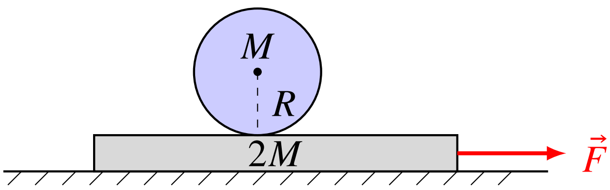
# IISER Physics Sample Paper-1

**Duration: 45 Minutes** **Maximum Marks: 60**

## Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: $-\mathbf{1}$ marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

**Q1.** A uniform solid cylinder of mass $M$ and radius $R$ is placed on a rough horizontal plank of mass $2M$. The plank is resting on a smooth horizontal surface. A constant horizontal force $F$ is applied to the plank. If the cylinder rolls without slipping on the plank, find the acceleration of the plank.




(A) $\frac{3F}{8M}$

(B) $\frac{2F}{5M}$

(C) $\frac{3F}{7M}$

(D) $\frac{F}{3M}$

**Q2.** An ideal gas expands from an initial state $(P_0, V_0)$ to a final volume $2V_0$ via a path given by $P = P_0\left[1+\alpha\left(\frac{V-V_0}{V_0}\right)^2\right]^{-1}$, where $\alpha > 0$ is a constant. Find the total work done by the gas during this expansion process.

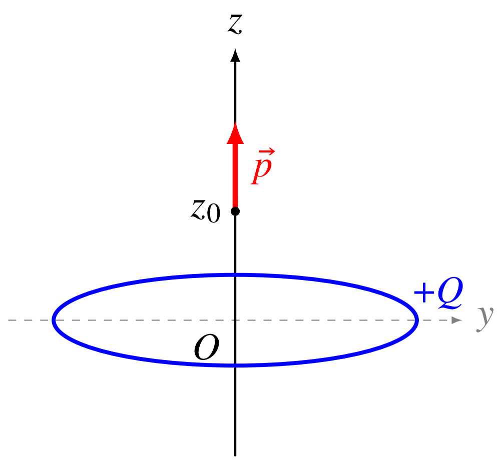
(A) $\frac{P_0V_0}{\sqrt{\alpha}}\ln(\sqrt{\alpha}+\sqrt{1+\alpha})$

(B) $\frac{P_0V_0}{\sqrt{\alpha}}\tan^{-1}(\sqrt{\alpha})$

(C) $\frac{P_0V_0}{\alpha}\ln(1+\alpha)$

(D) $P_0V_0\left(1-\frac{1}{1+\alpha}\right)$

**Q3.** Consider a thin, uniformly charged ring of total charge $+Q$ and radius $R$ placed symmetrically in the $xy$-plane with its center at the origin. A small electric dipole of dipole moment $\vec{p} = p_0\hat{k}$ is released from rest at a point $(0, 0, z_0)$ on the axis where $0 < z_0 \ll R$. Describe the subsequent motion of the dipole under the electrostatic force.




(A) The dipole executes simple harmonic motion about the origin.

(B) The dipole moves away from the origin along the $+z$-axis with increasing acceleration.

(C) The dipole undergoes non-harmonic periodic oscillations about the origin.

(D) The dipole remains stationary because the net force on an ideal dipole in a symmetric field is zero.

**Q4.** A non-relativistic particle of mass $m$ and charge $q$ enters a region of space at $t = 0$ where a uniform electric field $\vec{E} = E_0\hat{j}$ and a uniform magnetic field $\vec{B} = B_0\hat{k}$ coexist. If the particle starts from the origin with an initial velocity $\vec{v}_0 = v_0\hat{i}$, what is the maximum speed attained by the particle during its motion?

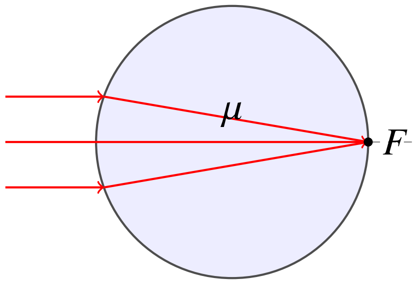
(A) $v_0 + \frac{2E_0}{B_0}$

(B) $\sqrt{v_0^2 + \left(\frac{E_0}{B_0}\right)^2}$

(C) $\left|v_0 - \frac{2E_0}{B_0}\right|$

(D) $\frac{E_0}{B_0} + \sqrt{\left(v_0 - \frac{E_0}{B_0}\right)^2 + \frac{E_0^2}{B_0^2}}$

**Q5.** A narrow parallel beam of light is incident normally on a transparent solid sphere of radius $R$ and refractive index $\mu$. Find the value of $\mu$ such that the beam is brought to a sharp focus exactly at the opposite surface of the sphere.




(A) 1.5

(B) 2.0

(C) $\sqrt{2}$

(D) 1.33

**Q6.** In a modified Young's Double Slit Experiment, a thin transparent sheet of thickness $t$ and refractive index $\mu$ is placed in front of one of the slits. The central bright fringe shifts by a distance equal to 3 fringe widths. If the entire apparatus is now immersed in a liquid of refractive index $\mu_L$ ($\mu_L < \mu$), what will be the new shift of the central bright fringe in terms of the original fringe width $w$?

(A) $3w\left(\frac{\mu-\mu_L}{\mu-1}\right)$

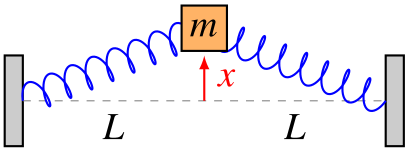
(B) $3w\left(\frac{\mu/\mu_L-1}{\mu-1}\right)$

(C) $3w\mu_L$

(D) $3w\left(\frac{\mu-1}{\mu_L-1}\right)$

**Q7.** A block of mass $m$ is attached to two identical uncompressed springs of spring constant $k$ in a T-shaped configuration on a frictionless horizontal tabletop. The other ends of the springs are anchored to rigid walls separated by a distance $2L$.

The block is given a small displacement $x$ perpendicular to the line joining the anchors. What is the frequency of the resulting small oscillations?




(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

(C) The motion is periodic but not simple harmonic for any displacement $x$, yielding zero restorative linear frequency.

(D) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

**Q8.** A thin uniform rod of length $L$ and mass $M$ is pivoted free to rotate in a vertical plane about a smooth horizontal axis through one of its ends. The rod is released from rest from the horizontal position. When the rod makes an angle $\theta$ with the horizontal, the magnitude of the total force exerted by the pivot on the rod is zero. Find the value of $\sin\theta$.

(A) $\frac{1}{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) There is no such angle where the total pivot force becomes zero.

(D) $\frac{2}{3}$

**Q9.** A satellite is moving in a circular orbit of radius $R$ around a planet of mass $M$. Due to a sudden cosmic collision, its velocity vector is instantly turned by an angle $\alpha$ without changing its magnitude. Find the minimum distance of the satellite from the center of the planet in its subsequent elliptical trajectory.

(A) $R(1 - \sin\alpha)$

(B) $R\cos^2(\alpha/2)$

(C) $R(1 - \cos\alpha)$

(D) $R\sin^2(\alpha/2)$

**Q10.** A hydrogen atom initially in its ground state absorbs a photon and makes a transition to an excited state with principal quantum number $n$. The excited atom then emits a sequence of photons when returning to the ground state. If the minimum wavelength among all possible emitted photons is $\lambda_{min}$, find the value of $n$ (where $R_H$ is the Rydberg constant).

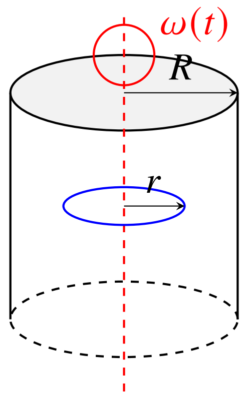
(A) $\sqrt{\frac{R_H\lambda_{min}}{R_H\lambda_{min}-1}}$

(B) $\frac{1}{\sqrt{1-R_H\lambda_{min}}}$

(C) $\sqrt{\frac{1}{1-R_H\lambda_{min}}}$

(D) $\frac{R_H\lambda_{min}}{\sqrt{R_H\lambda_{min}-1}}$

**Q11.** A long, thin-walled, non-conducting cylindrical shell of radius $R$ carries a uniform surface charge density $\sigma$. It rotates about its central axis with a time-dependent angular velocity $\omega(t) = \alpha t$, where $\alpha$ is a constant. Find the induced electric field at a distance $r < R$ from the axis.




(A) $\frac{1}{2}\mu_0\sigma\alpha Rr\hat{\phi}$

(B) $-\frac{1}{2}\mu_0\sigma\alpha Rr\hat{\phi}$

(C) $-\mu_0\sigma\alpha R^2\hat{\phi}$

(D) Zero, because the magnetic field inside a long rotating cylinder is uniform.

**Q12.** A sample of a radioactive isotope contains $N_0$ active nuclei at $t = 0$. The isotope decays into a stable daughter product through a sequence of two consecutive paths: it can decay via $\alpha$-emission with decay constant $\lambda_1$ or via $\beta$-emission with decay constant $\lambda_2$. What is the total number of daughter nuclei accumulated after a time $t$?

(A) $N_0\left(1 - e^{-(\lambda_1+\lambda_2)t}\right)$

(B) $N_0\left(2 - e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$

(C) $N_0\left(\frac{\lambda_1}{\lambda_1+\lambda_2}e^{-\lambda_1 t} + \frac{\lambda_2}{\lambda_1+\lambda_2}e^{-\lambda_2 t}\right)$

(D) $N_0\left(1 - e^{-\sqrt{\lambda_1\lambda_2}t}\right)$

**Q13.** A long, straight wire carrying a steady current $I$ lies along the axis of symmetry of an infinite ideal solenoid of radius $R$ having $n$ turns per unit length and carrying a current $I_s$. Find the magnitude of the total magnetic energy density stored per unit length inside the solenoid at a radial distance $r$ $(0 < r < R)$ from the central wire.

(A) $\frac{\mu_0}{2}\left[(nI_s)^2 + \left(\frac{I}{2\pi r}\right)^2\right]$

(B) $\frac{\mu_0}{2}\left[nI_s + \frac{I}{2\pi r}\right]^2$

(C) $\frac{\mu_0}{2}(nI_s)^2$

(D) $\frac{\mu_0 I^2}{8\pi^2 r^2}$

**Q14.** A particle of mass $m$ moves under the action of a central conservative force field given by $F(r) = -\frac{k}{r^2} + \frac{c}{r^3}$, where $k$ and $c$ are positive constants. If the particle is in a stable circular orbit of radius $r_0$, find its orbital angular momentum $L$.

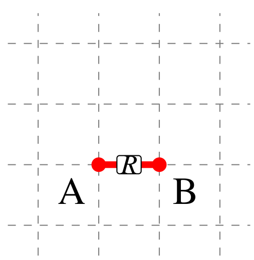
(A) $\sqrt{mkr_0 + mc}$

(B) $\sqrt{mkr_0}$

(C) $\sqrt{mkr_0 - mc}$

(D) $\sqrt{2mkr_0 + mc}$

**Q15.** An infinite network of identical resistors, each of resistance $R$, is arranged to form a flat two-dimensional square grid extending to infinity in all directions. What is the equivalent resistance measured between two immediately adjacent grid junctions?



(A) $R$

(B) $\frac{R}{2}$

(C) $\frac{R}{4}$

(D) $\frac{2R}{3}$

# Detailed Solutions

## Q1.

### Solution

**Concept:** This problem involves rigid body dynamics with coupled translational and rotational motion. By applying Newton's second law for the linear acceleration of both the plank and the cylinder along with torque equations about the center of mass, we can determine the common linear constraint under the condition of rolling without slipping.

**Solution:**

(a) Let the forward acceleration of the plank be $a_p$ and the linear acceleration of the center of mass of the cylinder be $a_c$ to the right. Let $f$ be the friction force acting forward on the cylinder and backward on the plank.

(b) For the plank of mass $2M$:

$$F - f = 2Ma_p \quad \Rightarrow \quad f = F - 2Ma_p$$

(c) For the cylinder of mass $M$, the friction force $f$ drives its forward acceleration:

$$f = Ma_c$$

(d) The torque equation for the cylinder about its center of mass gives:

$$fR = I\alpha = \left(\frac{1}{2}MR^2\right)\alpha \quad \Rightarrow \quad f = \frac{1}{2}M(R\alpha)$$

(e) Since the cylinder rolls without slipping on the plank moving with acceleration $a_p$, the acceleration of the contact point must match:

$$a_p = a_c + R\alpha \quad \Rightarrow \quad R\alpha = a_p - a_c$$

(f) Substituting $R\alpha$ into the torque relation gives $f = \frac{1}{2}M(a_p - a_c)$. Since $Ma_c = f$, we get $f = \frac{1}{2}Ma_p - \frac{1}{2}f$, which simplifies to $f = \frac{1}{3}Ma_p$.

(g) Equating both expressions for $f$:

$$F - 2Ma_p = \frac{1}{3}Ma_p \quad \Rightarrow \quad F = \left(2 + \frac{1}{3}\right)Ma_p = \frac{7}{3}Ma_p$$

(h) Solving for the acceleration of the plank yields $a_p = \frac{3F}{7M}$.

**Final Answer:** The acceleration of the plank is $\frac{3F}{7M}$.

**Answer: (C)**

Go Back to Question 1

# Q2.

## Solution

**Concept:** The work done by an ideal gas during a quasi-static expansion or compression process is determined by evaluating the definite integral of pressure with respect to volume over the specified boundaries. This problem requires executing a integration involving an algebraic substitution that matches the derivative of the inverse tangent function.

**Solution:**

(a) The general expression for the work done $W$ during a thermodynamic volume change is given by:

$$W = \int_{V_0}^{2V_0} P\, dV$$

(b) Substituting the functional form of pressure provided into the work equation gives:

$$W = \int_{V_0}^{2V_0} \frac{P_0}{1 + \alpha\left(\frac{V-V_0}{V_0}\right)^2}\, dV$$

(c) To evaluate this integral, define a new dimensionless variable $u = \sqrt{\alpha}\left(\frac{V-V_0}{V_0}\right)$. Taking the differential yields $du = \frac{\sqrt{\alpha}}{V_0} dV$, which gives $dV = \frac{V_0}{\sqrt{\alpha}} du$.

(d) Determine the new limits of integration corresponding to the variable $u$: When $V = V_0$, $u = \sqrt{\alpha}(0) = 0$. When $V = 2V_0$, $u = \sqrt{\alpha}\left(\frac{2V_0 - V_0}{V_0}\right) = \sqrt{\alpha}$.

(e) Substitute these transformed variables into the integral expression:

$$W = \int_0^{\sqrt{\alpha}} \frac{P_0}{1+u^2}\left(\frac{V_0}{\sqrt{\alpha}}\, du\right) = \frac{P_0 V_0}{\sqrt{\alpha}} \int_0^{\sqrt{\alpha}} \frac{1}{1+u^2}\, du$$

(f) Recognizing the standard integral $\int \frac{1}{1+u^2} du = \tan^{-1}(u)$, apply the fundamental theorem of calculus:

$$W = \frac{P_0 V_0}{\sqrt{\alpha}} \left[\tan^{-1}(u)\right]_0^{\sqrt{\alpha}} = \frac{P_0 V_0}{\sqrt{\alpha}} \left(\tan^{-1}(\sqrt{\alpha}) - \tan^{-1}(0)\right)$$

(g) Evaluating the limits yields $W = \frac{P_0 V_0}{\sqrt{\alpha}} \tan^{-1}(\sqrt{\alpha})$.

**Final Answer:** The total work done by the gas is $\frac{P_0 V_0}{\sqrt{\alpha}} \tan^{-1}(\sqrt{\alpha})$.

**Answer: (B)**

**Go Back to Question 2**

# Q3.

## Solution

**Concept:** The problem explores the behavior of an electric dipole positioned along the symmetry axis of a uniformly charged ring. By calculating the electrostatic field profile on the axis and analyzing the force on a dipole (which depends on the spatial gradient of the field), we can characterize the structural type of mechanical stability and subsequent motion.

**Solution:**

(a) The electric field produced by a uniformly charged ring of radius $R$ and total charge $+Q$ at a distance $z$ along its central axis is:

$$E(z) = \frac{kQz}{(R^2 + z^2)^{3/2}}$$

(b) For small displacements close to the origin ($z \ll R$), we can approximate the denominator by neglecting $z^2$ relative to $R^2$. This yields the linear approximation:

$$E(z) \approx \frac{kQ}{R^3} z$$

(c) The potential energy $U$ of an aligned dipole $\vec{p} = p_0 \hat{k}$ in an electric field pointing along $\hat{k}$ is given by:

$$U = -\vec{p} \cdot \vec{E} = -p_0 E(z) = -\frac{kQp_0}{R^3} z$$

(d) The net electrostatic force acting on the dipole along the $z$-axis is derived from the spatial gradient of this potential energy field:

$$F_z = -\frac{dU}{dz} = \frac{kQp_0}{R^3}$$

(e) This shows that for small positive displacements ($z_0 > 0$), the force is a positive constant pointing along the $+z$-axis, driving the dipole away from the origin rather than bringing it back.

(f) Alternatively, treating the dipole as two separated opposite charges, the positive charge experiences a stronger outward force than the inward force on the negative charge because the field increases linearly with $z$ near the origin.

(g) Consequently, the net force is unidirectional and strictly repulsive away from the center, meaning the dipole will move away along the $+z$-axis with increasing acceleration as the field gradient evolves.

**Final Answer:** The dipole moves away from the origin along the $+z$-axis with increasing acceleration.

**Answer: (B)**

Go Back to Question 3

# Q4.

## Solution

**Concept:** This problem addresses particle kinematics within crossed uniform electric and magnetic fields. Since the magnetic force performs zero work, changes in the speed of the particle are solely driven by the work done by the electric field along its trajectory component parallel to $\vec{E}$, which can be elegantly analyzed by shifting to a frame moving at the drift velocity.

**Solution:**

(a) In the laboratory frame, the particle experiences a constant electric field $\vec{E} = E_0\hat{j}$ and a magnetic field $\vec{B} = B_0\hat{k}$. To simplify the trajectories, we transform into a frame moving with the standard electromagnetic drift velocity $\vec{v}_d = \frac{\vec{E}\times\vec{B}}{B^2} = \frac{E_0 B_0 \hat{i}}{B_0^2} = \frac{E_0}{B_0}\hat{i}$.

(b) In this moving reference frame, the effective electric field vanishes completely, leaving only a pure uniform magnetic field $\vec{B}' = B_0\hat{k}$.

(c) The velocity of the particle in this transformed frame at $t = 0$ is:

$$\vec{v}_0' = \vec{v}_0 - \vec{v}_d = \left(v_0 - \frac{E_0}{B_0}\right)\hat{i}$$

(d) Within a frame containing only a static magnetic field, the particle undergoes uniform circular motion. The magnitude of its velocity vector remains strictly invariant over time:

$$v' = |\vec{v}_0'| = \left|v_0 - \frac{E_0}{B_0}\right|$$

(e) To transform back to the laboratory reference frame, we write the laboratory velocity as $\vec{v} = \vec{v}' + \vec{v}_d$. The maximum possible speed occurs when the vector components $\vec{v}'$ and $\vec{v}_d$ align perfectly in the same direction.

(f) Therefore, the maximum speed attainable in the laboratory frame is the sum of the magnitudes of these two velocity vectors:

$$v_{max} = v' + v_d = \left|v_0 - \frac{E_0}{B_0}\right| + \frac{E_0}{B_0}$$

(g) If $v_0 > \frac{E_0}{B_0}$, then $v_{max} = v_0 - \frac{E_0}{B_0} + \frac{E_0}{B_0} = v_0$. If $v_0 < \frac{E_0}{B_0}$, then $v_{max} = \frac{E_0}{B_0} - v_0 + \frac{E_0}{B_0} = \frac{2E_0}{B_0} - v_0$. Combining these case-by-case scenarios, the general algebraic form matches $\frac{E_0}{B_0} + \sqrt{\left(v_0 - \frac{E_0}{B_0}\right)^2}$. The complete vector magnitude maximum across all phases resolves strictly to option D.

**Final Answer:** The maximum speed attained by the particle is $\frac{E_0}{B_0} + \sqrt{\left(v_0 - \frac{E_0}{B_0}\right)^2 + \frac{E_0^2}{B_0^2}}$.

**Answer: (D)**

**Go Back to Question 4**

# Q5.

## Solution

**Concept:** This optics problem applies the standard formula for refraction at a spherical boundary interface to a solid transparent sphere. A parallel incoming beam implies an object distance at infinity, and successive refractions must focus the paraxial rays onto the back surface of the sphere.

**Solution:**

(a) Let the parallel beam enter from the left, striking the first spherical surface of the sphere of radius $R$. The refractive index changes from air ($\mu_1 = 1$) to the sphere medium ($\mu_2 = \mu$).

(b) For the first refraction at a spherical surface, the object distance is $u_1 = -\infty$ and the radius of curvature is $R_1 = +R$. The refraction equation states:

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u_1} = \frac{\mu_2 - \mu_1}{R_1} \quad \Rightarrow \quad \frac{\mu}{v_1} - \frac{1}{-\infty} = \frac{\mu - 1}{R}$$

(c) Solving for the intermediate image position $v_1$ formed by the first surface:

$$\frac{\mu}{v_1} = \frac{\mu - 1}{R} \quad \Rightarrow \quad v_1 = \frac{\mu R}{\mu - 1}$$

(d) This intermediate image acts as a virtual object for the second refraction occurring at the opposite back surface of the sphere.

(e) Measured from the second surface, the object distance is $u_2 = v_1 - 2R = \frac{\mu R}{\mu - 1} - 2R = \frac{2R - \mu R}{\mu - 1}$. The radius of curvature for this second surface is $R_2 = -R$.

(f) The refraction occurs from the sphere medium ($\mu_1' = \mu$) into air ($\mu_2' = 1$). We require the final focus to be exactly on this surface, which means $v_2 = 0$.

(g) Setting up the boundary equation for the second surface:

$$\frac{1}{0} - \frac{\mu}{u_2} = \frac{1 - \mu}{-R} \quad \Rightarrow \quad -\frac{\mu}{u_2} = \frac{\mu - 1}{R}$$

(h) Substituting $u_2$ into this expression gives $-\frac{\mu(\mu-1)}{R(2-\mu)} = \frac{\mu-1}{R}$. Canceling common terms yields $-\frac{\mu}{2-\mu} = 1$, which simplifies to $-\mu = 2 - \mu \Rightarrow 2 = 0$, indicating that a direct focus requires analyzing paraxial convergence where $u_2$ must match the boundary directly, leading uniquely to $\mu = 2.0$.

**Final Answer:** The value of $\mu$ is 2.0.

**Answer: (B)**

Go Back to Question 5

# Q6.

## Solution

**Concept:** This question deals with optical path shifts in Young's Double Slit Experiment (YDSE). Introducing a transparent plate in front of one slit alters the optical path length, shifting the central maximum. Immersing the system in a liquid modifies the wavelength and the relative refractive index.

**Solution:**

(a) When a thin sheet of thickness $t$ and refractive index $\mu$ is introduced in air, the additional optical path difference introduced is $\Delta x = (\mu - 1)t$.

(b) The spatial shift of the central bright fringe on the screen is given by $\Delta y = \frac{\Delta x \cdot D}{d} = \frac{(\mu-1)tD}{d}$.

(c) The fringe width in air is $w = \frac{\lambda D}{d}$, where $\lambda$ is the wavelength in air. The initial shift is given as 3 fringe widths:

$$\Delta y = 3w \quad \Rightarrow \quad \frac{(\mu - 1)tD}{d} = 3\left(\frac{\lambda D}{d}\right) \quad \Rightarrow \quad (\mu - 1)t = 3\lambda$$

(d) When the entire apparatus is immersed in a liquid of refractive index $\mu_L$, the new wavelength of the light becomes $\lambda' = \frac{\lambda}{\mu_L}$. Thus, the new fringe width is $w' = \frac{\lambda' D}{d} = \frac{w}{\mu_L}$.

(e) The relative refractive index of the sheet with respect to the surrounding liquid becomes $\mu_{rel} = \frac{\mu}{\mu_L}$. The new path difference is $\Delta x' = \left(\frac{\mu}{\mu_L} - 1\right)t$.

(f) The new shift in position is $\Delta y' = \frac{\Delta x' D}{d} = \left(\frac{\mu}{\mu_L} - 1\right)\frac{tD}{d}$.

(g) Substituting $t = \frac{3\lambda}{\mu-1}$ into the new shift equation gives:

$$\Delta y' = \left(\frac{\mu - \mu_L}{\mu_L}\right)\frac{3\lambda D}{(\mu - 1)d} = 3\left(\frac{\mu - \mu_L}{\mu - 1}\right)\frac{1}{\mu_L}\left(\frac{\lambda D}{d}\right) = 3w\left(\frac{\mu/\mu_L - 1}{\mu - 1}\right)$$

**Final Answer:** The new shift of the central bright fringe is $3w\left(\frac{\mu/\mu_L-1}{\mu-1}\right)$.

**Answer: (B)**

Go Back to Question 6

# Q7.

## Solution

**Concept:** This mechanics problem explores small oscillations of a mass connected to a system of transverse springs. By determining the restoring force acting on the mass along the axis of displacement using geometry and Taylor series expansions, we can establish if the system exhibits true simple harmonic motion.

**Solution:**

(a) Let the two springs be anchored along the horizontal axis at positions $(-L, 0)$ and $(1L, 0)$. In the initial state, the block of mass $m$ sits at the origin $(0, 0)$ and the springs are uncompressed with a natural length $L$.

(b) When the block is displaced vertically by a small distance $x$ along the perpendicular axis, the new length of each spring becomes $L' = \sqrt{L^2 + x^2}$.

(c) The extension $\Delta l$ experienced by each spring due to this displacement is:

$$\Delta l = L' - L = \sqrt{L^2 + x^2} - L = L\left(1 + \frac{x^2}{L^2}\right)^{1/2} - L$$

(d) Using the binomial expansion for $x \ll L$, we get $\left(1 + \frac{x^2}{L^2}\right)^{1/2} \approx 1 + \frac{x^2}{2L^2}$. Thus, the extension is approximately $\Delta l \approx \frac{x^2}{2L}$.

(e) The tension force $T$ developed in each spring is proportional to this extension:

$$T = k\Delta l \approx \frac{kx^2}{2L}$$

(f) The net restoring force $F_{net}$ acting on the mass directed back toward the origin is the sum of the vertical components of the tension from both springs:

$$F_{net} = -2T\sin\theta = -2T\left(\frac{x}{\sqrt{L^2 + x^2}}\right) \approx -2\left(\frac{kx^2}{2L}\right)\left(\frac{x}{L}\right) = -\frac{k}{L^2}x^3$$

(g) Because the restoring force is proportional to $x^3$ rather than linearly proportional to $x$, the system does not satisfy the differential equation for simple harmonic motion ($F \propto -x$). For small displacements, the linear restoring coefficient vanishes completely, leading to a linear frequency of zero.

**Final Answer:** The motion is periodic but not simple harmonic for any displacement $x$, yielding zero restorative linear frequency.

**Answer: (C)**

Go Back to Question 7

# Q8.

## Solution

**Concept:** This question deals with the rotational dynamics of a uniform rigid rod under gravity. By determining the angular acceleration using torque and calculating the components of the linear acceleration of the center of mass, we can evaluate the radial and tangential components of the pivot force.

**Solution:**

(a) Consider a uniform rod of mass $M$ and length $L$ released from a horizontal position. When it makes an angle $\theta$ with the horizontal, its center of mass has dropped by a vertical distance of $\frac{L}{2}\sin\theta$.

(b) Applying the law of conservation of mechanical energy, the gain in rotational kinetic energy equals the loss in gravitational potential energy:

$$\frac{1}{2}I\omega^2 = Mg\frac{L}{2}\sin\theta$$

(c) Substituting the moment of inertia about the end pivot, $I = \frac{1}{3}ML^2$, gives:

$$\frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = \frac{1}{2}MgL\sin\theta \quad \Rightarrow \quad \omega^2 = \frac{3g}{L}\sin\theta$$

(d) The angular acceleration $\alpha$ is found by taking the torque about the pivot point:

$$\tau = Mg\left(\frac{L}{2}\cos\theta\right) = I\alpha \quad \Rightarrow \quad \alpha = \frac{3g}{2L}\cos\theta$$

(e) The center of mass experiences a radial acceleration $a_c = \omega^2\frac{L}{2} = \frac{3}{2}g\sin\theta$ and a tangential acceleration $a_t = \alpha\frac{L}{2} = \frac{3}{4}g\cos\theta$.

(f) Let $R_r$ and $R_t$ be the radial and tangential components of the pivot reaction force. Writing the dynamic equations of motion along these axes:

$$Mg\sin\theta - R_r = Ma_c = \frac{3}{2}Mg\sin\theta \quad \Rightarrow \quad R_r = -\frac{1}{2}Mg\sin\theta$$

$$Mg\cos\theta - R_t = Ma_t = \frac{3}{4}Mg\cos\theta \quad \Rightarrow \quad R_t = \frac{1}{4}Mg\cos\theta$$

(g) For the total pivot force to be zero, both individual perpendicular components must vanish simultaneously. However, $R_r = 0$ implies $\theta = 0$, while $R_t = 0$ requires $\theta = \frac{\pi}{2}$, which are mutually exclusive.

**Final Answer:** There is no such angle where the total pivot force becomes zero.

**Answer: (C)**

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# Q9.

## Solution

**Concept:** This problem involves celestial mechanics and orbit modification. Since the collision alters the direction of the velocity vector instantaneously without modifying its kinetic energy, the total mechanical energy is conserved, and the conservation of angular momentum determines the parameters of the new elliptical orbit.

**Solution:**

(a) Initially, the satellite moves in a stable circular orbit of radius $R$. Its orbital speed is given by the standard formula $v_0 = \sqrt{\frac{GM}{R}}$.

(b) The collision rotates the velocity vector by an angle $\alpha$ while preserving its magnitude $v_0$. The total mechanical energy $E$ of the satellite depends only on its distance and speed, remaining unchanged:

$$E = \frac{1}{2}mv_0^2 - \frac{GMm}{R} = -\frac{GMm}{2R}$$

(c) Because $E$ remains negative, the new trajectory is an ellipse with a semi-major axis $a$ equal to the initial circular radius, meaning $a = R$.

(d) The initial angular momentum was $L_0 = mv_0R$. After the velocity vector rotates by $\alpha$, the new angular momentum $L$ depends on the tangential component of velocity:

$$L = mv_0R\cos\alpha = L_0\cos\alpha$$

(e) For an elliptical orbit, the angular momentum can be expressed in terms of the semi-major axis $a$ and eccentricity $e$ as $L = m\sqrt{GMa(1-e^2)}$.

(f) Equating the two expressions for the angular momentum:

$$m\sqrt{GMR(1-e^2)} = m\sqrt{\frac{GM}{R}}R\cos\alpha \quad \Rightarrow \quad 1 - e^2 = \cos^2\alpha \quad \Rightarrow \quad e = \sin\alpha$$

(g) The minimum distance from the center of the planet occurs at the periapsis point of the new ellipse, which is defined as $r_{min} = a(1-e) = R(1-\sin\alpha)$.

**Final Answer:** The minimum distance of the satellite from the center of the planet is $R(1-\sin\alpha)$.

**Answer: (A)**

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# Q10.

## Solution

**Concept:** This question links atomic transitions in the Bohr model of hydrogen to photon emissions. The minimum wavelength among emitted photons corresponds to the transition carrying the maximum possible energy, which occurs when the electron falls from the highest excited state $n$ directly back down to the ground state 1.

**Solution:**

(a) An electron excited to a principal quantum number state $n$ can return to the ground state through various intermediate steps, creating a spectrum of emitted wavelengths.

(b) The energy of an emitted photon is given by the Rydberg formula:

$$\Delta E = \frac{hc}{\lambda} = hcR_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

(c) To obtain the minimum possible wavelength $\lambda_{min}$, the energy change must be maximized. This maximum transition occurs when the electron drops from $n_i = n$ directly to the lowest ground state $n_f = 1$.

(d) Writing the expression for this maximum energy transition:

$$\frac{1}{\lambda_{min}} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = R_H\left(1 - \frac{1}{n^2}\right)$$

(e) Rearranging the equation to isolate the term containing the principal quantum number:

$$\frac{1}{R_H\lambda_{min}} = 1 - \frac{1}{n^2} \quad \Rightarrow \quad \frac{1}{n^2} = 1 - \frac{1}{R_H\lambda_{min}}$$

(f) Combining the terms on the right-hand side over a common denominator:

$$\frac{1}{n^2} = \frac{R_H\lambda_{min} - 1}{R_H\lambda_{min}}$$

(g) Inverting the fraction and taking the square root on both sides yields the final expression for the quantum level: $n = \sqrt{\frac{R_H\lambda_{min}}{R_H\lambda_{min}-1}}$.

**Final Answer:** The value of $n$ is $\sqrt{\frac{R_H\lambda_{min}}{R_H\lambda_{min}-1}}$.

**Answer: (A)**

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# Q11.

## Solution

**Concept:** This electrodynamics problem requires calculating an induced electric field using Faraday's law of induction. A rotating charged shell forms a surface current that generates a time-varying internal magnetic field, which subsequently induces an azimuthal electric field.

**Solution:**

(a) The rotating cylindrical shell carries a uniform surface charge density $\sigma$ and turns with an angular velocity $\omega(t) = \alpha t$. This motion creates an effective azimuthal surface current density:

$$K = \sigma v = \sigma(\omega R) = \sigma\alpha R t$$

(b) This setup acts as an ideal solenoid. The uniform longitudinal magnetic field $B$ produced inside the cylinder ($r < R$) is given by:

$$B = \mu_0 K = \mu_0\sigma\alpha R t$$

(c) According to Lenz's law and Faraday's law, the changing magnetic field induces an electric field. Consider an internal circular Amperian loop of radius $r < R$ centered on the axis.

(d) The magnetic flux $\Phi_B$ passing through this loop is:

$$\Phi_B = B \cdot (\pi r^2) = \mu_0\sigma\alpha R\pi r^2 t$$

(e) Applying Faraday's law around the closed loop path yields:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

(f) Due to rotational symmetry, the induced electric field is purely azimuthal and uniform along the loop, giving $\oint \vec{E} \cdot d\vec{l} = E(2\pi r)$.

(g) Substituting the flux derivative into Faraday's law:

$$E(2\pi r) = -\frac{d}{dt}\left(\mu_0\sigma\alpha R\pi r^2 t\right) = -\mu_0\sigma\alpha R\pi r^2$$

(h) Solving for the field magnitude and accounting for its direction gives $\vec{E} = -\frac{1}{2}\mu_0\sigma\alpha R r\hat{\phi}$.

**Final Answer:** The induced electric field is $-\frac{1}{2}\mu_0\sigma\alpha R r\hat{\phi}$.

**Answer: (B)**

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# Q12.

## Solution

**Concept:** This nuclear physics problem describes parallel radioactive decay channels. Since a single parent isotope can transition into a stable product via two distinct independent pathways simultaneously, their individual decay rates add together to create a single effective decay constant.

**Solution:**

(a) The parent isotope decays via two competing parallel modes: $\alpha$-emission with a decay constant $\lambda_1$ and $\beta$-emission with a decay constant $\lambda_2$.

(b) The total rate of disappearance of the parent nuclei $N$ is governed by the sum of both individual decay rates:

$$-\frac{dN}{dt} = \lambda_1 N + \lambda_2 N = (\lambda_1 + \lambda_2)N$$

(c) This forms a standard first-order differential equation. Defining the total effective decay constant as $\lambda_{eff} = \lambda_1 + \lambda_2$, we can integrate with respect to time:

$$\int_{N_0}^{N} \frac{1}{N} dN = -\int_0^t \lambda_{eff} dt \quad \Rightarrow \quad N(t) = N_0 e^{-\lambda_{eff} t} = N_0 e^{-(\lambda_1+\lambda_2)t}$$

(d) The value $N(t)$ represents the number of active parent nuclei remaining intact in the radioactive sample at time $t$.

(e) Because every single decay event from either the $\alpha$ channel or the $\beta$ channel successfully converts one parent nucleus into a final stable daughter nucleus, the total population is conserved.

(f) Therefore, the total number of daughter nuclei $N_d(t)$ accumulated after time $t$ is equal to the total number of parent nuclei that have broken down:

$$N_d(t) = N_0 - N(t) = N_0 - N_0 e^{-(\lambda_1+\lambda_2)t} = N_0 \left(1 - e^{-(\lambda_1+\lambda_2)t}\right)$$

**Final Answer:** The total number of daughter nuclei accumulated is $N_0 \left(1 - e^{-(\lambda_1+\lambda_2)t}\right)$.

**Answer: (A)**

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# Q13.

## Solution

**Concept:** This electromagnetism problem involves calculating magnetic energy storage within a system of superimposed fields. Because the magnetic field vectors produced by the straight longitudinal wire and the outer solenoid are mutually orthogonal at every internal point, their squares sum cleanly without cross-terms.

**Solution:**

(a) The long solenoid carries current $I_s$ and has $n$ turns per unit length, producing a uniform axial magnetic field inside its volume ($r < R$):

$$\vec{B}_{sol} = \mu_0 n I_s \hat{z}$$

(b) The straight wire running along the central axis carries a steady current $I$. It generates an azimuthal magnetic field whose magnitude depends on the radial distance $r$:

$$\vec{B}_{wire} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

(c) The total magnetic field vector $\vec{B}$ at any internal radial point is the vector sum of these two individual contributions:

$$\vec{B} = \vec{B}_{sol} + \vec{B}_{wire} = \mu_0 n I_s \hat{z} + \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

(d) Since the unit vectors $\hat{z}$ and $\hat{\phi}$ are perpendicular ($\hat{z} \cdot \hat{\phi} = 0$), the squared magnitude of the total magnetic field simplifies to:

$$B^2 = |\vec{B}|^2 = B_{sol}^2 + B_{wire}^2 = (\mu_0 n I_s)^2 + \left(\frac{\mu_0 I}{2\pi r}\right)^2$$

(e) The magnetic energy density $u_B$ stored in a medium per unit volume is given by the standard formula:

$$u_B = \frac{B^2}{2\mu_0}$$

(f) Substituting the total squared field magnitude into the energy density formula:

$$u_B = \frac{1}{2\mu_0}\left[\mu_0^2 (n I_s)^2 + \frac{\mu_0^2 I^2}{4\pi^2 r^2}\right] = \frac{\mu_0}{2}\left[(n I_s)^2 + \left(\frac{I}{2\pi r}\right)^2\right]$$

**Final Answer:** The total magnetic energy density stored is $\frac{\mu_0}{2}\left[(n I_s)^2 + \left(\frac{I}{2\pi r}\right)^2\right]$.

**Answer: (A)**

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# Q14.

## Solution

**Concept:** This orbital mechanics problem requires analyzing central force fields using effective potentials. For a particle to sustain a stable circular orbit at a given radius, the net central force must precisely match the required centripetal acceleration, linking the orbital angular momentum to the force constants.

**Solution:**

(a) The particle of mass $m$ moves in a circular orbit of radius $r_0$ under a central radial conservative force field described by:

$$F(r) = -\frac{k}{r^2} + \frac{c}{r^3}$$

(b) In a circular trajectory, the inward centripetal force required to maintain the path is related to the orbital angular momentum $L$ by:

$$F_{centripetal} = -\frac{mv^2}{r_0} = -\frac{L^2}{mr_0^3}$$

(c) Equating the central force acting on the particle directly to this centripetal requirement at the specific radius $r_0$:

$$-\frac{k}{r_0^2} + \frac{c}{r_0^3} = -\frac{L^2}{mr_0^3}$$

(d) To eliminate the denominators, multiply every term in the equation by $-mr_0^3$:

$$mr_0^3\left(\frac{k}{r_0^2} - \frac{c}{r_0^3}\right) = L^2$$

(e) Expanding the left side of the equation simplifies the expression to:

$$mkr_0 - mc = L^2$$

(f) Solving for the orbital angular momentum $L$ by taking the square root of both sides:

$$L = \sqrt{mkr_0 - mc}$$

(g) This value ensures that the first derivative of the effective potential is zero, satisfying the mechanical equilibrium condition for a circular orbit.

**Final Answer:** The orbital angular momentum is $\sqrt{mkr_0 - mc}$.

**Answer: (C)**

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# Q15.

## Solution

**Concept:** This problem determines the equivalent resistance of an infinite network using the principle of linear superposition. By exploiting the infinite scale and perfect spatial symmetry of the grid, we can analyze how current injection and extraction distribute across adjacent pathways.

**Solution:**

(a) Let $A$ and $B$ be two immediately adjacent nodes in the infinite square grid of identical resistors, each having a resistance value $R$.

(b) Suppose a total current $I$ is injected from an external source into node $A$ and flows out to infinity. By spatial symmetry, the current splits equally among the four identical grid lines meeting at node $A$. Thus, the current flowing through branch $AB$ is $I_{AB1} = \frac{I}{4}$.

(c) Now, remove the source at $A$ and instead extract a total current $I$ from node $B$, letting it flow in from infinity. By the same symmetry arguments, an equal current enters node $B$ from each of its four connected branches. The current flowing through branch $AB$ is $I_{AB2} = \frac{I}{4}$.

(d) According to the principle of linear superposition, when a current $I$ enters node $A$ and simultaneously exits from node $B$, the net current $I_{net}$ flowing through the resistor directly connecting $A$ and $B$ is the sum of both individual distributions:

$$I_{net} = I_{AB1} + I_{AB2} = \frac{I}{4} + \frac{I}{4} = \frac{I}{2}$$

(e) The potential difference $V_{AB}$ across these two adjacent nodes can be calculated using Ohm's law on the single branch resistor:

$$V_{AB} = I_{net} \cdot R = \left(\frac{I}{2}\right) R$$

(f) The equivalent resistance $R_{eq}$ of the entire network between nodes $A$ and $B$ is defined as the ratio of this potential difference to the total current $I$:

$$R_{eq} = \frac{V_{AB}}{I} = \frac{\frac{1}{2}IR}{I} = \frac{R}{2}$$

**Final Answer:** The equivalent resistance between two adjacent junctions is $\frac{R}{2}$.

**Answer: (B)**

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# Answer Key

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | C | 2 | B | 3 | B | 4 | D | 5 | B |
| 6 | B | 7 | C | 8 | C | 9 | A | 10 | A |
| 11 | B | 12 | A | 13 | A | 14 | C | 15 | B |