2\theta + a^2\sin^2\theta}$$

$$L = a\sqrt{\cos^2\theta + \sin^2\theta} = a$$

Step 3: State the final result.

The length of the segment of the tangent intercepted between the coordinate axes is constant and equal to a.

$$\boxed{a}$$

Quick Tip: A fundamental property of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is that the length of the tangent intercepted between the coordinate axes is always constant and equal to a.

---

5. Number of ways of distributing 10 identical chocolates among 3 children such that everyone gets at least one?

(A) $2^4 + 3\cdot 2^5 - 2$

(B) $2^4 - 3\cdot 2^5 - 2$

(C) $2^5 + 4$

(D) $2^5 - 4$

Correct Answer: (C) $2^5 + 4$

Solution:

Concept: When distributing identical objects among distinct people with each receiving at least one object, we use the **stars and bars** method.

The number of ways to distribute n identical objects among r persons such that each gets at least one is:

$$\binom{n-1}{r-1}$$

Step 1: Convert the condition into an equation.

Let the chocolates received by the three children be $x_1, x_2,$ and x_3:

$$x_1 + x_2 + x_3 = 10$$

with the condition that each child receives at least one chocolate:

$$x_1, x_2, x_3 \geq 1$$

Step 2: Apply the stars and bars formula.

Using $n = 10$ and $r = 3$:

$$\text{Number of ways} = \binom{10-1}{3-1} = \binom{9}{2}$$

$$\binom{9}{2} = \frac{9 \times 8}{2 \times 1} = 36$$

Step 3: Match with the given options.

We evaluate the provided expressions:

- (A) $16 + 3(32) - 2 = 16 + 96 - 2 = 110$
- (B) $16 - 96 - 2 = -82$
- (C) $2^5 + 4 = 32 + 4 = 36$
- (D) $2^5 - 4 = 32 - 4 = 28$

The value 36 matches exactly with option (C).

Quick Tip: For distributing identical objects where each recipient must get at least one, the formula is $\binom{n-1}{r-1}$. If recipients can receive zero, the formula is $\binom{n+r-1}{r-1}$.

---

6. A block of mass m is moving on a horizontal frictionless surface with velocity v. It hits a spring of constant k and compresses it by a distance x. If the initial velocity is doubled, what will be the new compression?

(A) $2x$

(B) $\sqrt{2}x$

(C) $4x$

(D) $x/2$

Correct Answer: (A) $2x$

Solution:

Concept: When a moving block compresses a spring on a frictionless surface, the kinetic energy of the block converts completely into the potential energy of the spring.

$$\text{Kinetic Energy} = \text{Spring Potential Energy}$$

Thus,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

Step 1: Relate compression with velocity.

From the energy conservation equation:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \implies x^2 = \frac{m}{k}v^2$$

Taking the square root on both sides:

$$x = v\sqrt{\frac{m}{k}}$$

Hence, the compression of the spring x is directly proportional to the velocity v ($x \propto v$).

Step 2: Consider the velocity is doubled.

If the new velocity $v' = 2v$, the new compression x' will be:

$$x' \propto 2v$$

Since $x \propto v$, it follows that:

$$x' = 2x$$

Step 3: State the final result.

The new compression of the spring becomes

$$\boxed{2x}$$

Quick Tip: In spring–block energy problems, equate kinetic energy with spring potential energy. Since $x^2 \propto v^2$, the compression x is directly proportional to the velocity v. If velocity doubles, compression doubles; if velocity quadruples, compression quadruples.

---

7. If P_A^0 and P_B^0 are the vapour pressures of pure liquids A and B, what is the total pressure of an ideal solution where the mole fraction of A is 0.4?

(A) $0.4P_A^0 + 0.6P_B^0$

(B) $0.6P_A^0 + 0.4P_B^0$

(C) $P_A^0 + P_B^0$

(D) $0.5(P_A^0 + P_B^0)$

Correct Answer: (A) $0.4P_A^0 + 0.6P_B^0$

Solution:

Concept: According to **Raoult's Law**, the partial vapour pressure of each component in an ideal solution is proportional to its mole fraction.

The total vapour pressure of the solution is:

$$P_{\text{total}} = P_A + P_B$$

where:

$$P_A = x_A P_A^0, \quad P_B = x_B P_B^0$$

Thus,

$$P_{\text{total}} = x_A P_A^0 + x_B P_B^0$$

Step 1: Use the given mole fraction of component A.

The problem states:

$$x_A = 0.4$$

Step 2: Find the mole fraction of component B.

Since the sum of mole fractions in a binary solution is always 1:

$$x_A + x_B = 1 \implies x_B = 1 - 0.4 = 0.6$$

Step 3: Substitute into Raoult's law.

Substituting the values into the total pressure formula:

$$P_{\text{total}} = (0.4)P_A^0 + (0.6)P_B^0$$

Hence, the total vapour pressure is:

$$\boxed{0.4P_A^0 + 0.6P_B^0}$$

Quick Tip: For ideal solutions, always apply Raoult's law: total vapour pressure equals the sum of mole fraction multiplied by vapour pressure of each pure component. Remember that $x_A + x_B = 1$.

---

8. If the n^{th} term of an A.P. is $2n + 5$, find the sum of the first 10 terms.

(A) 110

(B) 155

(C) 160

(D) 165

Correct Answer: (C) 160

Solution:

Concept: In an Arithmetic Progression (A.P.), the sum of the first n terms is given by

$$S_n = \frac{n}{2}(a + l)$$

where a = first term, l = last term (n^{th} term), n = number of terms.

Step 1: Find the first term.

The given n^{th} term is:

$$a_n = 2n + 5$$

For the first term $(n = 1)$:

$$a = 2(1) + 5 = 7$$

Step 2: Find the 10th term.

For the last term $(n = 10)$:

$$l = a_{10} = 2(10) + 5 = 25$$

Step 3: Calculate the sum of the first 10 terms.

Using the formula for S_{10}:

$$S_{10} = \frac{10}{2}(7 + 25)$$

$$S_{10} = 5(32) = 160$$

Hence, the sum is

$$\boxed{160}$$

Quick Tip: If the n^{th} term of an A.P. is given as a linear expression $An + B$, the common difference is always A. You can also find the sum using $S_n = \frac{n}{2}[2a + (n-1)d]$.

9. A pair of fair dice is rolled. What is the probability that the second die lands on a higher value than the first?

(A) $\frac{1}{36}$

(B) $\frac{5}{36}$

(C) $\frac{1}{6}$

(D) $\frac{5}{12}$

Correct Answer: (D) $\frac{5}{12}$

Solution:

Concept: When two fair dice are rolled, the total number of possible outcomes is:

$$6 \times 6 = 36$$

Each outcome can be written as an ordered pair (x, y), where $x =$ value on the first die, $y =$ value on the second die.

We need to find the probability that $y > x$.

Step 1: List favorable outcomes.

We count the pairs (x, y) where the second value is greater than the first:

- If $x = 1$, $y \in \{2, 3, 4, 5, 6\}$ (5 outcomes)
- If $x = 2$, $y \in \{3, 4, 5, 6\}$ (4 outcomes)
- If $x = 3$, $y \in \{4, 5, 6\}$ (3 outcomes)
- If $x = 4$, $y \in \{5, 6\}$ (2 outcomes)
- If $x = 5$, $y \in \{6\}$ (1 outcome)

Total favorable outcomes:

$$5 + 4 + 3 + 2 + 1 = 15$$

Step 2: Compute the probability.

$$P = \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{15}{36}$$

Simplifying the fraction by dividing by 3:

$$P = \frac{5}{12}$$

Step 3: State the final answer.

The probability is:

$$\boxed{\frac{5}{12}}$$

Quick Tip: By symmetry, $P(y > x) = P(x > y)$. Since there are 6 outcomes where $x = y$, the number of outcomes where $x \neq y$ is $36 - 6 = 30$. Thus, $P(y > x) = \frac{30/2}{36} = \frac{15}{36} = \frac{5}{12}$.

---

10. What is the maximum number of electrons that can be accommodated in a subshell with orbital angular momentum quantum number $l = 3$?

(A) 6

(B) 10

(C) 14

(D) 18

Correct Answer: (C) 14

Solution:

Concept: The maximum number of electrons that can be accommodated in a subshell is given by the formula:

$$2(2l + 1)$$

where l = azimuthal (orbital angular momentum) quantum number.

This formula accounts for the fact that there are $(2l + 1)$ orbitals in a subshell, and each orbital can hold a maximum of 2 electrons (with opposite spins).

Step 1: Substitute the given value of l.

Given:

$$l = 3$$

Calculate the number of orbitals:

$$2l + 1 = 2(3) + 1 = 7$$

Thus, there are 7 orbitals in this subshell.

Step 2: Calculate the maximum number of electrons.

Since each orbital can accommodate 2 electrons:

$$\text{Maximum electrons} = 2 \times 7 = 14$$

Step 3: Identify the subshell.

When $l = 3$, it corresponds to the **f-subshell**. The sequence is:

- $l = 0$: s-subshell (2 electrons)
- $l = 1$: p-subshell (6 electrons)
- $l = 2$: d-subshell (10 electrons)
- $l = 3$: f-subshell (14 electrons)

Quick Tip: To quickly find subshell capacities, remember they increase in steps of 4 starting from 2: $s = 2$, $p = 6$, $d = 10$, $f = 14$. This is consistent with the formula $2(2l + 1)$.

---

11. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is a prime number?

(A) $\frac{5}{12}$
(B) $\frac{7}{12}$
(C) $\frac{1}{2}$
(D) $\frac{13}{36}$

Correct Answer: (A) $\frac{5}{12}$

Solution:

Concept: When two dice are rolled, the total number of possible outcomes is:

$$6 \times 6 = 36$$

We need to find the probability that the **sum of the numbers is a prime number**. The possible sums for two dice range from 2 to 12. The prime numbers in this range are:

$$2, 3, 5, 7, 11$$

Step 1: Find the number of outcomes for each prime sum.

- Sum $= 2$: $(1,1) \rightarrow$ **1 way**
- Sum $= 3$: $(1,2),(2,1) \rightarrow$ **2 ways**
- Sum $= 5$: $(1,4),(4,1),(2,3),(3,2) \rightarrow$ **4 ways**
- Sum $= 7$: $(1,6),(6,1),(2,5),(5,2),(3,4),(4,3) \rightarrow$ **6 ways**
- Sum $= 11$: $(5,6),(6,5) \rightarrow$ **2 ways**

Step 2: Calculate total favourable outcomes.

$$\text{Total favourable} = 1 + 2 + 4 + 6 + 2 = 15$$

Step 3: Find the probability.

$$P = \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{15}{36}$$

Simplifying by dividing both numerator and denominator by 3:

$$P = \frac{5}{12}$$

Quick Tip: To quickly find the number of ways to get a sum S on two dice:

- If $S \leq 7$, ways $= S - 1$.
- If $S > 7$, ways $= 13 - S$.

Using this: $1(2) + 2(3) + 4(5) + 6(7) + 2(11) = 15$.