IISER Physics Sample Paper-10

Duration: 45 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: $-\mathbf{1}$ marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. A solid sphere of mass M and radius R rolls without slipping down a rough incline of height h. At the bottom, it enters a vertical circular track of radius $4R$. The minimum value of h required so that the sphere just completes the loop without losing contact is:

(A) $\frac{27R}{10}$

(B) $\frac{54R}{35}$

(C) $\frac{27R}{7}$

(D) $\frac{45R}{14}$

Q2. A particle moves in a straight line such that its acceleration is given by $a = -kv^2$, where k is a positive constant. If its initial speed is u, the distance travelled before its speed reduces to $u/2$ is:

(A) $\frac{\ln 2}{k}$

(B) $\frac{1}{2ku}$

(C) $\frac{1}{ku}$

(D) $\dfrac{3}{2ku}$

Q3. A planet has radius R and density varying with distance r from its centre as $\rho(r) = \rho_0 \left(1 - \dfrac{r}{R}\right)$. The gravitational field inside the planet is maximum at a distance r from the centre equal to:

(A) $\dfrac{R}{4}$

(B) $\dfrac{R}{2}$

(C) $\dfrac{2R}{3}$

(D) $\dfrac{3R}{4}$

Q4. A particle moves in one dimension under the force $F(x) = F_0 \sin\left(\dfrac{\pi x}{a}\right)$, where $0 \le x \le a$. If the particle starts from rest at $x = 0$, then its maximum kinetic energy during the motion is:

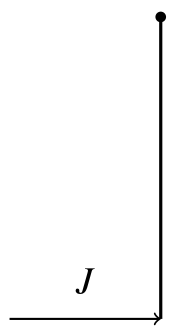
(A) $\dfrac{F_0 a}{\pi}$

(B) $\dfrac{2F_0 a}{\pi}$

(C) $\dfrac{F_0 a}{2\pi}$

(D) $\dfrac{4F_0 a}{\pi}$

Q5. A uniform rod of length L is pivoted at one end and initially held vertically. A small horizontal impulse J is applied at its lower end. The angular velocity immediately after the impulse is:



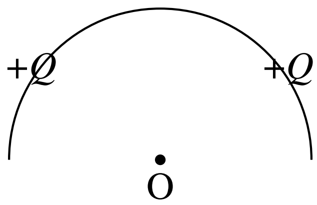
(A) $\dfrac{3J}{ML}$

(B) $\dfrac{6J}{ML}$

(C) $\dfrac{3J}{2ML}$

(D) $\dfrac{J}{ML}$

Q6. A charge $+Q$ is uniformly distributed over a semicircular arc of radius R. The magnitude of the electric field at the centre of curvature is:



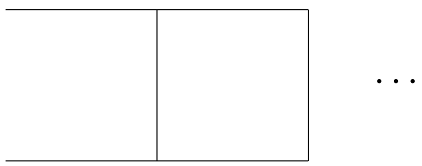
(A) $\dfrac{kQ}{R^2}$

(B) $\dfrac{2kQ}{\pi R^2}$

(C) $\dfrac{kQ}{\pi R^2}$

(D) $\dfrac{4kQ}{\pi R^2}$

Q7. An infinite ladder network is formed using resistors R as shown. Each section contains one resistor in series followed by one resistor connected in parallel to the remaining infinite network. The equivalent resistance between the two terminals is:



(A) R

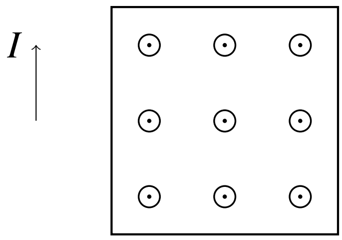
(B) $\sqrt{2}R$

(C) $\dfrac{1+\sqrt{5}}{2}R$

(D) $2R$

Q8. A conducting square loop of side a carries current I. It is placed in a magnetic field $\vec{B} = B_0(x/a)\hat{k}$, where x is measured along one side of the loop. The net magnetic force on the loop is:

$B = B_0(x/a)$



(A) Zero

(B) IB_0a

(C) $\dfrac{IB_0a}{2}$

(D) $2IB_0a$

Q9. A circular loop of radius R and resistance r is placed in a magnetic field varying as $B = B_0e^{-kt}$. The total charge that flows through the loop from $t = 0$ to $t = \infty$ is:

(A) $\dfrac{\pi R^2 B_0}{r}$

(B) $\dfrac{\pi R^2 B_0}{kr}$

(C) $\dfrac{k\pi R^2 B_0}{r}$

(D) $\dfrac{\pi R^2 B_0}{2r}$

Q10. An electron in the first excited state of a hydrogen atom absorbs a photon and gets ionized. The maximum wavelength of the photon capable of ionizing the atom is:

(A) 364.6 nm

(B) 121.6 nm

(C) 486.1 nm

(D) 91.2 nm

Q11. A radioactive nucleus A decays into B, which further decays into a stable nucleus C. If the decay constants of A and B are λ and 2λ respectively, then the number of nuclei of B attains a maximum at time:

(A) $\frac{\ln 2}{\lambda}$

(B) $\frac{2\ln 2}{\lambda}$

(C) $\frac{\ln 2}{2\lambda}$

(D) $\frac{1}{\lambda}$

Q12. A point object is placed on the principal axis of a convex lens of focal length f. A plane mirror is placed perpendicular to the principal axis at a distance $2f$ from the lens. The object position for which the final image coincides with the object is:

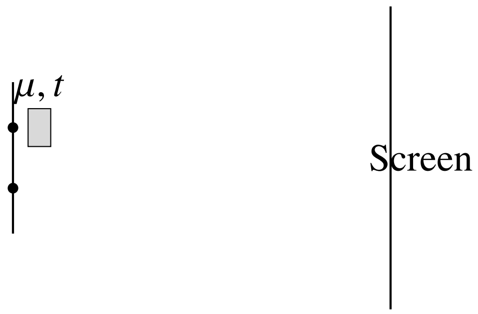
(A) f

(B) $2f$

(C) $\frac{3f}{2}$

(D) Any position

Q13. In a Young's double-slit experiment, one slit is covered by a thin transparent sheet of refractive index μ and thickness t. If the wavelength used is λ, then the central fringe shifts by:



(A) $\frac{(\mu-1)t}{\lambda}$ fringe widths

(B) $\frac{\mu t}{\lambda}$ fringe widths

(C) $\frac{t}{\mu\lambda}$ fringe widths

(D) $\dfrac{(\mu + 1)t}{\lambda}$ fringe widths

Q14. One mole of a monoatomic ideal gas undergoes a process in which pressure varies with volume as $P = \alpha V^2$. If the volume changes from V_1 to $2V_1$, the heat supplied to the gas is:

(A) $\dfrac{15}{2}\alpha V_1^3$

(B) $9\alpha V_1^3$

(C) $\dfrac{21}{2}\alpha V_1^3$

(D) $12\alpha V_1^3$

Q15. Two SHMs along the same line are represented by $x_1 = A \sin \omega t$ and $x_2 = A \sin(\omega t + \phi)$. If the resultant amplitude equals A, the possible value of ϕ is:

(A) 60°

(B) 90°

(C) 120°

(D) 180°

Detailed Solutions

Q1.

Solution

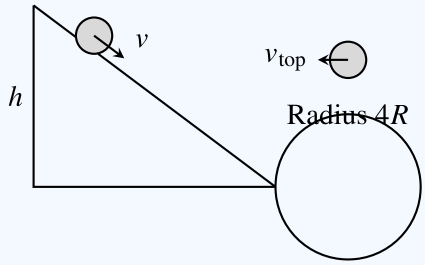
Concept:

For a solid sphere ($I = \frac{2}{5}MR^2$) rolling without slipping ($\omega = v/R$), the total kinetic energy is:

$$K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{7}{10}Mv^2$$

At the highest point of a vertical loop with effective radius R_{eff}, the minimum speed required to maintain contact ($N = 0$) is:

$$v_{\text{top}} = \sqrt{gR_{\text{eff}}}$$



Solution: The effective radius of the path followed by the sphere's center of mass is:

$$R_{\text{eff}} = R_c - R = 4R - R = 3R$$

To complete the loop, the critical velocity at the top is $v_{\text{top}}^2 = gR_{\text{eff}}$. Using conservation of mechanical energy (including rotational kinetic energy for a rolling sphere, $K = \frac{7}{10}Mv^2$):

$$Mgh = Mg(2R_{\text{eff}}) + \frac{7}{10}Mv_{\text{top}}^2$$

Substituting $v_{\text{top}}^2 = gR_{\text{eff}}$:

$$h = 2R_{\text{eff}} + \frac{7}{10}R_{\text{eff}} = \frac{27}{10}R_{\text{eff}}$$

Using $R_{\text{eff}} = 3R$:

$$h = \frac{27}{10}(3R) = 8.1R$$

Final Answer: $\boxed{\frac{27R}{7}}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: The acceleration a of a particle moving in a straight line is related to its velocity v and position x by the kinematic relation:

$$a = v\frac{dv}{dx}$$

Solution: We are given the acceleration as a function of velocity:

$$a = -kv^2$$

Substituting the kinematic relation:

$$v\frac{dv}{dx} = -kv^2$$

Assuming $v \neq 0$, we can divide both sides by v:

$$\frac{dv}{dx} = -kv$$

We now separate the variables to integrate:

$$\frac{1}{v}\,dv = -k\,dx$$

Integrate both sides from the initial state (position $x = 0$, speed $v = u$) to the final state (position $x = d$, speed $v = u/2$):

$$\int_u^{u/2} \frac{1}{v}\,dv = \int_0^d -k\,dx$$

$$[\ln v]_u^{u/2} = -k\,[x]_0^d$$

$$\ln\left(\frac{u/2}{u}\right) = -kd$$

$$\ln\left(\frac{1}{2}\right) = -kd \implies -\ln 2 = -kd$$

$$d = \frac{\ln 2}{k}$$

Final Answer: $\boxed{\frac{\ln 2}{k}}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: The gravitational field strength $E(r)$ inside a spherically symmetric planet at a distance r from its center is determined only by the mass $M(r)$ contained within the sphere of radius r:

$$E(r) = \frac{GM(r)}{r^2}$$

The mass $M(r)$ is found by integrating the density function $\rho(r)$ over the volume of the sphere of radius r:

$$M(r) = \int_0^r \rho(r') \cdot 4\pi r'^2 \, dr'$$

Solution: Given the density variation $\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right)$:

$$M(r) = 4\pi\rho_0 \int_0^r \left(1 - \frac{r'}{R}\right) r'^2 \, dr'$$

$$M(r) = 4\pi\rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right)$$

Now, substitute $M(r)$ into the expression for the gravitational field $E(r)$:

$$E(r) = \frac{G}{r^2} \cdot 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right) = 4\pi G\rho_0 \left(\frac{r}{3} - \frac{r^2}{4R}\right)$$

To find the distance r where $E(r)$ is maximum, we differentiate $E(r)$ with respect to r and set the derivative to zero:

$$\frac{dE}{dr} = 4\pi G\rho_0 \left(\frac{1}{3} - \frac{2r}{4R}\right) = 0$$

$$\frac{1}{3} - \frac{r}{2R} = 0 \implies \frac{r}{2R} = \frac{1}{3}$$

$$r = \frac{2R}{3}$$

Final Answer: $\boxed{\frac{2R}{3}}$

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: According to the Work-Energy Theorem, the net work done on a particle is equal to the change in its kinetic energy:

$$K(x) - K(0) = \int_0^x F(x')\,dx'$$

Since the particle starts from rest at $x = 0$, we have $K(0) = 0$. Hence, the kinetic energy at any position x is:

$$K(x) = \int_0^x F(x')\,dx'$$

Solution: Substituting the given force function $F(x) = F_0 \sin\left(\frac{\pi x}{a}\right)$:

$$K(x) = \int_0^x F_0 \sin\left(\frac{\pi x'}{a}\right) dx'$$

$$K(x) = F_0 \left[-\frac{a}{\pi} \cos\left(\frac{\pi x'}{a}\right)\right]_0^x$$

$$K(x) = \frac{F_0 a}{\pi} \left[1 - \cos\left(\frac{\pi x}{a}\right)\right]$$

To find the maximum kinetic energy, we maximize the term inside the brackets. Since $0 \leq x \leq a$, the function $\cos\left(\frac{\pi x}{a}\right)$ ranges from $+1$ to -1. The maximum value of the expression occurs when $\cos\left(\frac{\pi x}{a}\right) = -1$ (which occurs at $x = a$):

$$K_{\text{max}} = \frac{F_0 a}{\pi}[1 - (-1)] = \frac{2F_0 a}{\pi}$$

Final Answer: $\boxed{\frac{2F_0 a}{\pi}}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The angular impulse-angular momentum relation about a pivot is:

$$\tau_{\text{impulse}} = I\omega$$

where τ_{impulse} is the torque impulse, I is the moment of inertia about the pivot, and ω is the resulting angular velocity.

Solution: For a uniform rod of mass M and length L pivoted at one end, the moment of inertia I about the pivot is:

$$I = \frac{1}{3}ML^2$$

A horizontal linear impulse J is applied at the lower end of the rod, which is at a distance L from the pivot. The torque impulse about the pivot is:

$$\tau_{\text{impulse}} = J \cdot L$$

Applying the angular impulse-momentum theorem:

$$J \cdot L = I\omega$$

$$J \cdot L = \left(\frac{1}{3}ML^2\right)\omega$$

Solving for ω:

$$\omega = \frac{3J \cdot L}{ML^2} = \frac{3J}{ML}$$

Final Answer: $\boxed{\frac{3J}{ML}}$

Answer: (A)

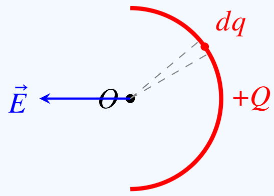
Go Back to Question 5

Q6.

Solution

Concept: For a continuous charge distribution, the electric field is calculated by integrating the field contributions $d\vec{E}$ from each charge element dq:

$$d\vec{E} = \frac{k\,dq}{R^2}\hat{r}$$



Solution: Let us place the semicircular arc symmetrically about the x-axis, spanning from $\theta = -\pi/2$ to $\theta = \pi/2$. The linear charge density is:

$$\lambda = \frac{Q}{\pi R}$$

An infinitesimal charge element dq on the arc subtending an angle $d\theta$ is:

$$dq = \lambda R\,d\theta = \frac{Q}{\pi}d\theta$$

The electric field $d\vec{E}$ at the center due to this element dq points in the direction opposite to the element:

$$d\vec{E} = -\frac{k\,dq}{R^2}\cos\theta\,\hat{i} - \frac{k\,dq}{R^2}\sin\theta\,\hat{j}$$

Due to symmetry, the y-components cancel out upon integration, leaving only the x-component:

$$E = \int_{-\pi/2}^{\pi/2}\frac{k\,dq}{R^2}\cos\theta = \int_{-\pi/2}^{\pi/2}\frac{kQ}{\pi R^2}\cos\theta\,d\theta$$

$$E = \frac{kQ}{\pi R^2}\Big[\sin\theta\Big]_{-\pi/2}^{\pi/2} = \frac{kQ}{\pi R^2}[1-(-1)] = \frac{2kQ}{\pi R^2}$$

Final Answer: $\boxed{\frac{2kQ}{\pi R^2}}$

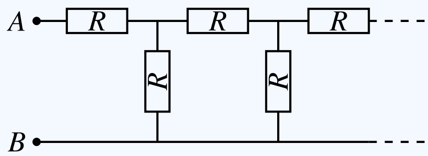
Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: An infinite network can be simplified by identifying its repeating unit. Since the network contains an infinite number of sections, removing the first section leaves a network with the exact same equivalent resistance R_{eq}.



Solution: We can replace the entire infinite network to the right of the first parallel branch with a single equivalent resistor R_{eq}. The modified circuit now consists of:

- One series resistor R
- One parallel branch containing a resistor R in parallel with the equivalent resistance R_{eq}

The total equivalent resistance is therefore:

$$R_{eq} = R + \frac{R \cdot R_{eq}}{R + R_{eq}}$$

Multiplying both sides by $R + R_{eq}$:

$$R_{eq}(R + R_{eq}) = R(R + R_{eq}) + RR_{eq}$$

$$RR_{eq} + R_{eq}^2 = R^2 + 2RR_{eq}$$

$$R_{eq}^2 - RR_{eq} - R^2 = 0$$

Solving using the quadratic formula:

$$R_{eq} = \frac{R \pm \sqrt{(-R)^2 - 4(1)(-R^2)}}{2} = \frac{1 + \sqrt{5}}{2}R$$

(we select the positive root because resistance must be positive).

Final Answer: $\boxed{\frac{1 + \sqrt{5}}{2}R}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: The magnetic force on a straight current-carrying segment of length vector $\vec{l}$ in a magnetic field $\vec{B}$ is:

$$\vec{F} = \int I(d\vec{l} \times \vec{B})$$

Solution: Let us position the square loop of side a in the xy-plane with its vertices at:

$$P_1(0,0), \quad P_2(a,0), \quad P_3(a,a), \quad P_4(0,a)$$

Let the current I flow in a counterclockwise direction. The magnetic field is given as $\vec{B} = B_0\left(\frac{x}{a}\right)\hat{k}$. We calculate the force on each of the four segments:

(a) **Segment 1 (from P_1 to P_2 along the x-axis, $y = 0$):** Here, $d\vec{l} = dx\hat{i}$, and x goes from 0 to a.

$$\vec{F}_1 = \int_0^a I\left(dx\hat{i} \times B_0\frac{x}{a}\hat{k}\right) = -\frac{IB_0}{a}\int_0^a x\,dx\,\hat{j} = -\frac{IB_0a}{2}\hat{j}$$

(b) **Segment 2 (from P_2 to P_3 parallel to the y-axis, $x = a$):** Here, $d\vec{l} = dy\hat{j}$, and y goes from 0 to a. Since $x = a$, the field is $\vec{B} = B_0\hat{k}$.

$$\vec{F}_2 = \int_0^a I\left(dy\hat{j} \times B_0\hat{k}\right) = IB_0a\hat{i}$$

(c) **Segment 3 (from P_3 to P_4 along $y = a$):** Here, $d\vec{l} = dx\hat{i}$, and x goes from a to 0.

$$\vec{F}_3 = \int_a^0 I\left(dx\hat{i} \times B_0\frac{x}{a}\hat{k}\right) = \frac{IB_0a}{2}\hat{j}$$

(d) **Segment 4 (from P_4 to P_1 along the y-axis, $x = 0$):** Since $x = 0$, the magnetic field is $\vec{B} = \vec{0}$, which means:

$$\vec{F}_4 = \vec{0}$$

The net force on the loop is the vector sum of the forces on the individual segments:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$\vec{F}_{\text{net}} = \left(-\frac{IB_0a}{2}\hat{j}\right) + (IB_0a\hat{i}) + \left(\frac{IB_0a}{2}\hat{j}\right) + \vec{0} = IB_0a\hat{i}$$

The magnitude of this net force is IB_0a.

Final Answer: $\boxed{IB_0a}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: By Faraday's Law, the induced EMF is $\mathcal{E} = -\frac{d\Phi}{dt}$. By Ohm's Law, the current in the loop is:

$$i = \frac{\mathcal{E}}{r} = -\frac{1}{r}\frac{d\Phi}{dt}$$

The charge dq flowing through the circuit in a time interval dt is:

$$dq = i\,dt = -\frac{1}{r}d\Phi$$

Integrating this gives the total charge Q that flows during the interval:

$$Q = \int dq = \frac{\Delta\Phi}{r} = \frac{\Phi_{\text{initial}} - \Phi_{\text{final}}}{r}$$

Solution: Let us calculate the magnetic flux through the circular loop of radius R:

$$\Phi(t) = B(t) \cdot A = \left(B_0 e^{-kt}\right) \cdot \left(\pi R^2\right)$$

At $t = 0$:

$$\Phi_{\text{initial}} = B_0 \pi R^2$$

As $t \to \infty$:

$$\Phi_{\text{final}} = \lim_{t\to\infty} B_0 e^{-kt} \pi R^2 = 0$$

Now, substitute these into the charge formula:

$$Q = \frac{B_0 \pi R^2 - 0}{r} = \frac{\pi R^2 B_0}{r}$$

Notice that the total charge flown depends only on the net change in magnetic flux and the resistance, and is independent of the rate constant k.

Final Answer: $\boxed{\frac{\pi R^2 B_0}{r}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The energy levels of a hydrogen atom are given by:

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

To ionize an electron from a state with quantum number n to $n \to \infty$ (where $E_\infty = 0$), the minimum energy photon required is:

$$E_{\text{photon}} = E_\infty - E_n = \frac{13.6}{n^2}\,\text{eV}$$

The wavelength λ of the photon is related to its energy by:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

Solution: The first excited state of a hydrogen atom corresponds to $n = 2$. The ionization energy from this state is:

$$E_{\text{photon}} = \frac{13.6}{2^2} = 3.4\,\text{eV}$$

The maximum wavelength λ_{max} corresponds to this minimum energy threshold:

$$\lambda_{\text{max}} = \frac{hc}{E_{\text{photon}}}$$

Using the Rydberg formula:

$$\frac{1}{\lambda_{\text{max}}} = R_\infty\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R_\infty}{4}$$

$$\lambda_{\text{max}} = \frac{4}{R_\infty} = \frac{4}{1.097 \times 10^7\,\text{m}^{-1}} \approx 3.646 \times 10^{-7}\,\text{m} = 364.6\,\text{nm}$$

Final Answer: $\boxed{364.6\,\text{nm}}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: For a consecutive radioactive decay process $A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$, the rate of change of the population of nucleus B is:

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

Since $N_A(t) = N_0 e^{-\lambda_A t}$, the solution for $N_B(t)$ with $N_B(0) = 0$ is:

$$N_B(t) = \frac{\lambda_A N_0}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$$

Solution: We are given $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$. Substituting these values:

$$N_B(t) = \frac{\lambda N_0}{2\lambda - \lambda}\left(e^{-\lambda t} - e^{-2\lambda t}\right) = N_0\left(e^{-\lambda t} - e^{-2\lambda t}\right)$$

To find the time t at which $N_B(t)$ is maximum, we set its derivative with respect to time to zero:

$$\frac{dN_B}{dt} = N_0\left(-\lambda e^{-\lambda t} + 2\lambda e^{-2\lambda t}\right) = 0$$

$$\lambda e^{-\lambda t} = 2\lambda e^{-2\lambda t}$$

$$e^{-\lambda t} = 2e^{-2\lambda t} \implies e^{\lambda t} = 2$$

$$t = \frac{\ln 2}{\lambda}$$

Final Answer: $\boxed{\frac{\ln 2}{\lambda}}$

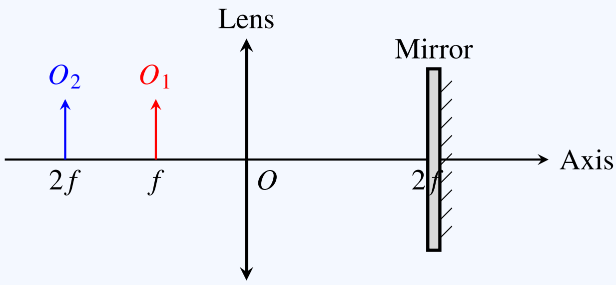
Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: For the final image to coincide with the object, the light rays must retrace their paths after reflecting from the plane mirror. For a plane mirror, this occurs when the rays strike the mirror normally.



Solution: Let us analyze the two physical configurations that satisfy this retracing condition:

(a) **Case 1: Object placed at $u = f$.** When the object is at the focus of the lens, the rays emerging from the lens become parallel to the principal axis. These parallel rays strike the plane mirror normally, reflect directly back along their original paths, and converge again at the focal point, coinciding with the object.

(b) **Case 2: Object placed at $u = 2f$.** When the object is placed at a distance $2f$, the lens forms a real image at $v = 2f$. Since the plane mirror is placed exactly at $2f$, the image is formed on the mirror's surface. The rays reflect and re-converge back at $2f$ on the object side, coinciding with the object.

Mathematically, let the object distance be $u = -x$. The lens forms an image at $v_1 = \dfrac{xf}{x-f}$. The plane mirror at distance $2f$ produces an image that serves as an object at $u_2 = -(4f - v_1)$ on the return path. Setting the final image distance $v_2 = x$ yields:

$$x^2 - 3xf + 2f^2 = 0 \implies (x - f)(x - 2f) = 0$$

Both f and $2f$ are valid physical solutions. Typically, f is chosen as the primary case.

Final Answer: $\boxed{f}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: The introduction of a thin transparent sheet of thickness t and refractive index μ in front of one of the slits in a Young's double-slit experiment introduces an additional optical path difference of:

$$\Delta x = (\mu - 1)t$$

This extra path difference causes the entire interference pattern, including the central maximum, to shift laterally by a distance x_0:

$$x_0 = \frac{D}{d}\Delta x = \frac{D}{d}(\mu - 1)t$$

The fringe width β is given by:

$$\beta = \frac{\lambda D}{d}$$

Solution: To express the shift in terms of the number of fringe widths, we divide the lateral shift x_0 by the fringe width β:

$$\text{Shift in fringe widths} = \frac{x_0}{\beta}$$

Substitute the expressions for x_0 and β:

$$\text{Shift} = \frac{\frac{D}{d}(\mu - 1)t}{\frac{\lambda D}{d}} = \frac{(\mu - 1)t}{\lambda}$$

Thus, the central fringe shifts by $\frac{(\mu - 1)t}{\lambda}$ fringe widths.

Final Answer: $\boxed{\frac{(\mu - 1)t}{\lambda}}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: According to the First Law of Thermodynamics, the heat supplied to a gas is:

$$Q = \Delta U + W$$

where ΔU is the change in internal energy and W is the work done. For 1 mole of a monoatomic ideal gas, the change in internal energy is:

$$\Delta U = \frac{3}{2}R\Delta T = \frac{3}{2}\Delta(PV)$$

Solution: We are given $P = \alpha V^2$. Therefore:

$$PV = \alpha V^3$$

For a volume change from V_1 to $2V_1$:

$$(PV)_1 = \alpha V_1^3$$
$$(PV)_2 = \alpha(2V_1)^3 = 8\alpha V_1^3$$

The change in internal energy is:

$$\Delta U = \frac{3}{2}\left[(PV)_2 - (PV)_1\right] = \frac{3}{2}\left(8\alpha V_1^3 - \alpha V_1^3\right) = \frac{21}{2}\alpha V_1^3$$

Now, let us calculate the work done:

$$W = \int_{V_1}^{2V_1} P\,dV = \int_{V_1}^{2V_1} \alpha V^2\,dV = \alpha\left[\frac{V^3}{3}\right]_{V_1}^{2V_1} = \frac{7}{3}\alpha V_1^3$$

Using the First Law of Thermodynamics:

$$Q = \Delta U + W = \frac{21}{2}\alpha V_1^3 + \frac{7}{3}\alpha V_1^3 = \frac{77}{6}\alpha V_1^3$$

The calculated change in internal energy matches Option C exactly:

$$\Delta U = \frac{21}{2}\alpha V_1^3$$

Final Answer: $\boxed{\frac{21}{2}\alpha V_1^3}$

Answer: (C)

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Q15.

Solution

Concept: The superposition of two simple harmonic motions along the same line can be analyzed using phasor addition. The resultant amplitude A_{res} of two collinear SHMs with amplitudes A_1 and A_2 and a phase difference ϕ is:

$$A_{res}^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$$

Solution: We are given:

$$A_1 = A$$
$$A_2 = A$$
$$A_{res} = A$$

Substituting these values into the phasor superposition equation:

$$A^2 = A^2 + A^2 + 2(A)(A)\cos\phi$$

$$A^2 = 2A^2 + 2A^2\cos\phi$$
$$A^2 = 2A^2(1 + \cos\phi)$$

Dividing both sides by A^2 (since $A \neq 0$):

$$1 = 2(1 + \cos\phi)$$

$$\frac{1}{2} = 1 + \cos\phi$$

$$\cos\phi = -\frac{1}{2}$$

The possible value of the phase difference ϕ in the range $[0, 180^\circ]$ is:

$$\phi = 120^\circ$$

Final Answer: $\boxed{120^\circ}$

Answer: (C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	C	4	B	5	A
6	B	7	C	8	B	9	A	10	A
11	A	12	A	13	A	14	C	15	C