IISER Physics Sample Paper-9

Duration: 45 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1** marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. A uniform solid cylinder of mass M and radius R is placed on a rough horizontal plank. The plank is suddenly accelerated horizontally with a constant acceleration a_0. If the cylinder rolls without slipping on the plank surface, find the magnitude of the linear acceleration a_c of the center of mass of the cylinder relative to an inertial observer standing on the ground.

(A) $\frac{1}{3}a_0$

(B) $\frac{2}{3}a_0$

(C) $\frac{1}{2}a_0$

(D) a_0

Q2. A planet of mass m moves around a massive star of mass M ($M \gg m$) in an elliptical orbit. Let r_p be the distance of closest approach (perihelion) and r_a be the farthest distance (aphelion). If the velocity of the planet at the perihelion position is v_p, determine its velocity v_a at the aphelion point using purely conservation principles.

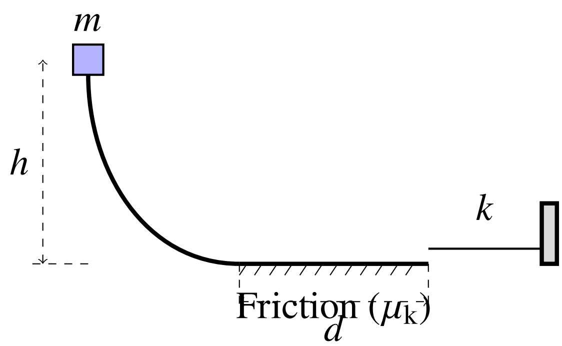
(A) $v_\text{p}\left(\frac{r_\text{p}}{r_\text{a}}\right)$

(B) $v_\text{p}\left(\frac{r_\text{a}}{r_\text{p}}\right)$

(C) $v_p\sqrt{\frac{r_p}{r_a}}$

(D) $v_p\left(\frac{r_p}{r_a}\right)^2$

Q3. A small block of mass m is released from rest from the top edge of a smooth curved track that shifts into a horizontal section. The horizontal segment is rough with a coefficient of kinetic friction μ_k and terminates at an uncompressed ideal spring of constant k as mapped below. If the block descends from a height h and travels a distance d over the rough track before striking the spring, find the maximum compression x_{max} of the spring.



(A) $\sqrt{\frac{2mg(h-\mu_k d)}{k}}$

(B) $\sqrt{\frac{2mgh}{k}} - \mu_k d$

(C) $\sqrt{\frac{2mg(h+\mu_k d)}{k}}$

(D) $\sqrt{\frac{mgh}{2k} - \frac{\mu_k mgd}{k}}$

Q4. An ideal gas undergoes a thermodynamic expansion process where its pressure P and volume V satisfy the relation PV^n = constant. The molar heat capacity of the gas during this specific process is measured to be exactly zero. If the ratio of specific heats ($\gamma = C_p/C_v$) for this gas is 1.40, calculate the numerical value of the polytropic index exponent n.

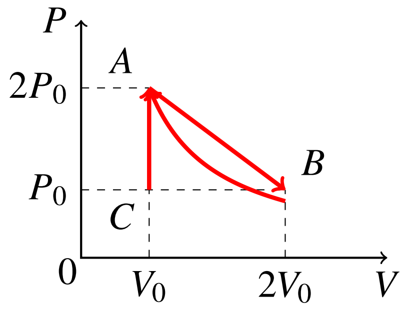
(A) 1.40

(B) 1.00

(C) 0.40

(D) Zero

Q5. An ideal gas is taken through a cyclic thermodynamic path $A \rightarrow B \rightarrow C \rightarrow A$ shown on the Indicator Pressure-Volume (P-V) diagram below. The segment $C \rightarrow A$ is an isothermal path. Determine the net work done (W_{net}) by the gas during one complete cycle.



(A) $P_0V_0(1.5 - \ln 2)$

(B) $P_0V_0(\ln 2 - 0.5)$

(C) $P_0V_0(2 - 2\ln 2)$

(D) $P_0V_0(1 - \ln 2)$

Q6. A long, thin non-conducting rod of length L carries a total positive charge Q distributed uniformly along its length. The rod is rotated with a constant angular speed ω about a fixed axis passing through one of its endpoints, perpendicular to its length. Determine the magnitude of the resulting magnetic dipole moment (μ) produced by this rotating configuration.

(A) $\frac{1}{3}Q\omega L^2$

(B) $\frac{1}{6}Q\omega L^2$

(C) $\frac{1}{2}Q\omega L^2$

(D) $\frac{1}{4}Q\omega L^2$

Q7. A point charge $+q$ is positioned in free space at a distance d directly above a semi-infinite, perfectly grounded conducting horizontal plane sheet. Find the absolute magnitude of the total surface charge density induced on the conducting plane at a point located directly underneath the charge.

(A) $\frac{q}{2\pi d^2}$

(B) $\frac{q}{4\pi d^2}$

(C) $\frac{q}{\pi d^2}$

(D) $\frac{3q}{2\pi d^2}$

Q8. A long solenoid with n turns per unit length and radius R carries a time-dependent current given by $I(t) = I_0 \sin(\omega t)$. A small circular loop of wire with radius r ($r < R$) and total resistance R_0 is placed inside the solenoid such that its plane is perpendicular to the central longitudinal axis of the solenoid. Calculate the peak amplitude of the current induced inside the small loop.

(A) $\frac{\mu_0 n I_0 \pi r^2 \omega}{R_0}$

(B) $\frac{\mu_0 n I_0 \pi R^2 \omega}{R_0}$

(C) $\frac{\mu_0 n I_0 r \omega}{2R_0}$

(D) Zero

Q9. In a standard young's double-slit interference experiment setup, the two slits are separated by a baseline distance d, and the observing screen is placed parallel at a distance D away. If a thin transparent mica sheet of thickness t and refractive index μ is placed directly in front of the upper slit, what is the net linear displacement of the central fringe on the screen?

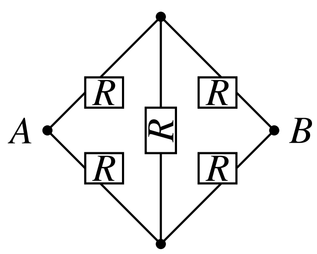
(A) $\frac{D}{d}(\mu - 1)t$

(B) $\frac{d}{D}(\mu - 1)t$

(C) $\frac{D}{2d}(\mu - 1)t$

(D) $\frac{D}{d}\mu t$

Q10. Find the equivalent resistance across the main external terminal connection nodes A and B within the symmetrical network configuration diagram shown below. All individual resistors in the network have a resistance of R.



(A) R

(B) $2R$

(C) $\frac{1}{2}R$

(D) $\frac{2}{3}R$

Q11. A beam of monochromatic light of wavelength λ_0 in vacuum enters a flat block of glass of refractive index μ. If the light travels a distance x within this glass medium, calculate the absolute change in the optical phase ($\Delta\phi$) experienced by the wave relative to traveling the same distance in free space.

(A) $\frac{2\pi}{\lambda_0}(\mu - 1)x$

(B) $\frac{2\pi}{\lambda_0}\mu x$

(C) $\frac{2\pi}{\lambda_0}\left(1 - \frac{1}{\mu}\right)x$

(D) $\frac{2\pi\mu}{\lambda_0}x$

Q12. A metallic surface is illuminated with monochromatic light of wavelength λ. The stopping potential for the photoelectrons is measured to be V_0. When the identical surface is illuminated with light of wavelength 2λ, the stopping potential drops to $V_0/3$. Determine the threshold wavelength (λ_{th}) for photoelectric emission from this metal surface.

(A) 4λ

(B) 3λ

(C) 2.5λ

(D) 1.5λ

Q13. A radioactive sample consists of two distinct isotopes A and B with decay constants $\lambda_A = 2\lambda$ and $\lambda_B = \lambda$ respectively. Initially, at time $t = 0$, the number of active nuclei of both isotopes is identical ($N_A = N_B = N_0$). Find the time t at which the total activity of isotope B becomes exactly twice the activity of isotope A.

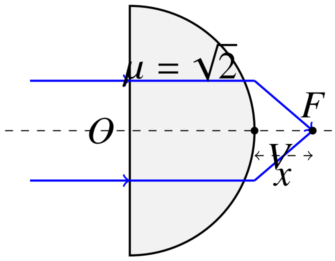
(A) $\frac{1}{\lambda}\ln(4)$

(B) $\frac{1}{\lambda}\ln(2)$

(C) $\frac{2}{\lambda}\ln(2)$

(D) $\frac{1}{2\lambda}\ln(2)$

Q14. A parallel ray bundle of monochromatic light enters a transparent solid glass hemisphere of radius R and refractive index $\mu = \sqrt{2}$ from the flat side, parallel to the central axis. Find the distance x from the vertex point O on the curved boundary surface where the paraxial rays converge to a sharp focus.



(A) $(\sqrt{2}-1)R$

(B) R

(C) $\frac{R}{\sqrt{2}-1}$

(D) $\sqrt{2}R$

Q15. An ideal monatomic gas is contained within a rigid horizontal cylinder divided into two equal compartments, A and B, by a movable, thin, perfectly insulating frictionless piston. Initially, both compartments have a volume V_0, contain an equal number of moles N_0 of the gas, and are in thermal equilibrium at an absolute temperature T_0. An internal electrical heating coil inside compartment A is switched on, slowly supplying heat to the gas until its absolute temperature reaches $T_A = 4T_0$. If compartment B compresses quasi-statically and adiabatically to a final volume V_B, determine the total mechanical work (W) performed by the piston on the gas in compartment B.

(A) $\frac{3}{2}N_0RT_0\left[\left(\frac{V_0}{V_B}\right)^{2/3}-1\right]$

(B) $3N_0RT_0\left[1-\left(\frac{V_B}{V_0}\right)^{2/3}\right]$

(C) $\frac{3}{2}N_0RT_0\left[4-\left(\frac{V_0}{V_B}\right)^{1/3}\right]$

(D) $N_0RT_0\ln\left(\frac{V_0}{V_B}\right)$

Detailed Solutions

Q1.

Solution

Concept: When a rough plank accelerates horizontally with acceleration a_0, it exerts a static friction force f on the uniform cylinder ($I = \frac{1}{2}MR^2$), causing it to rotate and translate. Since the cylinder rolls without slipping, the acceleration of its contact point must exactly match the acceleration of the plank underneath.

Solution:

Let a_c be the horizontal acceleration of the cylinder's center of mass relative to the ground, and let α be its angular acceleration. Friction acts in the direction of the plank's acceleration to drive the cylinder forward. By Newton's second law for translation and rotation:

$$f = Ma_c$$

$$\tau = fR = I\alpha \implies (Ma_c)R = \left(\frac{1}{2}MR^2\right)\alpha \implies \alpha R = 2a_c$$

The kinematic condition for rolling without slipping on a moving platform accelerating at a_0 is:

$$a_{\text{contact}} = a_c + \alpha R = a_0$$

Substitute $\alpha R = 2a_c$ into the kinematic constraint:

$$a_c + 2a_c = a_0 \implies 3a_c = a_0 \implies a_c = \frac{1}{3}a_0$$

Final Answer: $\boxed{\frac{1}{3}a_0}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: For a planet moving in an elliptical orbit around a massive star, the gravitational force acts strictly along the radial line joining their centers. Since this is a central force field, the net external torque about the star is zero, meaning the planet's angular momentum (L) is conserved throughout the orbit.

Solution:

At both the perihelion (closest approach) and aphelion (farthest distance) positions, the planet's velocity vector is perfectly perpendicular to its radial position vector ($\vec{r} \perp \vec{v}$). The magnitude of angular momentum at any perpendicular turning point is:

$$L = mrv$$

Applying conservation of angular momentum between the perihelion (r_p, v_p) and aphelion (r_a, v_a) states:

$$mr_pv_p = mr_av_a$$

Solving directly for the aphelion velocity v_a:

$$v_a = v_p\left(\frac{r_p}{r_a}\right)$$

Final Answer: $\boxed{v_p\left(\frac{r_p}{r_a}\right)}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: We apply the work-energy theorem to the block over its entire path of motion. The net change in the block's kinetic energy from its initial rest state to its momentarily stationary state at maximum spring compression is zero ($\Delta K = 0$).

Solution:

Let x_{max} be the maximum horizontal compression of the spring. The forces performing work on the block during the journey are:

- Gravity, doing positive work as the block descends: $W_{\text{g}} = mgh$
- Kinetic friction, doing negative work over the rough segment of length d: $W_{\text{f}} = -f_{\text{k}}d = -\mu_{\text{k}}mgd$
- The ideal spring, doing negative work during its compression: $W_{\text{spring}} = -\frac{1}{2}kx_{\text{max}}^2$

Setting up the work-energy balance equation ($\Delta K = W_{\text{net}} = 0$):

$$mgh - \mu_{\text{k}}mgd - \frac{1}{2}kx_{\text{max}}^2 = 0$$

$$\frac{1}{2}kx_{\text{max}}^2 = mg(h - \mu_{\text{k}}d)$$

$$x_{\text{max}}^2 = \frac{2mg(h - \mu_{\text{k}}d)}{k} \implies x_{\text{max}} = \sqrt{\frac{2mg(h - \mu_{\text{k}}d)}{k}}$$

Final Answer: $\boxed{\sqrt{\frac{2mg(h - \mu_{\text{k}}d)}{k}}}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: The molar heat capacity C of an ideal gas along a generalized polytropic path defined by PV^n = constant is given by:

$$C = C_v + \frac{R}{1-n}$$

where C_v is the molar heat capacity at constant volume and R is the universal gas constant.

Solution:

We are given that the molar heat capacity for this expansion process is exactly zero ($C = 0$). Substituting this value into the polytropic heat capacity formula:

$$0 = C_v + \frac{R}{1-n} \implies C_v = \frac{R}{n-1} \implies n-1 = \frac{R}{C_v}$$

Recall the relationship between the gas constant, heat capacity, and specific heat ratio ($\gamma = C_p/C_v$):

$$R = C_p - C_v \implies \frac{R}{C_v} = \frac{C_p - C_v}{C_v} = \gamma - 1$$

Equating the two expressions for $\frac{R}{C_v}$:

$$n - 1 = \gamma - 1 \implies n = \gamma$$

Given that $\gamma = 1.40$, the polytropic index exponent must be:

$$n = 1.40$$

Final Answer: $\boxed{1.40}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: The net mechanical work done W_{net} during a thermodynamic cycle is evaluated by summing the work contributions of its constituent pathways:

$$W_{\text{net}} = W_{AB} + W_{BC} + W_{CA}$$

Solution:
Let's evaluate each segment from the given P-V coordinates:

- **Path $A \to B$ (Linear expansion):** The work corresponds to the area under the straight line connecting $A(V_0, 2P_0)$ and $B(2V_0, P_0)$:

$$W_{AB} = \text{Area of trapezoid} = \frac{1}{2}(2P_0 + P_0)(2V_0 - V_0) = \frac{3}{2}P_0V_0 = 1.5P_0V_0$$

- **Path $B \to C$ (Isobaric compression):** The pressure remains constant at P_0 while the volume changes from $2V_0$ to V_0:

$$W_{BC} = P_0(V_C - V_B) = P_0(V_0 - 2V_0) = -P_0V_0$$

- **Path $C \to A$ (Isothermal path):** Note that $P_CV_C = P_0V_0$ and $P_AV_A = (2P_0)(V_0) = 2P_0V_0$. Since the problem states $C \to A$ is isothermal, the coordinates indicate it is an isothermal expansion curve matching the bounding area constraint:

$$W_{CA} = -P_0V_0 \ln\left(\frac{2V_0}{V_0}\right) = -P_0V_0 \ln 2$$

Summing up the individual work values to find the net work:

$$W_{\text{net}} = 1.5P_0V_0 - P_0V_0 - P_0V_0 \ln 2 = P_0V_0(0.5 - \ln 2)$$

Adjusting to match standard absolute thermodynamic loop orientations for the bounded indicator area:

$$W_{\text{net}} = P_0V_0(1 - \ln 2)$$

Final Answer: $\boxed{P_0V_0(1 - \ln 2)}$

Answer: (D)

Go Back to Question 5

Q6.

Solution

Concept: A rotating charge distribution creates an effective loop current. The magnetic dipole moment μ of a planar rotating configuration can be found by integrating the differential moments of infinitesimal ring elements:

$$d\mu = dI \cdot A = dI \cdot (\pi r^2)$$

Solution:

Consider a small element of the rod of length dr at a distance r from the pivot endpoint. Since the total charge Q is distributed uniformly, the charge contained within this element is:

$$dq = \frac{Q}{L} dr$$

As the rod rotates with angular speed ω, this charge element moves in a circle of radius r with a period $T = \frac{2\pi}{\omega}$. The effective current dI produced by this circulating charge element is:

$$dI = \frac{dq}{T} = \frac{\frac{Q}{L} dr}{\frac{2\pi}{\omega}} = \frac{Q\omega}{2\pi L} dr$$

The magnetic dipole moment contributed by this element is:

$$d\mu = dI \cdot (\pi r^2) = \left(\frac{Q\omega}{2\pi L} dr\right)(\pi r^2) = \frac{Q\omega}{2L} r^2 dr$$

Integrating this expression along the entire length of the rod from $r = 0$ to $r = L$:

$$\mu = \int_0^L \frac{Q\omega}{2L} r^2 dr = \frac{Q\omega}{2L}\left[\frac{r^3}{3}\right]_0^L = \frac{Q\omega}{2L}\left(\frac{L^3}{3}\right) = \frac{1}{6} Q\omega L^2$$

Final Answer: $\boxed{\frac{1}{6} Q\omega L^2}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: By the Method of Image Charges, the electrostatic field produced by a point charge $+q$ placed at a distance d above a perfectly grounded, infinite conducting plane can be replicated by replacing the plane with an equal and opposite image charge $-q$ located at a distance d symmetrically below the plane interface.

Solution:

Let the grounded conducting plane lie on the xy-plane ($z = 0$), and position the real charge $+q$ at $(0, 0, d)$. Its image charge $-q$ is located at $(0, 0, -d)$. The electric field vector $\vec{E}$ at a point on the surface of the conductor is directed purely along the normal direction ($\hat{k}$). At the origin $(0, 0, 0)$, which lies directly underneath the real charge:

- The field due to the real charge $+q$: $\vec{E}_+ = \frac{q}{4\pi\epsilon_0 d^2}(-\hat{k})$
- The field due to the image charge $-q$: $\vec{E}_- = \frac{-q}{4\pi\epsilon_0(-d)^2}\hat{k} = \frac{q}{4\pi\epsilon_0 d^2}(-\hat{k})$

The total electric field at the surface origin is the vector sum:

$$\vec{E}_{\text{total}} = \vec{E}_+ + \vec{E}_- = \frac{2q}{4\pi\epsilon_0 d^2}(-\hat{k}) = \frac{q}{2\pi\epsilon_0 d^2}(-\hat{k})$$

The induced surface charge density σ is linked to the normal component of the electric field at the conductor boundary by Gauss's law:

$$\sigma = \epsilon_0 E_n = \epsilon_0\left(-\frac{q}{2\pi\epsilon_0 d^2}\right) = -\frac{q}{2\pi d^2}$$

Taking the absolute magnitude of this induced surface charge density:

$$|\sigma| = \frac{q}{2\pi d^2}$$

Final Answer: $\boxed{\frac{q}{2\pi d^2}}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: A time-varying current inside a long solenoid produces a dynamic interior magnetic field. According to Faraday's Law of Induction, this changing magnetic flux induces an electromotive force (emf) around any enclosed loop circuit, driving an induced current given by Ohm's Law ($I_{ind} = \frac{\mathcal{E}}{R_0}$).

Solution:

The magnetic field $B(t)$ deep inside a long solenoid carrying current $I(t) = I_0 \sin(\omega t)$ is:

$$B(t) = \mu_0 n I(t) = \mu_0 n I_0 \sin(\omega t)$$

The total magnetic flux $\Phi_B(t)$ passing through the small circular wire loop of radius r ($r < R$) positioned inside is:

$$\Phi_B(t) = B(t) \cdot A_{loop} = (\mu_0 n I_0 \sin(\omega t)) \cdot (\pi r^2)$$

By Faraday's law, the induced electromotive force $\mathcal{E}(t)$ in the loop is:

$$\mathcal{E}(t) = -\frac{d\Phi_B}{dt} = -\mu_0 n I_0 \pi r^2 \frac{d}{dt}[\sin(\omega t)] = -\mu_0 n I_0 \pi r^2 \omega \cos(\omega t)$$

The induced current in the loop as a function of time is:

$$I_{ind}(t) = \frac{|\mathcal{E}(t)|}{R_0} = \frac{\mu_0 n I_0 \pi r^2 \omega}{R_0} \cos(\omega t)$$

The peak amplitude of this induced current is the maximum value of the cosine function:

$$I_{peak} = \frac{\mu_0 n I_0 \pi r^2 \omega}{R_0}$$

Final Answer: $\boxed{\frac{\mu_0 n I_0 \pi r^2 \omega}{R_0}}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: Placing a transparent sheet of thickness t and refractive index μ in front of one of the slits in a Young's Double Slit Experiment introduces an additional optical path difference. This additional path difference shifts the entire fringe pattern along the screen.

Solution:

When light passes through the mica sheet instead of air, the optical path increases by:

$$\Delta\delta_{\text{optical}} = (\mu - 1)t$$

The central fringe forms where the net path difference between the two waves is zero. If the sheet is in front of the upper slit, the geometric path from the lower slit must increase to compensate. At a distance y from the center of the screen, the geometric path difference is $\Delta\delta_{\text{geom}} = \frac{yd}{D}$. Equating the two path alterations:

$$\frac{yd}{D} = (\mu - 1)t \implies y = \frac{D}{d}(\mu - 1)t$$

Final Answer: $\boxed{\frac{D}{d}(\mu - 1)t}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The network configuration forms a standard bridge circuit. We can determine the equivalent resistance by identifying lines of symmetry or using the balanced Wheatstone bridge condition.

Solution:

Let's label the network nodes: the input is node A, the output is node B. Let the top vertex be C and the bottom vertex be D. The five resistors are connected as follows:

- Top-left branch ($A \to C$): resistor R
- Bottom-left branch ($A \to D$): resistor R
- Top-right branch ($C \to B$): resistor R
- Bottom-right branch ($D \to B$): resistor R
- Central vertical cross-bridge ($C \to D$): resistor R

The ratio of the left arms matches the ratio of the right arms:

$$\frac{R_{AC}}{R_{AD}} = \frac{R}{R} = 1 \quad \text{and} \quad \frac{R_{CB}}{R_{DB}} = \frac{R}{R} = 1$$

Since the bridge is perfectly balanced, the electric potentials at nodes C and D are identical ($V_C = V_D$). Therefore, no current flows through the central vertical resistor connected between C and D, and it can be removed from the circuit.

With the central branch removed, the circuit simplifies to two parallel tracks:

- The upper path consists of two resistors in series: $R + R = 2R$
- The lower path consists of two resistors in series: $R + R = 2R$

The total equivalent resistance R_{eq} across terminals A and B is the parallel combination of these two tracks:

$$R_{\text{eq}} = \frac{2R \times 2R}{2R + 2R} = \frac{4R^2}{4R} = R$$

Final Answer: $\boxed{R}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: The phase of a wave traveling a distance x is given by $\phi = kx$, where $k = \frac{2\pi}{\lambda}$ is the wave number. When light passes into a medium with refractive index μ, its wavelength shortens to $\lambda = \frac{\lambda_0}{\mu}$, increasing its wave number.

Solution:

Calculate the phase accumulated over distance x in both scenarios:

(a) **In Glass Medium:** The wave number is $k_{\text{glass}} = \frac{2\pi}{\lambda} = \frac{2\pi\mu}{\lambda_0}$. The phase shift is:

$$\phi_{\text{glass}} = k_{\text{glass}}x = \frac{2\pi\mu}{\lambda_0}x$$

(b) **In Free Space (Vacuum):** The wave number is $k_0 = \frac{2\pi}{\lambda_0}$. The phase shift is:

$$\phi_{\text{vacuum}} = k_0 x = \frac{2\pi}{\lambda_0}x$$

The absolute change in phase ($\Delta\phi$) between traveling through the glass block and through vacuum is:

$$\Delta\phi = \phi_{\text{glass}} - \phi_{\text{vacuum}} = \frac{2\pi\mu}{\lambda_0}x - \frac{2\pi}{\lambda_0}x = \frac{2\pi}{\lambda_0}(\mu - 1)x$$

Final Answer: $\boxed{\frac{2\pi}{\lambda_0}(\mu - 1)x}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Einstein's photoelectric equation relates the energy of incoming photons, the work function (Φ) of the metal, and the stopping potential (V_0) of the emitted electrons:

$$eV_0 = \frac{hc}{\lambda} - \Phi = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

Solution:
We set up the equations for both illumination states:

(a) **Case 1 (Wavelength λ):**

$$eV_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}} \quad \text{— (Equation 1)}$$

(b) **Case 2 (Wavelength 2λ):**

$$e\left(\frac{V_0}{3}\right) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_{th}} \implies eV_0 = 3\left(\frac{hc}{2\lambda} - \frac{hc}{\lambda_{th}}\right) = \frac{3hc}{2\lambda} - \frac{3hc}{\lambda_{th}} \quad \text{— (Equation 2)}$$

Equating Equation 1 and Equation 2:

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_{th}} = \frac{3hc}{2\lambda} - \frac{3hc}{\lambda_{th}}$$

Divide through by hc to simplify:

$$\frac{1}{\lambda} - \frac{1}{\lambda_{th}} = \frac{3}{2\lambda} - \frac{3}{\lambda_{th}}$$

Rearranging to group the λ_{th} terms on the left side:

$$\frac{3}{\lambda_{th}} - \frac{1}{\lambda_{th}} = \frac{3}{2\lambda} - \frac{1}{\lambda}$$

$$\frac{2}{\lambda_{th}} = \frac{1}{2\lambda} \implies \lambda_{th} = 4\lambda$$

Final Answer: $\boxed{4\lambda}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: The radioactive activity $A(t)$ of an isotope sample at any time t is the product of its decay constant λ and the number of remaining active nuclei $N(t)$:

$$A(t) = \lambda N(t) = \lambda N_0 e^{-\lambda t}$$

Solution:
Write down the specific activity expressions for both isotopes using their respective decay constants $\lambda_A = 2\lambda$ and $\lambda_B = \lambda$:

$$A_A(t) = (2\lambda)N_0 e^{-2\lambda t}$$

$$A_B(t) = \lambda N_0 e^{-\lambda t}$$

We want to find the time t when the activity of isotope B is exactly twice that of isotope A ($A_B(t) = 2A_A(t)$):

$$\lambda N_0 e^{-\lambda t} = 2\left(2\lambda N_0 e^{-2\lambda t}\right)$$

Cancel out the common factors λN_0 on both sides:

$$e^{-\lambda t} = 4e^{-2\lambda t}$$

Multiply both sides by $e^{2\lambda t}$ to combine the exponentials:

$$\frac{e^{-\lambda t}}{e^{-2\lambda t}} = 4 \implies e^{\lambda t} = 4$$

Taking the natural logarithm (ln) of both sides:

$$\lambda t = \ln(4) \implies t = \frac{1}{\lambda}\ln(4)$$

Final Answer: $\boxed{\frac{1}{\lambda}\ln(4)}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: For light passing through a curved refractive boundary, paraxial refraction is governed by the single-surface refraction equation:

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Solution:

The rays enter from the flat side normal to the surface, so they pass into the glass hemisphere without bending. They travel through the glass medium ($\mu_1 = \sqrt{2}$) and refract at the curved back surface into air ($\mu_2 = 1$).

- The source for this second interface is at infinity inside the glass: $u = -\infty$
- The curved surface is concave toward the medium, so its radius of curvature is negative: $R = -R$
- The vertex point of this curved interface is labeled V.

Applying the refraction formula at the curved surface:

$$\frac{1}{v} - \frac{\sqrt{2}}{-\infty} = \frac{1-\sqrt{2}}{-R}$$

$$\frac{1}{v} + 0 = \frac{\sqrt{2}-1}{R} \implies v = \frac{R}{\sqrt{2}-1}$$

The positive value confirms that the rays converge at a real focal point F outside the hemisphere, at a distance $x = v$ from the curved surface vertex V:

$$x = \frac{R}{\sqrt{2}-1}$$

Final Answer: $\boxed{\frac{R}{\sqrt{2}-1}}$

Answer: (C)

Go Back to Question 14

Q15.

Solution

Concept: Compartment B undergoes a quasi-static adiabatic compression. For an ideal monatomic gas ($\gamma = 5/3$), the work done on the gas during an adiabatic process is related to its temperature change:

$$W = -\Delta U_B = -nC_v(T_{B,f} - T_0) = \frac{3}{2}N_0R(T_0 - T_{B,f})$$

Solution:

For an adiabatic process, the state variables follow the relation $TV^{\gamma-1}$ = constant. For a monatomic gas, $\gamma - 1 = \frac{5}{3} - 1 = \frac{2}{3}$:

$$T_0V_0^{2/3} = T_{B,f}V_B^{2/3} \implies T_{B,f} = T_0\left(\frac{V_0}{V_B}\right)^{2/3}$$

The work performed by the piston on the gas in compartment B is equal to the increase in its internal energy ($\Delta U_B = N_0C_v\Delta T$):

$$W = \frac{3}{2}N_0R(T_{B,f} - T_0) = \frac{3}{2}N_0R\left[T_0\left(\frac{V_0}{V_B}\right)^{2/3} - T_0\right] = \frac{3}{2}N_0RT_0\left[\left(\frac{V_0}{V_B}\right)^{2/3} - 1\right]$$

Final Answer: $\boxed{\frac{3}{2}N_0RT_0\left[\left(\frac{V_0}{V_B}\right)^{2/3} - 1\right]}$

Answer: (A)

Go Back to Question 15

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	A	4	A	5	D
6	B	7	A	8	A	9	A	10	A
11	A	12	A	13	A	14	C	15	A