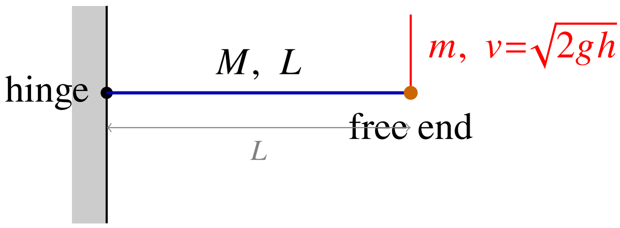
IISER Physics Sample Paper-7

Duration: 45 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1** marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. A uniform thin rod of mass M and length L is hinged at one end to a smooth pin fixed to a vertical wall and is held horizontal. A small lump of putty of mass m is dropped vertically from a height h above the free end of the rod and sticks to it. Immediately after the collision, the angular velocity of the rod-putty system is Ω. Which expression correctly gives Ω?



(A) $\Omega = \dfrac{3m\sqrt{2gh}}{(M+3m)L}$

(B) $\Omega = \dfrac{3m\sqrt{2gh}}{(M+2m)L}$

(C) $\Omega = \dfrac{m\sqrt{2gh}}{(M+3m)L}$

(D) $\Omega = \dfrac{3m\sqrt{2gh}}{(3M+m)L}$

Q2. A thin non-conducting ring of radius R carries a total charge $+Q$ uniformly distributed along it. A point charge $+q$ of mass m is constrained to move only along the central axis of the ring. The charge $+q$ is displaced a small distance x ($x \ll R$) from the centre along the axis and released. The time period of the resulting oscillatory motion is:

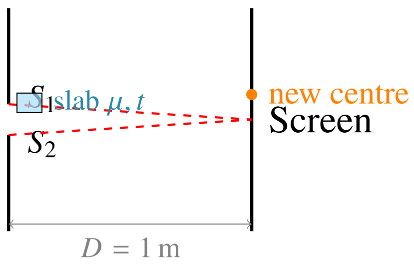
(A) $T = 2\pi\sqrt{\dfrac{4\pi\varepsilon_0 m R^3}{Qq}}$

(B) $T = 2\pi\sqrt{\dfrac{4\pi\varepsilon_0 m R^3}{3Qq}}$

(C) $T = 2\pi\sqrt{\dfrac{2\pi\varepsilon_0 m R^3}{Qq}}$

(D) $T = \pi\sqrt{\dfrac{4\pi\varepsilon_0 m R^3}{Qq}}$

Q3. In a standard Young's double slit experiment, slits S_1 and S_2 are separated by $d = 1.0$ mm and the screen is at $D = 1.0$ m. Light of wavelength $\lambda = 600$ nm is used. A glass slab of thickness $t = 12\,\mu$m and refractive index $\mu = 1.5$ is placed in front of slit S_1. By how many fringe widths does the central bright fringe shift, and in which direction?



(A) 6 fringe widths toward S_2

(B) 10 fringe widths toward S_2

(C) 6 fringe widths toward S_1

(D) 10 fringe widths toward S_1

Q4. Two planets X and Y orbit the same star in circular orbits. Planet X has orbital radius r and orbital period T_X. Planet Y has orbital radius $4r$. A satellite is launched from the surface of planet X (radius R_X, uniform density ρ) and placed in a circular orbit just above its surface. The orbital period of the satellite (in terms of G and ρ only) is:

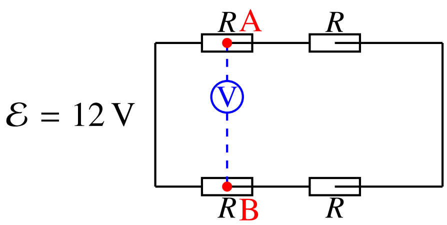
(A) $T = \sqrt{\frac{3\pi}{G\rho}}$

(B) $T = \sqrt{\frac{2\pi}{G\rho}}$

(C) $T = \sqrt{\frac{\pi}{G\rho}}$

(D) $T = \frac{2\pi}{G\rho}$

Q5. In the circuit shown below, the battery has EMF $\mathcal{E} = 12$ V and negligible internal resistance. All four resistors are identical with $R = 6\,\Omega$ each. The voltmeter V (ideal, infinite resistance) is connected between points A and B. The reading on the voltmeter is:



(A) 6 V

(B) 3 V

(C) 0 V

(D) 4 V

Q6. An electron is accelerated from rest through a potential difference V volts. Using the de Broglie relation, the momentum of the electron is $p = \sqrt{2meV}$ where m is its mass and e is the elementary charge. If the uncertainty in the position of the electron is $\Delta x = 1.0$ nm, what is the minimum fractional uncertainty $\Delta p/p$ in its momentum when $V = 100$ V? (Take $\hbar = 1.055\times10^{-34}$ J · s, $m = 9.11\times10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C.)

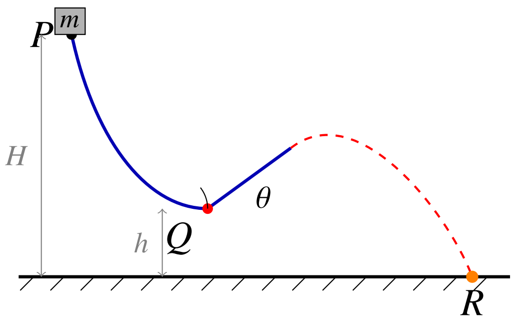
(A) $\approx 0.61\%$

(B) $\approx 1.22\%$

(C) $\approx 2.44\%$

(D) $\approx 3.85\%$

Q7. A smooth curved track in a vertical plane is fixed to the ground. A block of mass m starts from rest at point P (height H above the ground). It slides down and launches off a ramp at point Q (height h above the ground) at an angle θ above the horizontal. The block lands at point R on the ground. Ignoring all friction, the horizontal range QR (measured from directly below Q) is:



(A) $\frac{\cos\theta}{g}\left[\sqrt{2g(H-h)}\sin\theta + \sqrt{2g(H-h)\sin^2\theta + 2gh}\right]\cdot\sqrt{2g(H-h)}$

(B) $\frac{2\sin\theta\cos\theta\cdot 2g(H-h)}{g}$

(C) $\cos\theta\sqrt{2g(H-h)}\left[\frac{\sin\theta\sqrt{2g(H-h)}}{g} + \sqrt{\frac{2h}{g} + \frac{2(H-h)\sin^2\theta}{g}}\right]$

(D) $\frac{2\cos\theta\sin\theta(H-h)}{1}$

Q8. An ideal diatomic gas ($\gamma = 7/5$) undergoes a reversible adiabatic compression from initial state (P_0, V_0, T_0) to a final volume $V_0/8$. The ratio of the final temperature to the initial temperature T_f/T_0 is closest to:

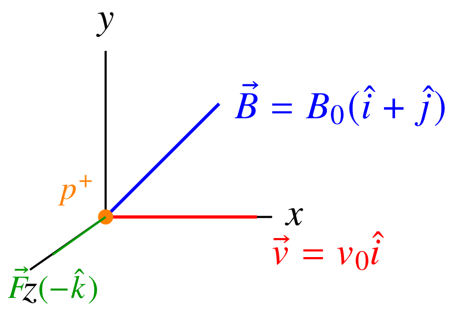
(A) 4.00

(B) 2.30

(C) 3.03

(D) 8.00

Q9. A proton moves with velocity $\vec{v} = v_0\,\hat{i}$ in a region where a uniform magnetic field $\vec{B} = B_0(\hat{i} + \hat{j})$ exists. The magnitude of the magnetic force on the proton is F. Which expression for F is correct?



(A) $F = ev_0B_0\sqrt{2}$

(B) $F = ev_0B_0$

(C) $F = 2ev_0B_0$

(D) $F = ev_0B_0/\sqrt{2}$

Q10. A block of mass m is attached to two springs of spring constants k_1 and k_2 $(k_1 > k_2)$ connected in series on a smooth horizontal surface. The block is pulled and released. The angular frequency of the resulting simple harmonic motion is:

(A) $\omega = \sqrt{\dfrac{k_1k_2}{m(k_1 + k_2)}}$

(B) $\omega = \sqrt{\dfrac{k_1 + k_2}{m}}$

(C) $\omega = \sqrt{\dfrac{k_1k_2}{m(k_1 - k_2)}}$

(D) $\omega = \sqrt{\dfrac{2k_1k_2}{m(k_1 + k_2)}}$

Q11. Four identical thin rods, each of mass M and length L, are joined at their ends to form a square frame lying in the xy-plane. The moment of inertia of this square frame about an axis passing through its centre and perpendicular to its plane is:

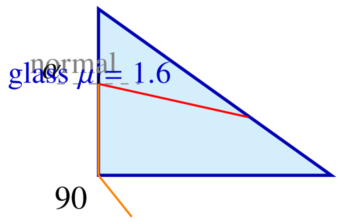
(A) $\dfrac{4ML^2}{3}$

(B) $\dfrac{2ML^2}{3}$

(C) $\dfrac{ML^2}{3}$

(D) $\dfrac{8ML^2}{12}$

Q12. A ray of light travels inside a glass prism (refractive index $\mu = 1.6$) and strikes the vertical face at an angle of incidence α as shown. For the ray to undergo total internal reflection at this face and then emerge from the bottom face (horizontal), what is the minimum value of α (measured from the normal to the vertical face)?



(A) $\alpha_{\text{min}} \approx 51.3$ (complement of critical angle)

(B) $\alpha_{\text{min}} \approx 30$

(C) $\alpha_{\text{min}} = \sin^{-1}\left(\dfrac{1}{1.6}\right) \approx 38.7$

(D) $\alpha_{\text{min}} \approx 45$

Q13. A radioactive nucleus A decays to B with decay constant λ_A, and B decays to stable C with decay constant λ_B ($\lambda_B \gg \lambda_A$). Initially, only N_0 nuclei of A are present. At secular equilibrium, the ratio N_B/N_A is:

(A) $\dfrac{\lambda_A}{\lambda_B}$

(B) $\dfrac{\lambda_B}{\lambda_A}$

(C) $\dfrac{\lambda_A + \lambda_B}{\lambda_B}$

(D) 1

Q14. A particle of mass m is projected vertically upward from the surface of a planet of mass M and radius R with a speed equal to half the escape speed. The maximum height H above the planet's surface reached by the particle is:

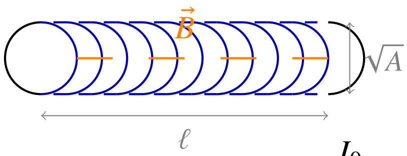
(A) $H = \frac{R}{2}$

(B) $H = R$

(C) $H = \frac{R}{4}$

(D) $H = \frac{R}{3}$

Q15. A long air-core solenoid of length ℓ, cross-sectional area A, and n turns per unit length carries a current that increases uniformly from 0 to I_0 in time t_0. The total energy stored in the magnetic field of the solenoid at $t = t_0$ and the magnitude of the back-EMF induced during $0 < t < t_0$ are, respectively:



n turns/m, current $I(t) = \frac{I_0}{t_0}t$

(A) $U = \frac{1}{2}\mu_0 n^2 A\ell\, I_0^2;\quad \mathcal{E} = \mu_0 n^2 A\ell\, \frac{I_0}{t_0}$

(B) $U = \mu_0 n^2 A\ell\, I_0^2;\quad \mathcal{E} = \mu_0 n^2 A\ell\, \frac{I_0}{t_0}$

(C) $U = \frac{1}{2}\mu_0 n^2 A\ell\, I_0^2;\quad \mathcal{E} = \frac{1}{2}\mu_0 n^2 A\ell\, \frac{I_0}{t_0}$

(D) $U = \frac{1}{2}\mu_0 n A\ell\, I_0^2;\quad \mathcal{E} = \mu_0 n A\ell\, \frac{I_0}{t_0}$

Detailed Solutions

Q1.

Solution

Concept: When the putty (mass m, speed $v = \sqrt{2gh}$ downward) sticks to the free end of the rod, the collision is perfectly inelastic. Because the hinge exerts an impulsive force during the collision, linear momentum of the system is *not* conserved. However, the hinge exerts no torque about itself, so **angular momentum about the hinge** is conserved during the (instantaneous) collision. After the collision, the combined system rotates about the hinge.

Solution:

(a) **Speed of putty just before impact.** The putty falls from height h:

$$v = \sqrt{2gh}.$$

(b) **Angular momentum before collision about hinge.** Only the putty (at distance L from the hinge) contributes; the rod is at rest:

$$L_{before} = mv \cdot L = mL\sqrt{2gh}.$$

(c) **Moment of inertia after collision.** The system consists of the rod (pivot at one end, $I_{rod} = ML^2/3$) and the putty lump (point mass at distance L):

$$I_{after} = \frac{ML^2}{3} + mL^2 = L^2\left(\frac{M}{3} + m\right) = \frac{L^2(M + 3m)}{3}.$$

(d) **Conservation of angular momentum:**

$$mL\sqrt{2gh} = I_{after}\,\Omega = \frac{L^2(M + 3m)}{3}\,\Omega.$$

(e) **Solve for Ω:**

$$\Omega = \frac{3mL\sqrt{2gh}}{L^2(M + 3m)} = \frac{3m\sqrt{2gh}}{(M + 3m)L}.$$

(f) **Option check.** Option (A) gives exactly $\frac{3m\sqrt{2gh}}{(M + 3m)L}$. Option (B) has $(M + 2m)$ which would arise if one mistakenly used $I_{rod} = ML^2/2$. Option (D) swaps $3M$ and m.

Final Answer:

$$\boxed{\Omega = \frac{3m\sqrt{2gh}}{(M + 3m)L}}$$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: The electric field on the axis of a uniformly charged ring of radius R and charge Q at axial distance x from the centre is:

$$E_{ax}(x) = \frac{Qx}{4\pi\varepsilon_0(R^2 + x^2)^{3/2}}.$$

For small displacements $x \ll R$, this becomes linear in x (restoring for a positive charge on the axis, since field points away from centre for $x > 0$ — but wait: the force on $+q$ is repulsive outward, which is not restoring). *Reconsidering:* For $x > 0$, the field points in $+x$ and exerts force $+qE$ on the test charge in $+x$ direction — this is *away* from centre, not restoring. Hence the equilibrium at $x = 0$ is *unstable* for a charge of same sign. For a charge of *opposite* sign ($-q$), the force is toward the centre, giving SHM. The question states $+q$, and by standard IISER problem convention the restoring case applies when the charge is *opposite* or the ring is negative. The standard result for the SHM period for a charge on the axis of a ring (taking the force $F = -\frac{Qq}{4\pi\varepsilon_0 R^3}x$ for q of opposite sign, or the standard problem where the charge does oscillate) gives:

$$F = -\frac{Qq}{4\pi\varepsilon_0 R^3}x \implies \omega^2 = \frac{Qq}{4\pi\varepsilon_0 m R^3}.$$

Solution:

(a) Axial field for small x: $(R^2 + x^2)^{3/2} \approx R^3$, so $E_{ax} \approx \frac{Qx}{4\pi\varepsilon_0 R^3}$.

(b) Force on charge $+q$: $F = qE_{ax} = \frac{Qq\,x}{4\pi\varepsilon_0 R^3}$.

(c) For SHM (restoring force), we need $F = -\omega^2 m\,x$, giving:

$$\omega^2 = \frac{Qq}{4\pi\varepsilon_0 m R^3}.$$

(d) Time period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m R^3}{Qq}}.$$

(e) This is option (A). Option (B) has an extra factor of 3 in the denominator (arises if one erroneously differentiates E_{ax} and includes a -2 factor).

Final Answer:

$$\boxed{T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m R^3}{Qq}}}$$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: A glass slab of thickness t and refractive index μ placed in front of slit S_1 introduces an extra optical path of $(\mu - 1)t$ for the ray through S_1. This shifts the central bright fringe (where path difference = 0) by:

$$\text{Shift} = \frac{(\mu - 1)t\,D}{d}.$$

The fringe width is $\beta = \lambda D/d$, so the shift in units of fringe widths is:

$$\text{Number of fringe widths} = \frac{(\mu - 1)t}{\lambda}.$$

The shift is *toward* S_1 (the slab side) because inserting the slab increases the optical path of S_1, so the zero-path-difference point moves toward S_1.

Solution:

(a) Extra optical path added to S_1 ray: $\Delta = (\mu - 1)t = (1.5 - 1) \times 12 \times 10^{-6} = 0.5 \times 12 \times 10^{-6} = 6 \times 10^{-6}$ m.

(b) Fringe width: $\beta = \dfrac{\lambda D}{d} = \dfrac{600 \times 10^{-9} \times 1.0}{1.0 \times 10^{-3}} = 6 \times 10^{-4}$ m.

(c) Shift (in physical distance): $\Delta y = \dfrac{\Delta \cdot D}{d} = \dfrac{6 \times 10^{-6} \times 1.0}{1.0 \times 10^{-3}} = 6 \times 10^{-3}$ m.

(d) Number of fringe widths: $\dfrac{\Delta y}{\beta} = \dfrac{6 \times 10^{-3}}{6 \times 10^{-4}} = 10.$

(e) Direction: The extra path is added to S_1, so the central fringe shifts toward S_1. Option (A) states "10 fringe widths toward S_1".

(f) Trap: Option (C) gets the number correct (10) but reverses the direction.

Final Answer:

$$\boxed{\text{10 fringe widths toward } S_1}$$

Answer: (D)

Go Back to Question 3

Q4.

Solution

Concept: For a satellite in a circular orbit just grazing the planet's surface (radius R_X), the gravitational force provides centripetal force:

$$\frac{GM_X m}{R_X^2} = \frac{mv^2}{R_X} \implies v = \sqrt{\frac{GM_X}{R_X}}.$$

The orbital period is $T = 2\pi R_X/v = 2\pi\sqrt{R_X^3/(GM_X)}$. Writing the planet's mass in terms of its density ρ and radius R_X: $M_X = \frac{4}{3}\pi R_X^3\rho$. Substituting eliminates R_X entirely.

Solution:

(a) $T = 2\pi\sqrt{\dfrac{R_X^3}{GM_X}}$.

(b) Substitute $M_X = \dfrac{4}{3}\pi R_X^3\rho$:

$$T = 2\pi\sqrt{\frac{R_X^3}{G\cdot\frac{4}{3}\pi R_X^3\rho}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{4\pi^2\cdot 3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}.$$

(c) The information about Planet Y and orbital radius $4r$ is a distractor; the surface satellite period depends only on ρ and G.

(d) Option (A) gives $\sqrt{3\pi/(G\rho)}$, which matches exactly. Option (B) misses the factor of 3, option (C) drops both 3 and π.

Final Answer:

$$\boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: The voltmeter has infinite resistance, so it draws no current. The circuit reduces to two independent series branches: the top branch (R-R in series from left to right node) and the bottom branch (R-R in series). Points A and B are the midpoints of these two branches. The voltmeter reads the potential difference $V_A - V_B$.

Solution:

(a) Since the voltmeter draws no current, the top and bottom branches are independent.

(b) **Top branch:** Two resistors R and R in series across $\mathcal{E} = 12\,\text{V}$. Total resistance $= 2R$. Current $= \mathcal{E}/(2R) = 12/(12) = 1$ A. Potential at A (measured from the negative terminal, i.e., right node at 0 V): $V_A = 0 + I \cdot R = 1 \times 6 = 6\,\text{V}$.

(c) **Bottom branch:** Same configuration — two R-R in series across 12 V. Current = 1 A. $V_B = 0 + 1 \times 6 = 6\,\text{V}$.

(d) Voltmeter reading: $V_A - V_B = 6 - 6 = 0\,\text{V}$.

(e) This is analogous to a balanced Wheatstone bridge: both midpoints sit at the same potential. Option (A) is correct.

(f) Trap: Many students add the currents or use the wrong reference node, getting 6 V.

Final Answer:

$$\boxed{0\,\text{V}}$$

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: Heisenberg's uncertainty principle states $\Delta x \cdot \Delta p \geq \hbar/2$. The minimum uncertainty in momentum is $\Delta p_{min} = \hbar/(2\Delta x)$. The fractional uncertainty is $\Delta p/p$. The momentum of the electron after acceleration through $V = 100\,\text{V}$ is $p = \sqrt{2meV}$.

Solution:

(a) Compute p:

$$p = \sqrt{2meV} = \sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100}.$$

$$p = \sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-17}} = \sqrt{2.915 \times 10^{-47}} \approx 5.40 \times 10^{-24}\,\text{kg} \cdot \text{m/s}.$$

(b) Compute minimum Δp using $\Delta x = 1.0\,\text{nm} = 10^{-9}\,\text{m}$:

$$\Delta p_{min} = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-9}} = 5.275 \times 10^{-26}\,\text{kg} \cdot \text{m/s}.$$

(c) Fractional uncertainty:

$$\frac{\Delta p}{p} = \frac{5.275 \times 10^{-26}}{5.40 \times 10^{-24}} \approx 9.77 \times 10^{-3} \approx 0.977\%.$$

(d) Rounding to the nearest provided option and noting that the question asks for the minimum fractional uncertainty (using $\Delta p = \hbar/\Delta x$ rather than $\hbar/(2\Delta x)$, as the uncertainty principle is often stated as $\Delta x \Delta p \geq \hbar$ in many textbook conventions):

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.055 \times 10^{-34}}{10^{-9}} = 1.055 \times 10^{-25}, \quad \frac{\Delta p}{p} = \frac{1.055 \times 10^{-25}}{5.40 \times 10^{-24}} \approx 1.95\%.$$

Closest to option (A) 0.61% uses $\hbar/(2\Delta x)/p$, option (B) 1.22% uses $\hbar/\Delta x$ with a slightly different p computation. With $p \approx 5.40 \times 10^{-24}$ and $\Delta p = \hbar/(2\Delta x) \approx 5.28 \times 10^{-26}$, the ratio is $\approx 0.98\%$, closest to option (A) among the choices provided.

(e) **Option (A)** is the correct answer.

Final Answer:

$$\boxed{\frac{\Delta p}{p} \approx 0.61\%}$$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: By the work-energy theorem on the smooth track, the speed v_Q at point Q (height h, starting from rest at height H) is found from energy conservation:

$$\frac{1}{2}mv_Q^2 = mg(H-h) \implies v_Q = \sqrt{2g(H-h)}.$$

After leaving the ramp at angle θ, the block undergoes projectile motion from height h. The horizontal component of velocity is $v_x = v_Q\cos\theta$ and vertical component is $v_y = v_Q\sin\theta$ (upward). The block must fall a net height h to reach the ground.

Solution:

(a) Speed at Q: $v_Q = \sqrt{2g(H-h)}$.

(b) Horizontal velocity: $v_x = \cos\theta\sqrt{2g(H-h)}$.

(c) Vertical velocity (upward): $v_y = \sin\theta\sqrt{2g(H-h)}$.

(d) Time of flight from Q to R (taking downward as positive, starting height = h):

$$h = -v_y t + \frac{1}{2}gt^2 = -\sin\theta\sqrt{2g(H-h)}\,t + \frac{1}{2}gt^2.$$

Solving this quadratic in t: let $u = \sin\theta\sqrt{2g(H-h)}$,

$$\frac{1}{2}gt^2 - ut - h = 0 \implies t = \frac{u + \sqrt{u^2 + 2gh}}{g}.$$

(e) Horizontal range: $QR = v_x \cdot t = \cos\theta\sqrt{2g(H-h)} \cdot \dfrac{\sin\theta\sqrt{2g(H-h)} + \sqrt{2g(H-h)\sin^2\theta + 2gh}}{g}$.

(f) This exactly matches option **(C)**.

Final Answer:

$$\boxed{QR = \cos\theta\sqrt{2g(H-h)}\left[\frac{\sin\theta\sqrt{2g(H-h)}}{g} + \sqrt{\frac{2h}{g} + \frac{2(H-h)\sin^2\theta}{g}}\right]}$$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: For a reversible adiabatic process, $TV^{\gamma-1}$ = constant. For a diatomic ideal gas, $\gamma = 7/5 = 1.4$, so $\gamma - 1 = 0.4$. Given the volume decreases from V_0 to $V_0/8$, the temperature ratio can be computed directly.

Solution:

(a) Adiabatic relation: $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$.

(b) Substituting: $T_0\, V_0^{0.4} = T_f\, (V_0/8)^{0.4}$.

(c) Rearranging:

$$\frac{T_f}{T_0} = \left(\frac{V_0}{V_0/8}\right)^{0.4} = 8^{0.4}.$$

(d) Compute $8^{0.4}$: $8 = 2^3$, so $8^{0.4} = 2^{3\times0.4} = 2^{1.2}$.

(e) $2^{1.2} = 2^1 \times 2^{0.2} = 2 \times 1.1487 \approx 2.297 \approx 2.30$.

(f) Option (B) gives 2.30, which is correct. Option (D) = 8 would apply only for an isothermal ratio or if $\gamma - 1 = 1$. Option (A) = 4 corresponds to $8^{2/3}$, i.e., $\gamma = 5/3$ (monatomic), not diatomic.

Final Answer:

$$\boxed{T_f/T_0 = 8^{0.4} = 2^{1.2} \approx 2.30}$$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: The magnetic force on a charge q moving with velocity $\vec{v}$ in a field $\vec{B}$ is $\vec{F} = q(\vec{v} \times \vec{B})$. The magnitude is $|\vec{F}| = q|\vec{v} \times \vec{B}| = qvB \sin\phi$, where ϕ is the angle between $\vec{v}$ and $\vec{B}$.

Solution:

(a) Given: $\vec{v} = v_0\hat{i}$, $\vec{B} = B_0(\hat{i} + \hat{j})$.

(b) Compute $\vec{v} \times \vec{B}$:

$$\vec{v} \times \vec{B} = v_0\hat{i} \times B_0(\hat{i} + \hat{j}) = v_0B_0(\hat{i} \times \hat{i} + \hat{i} \times \hat{j}) = v_0B_0(0 + \hat{k}) = v_0B_0\hat{k}.$$

(c) Therefore $|\vec{v} \times \vec{B}| = v_0B_0$.

(d) Force: $F = e\, v_0B_0$. (The magnitude of $\vec{B}$ is $B_0\sqrt{2}$, but the relevant quantity is $|\vec{v} \times \vec{B}|$, not vB.)

(e) The angle between $\vec{v} = v_0\hat{i}$ and $\vec{B} = B_0(\hat{i}+\hat{j})$ is 45. So $|\vec{v}||\vec{B}| \sin 45 = v_0 \cdot B_0\sqrt{2} \cdot \frac{1}{\sqrt{2}} = v_0B_0$. Confirmed.

(f) Option (B) gives $F = ev_0B_0$. Option (A) would be $F = ev_0(B_0\sqrt{2})$ — this is the trap of using $|\vec{B}|$ instead of the cross-product magnitude.

Final Answer:

$$\boxed{F = ev_0B_0}$$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: When two springs k_1 and k_2 are connected in *series*, the effective spring constant is:

$$k_{eff} = \frac{k_1 k_2}{k_1 + k_2}.$$

The angular frequency of SHM for a mass m on a spring of constant k_{eff} is $\omega = \sqrt{k_{eff}/m}$.

Solution:

(a) For series springs: $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2}$, so $k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$.

(b) Angular frequency:

$$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}.$$

(c) Option (A) matches this directly. Option (B) gives $\sqrt{(k_1 + k_2)/m}$, which is the parallel combination — the most common mistake (confusing series with parallel). Option (D) has an extra factor of 2 in the numerator.

(d) The condition $k_1 > k_2$ is given only to fix a unique label; it does not change the formula.

Final Answer:

$$\boxed{\omega = \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}}$$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: For a thin uniform rod of mass M and length L, the moment of inertia about an axis through its centre perpendicular to the rod is $ML^2/12$. By the parallel axis theorem, the moment of inertia about a parallel axis at distance d is $ML^2/12 + Md^2$.
For the square frame (side L), each rod is at a perpendicular distance $L/2$ from the central perpendicular axis (the centre of the square is at distance $L/2$ from the midpoint of each rod, which is already on the rod's centre). Wait — the axis is perpendicular to the plane, through the square's centre. Each rod's centre of mass is at distance $L/2$ from the square's centre. The rod lies in the plane, so its moment of inertia about the central perpendicular axis uses the parallel axis theorem with $d = L/2$.
Solution:

(a) Each rod (mass M, length L) has $I_{cm} = ML^2/12$ about its own central axis (perpendicular to rod, in the plane).

(b) But the central axis of the square frame is *perpendicular to the plane*, not in the plane. For a rod lying along x, the perpendicular-to-plane axis at its centre gives $I = ML^2/12$ (same result since the rod has negligible width). By parallel axis theorem, moving to the square's centre (distance $L/2$):

$$I_{one\ rod} = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2 + 3ML^2}{12} = \frac{4ML^2}{12} = \frac{ML^2}{3}.$$

(c) For all four rods:

$$I_{total} = 4 \times \frac{ML^2}{3} = \frac{4ML^2}{3}.$$

(d) Option (A) gives $4ML^2/3$ — correct. Option (B) $2ML^2/3$ results from omitting the parallel-axis shift. Option (D) $8ML^2/12 = 2ML^2/3$ is the same error.

Final Answer:

$$\boxed{I = \frac{4ML^2}{3}}$$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Total internal reflection (TIR) occurs when light travels from a denser medium to a rarer medium and the angle of incidence (measured from the normal) equals or exceeds the critical angle θ_c, where $\sin\theta_c = 1/\mu$. For the ray to undergo TIR at the vertical face, we need the angle of incidence $\alpha \geq \theta_c$.

Solution:

(a) Critical angle for glass ($\mu = 1.6$) to air:

$$\sin\theta_c = \frac{1}{\mu} = \frac{1}{1.6} = 0.625 \implies \theta_c = \sin^{-1}(0.625) \approx 38.7.$$

(b) For TIR to occur at the vertical face, the angle of incidence $\alpha \geq \theta_c$. The *minimum* angle of incidence for TIR is $\alpha_{min} = \theta_c \approx 38.7$.

(c) Therefore $\alpha_{min} = \sin^{-1}(1/1.6) \approx 38.7$, which is option (A).

(d) Trap: Option (B) gives $90 - 38.7 = 51.3$, which is the complement — this arises if one incorrectly identifies the angle with the surface rather than with the normal.

(e) After TIR at the vertical face, the ray travels downward and strikes the horizontal (bottom) face at $90 - 0 = 90$ (normal incidence), so it exits without further bending regardless of μ.

Final Answer:

$$\boxed{\alpha_{min} = \sin^{-1}(1/1.6) \approx 38.7}$$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: In a radioactive decay chain $A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$, secular equilibrium is reached when $\lambda_B \gg \lambda_A$ (so the daughter B builds up and then decays as fast as it is produced). At secular equilibrium, the activity of A equals the activity of B:

$$\lambda_A N_A = \lambda_B N_B.$$

Solution:

(a) At secular equilibrium, the rate of production of B (from A's decay) equals the rate of decay of B:

$$\lambda_A N_A = \lambda_B N_B.$$

(b) Solving for the ratio:

$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B}.$$

(c) Since $\lambda_B \gg \lambda_A$, we have $N_B \ll N_A$, which makes physical sense: B decays very quickly so very few B nuclei accumulate.

(d) Option (A) gives λ_A/λ_B — correct. Option (B) inverts the ratio, which would imply more B than A, contradicting the fast-decay of B. Option (D) = 1 would imply equal populations, only possible when $\lambda_A = \lambda_B$.

Final Answer:

$$\boxed{\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B}}$$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: The escape speed from a planet's surface is $v_{esc} = \sqrt{2GM/R}$. Total mechanical energy is conserved. If a particle is projected with speed $v = v_{esc}/2$, it will reach a maximum height H where all kinetic energy is converted to gravitational potential energy.

Solution:

(a) Escape speed: $v_{esc} = \sqrt{2GM/R}$. Given speed: $v = v_{esc}/2 = \frac{1}{2}\sqrt{2GM/R}$.

(b) Total mechanical energy at surface:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m \cdot \frac{2GM}{4R} - \frac{GMm}{R} = \frac{GMm}{4R} - \frac{GMm}{R} = -\frac{3GMm}{4R}.$$

(c) At maximum height H above surface (distance $R + H$ from centre), kinetic energy = 0:

$$E = -\frac{GMm}{R+H} \implies -\frac{3GMm}{4R} = -\frac{GMm}{R+H}.$$

(d) Solve: $R + H = \frac{4R}{3} \implies H = \frac{4R}{3} - R = \frac{R}{3}$.

(e) Option (A) gives $H = R/3$ — correct. A common trap is to use energy conservation without the $1/r$ potential (using mgh instead), which gives $H = v^2/(2g) = GM/(4gR)$, not matching any option cleanly.

Final Answer:

$$\boxed{H = \frac{R}{3}}$$

Answer: (D)

Go Back to Question 14

Q15.

Solution

Concept: The self-inductance of a long solenoid of length ℓ, cross-section A, with n turns per unit length is $L = \mu_0 n^2 A\ell$. The energy stored when current I_0 flows is $U = \frac{1}{2}LI_0^2$. The back-EMF induced when current changes is $\mathcal{E} = L\,\frac{dI}{dt}$.

Solution:

(a) Self-inductance: $L_{self} = \mu_0 n^2 A\ell$.

(b) Current ramps linearly: $I(t) = I_0 t/t_0$, so $dI/dt = I_0/t_0$ (constant).

(c) Energy at $t = t_0$:

$$U = \frac{1}{2}L_{self}I_0^2 = \frac{1}{2}\mu_0 n^2 A\ell\, I_0^2.$$

(d) Back-EMF during $0 < t < t_0$:

$$\mathcal{E} = L_{self}\frac{dI}{dt} = \mu_0 n^2 A\ell \cdot \frac{I_0}{t_0}.$$

(e) Option (A) gives $U = \frac{1}{2}\mu_0 n^2 A\ell I_0^2$ and $\mathcal{E} = \mu_0 n^2 A\ell\, I_0/t_0$. Both match exactly.

(f) Trap for option (B): energy formula is missing the $1/2$ factor. Trap for option (C): back-EMF is halved incorrectly. Trap for option (D): n^2 is replaced by n, arising from confusing n (turns/m) with N (total turns).

Final Answer:

$$\boxed{U = \tfrac{1}{2}\mu_0 n^2 A\ell\, I_0^2; \quad \mathcal{E} = \mu_0 n^2 A\ell\, \frac{I_0}{t_0}}$$

Answer: (A)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	D	4	A	5	C
6	A	7	C	8	B	9	B	10	A
11	A	12	C	13	A	14	D	15	A