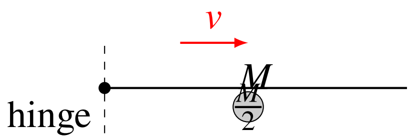
IISER Physics Sample Paper-8

Duration: 45 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1** marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. A uniform rod of mass M and length L is hinged at one end to a fixed support and is held horizontally. A small ball of mass $\frac{M}{2}$, moving with speed v in a direction perpendicular to the rod, strikes the rod at its midpoint and sticks to it. Find the angular speed of the system immediately after the collision.



(A) $\frac{3v}{5L}$

(B) $\frac{6v}{11L}$

(C) $\frac{2v}{3L}$

(D) $\frac{4v}{9L}$

Q2. A cube of side a has a point charge $+q$ placed at each of its eight vertices. What is the total electric flux through any one face of the cube?

(A) $\frac{q}{3\varepsilon_0}$

(B) $\frac{q}{6\varepsilon_0}$

(C) $\frac{q}{2\varepsilon_0}$

(D) $\frac{8q}{6\varepsilon_0}$

Q3. Electrons accelerated through a potential difference V produce first-order diffraction maxima at an angle θ when incident on a crystal with inter-atomic spacing d. If protons are accelerated through a potential difference $4V$ and incident on the same crystal, what is the ratio $\frac{\sin\theta_p}{\sin\theta_e}$ for the first-order maxima? (Take mass of proton as m_p and mass of electron as m_e.)

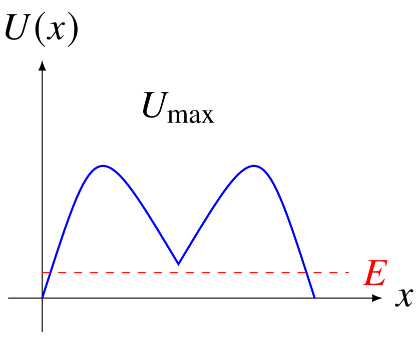
(A) $\frac{1}{2}\sqrt{\frac{m_p}{m_e}}$

(B) $\frac{1}{2}\sqrt{\frac{m_e}{m_p}}$

(C) $\frac{1}{4}\sqrt{\frac{m_e}{m_p}}$

(D) $2\sqrt{\frac{m_e}{m_p}}$

Q4. A particle of mass m moves in a one-dimensional conservative force field with potential energy $U(x) = U_0\left(\frac{x}{a}\right)^2\left(1-\frac{x}{a}\right)^2$, where $U_0 > 0$ and $a > 0$. If the total mechanical energy of the particle is $E = \frac{U_0}{8}$, what is its maximum speed?



(A) $\sqrt{\frac{U_0}{2m}}$

(B) $\sqrt{\frac{U_0}{4m}}$

(C) $\sqrt{\frac{U_0}{8m}}$

(D) $\frac{1}{2}\sqrt{\frac{U_0}{2m}}$

Q5. A thin transparent film of thickness t and refractive index $\mu = 1.5$ is coated on a glass substrate of refractive index $\mu_g = 1.8$. Light of wavelength λ in air is incident normally on the film. What is the minimum thickness t for which the reflected light has maximum intensity?

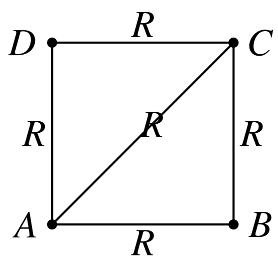
(A) $\frac{\lambda}{6}$

(B) $\frac{\lambda}{3}$

(C) $\frac{\lambda}{4}$

(D) $\frac{\lambda}{2}$

Q6. In the network shown in the figure, each resistor has resistance R. A square $ABCD$ has resistors along each side and a resistor along the diagonal AC. Find the equivalent resistance between terminals A and B.



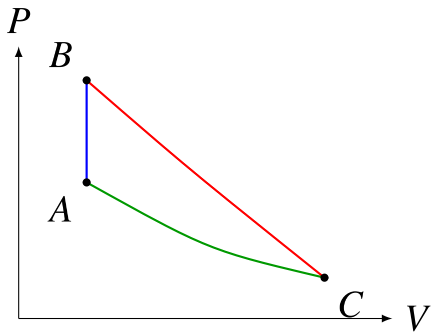
(A) $\frac{5R}{8}$

(B) $\frac{3R}{5}$

(C) $\frac{2R}{3}$

(D) $\frac{R}{2}$

Q7. One mole of a monatomic ideal gas undergoes a cyclic process $ABCA$ as shown. Process $A \rightarrow B$ is isochoric, $B \rightarrow C$ is adiabatic, and $C \rightarrow A$ is isothermal. Given $V_A = V_0$, $V_C = 8V_0$, and $T_A = T_0$, the efficiency η of the cycle is:



(A) $1 - \frac{\ln 2}{3}$

(B) $1 - \frac{2\ln 2}{3}$

(C) $\frac{2\ln 2}{3}$

(D) $\frac{1}{3}$

Q8. Two stars of masses M and $2M$ are separated by a distance d and revolve about their common center of mass in circular orbits. The minimum energy required to separate them to an infinite distance is:

(A) $\frac{GM^2}{d}$

(B) $\frac{GM^2}{2d}$

(C) $\frac{3GM^2}{2d}$

(D) $\frac{2GM^2}{d}$

Q9. A radioactive sample contains N_0 nuclei of isotope X at $t = 0$, which decays to a stable isotope Y with half-life T. The number of nuclei of Y produced during the time interval $t = \frac{T}{2}$ to $t = T$ is:

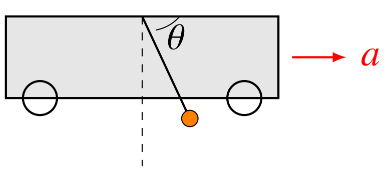
(A) $N_0\left(1 - \frac{1}{\sqrt{2}}\right)$

(B) $\frac{N_0}{2}$

(C) $\frac{N_0}{2\sqrt{2}}$

(D) $\frac{N_0(\sqrt{2}-1)}{2}$

Q10. A simple pendulum of length L is suspended from the ceiling of a trolley that moves horizontally with a constant acceleration a. For small oscillations, the time period of the pendulum is:



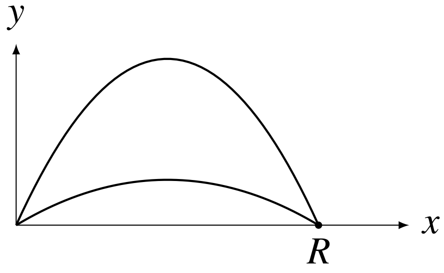
(A) $2\pi\sqrt{\frac{L}{g}}$

(B) $2\pi\sqrt{\frac{L}{\sqrt{g^2+a^2}}}$

(C) $2\pi\sqrt{\frac{L}{g+a}}$

(D) $2\pi\sqrt{\frac{L}{g-a}}$

Q11. Two projectiles are launched from the same point on horizontal ground with the same initial speed u but different angles of projection α and β (both acute). They land at the same point on the ground and satisfy $\alpha + \beta = 90°$. If their maximum heights are H_1 and H_2, the value of $H_1 + H_2$ is:



(A) $\frac{u^2}{g}$

(B) $\frac{u^2}{2g}$

(C) $\frac{u^2}{4g}$

(D) $\frac{u^2}{8g}$

Q12. A thin biconvex lens of focal length f in air is made of glass of refractive index $\mu = 1.5$. When the lens is completely immersed in a liquid, its focal length becomes $2f$. The refractive index of the liquid is:

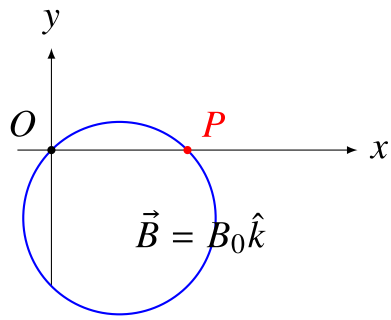
(A) 1.20

(B) 1.25

(C) 1.30

(D) 1.33

Q13. A particle of charge q and mass m is projected from the origin with velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ into a region of uniform magnetic field $\vec{B} = B_0\hat{k}$. The coordinates of the particle when it next crosses the x-axis are:

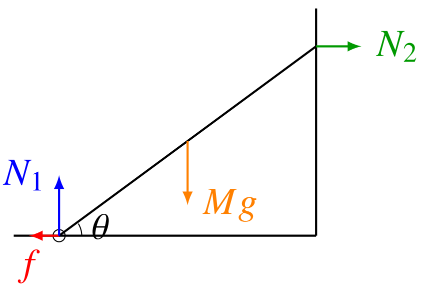


(A) $\left(\frac{\pi m v_0}{qB_0}, 0, 0\right)$
(B) $\left(\frac{m v_0}{qB_0}, 0, 0\right)$
(C) $\left(\frac{2m v_0}{qB_0}, 0, 0\right)$
(D) $(0, 0, 0)$

Q14. A wire of length L has a cross-sectional area A and a resistivity that varies linearly along its length as $\rho(x) = \rho_0\left(1 + \frac{x}{L}\right)$, where x is measured from one end. The resistance of the wire is:

(A) $\frac{\rho_0 L}{A}$
(B) $\frac{3\rho_0 L}{2A}$
(C) $\frac{2\rho_0 L}{A}$
(D) $\frac{\rho_0 L}{2A}$

Q15. A uniform rod of length L and mass M leans against a smooth vertical wall and stands on a rough horizontal floor with coefficient of static friction μ. The minimum angle θ that the rod can make with the horizontal without slipping is given by:



(A) $\tan\theta = \frac{1}{\mu}$

(B) $\tan\theta = \frac{\mu}{2}$

(C) $\tan\theta = 2\mu$

(D) $\tan\theta = \frac{1}{2\mu}$

Detailed Solutions

Q1.

Solution

Concept:
This problem involves a completely inelastic collision between a moving ball and a hinged rod. Since the rod is fixed at one end by a hinge, external linear forces act at the hinge during the impact. Therefore, linear momentum is not conserved. However, the external torque about the hinge is zero, meaning that the total angular momentum of the system about the hinge is conserved during the collision.

Solution:

(a) The initial angular momentum L_i of the system about the hinge just before the collision is solely due to the moving ball of mass $M/2$ traveling at velocity v. It strikes at the midpoint of the rod, which is at a distance of $L/2$ from the hinge. Thus, we have $L_i = (M/2) \cdot v \cdot (L/2) = MvL/4$.

(b) After the collision, the ball sticks to the rod at its midpoint. The final moment of inertia I_f of the combined system about the hinge is the sum of the moment of inertia of the uniform rod about its end and the moment of inertia of the stuck ball treated as a point mass.

(c) Calculating these values gives $I_{\text{rod}} = (1/3)ML^2$ and $I_{\text{ball}} = (M/2)(L/2)^2 = ML^2/8$. Adding them together yields the total moment of inertia: $I_f = (1/3)ML^2 + (1/8)ML^2 = (11/24)ML^2$.

(d) Applying the law of conservation of angular momentum ($L_i = L_f$), we set $MvL/4 = I_f \cdot \omega$. Substituting I_f into the equation gives $MvL/4 = (11/24)ML^2 \cdot \omega$.

(e) Solving for the final angular speed ω yields $\omega = (24/11) \cdot (v/4L) = 6v/11L$. This corresponds directly to option B.

Final Answer: 6v$_{\overline{11L}}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept:
This problem utilizes Gauss's Law and the principle of superposition in electrostatics. Gauss's Law states that the total electric flux passing through any closed surface is equal to the net charge enclosed divided by the permittivity of free space. When charges are located precisely on the boundary or vertices of a geometric shape, their contributions must be evaluated based on sharing symmetry among adjacent volumes.

Solution:

(a) Consider a single vertex of a cube. This vertex is symmetrically shared among 8 identical adjacent cubes meeting at that corner point. Therefore, only $1/8$ of the charge placed at a vertex resides inside any single cube.

(b) Since there are 8 vertices on a cube and each vertex contains a charge $+q$, the total effective charge enclosed inside the volume of our single cube is $Q_{\text{enclosed}} = 8 \cdot (q/8) = q$.

(c) According to Gauss's Law, the total electric flux Φ_{total} emerging through all 6 faces of this closed cube is given by $\Phi_{\text{total}} = Q_{\text{enclosed}}/\varepsilon_0 = q/\varepsilon_0$.

(d) By the inherent cubic symmetry of the configuration, the electric field distribution is identical with respect to each of the six flat faces. Thus, the total flux divides equally among all faces.

(e) The electric flux passing through any single face is exactly one-sixth of the total flux. This results in $\Phi_{\text{face}} = \Phi_{\text{total}}/6 = q/6\varepsilon_0$, matching option B.

Final Answer: q$_{\overline{6\varepsilon_0}}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept:
This problem connects the de Broglie wavelength of matter waves with Bragg's law for wave diffraction in crystals. Particles accelerated through an electric potential difference gain kinetic energy, which determines their momentum and wavelength. The diffraction condition then links this wavelength to the angular position of the observed maxima.

Solution:

(a) A particle of mass m and charge q accelerated from rest through a potential difference V gains a kinetic energy $K = qV$. The momentum is $p = \sqrt{2mK} = \sqrt{2mqV}$, so the de Broglie wavelength is $\lambda = h/p = h/\sqrt{2mqV}$.

(b) For first-order diffraction maxima ($n = 1$), Bragg's law states that $\lambda = 2d \sin\theta$, which means $\sin\theta = \lambda/2d$. Since the crystal lattice spacing d is identical for both runs, $\sin\theta$ is directly proportional to λ.

(c) We can write the proportionality relation as $\sin\theta \propto 1/\sqrt{mV}$ because both electrons and protons carry an elementary charge magnitude of $q = e$.

(d) Setting up the ratio for protons and electrons gives $\sin\theta_p/\sin\theta_e = \sqrt{m_e V_e}/\sqrt{m_p V_p}$. Substituting the given acceleration values $V_e = V$ and $V_p = 4V$ simplifies the relation inside the radical.

(e) This substitution yields $\sin\theta_p/\sin\theta_e = \sqrt{m_e \cdot V}/\sqrt{m_p \cdot 4V} = (1/2)\sqrt{m_e/m_p}$. This is option B.

Final Answer: $1\frac{}{2\sqrt{\frac{m_e}{m_p}}}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept:
This mechanics problem relies on the law of conservation of mechanical energy for a conservative force field. The total mechanical energy E is the sum of the particle's kinetic energy K and its potential energy $U(x)$, given as $E = K + U(x)$. The particle reaches its maximum speed where its kinetic energy is maximized, which occurs where the potential energy is minimized.

Solution:

(a) The total energy is constant at $E = (1/2)mv^2 + U(x)$. To maximize the velocity v, we must minimize the potential energy function $U(x) = U_0(x/a)^2(1 - x/a)^2$.

(b) Since $U_0 > 0$ and the terms are squared, the absolute minimum value that $U(x)$ can achieve is $U_{\text{min}} = 0$. This occurs at the stable position coordinates $x = 0$ and $x = a$.

(c) At these minimum potential energy positions, all of the available mechanical energy is converted completely into kinetic energy, meaning $E = K_{\text{max}} + 0$.

(d) We equate the given total energy to the maximum kinetic energy expression: $U_0/8 = (1/2)mv_{\text{max}}^2$.

(e) Isolating v_{max}^2 gives $v_{\text{max}}^2 = 2U_0/8m = U_0/4m$. Taking the square root of both sides leads to $v_{\text{max}} = \sqrt{U_0/4m}$, which matches option B.

Final Answer: $\sqrt{\frac{U_0}{4m}}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept:
This optics problem involves thin-film wave interference. When light traveling in air encounters a thin film coated on a substrate, it reflects from both the top surface (air-film boundary) and the bottom surface (film-substrate boundary). The relative values of the refractive indices determine whether phase shifts occur upon reflection, altering the condition for constructive interference.
Solution:

(a) The refractive indices given are $\mu_{\text{air}} = 1.0$, $\mu_{\text{film}} = 1.5$, and $\mu_{\text{glass}} = 1.8$. Since $\mu_{\text{air}} < \mu_{\text{film}}$, the wave reflecting from the first surface experiences a π phase change (hard reflection).

(b) Since $\mu_{\text{film}} < \mu_{\text{glass}}$, the wave reflecting from the second surface also moves from a lower to a higher index medium, experiencing an identical π phase change.

(c) Because both reflections undergo the exact same phase shift, their relative phase difference due to reflection is zero. The overall phase difference is caused entirely by the optical path difference traveled inside the film, which is $2\mu t$ for normal incidence.

(d) For maximum intensity in the reflected light, constructive interference must occur. The condition for constructive interference with no relative reflection phase shift is $2\mu t = n\lambda$, where $n = 1, 2, 3, \ldots$.

(e) To find the minimum thickness, we set $n = 1$, giving $2\mu t = \lambda$. Substituting $\mu = 1.5$ yields $2(1.5)t = \lambda$, which simplifies to $3t = \lambda$, or $t = \lambda/3$. This corresponds to option B.

Final Answer: $\lambda_{\overline{3}}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept:
This problem requires finding the equivalent resistance of a bridge network. Due to the asymmetric placement of the single diagonal resistor AC, the network cannot be simplified as a balanced Wheatstone bridge. Instead, we must exploit the geometric and electrical mirror symmetry of the circuit relative to an axis, or use delta-wye (Δ-Y) transformations.

Solution:

(a) Let us apply a potential difference across terminals A and B, setting $V_B = 0$ and $V_A = V$. The upper portion of the circuit forms a delta loop composed of nodes A, D, and C, with resistors along branches AD, DC, and AC, each having resistance R.

(b) We can transform this delta network (ADC) into an equivalent wye (Y) network centered at a new virtual node N. The resistances connected to nodes A, C, and D are given by $R_A = (R \cdot R)/(R + R + R) = R/3$, $R_C = R/3$, and $R_D = R/3$.

(c) Now, the original node D is isolated except for the series connection from N. The branch running through D is connected to B via resistor DB, but looking at the original circuit, there is no resistor along DB. Wait, let's re-examine the connections: the remaining original resistors not in the delta loop are branch BC (R) and branch AB (R). Node D is only connected to A and C, so transforming delta ADC completely removes node D.

(d) The transformed network consists of terminal A connected to branch resistor $R_A = R/3$ meeting at node N. From node N, one branch goes to node C with total resistance $R_C + R_{CB} = R/3 + R = 4R/3$, ending at terminal B. The other branch from node N goes through R_D, but node D has no other connections, meaning this branch is open and carries no current.

(e) Thus, the circuit simplifies to branch AN ($R/3$) in series with branch NB (which is $4R/3$). This combined upper path has a total resistance of $R/3 + 4R/3 = 5R/3$. This path is in parallel with the direct resistor along branch AB (R). The equivalent resistance is $R_{\text{eq}} = (R \cdot 5R/3)/(R + 5R/3) = (5R^2/3)/(8R/3) = 5R/8$, matching option A.

Final Answer: $5R_{\overline{8}}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:
This thermodynamics problem focuses on calculating the thermal efficiency of a cyclic process. The efficiency η of any heat engine cycle is defined as the ratio of the net work done by the gas during the cycle to the total heat energy absorbed by the gas from external sources, expressed as $\eta = W_{\text{net}}/Q_{\text{absorbed}} = 1 - |Q_{\text{released}}|/Q_{\text{absorbed}}$.

Solution:

(a) We identify the states and processes. For state A, $V_A = V_0$ and $T_A = T_0$. Process $C \to A$ is isothermal, so $T_C = T_A = T_0$, and $V_C = 8V_0$. Process $B \to C$ is adiabatic, meaning $T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$. Since $A \to B$ is isochoric, $V_B = V_A = V_0$.

(b) For a monatomic gas, $\gamma = 5/3$, so $\gamma - 1 = 2/3$. Using the adiabatic relation: $T_B(V_0)^{2/3} = T_0(8V_0)^{2/3}$, which simplifies to $T_B = T_0 \cdot (8)^{2/3} = 4T_0$.

(c) Heat exchange occurs in two processes. In the isochoric process $A \to B$, temperature increases from T_0 to $4T_0$, so heat is absorbed: $Q_{\text{in}} = nC_v\Delta T = (1)(3/2R)(4T_0 - T_0) = 9/2RT_0$.

(d) In the isothermal process $C \to A$, heat is released as the gas is compressed from $8V_0$ to V_0: $Q_{\text{out}} = nRT_0 \ln(V_A/V_C) = RT_0 \ln(1/8) = -3RT_0 \ln 2$. The magnitude of heat rejected is $|Q_{\text{out}}| = 3RT_0 \ln 2$. No heat exchange happens during the adiabatic leg $B \to C$.

(e) The efficiency is $\eta = 1 - |Q_{\text{out}}|/Q_{\text{in}} = 1 - (3RT_0 \ln 2)/(9/2RT_0) = 1 - (6 \ln 2/9) = 1 - (2 \ln 2/3)$. This corresponds perfectly to option B.

Final Answer: 1 - 2ln 2_3

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:

This problem addresses a two-body gravitationally bound system consisting of two stars orbiting their common center of mass. The minimum energy required to completely separate the components of a bound system to an infinite distance where they are at rest is known as the binding energy. This energy equals the negative of the total mechanical energy of the system.

Solution:

(a) Let the stars have masses $m_1 = M$ and $m_2 = 2M$, separated by a distance d. They revolve in circular paths around their center of mass. The gravitational force between them supplies the necessary centripetal force for each star.

(b) For a gravitationally bound binary system in stable circular orbits, the virial theorem state applies, which means the total kinetic energy K of the system is equal to half the magnitude of the gravitational potential energy U. That is, $K = -U/2$.

(c) The gravitational potential energy of the two-mass configuration separated by distance d is given by the formula $U = -Gm_1m_2/d = -G(M)(2M)/d = -2GM^2/d$.

(d) We calculate the total mechanical energy E of the binary system by summing the total kinetic and potential energies: $E = K + U = -U/2 + U = U/2$. Substituting the value of U yields $E = (-2GM^2/d)/2 = -GM^2/d$.

(e) The minimum external energy E_{req} required to separate them completely to infinity is equal to $-E$. Therefore, $E_{\text{req}} = -(-GM^2/d) = GM^2/d$, which corresponds to option A.

Final Answer: GM$^2\overline{d}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept:
This problem tracks radioactive decay kinetics using the standard exponential decay law. The number of parent nuclei remaining in a sample decreases exponentially over time. Every parent nucleus that decays is converted directly into a stable daughter nucleus, allowing us to find the production yield by computing the change in parent quantities.

Solution:

(a) Let $N(t)$ be the number of active radioactive nuclei of isotope X remaining at time t. According to the radioactive decay law, the population decays from its initial count N_0 following the expression $N(t) = N_0(1/2)^{t/T}$, where T is the half-life.

(b) We calculate the remaining number of parent nuclei X at the first timestamp, $t_1 = T/2$. Substituting this into the formula gives $N(T/2) = N_0(1/2)^{(T/2)/T} = N_0(1/2)^{1/2} = N_0/\sqrt{2}$.

(c) Next, we determine the remaining number of parent nuclei X at the second timestamp, $t_2 = T$. By definition of the half-life, exactly half of the initial population remains, so $N(T) = N_0/2$.

(d) The number of parent nuclei X that decayed during this specific time interval from $t = T/2$ to $t = T$ is the difference between these two values: $\Delta N = N(T/2) - N(T) = N_0/\sqrt{2} - N_0/2$.

(e) Since each decaying nucleus of X produces exactly one stable nucleus of Y, the number of Y nuclei produced is $N_Y = N_0(1/\sqrt{2} - 1/2) = N_0(\sqrt{2} - 1)/2$. This matches option D.

Final Answer: $\mathrm{N}_0(\sqrt{2} - 1)_{\frac{}{2}}$

Answer: (D)

Go Back to Question 9

Q10.

Solution

Concept:
This problem addresses the behavior of a simple pendulum inside an accelerating reference frame. Because the trolley accelerates horizontally, it constitutes a non-inertial frame of reference. To analyze the pendulum's motion from the trolley's perspective, a horizontal inertial pseudo-force must be introduced, altering the effective acceleration experienced by the pendulum bob.

Solution:

(a) Consider the bob of mass m suspended inside the trolley. In the accelerating frame of the trolley, the bob experiences two constant acceleration fields: the downward acceleration due to gravity $\vec{g}$ and a horizontal pseudo-acceleration $-\vec{a}$ directed opposite to the trolley's motion.

(b) Since these two acceleration vectors are perpendicular to each other, we can find the net effective gravitational acceleration $\vec{g}_{\text{eff}}$ acting on the bob by calculating their vector sum.

(c) The magnitude of this effective acceleration field is determined using the Pythagorean theorem, which gives $g_{\text{eff}} = |\vec{g} - \vec{a}| = \sqrt{g^2 + a^2}$.

(d) At the equilibrium position, the pendulum string aligns itself parallel to the direction of this net effective acceleration, tilting at an angle $\theta = \tan^{-1}(a/g)$ with the vertical.

(e) For small angular displacements away from this new equilibrium position, the pendulum executes simple harmonic motion. The time period is given by modifying the standard formula to $T = 2\pi\sqrt{L/g_{\text{eff}}} = 2\pi\sqrt{L/\sqrt{g^2 + a^2}}$, matching option B.

Final Answer: $2\pi\sqrt{\frac{L}{\sqrt{g^2+a^2}}}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept:
This problem examines two projectiles launched with identical speeds but complementary angles of projection, meaning $\alpha + \beta = 90^\circ$. Complementary projection angles ensure that both projectiles have identical horizontal ranges, but their trajectories differ, resulting in different peak vertical heights.

Solution:

(a) The maximum height attained by a projectile launched with speed u at an angle θ relative to the horizontal is given by the standard kinematic formula $H = u^2 \sin^2 \theta/2g$.

(b) For the first projectile launched at an angle α, the maximum height is $H_1 = u^2 \sin^2 \alpha/2g$.

(c) For the second projectile launched at an angle β, the maximum height is $H_2 = u^2 \sin^2 \beta/2g$. Since the angles are complementary, we substitute $\beta = 90^\circ - \alpha$, giving $\sin\beta = \sin(90^\circ - \alpha) = \cos\alpha$.

(d) Rewriting the maximum height for the second projectile using this trigonometric identity yields $H_2 = u^2 \cos^2 \alpha/2g$.

(e) We find the sum of their maximum heights by adding the two expressions together: $H_1 + H_2 = (u^2/2g)\sin^2\alpha + (u^2/2g)\cos^2\alpha = (u^2/2g)(\sin^2\alpha + \cos^2\alpha)$. Using the identity $\sin^2\alpha + \cos^2\alpha = 1$, the expression simplifies to $H_1 + H_2 = u^2/2g$, matching option B.

Final Answer: $\mathrm{u}^2 \overline{2g}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:
This optics problem analyzes how surrounding media impact a lens's focal length using the Lens Maker's Formula. The refractive power of a lens depends on its geometry and the relative refractive index between the lens material and the surrounding environment. Immersing a lens in a liquid modifies this relative index, altering its focal length.
Solution:

(a) Let the biconvex lens have radii of curvature R_1 and R_2. The Lens Maker's Formula states that $1/f = (\mu_{\text{relative}} - 1)(1/R_1 - 1/R_2)$, where μ_{relative} is the ratio of the lens index to the medium index.

(b) In air, the surrounding medium has a refractive index of 1.0. Given the glass refractive index is $\mu_g = 1.5$, the formula becomes $1/f = (1.5/1 - 1)(1/R_1 - 1/R_2) = 0.5(1/R_1 - 1/R_2)$. This means $(1/R_1 - 1/R_2) = 2/f$.

(c) When the lens is completely immersed in a liquid of refractive index μ_l, the new focal length is $f' = 2f$. The modified Lens Maker's Equation becomes $1/(2f) = (\mu_g/\mu_l - 1)(1/R_1 - 1/R_2)$.

(d) Substituting the geometric factor $(1/R_1 - 1/R_2) = 2/f$ into the immersion equation gives $1/(2f) = (1.5/\mu_l - 1)(2/f)$.

(e) Canceling the focal length terms from both sides yields $1/4 = 1.5/\mu_l - 1$. Adding 1 to both sides gives $5/4 = 1.5/\mu_l$. Solving for μ_l gives $\mu_l = 1.5 \cdot (4/5) = 6/5 = 1.20$, which matches option A.

Final Answer: 1.20

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept:

This problem analyzes the motion of a charged particle moving through a uniform magnetic field. The magnetic Lorentz force acts perpendicular to the velocity vector, serving as a centripetal force that bends the particle's path into a circle within the plane perpendicular to the field lines.

Solution:

(a) The uniform magnetic field is directed along the z-axis, $\vec{B} = B_0\hat{k}$. The particle is launched from the origin $(0, 0, 0)$ with a velocity vector lying completely within the xy-plane, $\vec{v} = v_0\hat{i} + v_0\hat{j}$. The magnitude of this initial speed is $v = \sqrt{v_0^2 + v_0^2} = v_0\sqrt{2}$.

(b) The magnetic force acting at the origin is $\vec{F} = q(\vec{v} \times \vec{B}) = q[(v_0\hat{i} + v_0\hat{j}) \times B_0\hat{k}] = qv_0B_0(-\hat{j} + \hat{i}) = qv_0B_0(\hat{i} - \hat{j})$. This force guides the particle along a circular trajectory in the xy-plane.

(c) The radius of this circular path is determined by balancing the centripetal and magnetic forces: $R = mv/qB_0 = m(v_0\sqrt{2})/qB_0$.

(d) The center of the circular path (x_c, y_c) can be found by moving a distance R from the origin along the direction of the initial force vector. This gives the coordinates $x_c = mv_0/qB_0$ and $y_c = -mv_0/qB_0$.

(e) The equation of the circular path is $(x - x_c)^2 + (y - y_c)^2 = R^2$. To find where the particle next crosses the x-axis, we set $y = 0$, giving $(x - x_c)^2 + (-y_c)^2 = R^2$. Substituting the values reveals the next intersection occurs at $x = 2x_c = 2mv_0/qB_0$, corresponding to option C.

Final Answer: $\left(\frac{2mv_0}{qB_0}, , 0, , 0\right)$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept:
This problem addresses the electrical resistance of a non-uniform conductor where the material resistivity varies continuously along its length. Because the cross-sectional area remains uniform but the resistivity changes as a function of position x, we must integrate the differential resistance of thin slices along the wire's length.

Solution:

(a) Consider an infinitesimal segment of the conducting wire located at a distance x from one end, possessing a differential length dx. The uniform cross-sectional area of the wire is given as A.

(b) The resistivity of this small slice depends on its linear position according to the function $\rho(x) = \rho_0(1 + x/L)$. The differential resistance dR of this thin slice is described by the relation $dR = \rho(x) \cdot dx/A$.

(c) Substituting the linear resistivity profile into our differential equation yields the expression $dR = (\rho_0/A)(1 + x/L)dx$.

(d) To determine the total resistance R of the entire wire, we integrate this differential expression over the full length of the conductor, from $x = 0$ to $x = L$: $R = \int_0^L (\rho_0/A)(1 + x/L)dx$.

(e) Factoring out the constant terms and integrating gives $R = (\rho_0/A)\cdot[x+x^2/2L]_0^L$. Evaluating this expression at the limits yields $R = (\rho_0/A)\cdot(L+L^2/2L) = (\rho_0/A)\cdot(3L/2) = 3\rho_0 L/2A$, matching option B.

Final Answer: $3\rho_0 L_{\overline{2A}}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept:

This static equilibrium problem involves a rigid rod leaning against a wall. For the rod to remain stable without slipping, it must satisfy two equilibrium conditions: the net vector sum of all external forces must equal zero, and the net torque about any chosen pivot point must equal zero.

Solution:

(a) Let the uniform rod have mass M and length L, making an angle θ with the horizontal floor. The forces acting on it are its weight Mg downward at the midpoint, a vertical normal force N_1 and horizontal friction force f from the floor, and a horizontal normal force N_2 from the smooth wall.

(b) Balancing horizontal and vertical forces gives $N_1 = Mg$ and $N_2 = f$. At the minimum angle before slipping, static friction reaches its maximum threshold: $f = f_{\max} = \mu N_1 = \mu Mg$. Therefore, $N_2 = \mu Mg$.

(c) Next, we calculate the net torque about the base contact point on the floor to eliminate N_1 and f. The clockwise torque from gravity is $\tau_g = Mg \cdot (L/2)\cos\theta$.

(d) The counter-clockwise torque exerted by the normal force from the wall is $\tau_w = N_2 \cdot L\sin\theta$. Setting the net torque to zero gives $Mg(L/2)\cos\theta = N_2 L\sin\theta$.

(e) Canceling L and substituting $N_2 = \mu Mg$ yields $Mg(1/2)\cos\theta = \mu Mg\sin\theta$. Dividing both sides by $Mg\cos\theta$ simplifies the equation to $1/2 = \mu\tan\theta$. Solving for $\tan\theta$ gives $\tan\theta = 1/2\mu$, matching option D.

Final Answer: $\tan\theta = \frac{1}{2\mu}$

Answer: (D)

Go Back to Question 15

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	B	5	B
6	A	7	B	8	A	9	D	10	B
11	B	12	A	13	C	14	B	15	D