IIT JAM 2017 Mathematics (MA) Question Paper

Time Allowed :3 Hours	Maximum Marks :100	Total questions :60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

1. Consider the function $f(x, y) = 5 - 4\sin x + y^2$ for $0 < x < 2\pi$ and $y \in \mathbb{R}$. The set of critical points of $f(x, y)$ consists of

(A) a point of local maximum and a point of local minimum

(B) a point of local maximum and a saddle point

(C) a point of local maximum, a point of local minimum and a saddle point

(D) a point of local minimum and a saddle point

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2. Let $\varphi : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $\varphi'(1) = 0$. Let α and β denote the minimum and maximum values of $\varphi(x)$ on the interval [2, 3], respectively. Then which one of the following is TRUE?

(A) $\beta = \varphi(3)$

(B) $\alpha = \varphi(2.5)$

(C) $\beta = \varphi(2.5)$

(D) $\alpha = \varphi(3)$

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3. The number of generators of the additive group $\mathbb{Z}_{36}$ is equal to

(A) 6

(B) 12

(C) 18

(D) 36

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4. Find the limit:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\frac{\pi}{2} + \frac{5\pi}{2} \cdot \frac{k}{n}\right) =$$

(A) $\frac{2\pi}{5}$

(B) $\frac{5}{2}$

(C) $\frac{2}{5}$

(D) $\frac{5\pi}{2}$

---

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $g(u, v) = f(u^2 - v^2)$, then

$$\frac{\partial^2 g}{\partial u^2} + \frac{\partial^2 g}{\partial v^2} =$$

(A) $4(u^2 - v^2)f''(u^2 - v^2)$

(B) $4(u^2 + v^2)f''(u^2 - v^2)$

(C) $2f'(u^2 - v^2) + 4(u^2 - v^2)f''(u^2 - v^2)$

(D) $2(u - v)f''(u^2 - v^2)$

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6. Evaluate the integral:

$$\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$$

(A) $\frac{1+\cos 1}{2}$

(B) $1 - \cos 1$

(C) $1 + \cos 1$

(D) $\frac{1-\cos 1}{2}$

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7. Let $f_1(x), f_2(x), g_1(x), g_2(x)$ be differentiable functions on $\mathbb{R}$. Let

$$F(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}.$$

Then $F'(x)$ is equal to

(A) $f_1'(x)f_2'(x) + f_1(x)g_1'(x)$

(B) $f_1'(x)f_2'(x) + f_1(x)g_1'(x)$

(C) $f_1'(x)f_2'(x) - |f_1(x)g_1'(x)|$

(D) $f_1'(x)g_2'(x) - f_2'(x)g_2'(x)$

8. Let $f(x) = \frac{x+|x|(1+x)}{x} \sin\left(\frac{1}{x}\right)$, for $x \neq 0$. Write $L = \lim_{x\to 0^-} f(x)$ and $R = \lim_{x\to 0^+} f(x)$. Then which one of the following is TRUE?

(A) L exists but R does not exist

(B) L does not exist but R exists

(C) Both L and R exist

(D) Neither L nor R exists

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9. If $\lim_{t\to\infty} \int_0^t e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then

$$\lim_{t\to\infty} \int_0^t x^2 e^{-x^2} dx =$$

(A) $\frac{\sqrt{\pi}}{4}$

(B) $\frac{\sqrt{\pi}}{2}$

(C) $\sqrt{2\pi}$

(D) $2\sqrt{\pi}$

---

10. If

$$f(x) = \begin{cases} 1+x & \text{if } x < 0, \\ (1-x)(px+q) & \text{if } x \geq 0, \end{cases}$$

satisfies the assumptions of Rolle's theorem in the interval $[-1, 1]$, then the ordered pair (p, q) is

(A) $(1, 1)$

(B) $(2, 1)$

(C) $(0, 1)$

(D) $(1, 0)$

---

11. The flux of the vector field

$$\mathbf{F} = \left(\frac{2\pi x + 2x^2y^2}{\pi}\right)\hat{i} + \left(\frac{2\pi xy - 4y}{\pi}\right)\hat{j}$$

along the outward normal, across the ellipse $x^2 + 16y^2 = 4$ is equal to

(A) $4\pi^2 - 2$

(B) $2\pi^2 - 4$

(C) $\pi^2 - 2$

(D) 2π

---

12. Let $\mathcal{M}$ be the set of all invertible 5×5 matrices with entries 0 and 1. For each $M \in \mathcal{M}$, let $n_1(M)$ and $n_0(M)$ denote the number of 1's and 0's in M, respectively. Then

$$\min_{M\in\mathcal{M}} |n_1(M) - n_0(M)| =$$

(A) 1

(B) 3

(C) 5

(D) 15

---

13. Let

$$M = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

Then

$$\lim_{n\to\infty} M^n x$$

(A) does not exist

(B) is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(C) is $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

(D) is $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

---

14. Let

$$\mathbf{F} = (3 + 2xy)\hat{i} + (x^2 - 3y^2)\hat{j}$$

and let L be the curve

$$\mathbf{r}(t) = e^t \sin t\, \hat{i} + e^t \cos t\, \hat{j}, \quad 0 \leq t \leq \pi.$$

Then

$$\int_{\mathbf{L}} \mathbf{F} \cdot d\mathbf{r} =$$

(A) $e^{-3t} + 1$

(B) $e^{-6t} + 2$

(C) $e^{6t} + 2$

(D) $e^{3t} + 1$

---

15. The line integral of the vector field

$$\mathbf{F} = zx\hat{i} + xy\hat{j} + yz\hat{k}$$

along the boundary of the triangle with vertices $(1, 0, 0), (0, 1, 0)$ **and** $(0, 0, 1)$**, oriented anti-clockwise, when viewed from the point** $(2, 2, 2)$**, is**

$$\int_{C} \mathbf{F} \cdot d\mathbf{r} =$$

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

---

16. The area of the surface

$$z = \frac{xy}{3}$$

intercepted by the cylinder

$$x^2 + y^2 \leq 16$$

lies in the interval

$$(20\pi, 22\pi)$$

(A) $(20\pi, 22\pi)$

(B) $(22\pi, 24\pi)$

(C) $(24\pi, 26\pi)$

(D) $(26\pi, 28\pi)$

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17. For $a > 0, b > 0$, let

$$\mathbf{F} = \frac{xj - yk}{bx^2 + ay^2}.$$

Let

$$C = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 = a^2 + b^2\}.$$

Then the line integral

$$\oint_C \mathbf{F} \cdot d\mathbf{r} =$$

(A) $\frac{2\pi}{ab}$

(B) 2π

(C) $2nab$

(D) 0

---

18. The flux of

$$\mathbf{F} = y\hat{i} - x\hat{j} + 2z\hat{k}$$

along the outward normal, across the surface of the solid

$$\left\{(x, y, z) \in \mathbb{R}^3 \mid 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq \sqrt{2 - x^2 - y^2}\right\}$$

is equal to

$$\iint_S \mathbf{F} \cdot \hat{n}\, dS =$$

(A) $\frac{2}{3}$

(B) $\frac{5}{3}$

(C) $\frac{8}{3}$

(D) $\frac{4}{3}$

---

19. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(2) = 2$ and

$$|f(x) - f(y)| \leq 5||x - y|^{3/2}.$$

For all $x \in \mathbb{R}, y \in \mathbb{R}$, let $g(x) = x^3 f(x)$. Then

$$g'(2) =$$

(A) 5

(B) $\frac{15}{2}$

(C) 12

(D) 24

---

20. Let $f : \mathbb{R} \to [0, \infty)$ be a continuous function. Then which one of the following is NOT TRUE?

$$\text{There exists } x \in \mathbb{R} \text{ such that } f(x) = \frac{f(0) + f(1)}{2}.$$

(A) There exists $x \in \mathbb{R}$ such that $f(x) = \frac{f(0)+f(1)}{2}$

(B) There exists $x \in \mathbb{R}$ such that $f(x) = f(0)$

(C) There exists $x \in \mathbb{R}$ such that $f(x) = f(1)$

(D) There exists $x \in \mathbb{R}$ such that $f(x) = \int_0^1 f(t)dt$

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21. The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1}{(-3)^n + 2} \frac{(4x-12)^n}{n^2+1}$$

is

(A) $\frac{10}{4} \leq x \leq \frac{14}{4}$

(B) $\frac{9}{4} \leq x \leq \frac{15}{4}$

(C) $\frac{10}{4} \leq x \leq \frac{14}{4}$

(D) $\frac{9}{4} \leq x \leq \frac{15}{4}$

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22. Let P_3 denote the real vector space of all polynomials with real coefficients of degree at most 3. Consider the map

$$T : P_3 \to P_3 \quad \text{given by} \quad T(p(x)) = p'(x) + p(x).$$

Then

(A) T is neither one-one nor onto

(B) T is both one-one and onto

(C) T is one-one but not onto

(D) T is onto but not one-one

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23. Let

$$f(x,y) = \frac{xzy}{x^2+y^2+z^2}, \quad (x,y) \neq (0,0).$$

Then

$$\frac{\partial f}{\partial x} \text{ and } f \text{ are bounded and unbounded.}$$

(A) $\frac{\partial f}{\partial x}$ and f are bounded

(B) $\frac{\partial f}{\partial x}$ is bounded and f is unbounded

(C) $\frac{\partial f}{\partial x}$ is unbounded and f is bounded

(D) $\frac{\partial f}{\partial x}$ and f are unbounded

---

24. Let S be an infinite subset of $\mathbb{R}$ such that $S \setminus \{a\}$ is compact for some $a \in S$. Then which one of the following is TRUE?

(A) S is a connected set

(B) S contains no limit points

(C) S is a union of open intervals

(D) Every sequence in S has a subsequence converging to an element in S

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25. The sum of the series

$$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right)$$

is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) π

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26. Let $0 < a_1 < b_1$, For $n \geq 1$, define

$$a_{n+1} = \sqrt{a_n b_n} \quad \text{and} \quad b_{n+1} = \frac{a_n + b_n}{2}.$$

Then which one of the following is NOT TRUE?

(A) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are not equal

(B) Both $\{a_n\}$ and $\{b_n\}$ converge and the limits are equal

(C) $\{b_n\}$ is a decreasing sequence

(D) $\{a_n\}$ is an increasing sequence

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27. Evaluate the limit

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{3}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{9}} + \cdots + \frac{1}{\sqrt{3n}+\sqrt{3n+3}} \right).$$

(A) $1 + \sqrt{3}$

(B) $\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{1+\sqrt{3}}$

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28. Which of the following is TRUE?

(A) Every sequence that has a convergent subsequence is a Cauchy sequence

(B) Every sequence that has a convergent subsequence is a bounded sequence

(C) The sequence $\{sinn\}$ has a convergent subsequence

(D) The sequence $\{n \cos \frac{1}{n}\}$ has a convergent subsequence

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29. A particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x} \sin x$$

is

(A) $\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(B) $-\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(C) $-\frac{e^{2x}}{5}(2\cos x + \sin x)$

(D) $\frac{e^{2x}}{5}(2\cos x - \sin x)$

---

30. Let $y(x)$ be the solution of the differential equation

$$(xy + y + e^{-x})\, dx + (x + e^{-x})\, dy = 0$$

satisfying $y(0) = 1$. Then $y(-1)$ is equal to

(A) $\frac{e}{e-1}$

(B) $\frac{2e}{e-1}$

(C) $\frac{e}{1-e}$

(D) 0

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31. For $a, \beta \in \mathbb{R}$, define the map $\varphi_{a,\beta} : \mathbb{R} \to \mathbb{R}$ by

$$\varphi_{a,\beta}(x) = ax + \beta.$$

Let

$$G = \{\varphi_{a,\beta} \mid (a, \beta) \in \mathbb{R}^2\}.$$

For $f, g \in G$, define $g \circ f \in G$ by

$$(g \circ f)(x) = g(f(x)).$$

Then which of the following statements is/are TRUE?

(A) The binary operation $\circ$ is associative.

(B) The binary operation $\circ$ is commutative.

(C) For every $(a, \beta) \in \mathbb{R}^2, a \neq 0$, there exists $(a', \beta') \in \mathbb{R}^2$ such that $\varphi_{a,\beta} \circ \varphi_{a',\beta'} = \varphi_{1,0}$.

(D) $(G, \circ)$ is a group.

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32. The volume of the solid

$$\left\{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x \leq 2, 0 \leq y \leq 2/x, 0 \leq z \leq x\right\}$$

is expressible as

(A) $\int_1^2 \int_0^{2/x} \int_0^x \, dz \, dy \, dx$

(B) $\int_1^2 \int_0^x \int_0^{x^2} \, dy \, dz \, dx$

(C) $\int_1^2 \int_1^2 \int_0^x \, dz \, dy \, dx$

(D) $\int_1^2 \int_0^1 \int_0^{x^2} \, dy \, dz \, dx$

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33. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function. Then which of the following statements is/are TRUE?

(A) If f is differentiable at $(0, 0)$, then all directional derivatives of f exist at $(0, 0)$.

(B) If all directional derivatives of f exist at $(0,0)$, then f is differentiable at $(0,0)$.

(C) If all directional derivatives of f exist at $(0,0)$, then f is continuous at $(0,0)$.

(D) If the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous in a disc centered at $(0,0)$, then f is differentiable at $(0,0)$.

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34. If X and Y are $n \times n$ matrices with real entries, then which of the following is/are TRUE?

(A) If $P^{-1}XP$ is diagonal for some real invertible matrix P, then there exists a basis for $\mathbb{R}^n$ consisting of eigenvectors of X.

(B) If X is diagonal with distinct diagonal entries and $XY = YX$, then Y is also diagonal.

(C) If X^2 is diagonal, then X is diagonal.

(D) If X is diagonal and $XY = YX$ for all Y, then $X = \lambda I$ for some $\lambda \in \mathbb{R}$.

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35. Let G be a group of order 20 in which the conjugacy classes have sizes 1, 4, 5, 5, 5. Then which of the following is/are TRUE?

(A) G contains a normal subgroup of order 5

(B) G contains a non-normal subgroup of order 5

(C) G contains a subgroup of order 10

(D) G contains a normal subgroup of order 4

---

36. Let $\{x_n\}$ be a real sequence such that $7x_{n+1} = x_n^3 + 6$ for $n \geq 1$. Then which of the following statements is/are TRUE?

(A) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 1.

(B) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 2.

(C) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to 1.

(D) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to -3.

---

37. Let S be the set of all rational numbers in $(0,1)$. Then which of the following statements is/are TRUE?

(A) S is a closed subset of $\mathbb{R}$.

(B) S is not a closed subset of $\mathbb{R}$.

(C) S is an open subset of $\mathbb{R}$.

(D) S is a limit point of S.

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38. Let M be an $n \times n$ matrix with real entries such that $M^3 = I$. Suppose that $Mv \neq v$ for any non-zero vector v. Then which of the following statements is/are TRUE?

(A) M has real eigenvalues

(B) $M + M^{-1}$ has real eigenvalues

(C) n is divisible by 2

(D) n is divisible by 3

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39. Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = (y-1)(y-3)$$

satisfying the condition $y(0) = 2$. Then which of the following is/are TRUE?

(A) The function $y(x)$ is not bounded above.

(B) The function $y(x)$ is bounded.

(C) $\lim_{x\to\infty} y(x) = 1$

(D) $\lim_{x\to\infty} y(x) = 3$

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40. Let $k, \ell \in \mathbb{R}$ be such that every solution of

$$\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + \ell y = 0$$

satisfies $\lim_{x\to\infty} y(x) = 0$. Then which of the following is/are TRUE?

(A) $3k^2 + \ell < 0$ and $k > 0$

(B) $k^2 + \ell > 0$ and $k < 0$

(C) $k^2 - \ell \leq 0$ and $k > 0$

(D) $k^2 - \ell > 0$, $k > 0$, and $\ell > 0$

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41. If the orthogonal trajectories of the family of ellipses

$$x^2 + 2y^2 = c_1, \quad c_1 > 0,$$

are given by

$$y = c_2 x^\alpha, \quad c_2 \in \mathbb{R},$$

then $\alpha =$ ....

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42. Let G be a subgroup of $GL_2(\mathbb{R})$ generated by

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}.$$

Then the order of G is ....

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43. Consider the permutations

$$\sigma = (12345678), \quad \tau = (12345678)(45378612),$$

in S_8. The number of $\eta \in S_8$ such that $\eta^{-1}\sigma\eta = \tau$ is equal to ....

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44. Let P be the point on the surface

$$z = \sqrt{x^2 + y^2}$$

closest to the point $(4, 2, 0)$. Then the square of the distance between the origin and P is ....

45. Evaluate

$$\left(\int_0^1 x^4(1-x)^5\, dx\right)^{-1}.$$

46. Let $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **and** $v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. **Let** M **be the matrix whose columns are** $v_1, v_2, 2v_1 - v_2, v_1 + 2v_2$ **in that order. Then the number of linearly independent solutions of the homogeneous system of linear equations** $Mx = 0$ **is** ....

47. Evaluate

$$\frac{1}{2\pi}\left(\frac{\pi^3}{1\cdot 3} - \frac{\pi^5}{3\cdot 5} + \frac{\pi^7}{5\cdot 7} - \cdots + (-1)^{n-1}\frac{\pi^{2n+1}}{(2n-1)!}\,(2n+1)\right).$$

48. Let P **be a** 7×7 **matrix of rank 4 with real entries. Let** $a \in \mathbb{R}^7$ **be a column vector. Then the rank of** $P + aa^T$ **is at least** ....

49. For $x > 0$**, let** $\lfloor x \rfloor$ **denote the greatest integer less than or equal to** x**. Then**

$$\lim_{x\to\infty} x\left(\lfloor x\rfloor + \lfloor x/2\rfloor + \lfloor x/3\rfloor + \cdots + \lfloor x/10\rfloor\right)$$

is ....

50. The number of subgroups of $\mathbb{Z}_7 \times \mathbb{Z}_7$ **of order 7 is** ....

51. Let $y(x), x > 0$ **be the solution of the differential equation**

$$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = 0$$

satisfying the conditions $y(1) = 1$ **and** $y'(1) = 0$. **Then the value of** $e^2 y(e)$ **is** ....

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52. Let T **be the smallest positive real number such that the tangent to the helix**

$$x = \cos t, \quad y = \sin t, \quad z = \frac{t}{\sqrt{2}}$$

at $t = T$ **is orthogonal to the tangent at** $t = 0$. **Then the line integral of** $\mathbf{F} = xj - y\mathrm{i}$ **along the section of the helix from** $t = 0$ **to** $t = T$ **is** ....

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53. Let $f(x) = \frac{\sin\left(\frac{n\pi x}{\pi \sin x}\right)}{\sin x}$**, Let** $x_0 \in (0, \pi)$ **and let** $f'(x_0) = 0$**. Then**

$$(f(x_0))^2 \left(1 + (\pi^2 - 1)\sin^2 x_0\right) = \text{.........}$$

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54. The maximum order of a permutation σ **in the symmetric group** S_{10} **is** ....

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55. Let $a_n = \sqrt{n}, n \geq 1$**, and let**

$$s_n = a_1 + a_2 + \cdots + a_n. \text{ Then}$$

$$\lim_{n \to \infty} \left(\frac{a_n}{s_n}\right)\left(-\ln\left(1 - \frac{a_n}{s_n}\right)\right) = \text{............}$$

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56. For a real number x**, define** $\lceil x \rceil$ **to be the smallest integer greater than or equal to** x**. Then**

$$\int_0^1 \int_0^1 (\lceil x \rceil + \lceil y \rceil + |z|)\, dx\, dy\, dz = \text{.............}$$

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57. For $x > 1$**, let**

$$f(x) = \int_1^x \left(\sqrt{\log t - \frac{1}{2}\log\sqrt{t}}\right) dt.$$

The number of tangents to the curve $y = f(x)$ **parallel to the line** $x + y = 0$ **is** ....

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58. Let $\alpha, \beta, \gamma, \delta$ **be the eigenvalues of the matrix**

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 =$ ....

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59. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} n! x^{n^2}$$

is ....

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60. If

$$y(x) = \int_{\sqrt{x}}^{x} e^t \, dt, \quad x > 0$$

then $y'(1) =$ ....
