IISER Mathematics Sample Paper-2

Duration: 45 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1** marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. 1 The value of $\lim_{x\to 0} \dfrac{\sin^{-1} x - x}{x^3}$ is

(A) $\frac{1}{6}$

(B) $-\frac{1}{6}$

(C) $\frac{1}{3}$

(D) 0

Q2. 2 If z is a complex number such that $|z| = 1$ and $\arg(z) = \frac{\pi}{4}$, then $\text{Re}(z^2)$ equals

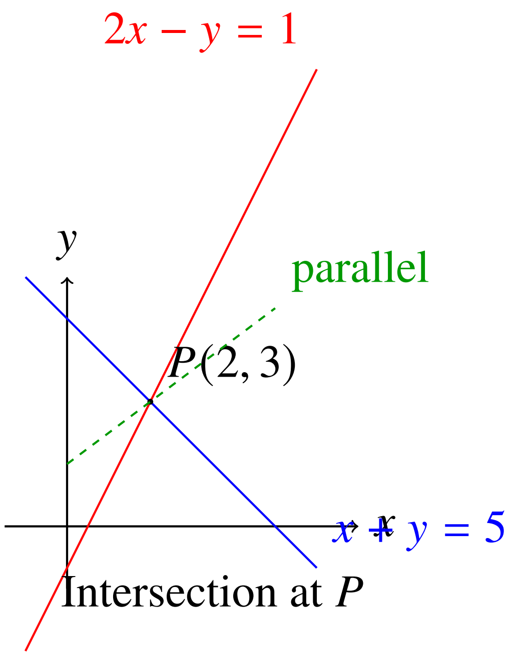
(A) $\frac{1}{\sqrt{2}}$

(B) 1

(C) 0

(D) $\frac{\sqrt{2}}{2}$

Q3. 3 The lines $x + y = 5$ and $2x - y = 1$ intersect at the point P. The equation of the line passing through P and parallel to the line $3x - 4y = 7$ is



(A) $3x - 4y - 6 = 0$

(B) $3x - 4y + 6 = 0$

(C) $3x + 4y + 6 = 0$

(D) $4x - 3y + 6 = 0$

Q4. 4 The value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{6}$

Q5. 5 If $x^2 + y^2 = 2xy$, then the value of $\frac{dy}{dx}$ at the point $(1, 1)$ is

(A) -1

(B) 0

(C) 2

(D) 1

Q6. 6 The number of 3-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 (without repetition) and which are divisible by 4 is

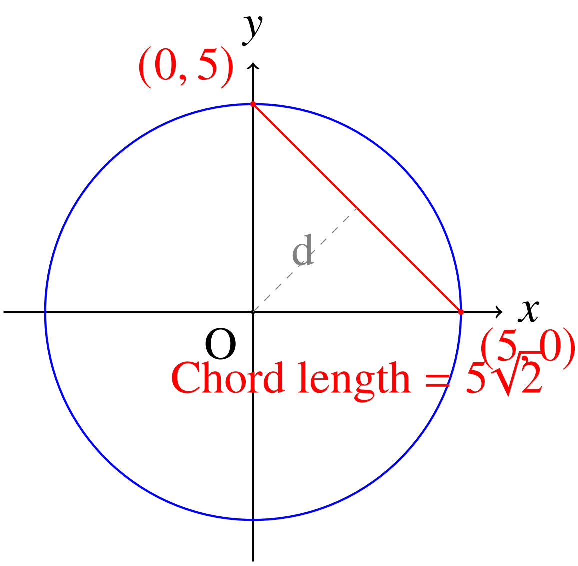
(A) 20

(B) 12

(C) 24

(D) 30

Q7. 7 The circle $x^2 + y^2 = 25$ and the straight line $x + y = 5$ intersect at two points. The length of the chord of intersection is



(A) $5\sqrt{2}$

(B) 10

(C) 5

(D) $5\sqrt{2}/2$

Q8. 8 A bag contains 5 red balls and 3 blue balls. Two balls are drawn at random without replacement. The probability that both balls drawn are red is

(A) $\frac{5}{14}$

(B) $\frac{5}{28}$

(C) $\frac{10}{56}$

(D) $\frac{1}{2}$

Q9. 9 The number of solutions of the equation $\sin 2x = \sin x$ in the closed interval $[0, 2\pi]$ is

(A) 4

(B) 5

(C) 3

(D) 6

$x y \pi/2 \pi 3\pi/2 2\pi \sin 2x \sin x$

Q10. 10 $\int_0^1 \frac{x}{1+x^2}\,dx$ equals

(A) $\ln 2$

(B) $\frac{\pi}{4}$

(C) 1

(D) $\frac{1}{2}\ln 2$

Q11. 11 If A is a 2×2 matrix with $\det(A) = 5$, then $\det(\text{adj}(A))$ equals

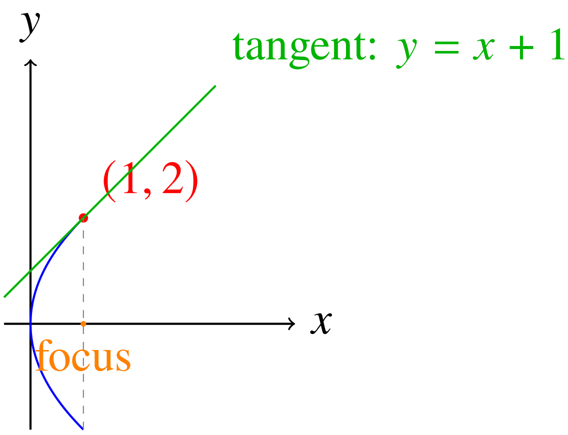
(A) 5

(B) 25

(C) 1

(D) 0

Q12. 12 For the parabola $y^2 = 4x$, the equation of the tangent at the point $(1, 2)$ is



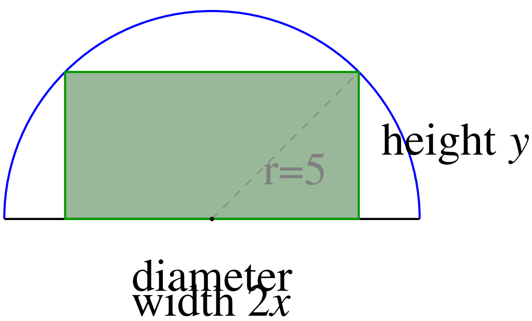
(A) $y = x - 1$

(B) $y = 2x + 1$

(C) $y = x + 1$

(D) $x + y = 2$

Q13. 13 A rectangle is inscribed in a semicircle of radius 5 with its base along the diameter. The maximum area of such a rectangle is



(A) 50

(B) 12.5

(C) 62.5

(D) 25

Q14. 14 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ equals

$\vec{b}\vec{c}$ $\vec{a}$

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

(A) $-\frac{3}{2}$

(B) -1

(C) 0

(D) $\frac{3}{2}$

Q15. 15 The general solution of the differential equation $\frac{dy}{dx} + y = e^x$ is

(A) $y = \frac{1}{2}e^x + ce^{-x}$

(B) $y = e^x + ce^{-x}$

(C) $y = \frac{1}{2}e^{2x} + ce^{-x}$

(D) $y = \frac{1}{2}e^x + c$

Detailed Solutions

Q1.

Solution

Concept:
To evaluate limits exhibiting indeterminate forms like $\frac{0}{0}$, series expansion is a powerful analytical method. By substituting the Maclaurin series expansion of the inverse trigonometric term into the expression, the terms can be simplified algebraically to eliminate the indeterminacy.

Solution:

(a) The given expression is $\lim_{x\to 0} \frac{\sin^{-1} x - x}{x^3}$. Substituting $x = 0$ yields the indeterminate form $\frac{0}{0}$.

(b) We recall the standard Maclaurin power series expansion for the inverse sine function:

$$\sin^{-1} x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \dots$$

(c) Substitute this series expansion into the original limit expression:

$$\lim_{x\to 0} \frac{\left(x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots\right) - x}{x^3}$$

(d) Simplify the numerator by canceling out the linear x terms:

$$\lim_{x\to 0} \frac{\frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots}{x^3}$$

(e) Factor out and divide each term by x^3 to evaluate the limit as x approaches zero:

$$\lim_{x\to 0} \left(\frac{1}{6} + \frac{3}{40}x^2 + \dots\right) = \frac{1}{6}$$

Final Answer: The value of the limit is $1_{\overline{6.}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept:
Any complex number can be expressed in polar coordinates as $z = |z|(\cos\theta + i\sin\theta)$, where $|z|$ is the modulus and θ represents the principal argument. De Moivre's Theorem states that for any integer n, $z^n = |z|^n(\cos n\theta + i\sin n\theta)$.
Solution:

(a) We are given a complex number z with a magnitude $|z| = 1$ and a principal argument $\arg(z) = \frac{\pi}{4}$.

(b) Expressing the complex number z in its standard polar form yields:

$$z = 1\cdot\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

(c) To compute z^2, we apply De Moivre's Theorem by squaring the magnitude and multiplying the argument by 2:

$$z^2 = 1^2\cdot\left(\cos\left(2\cdot\frac{\pi}{4}\right) + i\sin\left(2\cdot\frac{\pi}{4}\right)\right)$$

(d) Simplifying the resulting angle inside the trigonometric functions gives:

$$z^2 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

(e) Evaluating these standard trigonometric values gives $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$, which simplifies to $z^2 = 0 + i = i$.

(f) The real part of this number, denoted by $\mathrm{Re}(z^2)$, is the component without the imaginary unit, which equals 0.

Final Answer: The real part of z$^2 is 0$.

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept:
The coordinates of the intersection point P of two straight lines are found by solving their linear equations simultaneously. Parallel lines share identical slopes. The equation of a straight line with a known slope m passing through a specific coordinate point (x_1, y_1) is derived using the point-slope formula.

Solution:

(a) Find the intersection point P by solving the two given line equations: $x + y = 5$ and $2x - y = 1$.

(b) Adding these equations eliminates the variable y, resulting in $3x = 6$, which simplifies to $x = 2$.

(c) Substitute $x = 2$ back into the first linear equation to solve for y: $2 + y = 5 \implies y = 3$. Thus, the intersection point is exactly $P(2, 3)$.

(d) The required line must be parallel to $3x - 4y = 7$. Parallel lines share the same linear coefficients for x and y, so its equation can be written in the general form $3x - 4y + k = 0$.

(e) To solve for the unknown constant k, substitute the coordinates of point $P(2, 3)$ into this equation:

$$3(2) - 4(3) + k = 0 \implies 6 - 12 + k = 0 \implies k = 6$$

(f) Substituting $k = 6$ back into the general form gives the exact line equation: $3x - 4y + 6 = 0$.

Final Answer: The equation of the line is 3x - 4y + 6 = 0.

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept:
The sum of two inverse tangent functions can be simplified into a single inverse trigonometric term using the fundamental identity: $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$, which remains valid as long as the product of the arguments satisfies the inequality $AB < 1$.

Solution:

(a) Identify the parameters from the problem statement as $A = \frac{1}{2}$ and $B = \frac{1}{3}$.

(b) Verify the validity condition for the identity by checking the product of A and B:

$$AB = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

Since $\frac{1}{6} < 1$, the standard addition identity can be safely applied without adjustments.

(c) Substitute the values directly into the inverse tangent addition formula:

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)$$

(d) Simplify the fraction inside the argument by finding a common denominator for the numerator and denominator terms:

$$\text{Numerator} = \frac{3+2}{6} = \frac{5}{6}, \quad \text{Denominator} = 1 - \frac{1}{6} = \frac{5}{6}$$

(e) This reduces the expression to a basic inverse trigonometric value: $\tan^{-1}\left(\frac{5/6}{5/6}\right) = \tan^{-1}(1)$.

(f) The principal value for $\tan^{-1}(1)$ is known to be $\frac{\pi}{4}$.

Final Answer: The value of the expression is $\frac{\pi}{4}$.

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept:
Implicit differentiation is used when a curve is defined by an equation relating x and y implicitly. We differentiate both sides with respect to x, applying the chain rule to terms involving y and the product rule to mixed terms, then isolate the derivative term.

Solution:

(a) The given implicit equation represents a geometric curve: $x^2 + y^2 = 2xy$.

(b) Differentiate both sides of the equation with respect to x:

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(2xy)$$

(c) Apply the power rule to the first term, the chain rule to the second term, and the product rule to the right-hand side expression:

$$2x + 2y\frac{dy}{dx} = 2y(1) + 2x\frac{dy}{dx}$$

(d) Divide the entire equation by 2 to reduce the coefficients:

$$x + y\frac{dy}{dx} = y + x\frac{dy}{dx}$$

(e) Group the derivative terms together on one side of the equation:

$$y\frac{dy}{dx} - x\frac{dy}{dx} = y - x \implies (y - x)\frac{dy}{dx} = y - x$$

(f) For any point where $x \neq y$, dividing both sides by $(y - x)$ yields $\frac{dy}{dx} = 1$. Alternatively, notice that the original equation can be factored as $(x - y)^2 = 0$, which defines the straight line $y = x$. The derivative of this line at any point, including $(1, 1)$, is a constant equal to 1.

Final Answer: The value of the derivative is 1.

Answer: (D)

Go Back to Question 5

Q6.

Solution

Concept:
A multi-digit integer is divisible by 4 if and only if the sub-number formed by its last two digits is a multiple of 4. To find the total count of such numbers, we identify all valid ending pairs from the given set and calculate the remaining permutations.

Solution:

(a) We need to form a 3-digit number using digits from the set $\{1, 2, 3, 4, 5\}$ without repeating any digit.

(b) Analyze all possible two-digit combinations from this specific set that form a number divisible by 4. The valid pairs for the tens and units places are: 12, 24, 32, and 52.

(c) Case 1: The number ends in 12. The hundreds place can be filled by any of the remaining 3 digits from the set $\{3, 4, 5\}$. This gives 3 unique numbers.

(d) Case 2: The number ends in 24. The hundreds place can be filled by any of the remaining 3 digits from the set $\{1, 3, 5\}$. This gives 3 unique numbers.

(e) Case 3: The number ends in 32. The hundreds place can be filled by any of the remaining 3 digits from the set $\{1, 4, 5\}$. This gives 3 unique numbers.

(f) Case 4: The number ends in 52. The hundreds place can be filled by any of the remaining 3 digits from the set $\{1, 3, 4\}$. This gives 3 unique numbers.

(g) Since these cases are mutually exclusive, add the numbers together: $3 + 3 + 3 + 3 = 12$.

Final Answer: The number of 3-digit numbers is 12.

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:
The chord of intersection between a line and a circle is a line segment whose endpoints lie on the circle. The length of this chord can be computed by explicitly finding the coordinates of these intersection points and using the distance formula.

Solution:

(a) The equation of the circle is $x^2 + y^2 = 25$, which represents a circle centered at the origin $O(0, 0)$ with radius $R = 5$.

(b) The intersecting straight line is given by the linear equation $x + y = 5$.

(c) Express y in terms of x from the line equation: $y = 5 - x$. Substitute this into the circle equation to find the intersection points:

$$x^2 + (5 - x)^2 = 25$$

(d) Expand the squared term and combine like terms to simplify the quadratic equation:

$$x^2 + 25 - 10x + x^2 = 25 \implies 2x^2 - 10x = 0$$

(e) Factor out the common terms: $2x(x - 5) = 0$. This yields two distinct coordinates: $x = 0$ or $x = 5$.

(f) If $x = 0$, substituting back gives $y = 5$, establishing the first point $A(0, 5)$. If $x = 5$, substituting back gives $y = 0$, establishing the second point $B(5, 0)$.

(g) Find the distance between $A(0, 5)$ and $B(5, 0)$ using the standard coordinate distance formula:

$$\text{Length} = \sqrt{(5 - 0)^2 + (0 - 5)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

Final Answer: The length of the chord is $5\sqrt{2}$.

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept:
The probability of dependent sequential events is computed using conditional probability. When items are drawn without replacement, the total number of available items and the number of favorable items decrease with each successive selection.

Solution:

(a) The problem states that a bag contains 5 red balls and 3 blue balls, making the total number of balls in the bag initially $5 + 3 = 8$.

(b) Let R_1 represent the event that the first ball drawn from the bag is red. The probability of this first event is given by the ratio of favorable red balls to the total number of balls:

$$P(R_1) = \frac{5}{8}$$

(c) Since the selection is made without replacement, the remaining number of red balls in the bag becomes $5 - 1 = 4$, and the total number of remaining balls in the bag drops to $8 - 1 = 7$.

(d) Let R_2 represent the event that the second ball drawn from the bag is red. The conditional probability that the second ball is red, given that the first was red, is:

$$P(R_2 \mid R_1) = \frac{4}{7}$$

(e) To find the joint probability that both balls drawn are red, we apply the multiplication rule for dependent events:

$$P(R_1 \cap R_2) = P(R_1) \cdot P(R_2 \mid R_1) = \frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56}$$

(f) Simplifying the resulting fraction by dividing both the numerator and the denominator by their greatest common divisor, 4, yields $\frac{5}{14}$.

Final Answer: The probability that both balls drawn are red is $\frac{5}{14}$.

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept:
Trigonometric equations are solved by using standard formulas to transform expressions into factored algebraic forms. For general solutions of sine functions, $\sin\theta = \sin\alpha$ implies specific periodic relationships, which are then restricted to the designated closed interval.

Solution:

(a) The given trigonometric equation to solve within the interval $[0, 2\pi]$ is $\sin 2x = \sin x$.

(b) Apply the double-angle identity for the sine function, which states that $\sin 2x = 2\sin x \cos x$, and substitute it directly:

$$2\sin x \cos x = \sin x$$

(c) Move all terms to one side of the equation to avoid losing solutions during division:

$$2\sin x \cos x - \sin x = 0$$

(d) Factor out the common trigonometric term $\sin x$ from the expression:

$$\sin x(2\cos x - 1) = 0$$

(e) This factorization produces two independent linear trigonometric branches to solve: $\sin x = 0$ or $2\cos x - 1 = 0 \implies \cos x = \frac{1}{2}$.

(f) From the first branch $\sin x = 0$, the valid solutions inside the closed interval $[0, 2\pi]$ are $x = 0$, $x = \pi$, and $x = 2\pi$.

(g) From the second branch $\cos x = \frac{1}{2}$, the cosine function is positive in the first and fourth quadrants. The valid solutions are $x = \frac{\pi}{3}$ and $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

(h) Combining all distinct values gives the complete solution set: $\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi\}$, which contains exactly 5 elements.

Final Answer: The number of solutions of the equation is 5.

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept:
Definite integrals containing a fractional function where the numerator is a scalar multiple of the derivative of the denominator can be solved directly using integration by substitution. The limits of integration must be converted accordingly.

Solution:

(a) The problem requires evaluating the following definite integral:

$$I = \int_0^1 \frac{x}{1+x^2}\,dx$$

(b) Let us introduce a variable substitution by setting $u = 1 + x^2$.

(c) Differentiate both sides of the substitution equation with respect to x to find the differential relationship:

$$du = 2x\,dx \implies x\,dx = \frac{1}{2}\,du$$

(d) Determine the new integration limits for the variable u corresponding to the original limits of x:

$$\text{When } x = 0 : u = 1 + (0)^2 = 1$$

$$\text{When } x = 1 : u = 1 + (1)^2 = 2$$

(e) Substitute u, the modified differential element, and the updated limits back into the integral expression:

$$I = \int_1^2 \frac{\frac{1}{2}\,du}{u} = \frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

(f) Integrate the function using the standard natural logarithm integration rule:

$$I = \frac{1}{2}\left[\ln|u|\right]_1^2 = \frac{1}{2}(\ln 2 - \ln 1)$$

(g) Since the natural logarithm of 1 is equal to 0, the final simplified expression reduces directly to $\frac{1}{2}\ln 2$.

Final Answer: The value of the definite integral is $\frac{1}{2}\ln 2$.

Answer: (D)

Go Back to Question 10

Q11.

Solution

Concept:
For any square matrix A of dimension $n \times n$, there exists an established algebraic identity linking its determinant and the determinant of its adjugate matrix, which is derived from the property $A \cdot \text{adj}(A) = \det(A) \cdot I$. The identity is given by $\det(\text{adj}(A)) = (\det(A))^{n-1}$.
Solution:

(a) Identify the given properties of the matrix from the problem statement: matrix A is a square matrix with dimensions 2×2, which implies that its order is $n = 2$.

(b) The determinant value of the matrix is explicitly provided as $\det(A) = 5$.

(c) Recall the general formula that computes the determinant of an adjugate matrix for a square matrix of order n:
$$\det(\text{adj}(A)) = (\det(A))^{n-1}$$

(d) Substitute the order $n = 2$ and the determinant value $\det(A) = 5$ directly into this mathematical relationship:
$$\det(\text{adj}(A)) = 5^{2-1}$$

(e) Simplify the exponent in the equation:
$$\det(\text{adj}(A)) = 5^1 = 5$$

(f) This shows that for any second-order square matrix, the determinant of its adjugate matrix is identically equal to the determinant of the original matrix itself.

Final Answer: The value of $\det(\text{adj}(A)) is 5$.

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept:
The equation of a tangent line to a standard conic section like a parabola $y^2 = 4ax$ at a specific point (x_1, y_1) can be constructed directly by applying the substitution rule of coordinate geometry, where y^2 is replaced by yy_1 and $2x$ is replaced by $(x + x_1)$.
Solution:

(a) The given equation of the parabola is $y^2 = 4x$. Comparing this with the standard equation $y^2 = 4ax$ reveals that the parameter $a = 1$.

(b) The point of tangency is specified on the curve as $(x_1, y_1) = (1, 2)$.

(c) The general equation of a tangent line to the parabola $y^2 = 4ax$ at a point (x_1, y_1) is derived using calculus or substitution as:

$$yy_1 = 2a(x + x_1)$$

(d) Substitute the value of $a = 1$ and the coordinates $x_1 = 1$ and $y_1 = 2$ directly into the general tangent line formula:

$$y(2) = 2(1)(x + 1)$$

(e) Simplify the resulting linear algebraic equation:

$$2y = 2(x + 1)$$

(f) Divide both sides of the expression by 2 to isolate the variable terms and write the line equation in explicit form:

$$y = x + 1$$

Final Answer: The equation of the tangent is y = x + 1.

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept:
Optimization problems involving geometric shapes inscribed inside boundaries are solved by expressing the total area as a single-variable function using the boundary constraints, and then applying differential calculus to locate the global maximum point.
Solution:

(a) Let the rectangle be inscribed in a semicircle centered at the origin with radius $R = 5$. Let the vertices on the diameter along the x-axis be $(-x, 0)$ and $(x, 0)$, so the total width is $2x$.

(b) The upper vertices lie on the circular boundary arc defined by the equation $x^2 + y^2 = 5^2 \implies y = \sqrt{25 - x^2}$, where y represents the height of the rectangle.

(c) Write the objective function for the area A of the rectangle in terms of the variable x:

$$A = \text{width} \cdot \text{height} = 2x \cdot \sqrt{25 - x^2}$$

(d) To maximize the area, it is mathematically simpler to maximize the square of the area, A^2, which we define as a function $f(x)$:

$$f(x) = A^2 = 4x^2(25 - x^2) = 100x^2 - 4x^4$$

(e) Differentiate $f(x)$ with respect to x and set the derivative equal to zero to find the critical points:

$$f'(x) = 200x - 16x^3 = 0 \implies 8x(25 - 2x^2) = 0$$

(f) Since width must be positive, $x \neq 0$, leaving $25 - 2x^2 = 0 \implies x^2 = \frac{25}{2} \implies x = \frac{5}{\sqrt{2}}$.

(g) Substitute $x^2 = \frac{25}{2}$ back into the area equation to compute the maximum value:

$$A = 2\left(\frac{5}{\sqrt{2}}\right)\sqrt{25 - \frac{25}{2}} = \frac{10}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = \frac{50}{2} = 25$$

Final Answer: The maximum area of such a rectangle is 25.

Answer: (D)

Go Back to Question 13

Q14.

Solution

Concept:
Vector dot products and vector addition properties can be exploited to evaluate scalar combinations. Squaring a vector sum identities relates individual vector magnitudes, which are equal to 1 for unit vectors, to their mutual dot products.

Solution:

(a) We are given that $\vec{a}$, $\vec{b}$, and $\vec{c}$ are unit vectors, which mathematically means that their magnitudes satisfy $|\vec{a}| = 1$, $|\vec{b}| = 1$, and $|\vec{c}| = 1$.

(b) The vectors satisfy the linear equilibrium equation:

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

(c) Take the dot product of the vector sum with itself, or square both sides of the equation:

$$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = \vec{0} \cdot \vec{0}$$

(d) Expand the left-hand side expression using the distributive property of vector dot products:

$$\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

(e) Substitute the known squared magnitudes $|\vec{a}|^2 = 1$, $|\vec{b}|^2 = 1$, and $|\vec{c}|^2 = 1$ into the expanded relation:

$$1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

(f) Simplify the constants and isolate the required scalar sum expression:

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \implies 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$$

(g) Dividing by 2 yields the final value: $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$.

Final Answer: The value of the expression is - $\frac{3}{2}$.

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:

A first-order linear differential equation of the general mathematical form $\frac{dy}{dx} + P(x)y = Q(x)$ is solved by calculating an integrating factor, defined as $I.F. = e^{\int P(x)\,dx}$. The general solution is then given by the formula $y \cdot (I.F.) = \int Q(x) \cdot (I.F.)\,dx + c$.

Solution:

(a) The given first-order differential equation is expressed as:

$$\frac{dy}{dx} + y = e^x$$

(b) By comparing this expression with the standard linear differential form, we identify the functional coefficients as $P(x) = 1$ and $Q(x) = e^x$.

(c) Calculate the integrating factor ($I.F.$) by evaluating the exponential integral of $P(x)$:

$$I.F. = e^{\int 1\,dx} = e^x$$

(d) Set up the standard general solution formula by multiplying the dependent variable by the integrating factor:

$$y \cdot e^x = \int e^x \cdot e^x\,dx + c$$

(e) Simplify the integrand on the right side by combining the exponential terms using regular exponent laws:

$$y \cdot e^x = \int e^{2x}\,dx + c$$

(f) Perform the integration step on the exponential function:

$$y \cdot e^x = \frac{1}{2}e^{2x} + c$$

(g) Isolate the variable y by multiplying the entire equation by e^{-x} to find the explicit solution:

$$y = \frac{1}{2}e^{2x} \cdot e^{-x} + c \cdot e^{-x} \implies y = \frac{1}{2}e^x + ce^{-x}$$

Final Answer: The general solution of the differential equation is y = $\frac{1}{2}e^x + ce^{-x}$.

Answer: (A)

Go Back to Question 15

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	B	4	A	5	D
6	B	7	A	8	A	9	B	10	D
11	A	12	C	13	D	14	A	15	A