IIT JAM 2017 Mathematical Statistics (MS) Question Paper

Time Allowed :3 Hours	Maximum Marks :100	Total questions :60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

1. The imaginary parts of the eigenvalues of the matrix

$$P = \begin{pmatrix} 3 & 2 & 5 \\ 2 & -3 & 6 \\ 0 & 0 & -3 \end{pmatrix}$$

are

(A) 0, 0, 0

(B) 2, -2, 0

(C) 1, -1, 0

(D) 3, -3, 0

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2. Let $u, v \in \mathbb{R}^4$ **be such that** $u = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \end{pmatrix}$ **and** $v = \begin{pmatrix} 5 \\ 3 \\ 2 \\ 1 \end{pmatrix}$**. Then the equation** $u^T x = v$ **has**

(A) infinitely many solutions

(B) no solution

(C) exactly one solution

(D) exactly two solutions

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3. Let $u_n = \frac{(4-n)}{n}$**,** $n \in \mathbb{N}$**, and let** $l = \lim_{n\to\infty} u_n$**.**

Which of the following statements is TRUE?

(A) $l = 0$ and $\sum_{n=1}^{\infty} u_n$ is convergent

(B) $l = \frac{1}{4}$ and $\sum_{n=1}^{\infty} u_n$ is divergent

(C) $l = \frac{1}{4}$ and $\sum_{n=1}^{\infty} u_n$ is oscillatory

(D) $l = 1$ and $\sum_{n=1}^{\infty} u_n$ is divergent

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4. Let $\{a_n\}$ be a sequence defined as follows:

$$a_1 = 1 \quad \text{and} \quad a_{n+1} = \frac{7a_n + 11}{21}, \quad n \in \mathbb{N}.$$

Which of the following statements is TRUE?

(A) $\{a_n\}$ is an increasing sequence which diverges

(B) $\{a_n\}$ is an increasing sequence with $\lim_{n\to\infty} a_n = \frac{11}{14}$

(C) $\{a_n\}$ is a decreasing sequence which diverges

(D) $\{a_n\}$ is a decreasing sequence with $\lim_{n\to\infty} a_n = \frac{11}{14}$

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5. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} x^3, & 0 < x \leq 1 \\ 3x^5, & x > 1 \\ 0, & \text{otherwise} \end{cases}$$

Then $P\left(\frac{1}{2} \leq X \leq 2\right)$ equals

(A) $\frac{15}{16}$

(B) $\frac{11}{16}$

(C) $\frac{7}{12}$

(D) $\frac{3}{8}$

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6. Let X be a random variable with the moment generating function

$$M_X(t) = \frac{1}{216}\left(5 + e^t\right)^3, \quad t \in \mathbb{R}.$$

Then $P(X > 1)$ equals

(A) $\frac{1}{27}$

(B) $\frac{1}{12}$

(C) $\frac{1}{216}$

(D) $\frac{2}{9}$

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7. Let X be a discrete random variable with the probability mass function

$$p(x) = k(1 + |x|)^2, \quad x = -2, -1, 0, 1, 2,$$

where k is a real constant. Then $P(X = 0)$ equals

(A) $\frac{2}{9}$

(B) $\frac{2}{27}$

(C) $\frac{1}{27}$

(D) $\frac{1}{81}$

---

8. Let the random variable X have uniform distribution on the interval $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$. Then

$$P(\cos X > \sin X)$$

is

(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

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9. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables having common probability density function

$$f(x) = \begin{cases} xe^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, $n = 1, 2, \ldots$. Then

$$\lim_{n\to\infty} P(\bar{X}_n = 2)$$

equals

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

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10. Let X_1, X_2, X_3 be a random sample from a distribution with the probability density function

$$f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, \quad x > 0,\ \theta > 0$$

Which of the following estimators of θ has the smallest variance for all $\theta > 0$?

(A) $\frac{X_1+X_3+X_5}{3}$

(B) $\frac{X_1+X_2+X_3}{4}$

(C) $\frac{X_1+X_2+X_3}{3}$

(D) $\frac{X_1+X_2+X_3}{6}$

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11. Player P_1 tosses 4 fair coins and player P_2 tosses a fair die independently of P_1. The probability that the number of heads observed is more than the number on the upper face of the die, equals

(A) $\frac{7}{16}$

(B) $\frac{5}{32}$

(C) $\frac{17}{96}$

(D) $\frac{21}{64}$

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12. Let X_1 and X_2 be i.i.d. continuous random variables with the probability density function

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Using Chebyshev's inequality, the lower bound of $P\left(|X_1 + X_2 - 1| \leq \frac{1}{2}\right)$ **is**

(A) $\frac{5}{6}$

(B) $\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $\frac{1}{3}$

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13. Let X_1, X_2, X_3 **be i.i.d. discrete random variables with the probability mass function**

$$p(k) = \left(\frac{2}{3}\right)^{k-1}\left(\frac{1}{3}\right), \quad k = 1, 2, 3, \ldots$$

Let $Y = X_1 + X_2 + X_3$**. Then** $P(Y \geq 5)$ **equals**

(A) $\frac{1}{9}$

(B) $\frac{8}{9}$

(C) $\frac{2}{27}$

(D) $\frac{25}{27}$

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14. Let X **and** Y **be continuous random variables with the joint probability density function**

$$f(x, y) = \begin{cases} cx(1-x), & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

where c **is a positive real constant. Then** $E(X)$ **equals**

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{2}{5}$

(D) $\frac{1}{3}$

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15. Let X and Y be continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} x+y, & \text{if } 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then $P(X+Y > \frac{1}{2})$ equals

(A) $\frac{23}{24}$

(B) $\frac{1}{12}$

(C) $\frac{11}{12}$

(D) $\frac{1}{24}$

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16. Let $X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_n$ be i.i.d. $N(0,1)$ random variables. Then

$$W = \frac{n\sum_{i=1}^{m} X_i^2}{m\sum_{j=1}^{n} Y_j^2}$$

has

(A) X^2_{n+m} distribution

(B) t_n distribution

(C) $F_{m,n}$ distribution

(D) $F_{1,n}$ distribution

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17. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables with the probability mass function

$$f(x) = \begin{cases} \frac{1}{4}, & \text{if } x = 4 \\ \frac{3}{4}, & \text{if } x = 8 \\ 0, & \text{otherwise} \end{cases}$$

Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i, n = 1, 2, \ldots$. If $\lim_{n\to\infty} P(m \leq \bar{X}_n \leq M) = 1$, then possible values of m and M are

(A) $m = 2.1, M = 3.1$

(B) $m = 3.2, M = 4.1$

(C) $m = 4.2, M = 5.7$

(D) $m = 6.1, M = 7.1$

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18. Let $x_1 = 1.1, x_2 = 0.5, x_3 = 1.4, x_4 = 1.2$ be the observed values of a random sample of size four from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} e^{-\theta x}, & \text{if } x \geq \theta \\ 0, & \text{otherwise}, \quad \theta \in (-\infty, \infty) \end{cases}$$

Then the maximum likelihood estimate of θ^2 is

(A) 0.5

(B) 0.25

(C) 1.21

(D) 1.44

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19. Let $x_1 = 2, x_2 = 1, x_3 = \sqrt{5}, x_4 = \sqrt{2}$ be the observed values of a random sample of size four from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} \frac{1}{2\theta}, & \text{if } -\theta \leq x \leq \theta \\ 0, & \text{otherwise}, \quad \theta > 0 \end{cases}$$

Then the method of moments estimate of θ is

(A) 1

(B) 2

(C) 3

(D) 4

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20. Let X_1, X_2 be a random sample from an $N(0, \theta)$ distribution, where $\theta > 0$. Then the value of k, for which the interval

$$\left(0, \frac{X_1^2 + X_2^2}{k}\right)$$

is a 95% confidence interval for θ, equals

(A) $1 - \log_e(0.95)$

(B) $2 \log_e(0.95)$

(C) $\frac{1}{2} \log_e(0.95)$

(D) 2

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21. Let X_1, X_2, X_3, X_4 be a random sample from $N(\theta_1, \sigma^2)$ distribution and Y_1, Y_2, Y_3, Y_4 be a random sample from $N(\theta_2, \sigma^2)$ distribution, where $\theta_1, \theta_2 \in (-\infty, \infty)$ and $\sigma > 0$. Further suppose that the two random samples are independent. For testing the null hypothesis $H_0 : \theta_1 = \theta_2$ against the alternative hypothesis $H_1 : \theta_1 \neq \theta_2$, suppose that a test ψ rejects H_0 if and only if $\sum_{i=1}^{4} X_i > \sum_{i=1}^{4} Y_i$. The power of the test ψ at $\theta_1 = 1 + \sqrt{2}, \theta_2 = 1$ and $\sigma^2 = 4$ is

(A) 0.5987

(B) 0.7341

(C) 0.7612

(D) 0.8413

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22. Let X be a random variable having a probability density function $f \in \{f_0, f_1\}$, where

$$f_0(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad f_1(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

For testing the null hypothesis $H_0 : f = f_0$ against $H_1 : f = f_1$, based on a single observation on X, the power of the most powerful test of size $\alpha = 0.05$ equals

(A) 0.425

(B) 0.525

(C) 0.625

(D) 0.725

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23. If

$$\int_0^1 \int_0^{\sqrt{1-(y-1)^2}} f(x, y)\, dx\, dy$$

equals

$$\int_0^1 \int_0^x f(x, y)\, dy\, dx,$$

then $\alpha(x)$ and $\beta(x)$ are

(A) $\alpha(x) = x, \beta(x) = 1 + \sqrt{1 - (x-2)^2}$

(B) $\alpha(x) = x, \beta(x) = 1 - \sqrt{1 - (x-2)^2}$

(C) $\alpha(x) = 1 + \sqrt{1 - (x-2)^2}, \beta(x) = x$

(D) $\alpha(x) = 1 + \sqrt{1 - (x-2)^2}, \beta(x) = 1 - \sqrt{1 - (x-2)^2}$

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24. Let $f : [0, 1] \to \mathbb{R}$ be a function defined as

$$f(t) = \begin{cases} t^3 \left(1 + \frac{1}{5}\cos(\log(e^t))\right), & \textbf{if } t \in (0, 1] \\ 0, & \textbf{if } t = 0 \end{cases}$$

Let $F : [0, 1] \to \mathbb{R}$ be defined as

$$F(x) = \int_0^x f(t)\, dt$$

Then $F''(0)$ equals

(A) 0

(B) $\frac{3}{5}$

(C) $-\frac{5}{3}$

(D) $\frac{1}{5}$

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25. Consider the function

$$f(x, y) = x^3 - y^3 - 3x^2 + 3y^2 + 7,\ x, y \in \mathbb{R}.$$

Then the local minimum (m) and the local maximum (M) of f are given by

(A) $m = 3, M = 7$

(B) $m = 4, M = 11$

(C) $m = 7, M = 11$

(D) $m = 3, M = 11$

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26. For $c \in \mathbb{R}$, let the sequence $\{u_n\}$ be defined by

$$u_n = \frac{(1 + \frac{c}{n})^{n^2}}{(3 - \frac{1}{n})^n}$$

Then the values of c for which the series

$$\sum_{n=1}^{\infty} u_n$$

converges are

(A) $\log_e 6 < c < \log_e 9$

(B) $c < \log_e 3$

(C) $\log_e 9 < c < \log_e 12$

(D) $\log_e 3 < c < \log_e 6$

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27. If for a suitable $\alpha > 0$,

$$\lim_{x \to 0} \left(\frac{1}{e^{2x} - 1} - \frac{1}{\alpha x} \right)$$

exists and is equal to l ($|l| < \infty$), then $\alpha = 2, l = -\frac{1}{2}$ is given by

(A) $\alpha = 2, l = 2$

(B) $\alpha = 2, l = -\frac{1}{2}$

(C) $\alpha = \frac{1}{2}, l = -2$

(D) $\alpha = \frac{1}{2}, l = \frac{1}{2}$

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28. Let

$$P = \int_0^1 \frac{dx}{\sqrt{8 - x^2 - x^3}}.$$

Which of the following statements is TRUE?

(A) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) < P < \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(B) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) < P < \sin^{-1}\left(\frac{1}{2}\right)$

(C) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) < P < \sin^{-1}\left(\frac{1}{2}\right)$

(D) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) < P < \frac{\sqrt{3}}{2}\sin^{-1}\left(\frac{1}{2}\right)$

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29. Let Q, A, B be matrices of order $n \times n$ with real entries such that Q is orthogonal and A is invertible. Then the eigenvalues of $Q^T A^{-1} B Q$ are always the same as those of

(A) AB

(B) $Q^T A^{-1} B Q$

(C) $A^{-1} B Q^T$

(D) BA^{-1}

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30. Let $x(t), y(t), 1 \le t \le \pi$, be the curve defined by

$$x(t) = \int_1^t \frac{\cos z}{z^2}\, dz \quad \text{and} \quad y(t) = \int_1^t \frac{\sin z}{z^2}\, dz.$$

Let L be the length of the arc of this curve from the origin to the point P on the curve at which the tangent is perpendicular to the x-axis. Then L equals

(A) $\sqrt{2}$

(B) $\frac{\pi}{\sqrt{2}}$

(C) $1-\frac{2}{\pi}$

(D) $\frac{\pi}{2}+\sqrt{2}$

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31. Let $\mathbf{v} \in \mathbb{R}^k$ with $\mathbf{v}^T\mathbf{v} \neq 0$. Let

$$P = I - 2\frac{\mathbf{v}\mathbf{v}^T}{\mathbf{v}^T\mathbf{v}},$$

where I is the $k \times k$ identity matrix. Then which of the following statements is (are) TRUE?

(A) $P^{-1} = I - P$

(B) -1 and 1 are eigenvalues of P

(C) $P^{-1} = P$

(D) $(I + P)\mathbf{v} = \mathbf{v}$

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32. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers such that $\{a_n\}$ is increasing and $\{b_n\}$ is decreasing. Under which of the following conditions, the sequence $\{a_n + b_n\}$ is always convergent?

(A) $\{a_n\}$ and $\{b_n\}$ are bounded sequences

(B) $\{a_n\}$ is bounded above

(C) $\{a_n\}$ is bounded above and $\{b_n\}$ is bounded below

(D) $a_n \to \infty$ and $b_n \to -\infty$

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33. Let $f : [0, 1] \to [0, 1]$ be defined as follows:

$$f(x) = \begin{cases} x, & \text{if } x \in \mathbb{Q} \cap [0, 1] \\ x + \frac{2}{3}, & \text{if } x \in \mathbb{Q}^c \cap (0, \frac{1}{3}) \\ x - \frac{1}{3}, & \text{if } x \in \mathbb{Q}^c \cap (\frac{1}{3}, 1] \end{cases}$$

Which of the following statements is (are) TRUE?

(A) f is one-to-one and onto

(B) f is not one-to-one but onto

(C) f is continuous on $\mathbb{Q} \cap [0, 1]$

(D) f is discontinuous everywhere on $[0, 1]$

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34. Let $f(x)$ be a nonnegative differentiable function on $[a, b] \subset \mathbb{R}$ such that $f(a) = 0 = f(b)$ and $|f'(x)| \leq 4$. Let L_1 and L_2 be the straight lines given by the equations

$y = 4(x-a)$ and $y = -4(x-b)$, respectively. Then which of the following statements is (are) TRUE?

(A) The curve $y = f(x)$ will always lie below the lines L_1 and L_2

(B) The curve $y = f(x)$ will always lie above the lines L_1 and L_2

(C) $\int_a^b f(x)dx \leq (b-a)^2$

(D) The point of intersection of the lines L_1 and L_2 lie on the curve $y = f(x)$

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35. Let E and F be two events with $0 < P(E) < 1$, $0 < P(F) < 1$ and $P(E) + P(F) \geq 1$. Which of the following statements is (are) TRUE?

(A) $P(E \cap F) \leq P(E)$

(B) $P(E \cup F) < P(E^C \cup F^C)$

(C) $P(E|F^C) \geq P(F^C|E)$

(D) $P(E^C|F) \leq P(F|E^C)$

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36. The cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{4}{9}, & \text{if } 0 \leq x < 1 \\ \frac{8}{9}, & \text{if } 1 \leq x < 2 \\ 1, & \text{if } x \geq 2 \end{cases}$$

Which of the following statements is (are) TRUE?

(A) The random variable X takes positive probability only at two points

(B) $P(1 \leq X \leq 2) = \frac{5}{9}$

(C) $E(X) = \frac{2}{3}$

(D) $P(0 \leq X \leq 1) = \frac{4}{9}$

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37. Let X_1, X_2 be a random sample from a distribution with the probability mass function

$$f(x|\theta) = \begin{cases} 1-\theta, & \text{if } x = 0 \\ \theta, & \text{if } x = 1 \\ 0, & \text{otherwise}, \quad 0 < \theta < 1. \end{cases}$$

Which of the following is (are) unbiased estimator(s) of θ?

(A) $\frac{X_1+X_2}{2}$

(B) $\frac{X_1^2+X_2}{2}$

(C) $\frac{X_1^2+X_2^2}{2}$

(D) $\frac{X_1+X_2-X_1^2}{2}$

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38. Let X_1, X_2, X_3 be a random sample from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} \frac{1}{\theta}e^{-x/\theta}, & \text{if } x > 0,\, \theta > 0 \\ 0, & \text{otherwise} \end{cases}$$

If $\hat{X}(X_1, X_2, X_3)$ is an unbiased estimator of θ, which of the following CANNOT be attained as a value of the variance of $\hat{X}$ at $\theta = 1$?

(A) 0.1

(B) 0.2

(C) 0.3

(D) 0.5

39. Let $X_1, X_2, \ldots, X_n$ (where $n \geq 2$) be a random sample from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} \frac{x}{\theta^2} e^{-x/\theta}, & x > 0,\, \theta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Which of the following statistics is (are) sufficient but NOT complete?

(A) $\bar{X}$

(B) $\bar{X}^2 + 3$

(C) $(X_1, \sum_{i=2}^{n} X_i)$

(D) $(X_1, \bar{X})$

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40. Let X_1, X_2, X_3, X_4 be a random sample from an $N(\theta, 1)$ distribution, where $\theta \in (-\infty, \infty)$. Suppose the null hypothesis $H_0 : \theta = 1$ is to be tested against the hypothesis $H_1 : \theta < 1$ at $\alpha = 0.05$ level of significance. For what observed values of $\sum_{i=1}^{4} X_i$, the uniformly most powerful test would reject H_0?

(A) -1

(B) 0

(C) 0.5

(D) 0.8

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41. Let the random variable X have uniform distribution on the interval $(0, 1)$ and $Y = -2\log X$. Then $E(Y)$ equals

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42. If $Y = \log_{10} X$ has $N(\mu, \sigma^2)$ distribution with moment generating function $M_Y(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$, then $P(X < 1000)$ equals

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43. Let X_1, X_2, X_3, X_4, X_5 be independent random variables with

$$X_1 \sim N(200, 8),\ X_2 \sim N(104, 8),\ X_3 \sim N(108, 15),\ X_4 \sim N(120, 15),\ X_5 \sim N(210, 15).$$

Let

$$U = \frac{X_1 + X_2}{2}, \quad V = \frac{X_3 + X_4 + X_5}{3}.$$

Then $P(U > V)$ equals

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44. Let X and Y be discrete random variables with the joint probability mass function

$$p(x, y) = \frac{1}{25}(x^2 + y^2), \quad \text{if } x = 1, 2;\ y = 0, 1, 2.$$

Then $P(Y = 1|X = 1)$ equals

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45. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then $9\,\mathbf{Cov}(X, Y)$ equals

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46. Let X_1, X_2, X_3, X_4 be a random sample from an $N(\mu, \sigma^2)$ distribution. Let $\bar{X} = \frac{1}{4}\sum_{i=1}^{4} X_i$ and

$$\tilde{Y} = \frac{15}{7}\sum_{i=1}^{4} X_i.$$

If $\bar{X}$ has a t-distribution, then $(\nu - k)$ equals

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47. Let $f : [0, \frac{\pi}{2}] \to \mathbb{R}$ be defined as

$$f(x) = ax + \beta \sin x,$$

where $a, \beta \in \mathbb{R}$. Let f have a local minimum at $x = \frac{\pi}{4}$ with

$$f'\left(\frac{\pi}{4}\right) = -\frac{4}{\sqrt{2}}.$$

Then $8\sqrt{2a} + 4\beta$ equals

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48. The area bounded between two parabolas $y = x^2 + 4$ and $y = -x^2 + 6$ is

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49. For $j = 1, 2, \ldots, 5$, let P_j be the matrix of order 5×5 obtained by replacing the j^{th} column of the identity matrix of order 5×5 with the column vector $v = (5, 4, 3, 2, 1)^T$. Then the determinant of the matrix product $P_1 P_2 P_3 P_4 P_5$ is

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50. Let $u_n = \frac{18n+3}{(3n-1)^2(3n+2)^2}$, $n \in \mathbb{N}$. Then

$$\sum_{n=1}^{\infty} u_n \quad \text{equals}$$

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51. Let a unit vector $\mathrm{v} = (v_1, v_2, v_3)^T$ be such that $A\mathrm{v} = 0$, where

$$A = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{5}{6} \end{pmatrix}.$$

Then the value of $\sqrt{6}\left(|v_1| + |v_2| + |v_3|\right)$ equals

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52. Let

$$F(x) = \int_0^x e^{t^2-3t-5}\, dt, \quad x > 0.$$

Then the number of roots of $F(x) = 0$ in the interval $(0, 4)$ is

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53. A tangent is drawn on the curve $y = \frac{1}{3}\sqrt{x^3}$, $x > 0$ at the point $P\left(1, \frac{1}{3}\right)$, which meets the x-axis at Q. Then the length of the closed curve $OQPO$, where O is the origin, is

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54. The volume of the region

$$R = \left\{(x, y, z) \in \mathbb{R}^3 : x + y + z \leq 3, y^2 \leq 4x, 0 \leq x \leq 1, y \geq 0, z \geq 0\right\}$$

is

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55. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x}{8}, & \text{if } 0 < x < 2, \\ k, & \text{if } 2 \leq x \leq 4, \\ \frac{6-x}{8}, & \text{if } 4 < x < 6, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(1 < X < 5)$ equals

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56. Let X_1, X_2, X_3 be independent random variables with the common probability density function

$$f(x) = \begin{cases} 2e^{-2x}, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y = \min\{X_1, X_2, X_3\}$, $E(Y) = \mu_y$ and $\text{Var}(Y) = \sigma_y^2$. Then $P(Y > \mu_y + \sigma_y)$ equals

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57. Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{2}e^{-x}, & \text{if } |y| \leq x,\ x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then $E(X|Y = -1)$ equals

58. Let X and Y be discrete random variables with $P(Y \in \{0, 1\}) = 1$,

$$P(X = 0) = \frac{3}{4},\ P(X = 1) = \frac{1}{4},\ P(Y = 1|X = 1) = \frac{3}{4},\ P(Y = 0|X = 0) = \frac{7}{8}.$$

Then $3P(Y = 1) - P(Y = 0)$ equals

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59. Let $X_1, X_2, \ldots, X_{100}$ **be i.i.d. random variables with** $E(X_1) = 0, E(X_1^2) = \sigma^2$**, where** $\sigma > 0$**. Let**

$S = \sum_{i=1}^{100} X_i$. If an approximate value of $P(S \leq 30)$ is 0.9332, then σ^2 equals.

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60. Let X **be a random variable with the probability density function**

$$f(x|r, \lambda) = \frac{x^{r-1}e^{-x/\lambda}}{\lambda^r(r-1)!}, \quad x > 0,\ \lambda > 0,\ r > 0.$$

If $E(X) = 2$ **and Var**$(X) = 2$**, then** $P(X < 1)$ **equals**
