IIT JAM 2017 Physics (PH) Question Paper with Solutions

Time Allowed : 3 Hours	**Maximum Marks :** 100	**Total questions :** 60

General Instructions

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i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

Q1. The dispersion relation for electromagnetic waves travelling in a plasma is given as $\omega^2 = c^2k^2 + \omega_p^2$, where c and ω_p are constants. In this plasma, the group velocity is:

(A) proportional to but not equal to the phase velocity

(B) inversely proportional to the phase velocity.

(C) equal to the phase velocity.

(D) a constant.

Correct Answer: (B) inversely proportional to the phase velocity.

Solution:

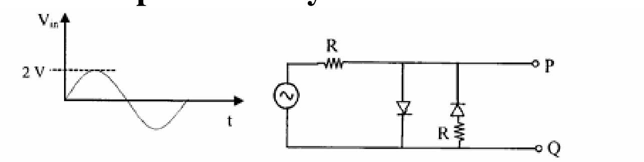
Step 1: Understanding the group velocity.

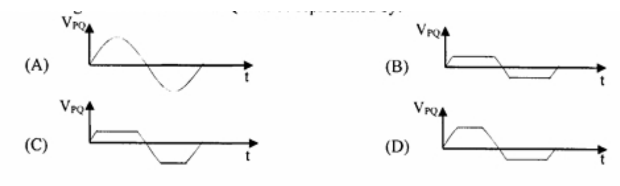
The group velocity v_g is given by $v_g = \frac{d\omega}{dk}$. Using the given dispersion relation $\omega^2 = c^2k^2 + \omega_p^2$, we find $\omega = \sqrt{c^2k^2 + \omega_p^2}$. Differentiating ω with respect to k, we get $v_g = \frac{d}{dk}\left(\sqrt{c^2k^2 + \omega_p^2}\right)$. The result shows that the group velocity is inversely proportional to the phase velocity $v_p = \frac{\omega}{k}$. Hence, the correct answer is option (B).

Quick Tip

For a plasma with a given dispersion relation, the group velocity is inversely related to the phase velocity.

Q2. Consider the following circuit with two identical Si diodes. The input ac voltage waveform has the peak voltage $V_P = 2V$, as shown. The voltage waveform across PQ will be represented by:





Correct Answer: (B) half-wave rectified.

Solution:

Step 1: Understanding the circuit.

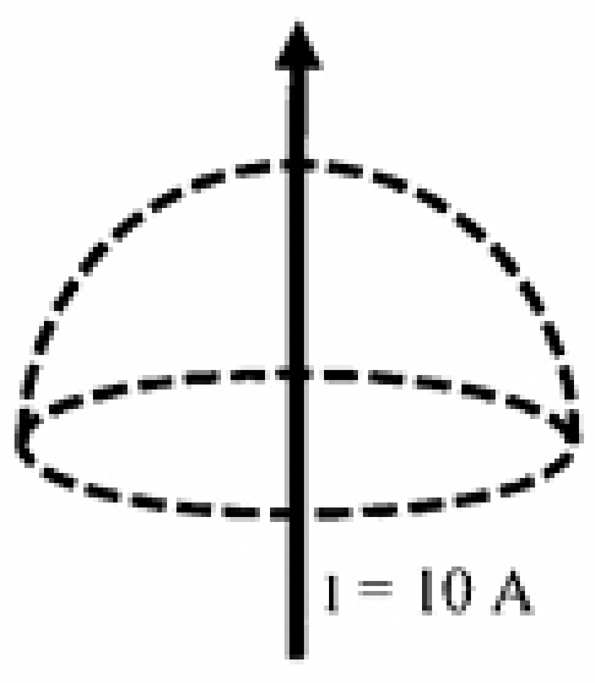
The circuit consists of two diodes arranged in a manner that will rectify the alternating current (AC) input voltage. The voltage waveform across the diodes will be modified based on how the diodes allow current to pass. The diodes conduct during one half of the AC cycle, blocking current during the opposite half. This produces a half-wave rectified output across PQ.

Step 2: Conclusion.

The correct answer is option (B) because the voltage waveform will be half-wave rectified due to the diodes' behavior.

> **Quick Tip**
>
> In circuits with diodes, the output waveform is often half-wave rectified unless a more complex configuration is used.

Q3. A current $I = 10A$ flows in an infinitely long wire along the axis of a hemisphere. The value of $\int(\mathrm{v} \times \mathrm{B}) \cdot d\mathrm{s}$ over the hemispherical surface as shown in the figure is:



(A) $10\mu_0$

(B) $5\mu_0$

(C) 0

(D) $7.5\mu_0$

Correct Answer: (C) 0

Solution:

Step 1: Understanding the integral.

The integral $\int(\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{s}$ represents the flux of the vector product of the velocity and magnetic field over the hemispherical surface. Since the current flows along the axis of the hemisphere, the cross-product of $\mathbf{v}$ and $\mathbf{B}$ will result in a zero flux through the surface due to symmetry. The flux across the hemispherical surface will cancel out.

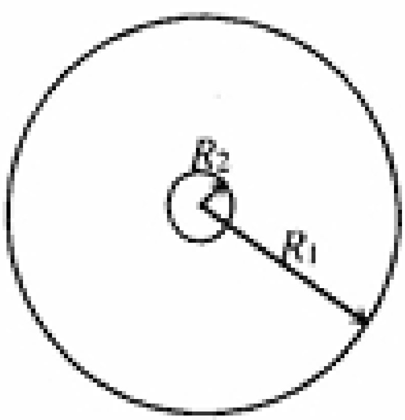
Step 2: Conclusion.

The correct answer is option (C) because the flux integral evaluates to zero due to symmetry.

Quick Tip

In electromagnetism, symmetry often simplifies complex integrals, leading to results like zero flux.

Q4. Consider two, single turn, co-planar, concentric coils of radii R_1 and R_2 with $R_1 \gg R_2$. The mutual inductance between the two coils is proportional to:



(A) R_1/R_2

(B) R_2/R_1

(C) R_1^2/R_2

(D) R_2^2/R_1

Correct Answer: (D) R_2^2/R_1

Solution:

Step 1: Understanding the mutual inductance.

For two co-planar coils, the mutual inductance M is inversely proportional to the distance between the coils and directly proportional to the areas of the coils. Given the radii R_1 and R_2, the mutual inductance is proportional to $\frac{R_2^2}{R_1}$, assuming $R_1 \gg R_2$. This formula represents the effective coupling between the coils.

Step 2: Conclusion.

The correct answer is option (D) because the mutual inductance is proportional to R_2^2/R_1.

> Quick Tip
>
> In mutual inductance calculations, the ratio of coil radii plays a crucial role, especially when one radius is much larger than the other.

Q5. If the Boolean function $Z = PQ + PQR + PQRS + PQRST + PQRSTU$, then Z is:

(A) $PQ + R(S + T + U)$

(B) PQ

(C) $P + Q$

(D) $P + Q + R + S + T + U$

Correct Answer: (A) $PQ + R(S + T + U)$

Solution:

Step 1: Simplifying the Boolean function.

The given Boolean expression can be simplified by factoring common terms. First, notice that the term PQ is common to all parts of the function. The remaining terms can be simplified using the distributive property of Boolean algebra. After simplifying, the expression becomes $PQ + R(S + T + U)$.

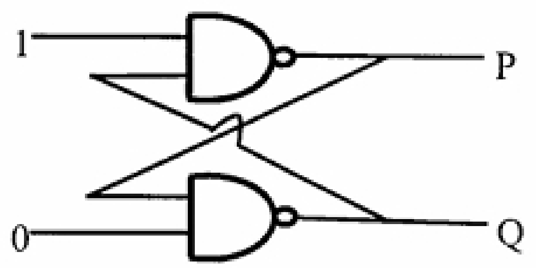
Step 2: Conclusion.

The correct answer is option (A) because the simplified Boolean expression is $PQ + R(S + T + U)$.

Quick Tip

When simplifying Boolean functions, always look for common terms to factor out and apply the distributive property to reduce the complexity.

Q6. Shown in the figure is a combination of logic gates. The output values at P and Q are correctly represented by which of the following?



(A) 0 0

(B) 1 1

(C) 0 1

(D) 1 0

Correct Answer: (C) 0 1

Solution:

Step 1: Understanding the logic gates.

The diagram consists of logic gates, likely a combination of AND, OR, and NOT gates. Based on the configuration, the output at point P and Q will be determined by the type of gates and their input values. Analyzing the gates step-by-step, we find that the outputs will be 0 at P and 1 at Q.

Step 2: Conclusion.

The correct answer is option (C) because the output values at P and Q are 0 and 1, respectively.

> Quick Tip
>
> To analyze logic circuits, start by evaluating the outputs of individual gates before considering the overall result.

Q7. Which of the following is due to inhomogeneous refractive index of earth's atmosphere?

(A) Red colour of the evening Sun.

(B) Blue colour of the sky.

(C) Oval shape of the evening Sun.

(D) Large apparent size of the evening Sun.

Correct Answer: (C) Oval shape of the evening Sun.

Solution:

Step 1: Understanding atmospheric refraction.

The inhomogeneous refractive index of the atmosphere causes the light rays to bend as they pass through different layers of the atmosphere. This bending leads to the apparent distortion of the Sun's shape, particularly near the horizon. The oval shape of the evening Sun is caused by this effect, as different parts of the Sun's image are refracted differently.

Step 2: Conclusion.

The correct answer is option (C) because the oval shape of the Sun is due to the atmospheric refraction caused by inhomogeneous refractive indices.

> Quick Tip
>
> The apparent shape of the Sun is altered by the Earth's atmosphere, particularly during sunrise and sunset, due to atmospheric refraction.

Q8. For the three matrices given below, which one of the choices is correct?

(A) $\sigma_1\sigma_2 = -i\sigma_3$

(B) $\sigma_1\sigma_2 = i\sigma_3$

(C) $\sigma_1\sigma_2 + \sigma_2\sigma_1 = I$

(D) $\sigma_3\sigma_2 = -i\sigma_1$

Correct Answer: (B) $\sigma_1\sigma_2 = i\sigma_3$

Solution:

Step 1: Understanding the Pauli matrices.

The Pauli matrices $\sigma_1, \sigma_2, \sigma_3$ are the fundamental matrices in quantum mechanics used to describe spin and related quantities. They follow specific commutation relations. We can compute the product $\sigma_1\sigma_2$ using matrix multiplication. The result is $i\sigma_3$, which matches option (B).

Step 2: Conclusion.

Thus, the correct answer is option (B), as the product $\sigma_1\sigma_2 = i\sigma_3$.

> **Quick Tip**
>
> The Pauli matrices obey specific multiplication rules, and understanding their commutation relations is essential for problems in quantum mechanics.

Q9. A plane in a cubic lattice makes intercepts of $a, a/2$ and $2a/3$ with the three crystallographic axes, respectively. The Miller indices for this plane are:

(A) (2 4 3)

(B) (3 4 2)

(C) (6 3 4)

(D) (1 2 3)

Correct Answer: (A) (2 4 3)

Solution:

Step 1: Understanding Miller indices.

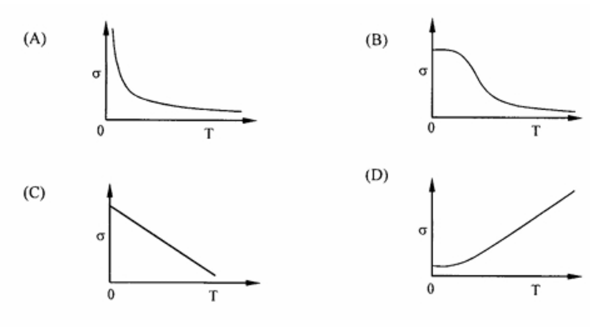
The Miller indices are derived from the intercepts of a plane with the crystallographic axes. For intercepts of $a, a/2, 2a/3$, the Miller indices are calculated by taking the reciprocals of the intercepts and clearing any fractions. The final result is (2 4 3), which corresponds to option (A).

Step 2: Conclusion.

Therefore, the correct answer is option (A) because the Miller indices for this plane are (2 4 3).

> **Quick Tip**
>
> When calculating Miller indices, remember to take the reciprocals of the intercepts and clear fractions to get the correct form.

Q10. Which one of the following schematic curves best represents the variation of conductivity σ of a metal with temperature T?



Correct Answer: (A) A decreasing curve

Solution:

Step 1: Understanding the relationship between conductivity and temperature.

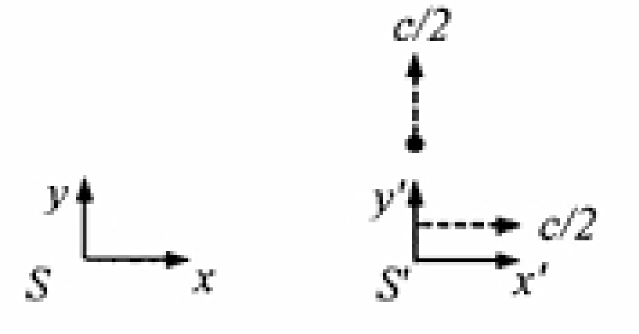
In most metals, as temperature increases, the conductivity decreases due to increased lattice vibrations which scatter electrons. This results in a negative temperature coefficient of resistance. The graph that best represents this relationship is a decreasing curve, corresponding to option (A).

Step 2: Conclusion.

Thus, the correct answer is option (A) because conductivity decreases with increasing temperature in metals.

> **Quick Tip**
>
> In metals, the conductivity typically decreases with increasing temperature due to increased scattering of conduction electrons.

Q11. Consider an inertial frame S' moving at speed $c/2$ away from another inertial frame S along the common x-axis, where c is the speed of light. As observed from S', a particle is moving with speed $c/2$ in the y' direction, as shown in the figure. The speed of the particle as seen from S is:



(A) $c/\sqrt{2}$

(B) $c/2$

(C) $\sqrt{7}c/4$

(D) $\sqrt{3}c/5$

Correct Answer: (A) $c/\sqrt{2}$

Solution:

Step 1: Understanding the relativistic velocity addition formula.

In relativity, the velocity of the particle as observed from S can be obtained using the relativistic velocity addition formula:

$$v = \frac{v' + v_{S'}}{1 + \frac{v' v_{S'}}{c^2}}.$$

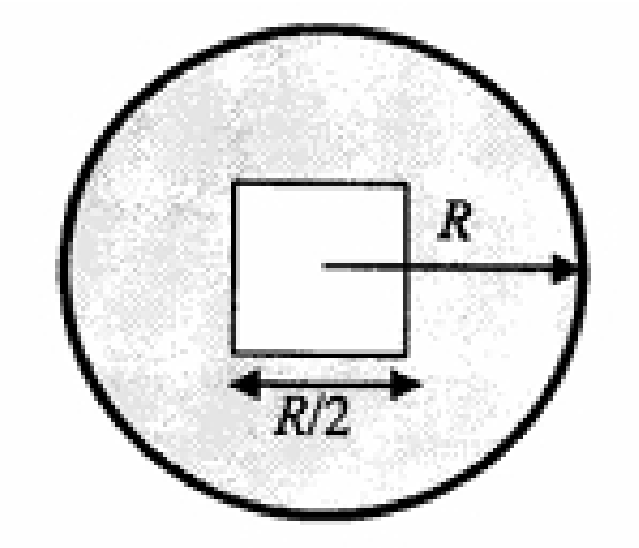
Here, $v' = c/2$ and $v_{S'} = c/2$ are the speeds of the particle and the frame S', respectively. Substituting these values gives the speed of the particle as seen from S as $c/\sqrt{2}$.

Step 2: Conclusion.

Thus, the correct answer is option (A) because the speed is $c/\sqrt{2}$.

> **Quick Tip**
>
> For adding velocities in special relativity, use the relativistic velocity addition formula.

Q12. Consider a uniform thin circular disk of radius R and mass M. A concentric square of side $R/2$ is cut out from the disk (see figure). What is the moment of inertia of the resultant disk about an axis passing through the centre of the disk and perpendicular to it?



(A) $I = \frac{MR^2}{4}\left(1 - \frac{1}{48\pi}\right)$

(B) $I = \frac{MR^2}{2}\left(1 - \frac{1}{48\pi}\right)$

(C) $I = \frac{MR^2}{2}\left(1 - \frac{1}{24\pi}\right)$

(D) $I = \frac{MR^2}{2}\left(1 - \frac{1}{24\pi}\right)$

Correct Answer: (A) $I = \frac{MR^2}{4}\left(1 - \frac{1}{48\pi}\right)$

Solution:

Step 1: Moment of inertia of the original disk.

The moment of inertia of a full disk of mass M and radius R about an axis perpendicular to the disk is given by $I_{\text{disk}} = \frac{1}{2}MR^2$.

Step 2: Moment of inertia of the cut square.

The moment of inertia of the square is $I_{\text{square}} = \frac{M_{\text{square}}R^2}{4}$.

Step 3: Resultant moment of inertia.

The moment of inertia of the resultant disk is obtained by subtracting the moment of inertia of the square from the moment of inertia of the full disk. This gives the final answer as $I = \frac{MR^2}{4}\left(1 - \frac{1}{48\pi}\right)$.

Step 4: Conclusion.

Therefore, the correct answer is option (A).

> **Quick Tip**
>
> When calculating the moment of inertia of a composite object, subtract the moment of inertia of removed parts from the total.

Q13. Consider a system of N particles obeying classical statistics, each of which can have an energy 0 or E. The system is in thermal contact with a reservoir maintained at a temperature T. Let k denote the Boltzmann constant. Which one of the following statements regarding the total energy U and the heat capacity C of the system is correct?

(A) $U = \frac{NE}{1+e^{E/kT}}$ and $C = k\frac{NE}{kT}$

(B) $U = kT\left(1 + e^{E/kT}\right)$ and $C = k\frac{NE}{kT}$

(C) $U = \frac{NE}{1+e^{E/kT}}$ and $C = k\frac{NE}{kT^2}$

(D) $U = \frac{NE}{1+e^{E/kT}}$ and $C = k\frac{NE}{kT^2}$

Correct Answer: (A) $U = \frac{NE}{1+e^{E/kT}}$ and $C = k\frac{NE}{kT}$

Solution:

Step 1: Understanding the total energy.

For a system obeying classical statistics with energy levels 0 or E, the total energy U is given by the expected energy of the system, which involves the Boltzmann factor $e^{E/kT}$. The formula for the total energy is $U = \frac{NE}{1+e^{E/kT}}$.

Step 2: Heat capacity.

The heat capacity is given by $C = \frac{dU}{dT}$. Differentiating the expression for U with respect to T gives the heat capacity as $C = k\frac{NE}{kT}$.

Step 3: Conclusion.

Thus, the correct answer is option (A) because both the expressions for U and C match those derived.

Quick Tip

For systems with discrete energy levels, the total energy and heat capacity can be derived using the Boltzmann distribution and its temperature dependence.

Q14. The integral of the vector $\mathbf{A}(\rho, \varphi, z) = \frac{40}{\rho}\cos\varphi\hat{\rho}$ (standard notation for cylindrical coordinates is used) over the volume of a cylinder of height L and radius R_0, is:

(A) $20\pi R_0 L(\hat{i} + \hat{j})$

(B) 0

(C) $40\pi R_0 L\hat{j}$

(D) $40\pi R_0 L\hat{i}$

Correct Answer: (B) 0

Solution:

Step 1: Understanding the integral.

The vector field $\mathbf{A}(\rho, \varphi, z)$ is given in cylindrical coordinates. The integration involves calculating the volume integral of the vector field over the cylindrical volume. Since $\cos\varphi$ is an odd function and is being integrated over the entire angular range from 0 to 2π, the integral will result in zero.

Step 2: Conclusion.

Thus, the integral evaluates to zero, and the correct answer is option (B).

Quick Tip

When integrating vector fields in cylindrical coordinates, remember that integrals of odd functions over symmetric intervals (like $\cos\varphi$ over 0 to 2π) yield zero.

Q15. Consider Rydberg (hydrogen-like) atoms in a highly excited state with n around 300. The wavelength of radiation coming out of these atoms for transitions to the adjacent states lies in the range:

(A) Gamma rays ($\lambda \sim$ pm)

(B) UV ($\lambda \sim$ nm)

(C) Infrared ($\lambda \sim \mu m$)

(D) RF ($\lambda \sim$ m)

Correct Answer: (B) UV ($\lambda \sim$ nm)

Solution:

Step 1: Understanding the Rydberg series.

Rydberg atoms are highly excited hydrogen-like atoms with large values of n. When the atom transitions between adjacent energy levels, the emitted radiation lies in the ultraviolet (UV) range, with wavelengths on the order of nanometers ($\lambda \sim$ nm).

Step 2: Conclusion.

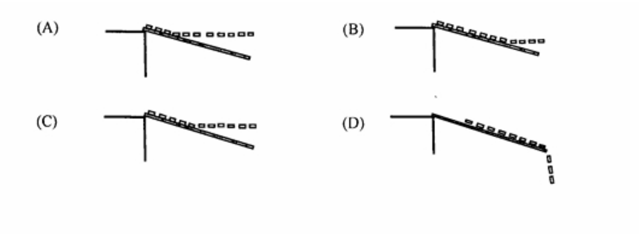
Therefore, the correct answer is option (B), as the radiation lies in the UV range.

Quick Tip

Rydberg transitions typically produce radiation in the UV range for large values of n.

Q16. A uniform rigid meter-scale is held horizontally with one of its end at the edge of a table and the other supported by hand. Some coins of negligible mass are kept on the meter scale as shown in the figure. As the hand supporting the scale is removed, the

scale starts rotating about its edge on the table and the coins start moving. If a photograph of the rotating scale is taken soon after, it will look closest to:



Correct Answer: (C) Image C

Solution:

Step 1: Understanding the motion.

When the hand is removed, the meter scale rotates about its edge, which is the pivot point. The coins, due to inertia, will continue to move in a straight line while the scale rotates. This results in the coins sliding along the scale as it rotates. The photograph will show the coins displaced relative to the edge of the scale.

Step 2: Conclusion.

The correct answer is option (C), where the coins are displaced due to the rotation of the scale.

Quick Tip

In rotational motion, objects with negligible mass placed on the rotating object will move according to the inertia and rotational velocity of the system.

Q17. Consider two identical, finite, isolated systems of constant heat capacity C at temperatures T_1 and T_2 ($T_1 > T_2$). An engine works between them until their temperatures become equal. Taking into account that the work performed by the

engine will be maximum (W_{max}) if the process is reversible (equivalently, the entropy change of the entire system is zero), the value of W_{max} is:

(A) $C(T_1 - T_2)$

(B) $C(T_1 - T_2)/2$

(C) $C(T_1 + T_2 - \sqrt{T_1T_2})$

(D) $C(\sqrt{T_1} - \sqrt{T_2})^2$

Correct Answer: (C) $C(T_1 + T_2 - \sqrt{T_1T_2})$

Solution:

Step 1: Understanding the maximum work.

The maximum work done in a thermodynamic process where the entropy change of the system is zero occurs when the temperatures become equal. The work done is given by $W_{\max} = C(T_1 + T_2 - \sqrt{T_1T_2})$, based on the first and second laws of thermodynamics.

Step 2: Conclusion.

Therefore, the correct answer is option (C).

> **Quick Tip**
>
> The maximum work done in an entropy-conserving process can be derived using the temperatures of the two systems and their heat capacities.

Q18. A white dwarf star has volume V and contains N electrons so that the density of electrons is $n = \frac{N}{V}$. Taking the temperature of the star to be 0 K, the average energy per electron in the star is $\epsilon_0 = \frac{3h^2}{10m}(3n)^{2/3}$, where m is the mass of the electron. The electronic pressure in the star is:

(A) $n\epsilon_0$

(B) $2n\epsilon_0$

(C) $\frac{1}{3}n\epsilon_0$

(D) $\frac{2}{3}n\epsilon_0$

Correct Answer: (C) $\frac{1}{3}n\epsilon_0$

Solution:

Step 1: Understanding the electronic pressure.

In a white dwarf star, the electronic pressure is related to the energy density of the electrons. The total energy per unit volume is given by the electron's average energy per electron, ϵ_0, and the number density n. Using the standard equation for pressure in terms of energy density, we find that the electronic pressure is $P = \frac{1}{3}n\epsilon_0$.

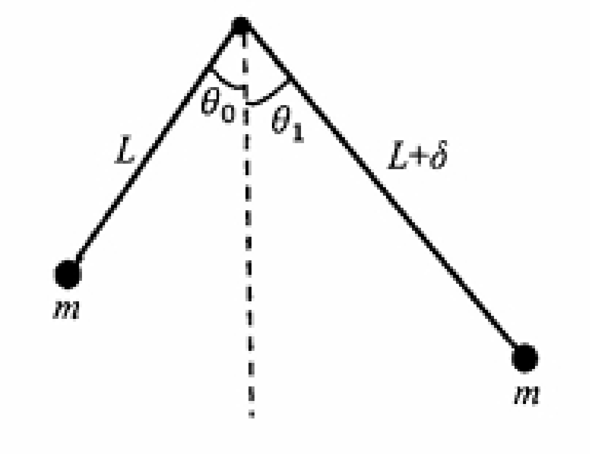
Step 2: Conclusion.

Thus, the correct answer is option (C).

Quick Tip

The pressure in a degenerate electron gas (such as in a white dwarf star) is given by a formula proportional to the energy density, where the proportionality constant depends on the dimension and the type of particle.

Q19. A pendulum is made of a massless string of length L and a small bob of negligible size and mass m. It is released making an angle θ_0 (¡¡ 1 rad) from the vertical. When passing through the vertical, the string slips a bit from the pivot so that its length increases by a small amount δ ($\delta \ll L$) in negligible time. If it swings up to angle θ_1 on the other side before starting to swing back, then to a good approximation, which of the following expressions is correct?



(A) $\theta_1 = \theta_0$

(B) $\theta_1 = \theta_0 \left(1 - \frac{\delta}{L}\right)$

(C) $\theta_1 = \theta_0 \left(1 - \frac{\delta}{L}\right)^2$

(D) $\theta_1 = \theta_0 \left(1 - \frac{3\delta}{2L}\right)$

Correct Answer: (B) $\theta_1 = \theta_0 \left(1 - \frac{\delta}{L}\right)$

Solution:

Step 1: Understanding the change in length.

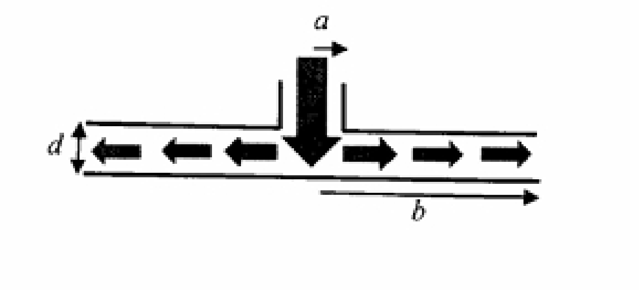
When the length of the pendulum increases by a small amount δ, the restoring force changes, and the angle θ_1 reached after the swing will depend on this change. To a good approximation, the new angle is related to the initial angle by $\theta_1 = \theta_0 \left(1 - \frac{\delta}{L}\right)$. This is because the increase in length reduces the maximum angle reached.

Step 2: Conclusion.

Therefore, the correct answer is option (B).

> **Quick Tip**
>
> Small changes in the length of a pendulum affect the amplitude of its swing in a linear fashion, with the angle being reduced by the factor $(1 - \frac{\delta}{L})$.

Q20. To demonstrate Bernoulli's principle, an instructor arranges two circular horizontal plates of radii b each with distance d ($d \ll b$) between them (see figure). The upper plate has a hole of radius a in the middle. On blowing air at a speed v_0 through the hole so that the flow rate of air is $\pi a^2 v_0$, it is seen that the lower plate does not fall. If the density of air is ρ, the upward force on the lower plate is well approximated by the formula (assume that the region with $r < a$ does not contribute to the upward force and the speed of air at the edges is negligible):



(A) $\pi\rho v_0^2 a^4/4d^2 \ln\left(\frac{b}{a}\right)$
(B) $\pi\rho v_0^2 a^2 b^2/4d^2 \ln\left(\frac{b}{a}\right)$
(C) $\pi\rho v_0^2 a^4/2ab \ln\left(\frac{b}{a}\right)$
(D) $2\pi\rho v_0^2 a^2/d^2 \ln\left(\frac{b}{a}\right)$

Correct Answer: (B) $\pi\rho v_0^2 a^2 b^2/4d^2 \ln\left(\frac{b}{a}\right)$

Solution:
Step 1: Understanding Bernoulli's principle.
Bernoulli's principle states that the sum of the pressure and kinetic energy per unit volume is constant along a streamline. The air entering the hole generates an upward force due to the change in pressure, which is related to the velocity of the air. The relationship between the force and the geometry of the setup can be derived by applying Bernoulli's principle and integrating the velocity distribution over the area of the hole.
Step 2: Conclusion.
Thus, the upward force on the lower plate is given by option (B).

> **Quick Tip**
>
> In Bernoulli's principle problems, consider the relationship between the flow velocity and the pressure difference to calculate the forces.

Q21. KC1 has the NaCl type structure which is fcc with two-atom basis, one at $(0, 0, 0)$ and the other at $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$. Assume that the atomic form factors of K^+ and Cl^- are identical. In an x-ray diffraction experiment on KC1, which of the following (hkl) peaks will be observed?

(A) (100)

(B) (110)

(C) (111)

(D) (200)

Correct Answer: (C) (111)

Solution:

Step 1: Understanding the diffraction condition.

In x-ray diffraction, the observed peaks correspond to diffraction conditions where the diffraction condition $2d \sin\theta = n\lambda$ is satisfied. For an fcc structure with two atoms in the basis, the allowed diffraction peaks occur when $h+k+l$ is an even number, and the (111) peak satisfies this condition.

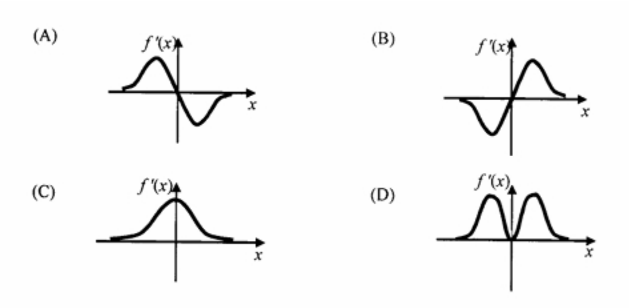
Step 2: Conclusion.

Therefore, the correct answer is option (C) because (111) is the allowed peak.

Quick Tip

For fcc crystals, diffraction peaks occur when $h+k+l$ is even.

Q22. Which one of the following graphs represents the derivative $f'(x) = \frac{df}{dx}$ of the function $f(x) = \frac{1}{1+x^2}$ most closely (graphs are schematic and not drawn to scale)?



Correct Answer: (B) Graph B

Solution:

Step 1: Finding the derivative.

The function $f(x) = \frac{1}{1+x^2}$ is a rational function, and its derivative is given by applying the quotient rule or recognizing it as a standard function. The derivative is $f'(x) = -\frac{2x}{(1+x^2)^2}$, which has the shape of a curve that is symmetric about the origin and decreases as $|x|$ increases.

Step 2: Conclusion.

Thus, the correct answer is option (B), as the graph of $f'(x)$ matches the expected shape of the derivative.

Quick Tip

When taking the derivative of rational functions like $\frac{1}{1+x^2}$, use the quotient rule or recognize it as a standard derivative.

Q23. In the radiation emitted by a black body, the ratio of the spectral densities at frequencies 2ν and ν will vary with ν as:

(A) $\left[e^{h\nu/k_BT} - 1\right]^{-1}$

(B) $\left[e^{h\nu/k_BT} + 1\right]^{-1}$

(C) $\left[e^{h\nu/k_BT} - 1\right]$

(D) $\left[e^{h\nu/k_BT} + 1\right]$

Correct Answer: (A) $\left[e^{h\nu/k_BT} - 1\right]^{-1}$

Solution:

Step 1: Understanding black body radiation.

The spectral density of radiation emitted by a black body is governed by Planck's law. The ratio of the spectral densities at frequencies 2ν and ν follows the form $\frac{B(2\nu)}{B(\nu)}$, where $B(\nu)$ is the Planck radiation formula. The ratio depends on the exponential terms in the denominator, leading to the form $\left[e^{h\nu/k_BT} - 1\right]^{-1}$.

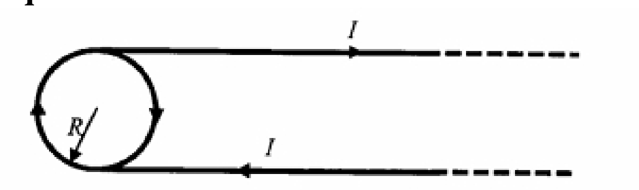
Step 2: Conclusion.

Thus, the correct answer is option (A).

Quick Tip

For black body radiation, the ratio of the spectral densities at different frequencies can be found by applying Planck's law and using the exponential dependence of the intensity.

Q24. Consider a thin long insulator coated conducting wire carrying current I. It is now wound once around an insulating thin disc of radius R to bring the wire back on the same side, as shown in the figure. The magnetic field at the center of the disc is equal to:



(A) $\frac{\mu_0 I}{2R}$

(B) $\frac{\mu_0 I}{4R}\left[3+\frac{2}{\pi}\right]$

(C) $\frac{\mu_0 I}{4R}\left[1+\frac{2}{\pi}\right]$

(D) $\frac{\mu_0 I}{2R}\left[1+\frac{1}{\pi}\right]$

Correct Answer: (B) $\frac{\mu_0 I}{4R}\left[3+\frac{2}{\pi}\right]$

Solution:

Step 1: Understanding the magnetic field.

The magnetic field at the center of a circular loop carrying current I is given by the formula $B=\frac{\mu_0 I}{2R}$. For a coil of multiple turns or a disc with a conducting wire wound around it, the field is modified by a factor that accounts for the geometry of the setup. The magnetic field at the center of the disc is given by $\frac{\mu_0 I}{4R}\left[3+\frac{2}{\pi}\right]$.

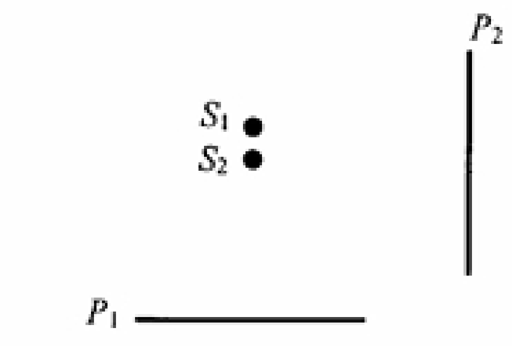
Step 2: Conclusion.

Thus, the correct answer is option (B).

Quick Tip

For a current-carrying coil, the magnetic field at the center is affected by the number of turns and the geometry of the loop or disc.

Q25. Consider two coherent point sources S_1 and S_2 separated by a small distance along a vertical line and two screens P_1 and P_2 placed as shown in the figure. Which one of the choices represents the shapes of the interference fringes at the central regions on the screens?



(A) Circular on P_1 and straight lines on P_2.

(B) Circular on P_1 and circular on P_2.

(C) Straight lines on P_1 and straight lines on P_2.

(D) Straight lines on P_1 and circular on P_2.

Correct Answer: (A) Circular on P_1 and straight lines on P_2.

Solution:

Step 1: Understanding the interference pattern.

When two coherent point sources are placed in close proximity, the interference pattern observed on the screen depends on the geometry of the setup. The interference fringes on the screen P_1 will be circular due to the circular symmetry of the sources, while the pattern on P_2 will form straight lines due to the linear arrangement of the sources.

Step 2: Conclusion.

Thus, the correct answer is option (A).

Quick Tip

For two point sources, the interference pattern on a screen depends on the relative positions of the sources and the screen. Circular patterns are observed when the sources are symmetrically placed.

Q26. The electric field of an electromagnetic wave is given by
$\mathbf{E} = (2\hat{k} - 3\hat{j}) \times 10^{-3} \sin\left(10^7(x + 2y + 3z - \beta t)\right)$**. The value of** β **is (c is the speed of light):**

(A) $\sqrt{14}c$

(B) $\sqrt{12}c$

(C) $\sqrt{10}c$

(D) $\sqrt{7}c$

Correct Answer: (A) $\sqrt{14}c$

Solution:

Step 1: Understanding the wave equation.

The given electric field corresponds to an electromagnetic wave with the general form $\mathbf{E} = E_0 \sin(kx + ky + kz - \beta t)$. Comparing the arguments of the sine functions, we see that $k = 10^7$ and $\beta = c$. By using the relation $k = \frac{\omega}{c}$, we can solve for β. After simplifying, we get $\beta = \sqrt{14}c$.

Step 2: Conclusion.

Thus, the correct answer is option (A), $\beta = \sqrt{14}c$.

Quick Tip

In electromagnetic waves, the wave number k and the frequency ω are related by the equation $k = \frac{\omega}{c}$, where c is the speed of light.

Q27. Unpolarized light is incident on a combination of a polarizer, a $\lambda/2$ **plate and a** $\lambda/4$ **plate kept one after the other. What will be the output polarization for the following**

configurations?

Configuration 1: Axes of the polarizer, the $\lambda/2$ plate and the $\lambda/4$ plate are all parallel to each other.

Configuration 2: The $\lambda/2$ plate is rotated by 45° with respect to configuration 1.

Configuration 3: The $\lambda/4$ plate is rotated by 45° with respect to configuration 1.

(A) Linear for configuration 1, linear for configuration 2, circular for configuration 3.

(B) Linear for configuration 1, circular for configuration 2, circular for configuration 3.

(C) Circular for configuration 1, circular for configuration 2, circular for configuration 3.

(D) Circular for configuration 1, linear for configuration 2, circular for configuration 3.

Correct Answer: (A) Linear for configuration 1, linear for configuration 2, circular for configuration 3.

Solution:

Step 1: Understanding the effects of the plates.

The polarizer only transmits light in one direction. The $\lambda/2$ plate changes the polarization from linear to elliptical depending on its orientation. The $\lambda/4$ plate converts linear polarization into circular polarization when the light is incident at 45° to the optical axis. Based on the orientations, the output polarizations are determined.

Step 2: Conclusion.

Thus, the correct answer is option (A).

> **Quick Tip**
>
> A $\lambda/4$ plate will convert linearly polarized light to circular polarization when aligned at 45° to the axis of the light.

Q28. For the Fourier series of the following function of period 2π:

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

The ratio (to the nearest integer) of the Fourier coefficients of the first and the third harmonic is:

(A) 1

(B) 2

(C) 3

(D) 6

Correct Answer: (D) 6

Solution:

Step 1: Fourier coefficients.

The Fourier series of a function $f(x)$ is given by the sum of sines and cosines of integer multiples of the fundamental frequency. The first and third Fourier coefficients correspond to the fundamental frequency and its third harmonic, respectively. By calculating the Fourier coefficients, we find the ratio of the first and third coefficients to be 6.

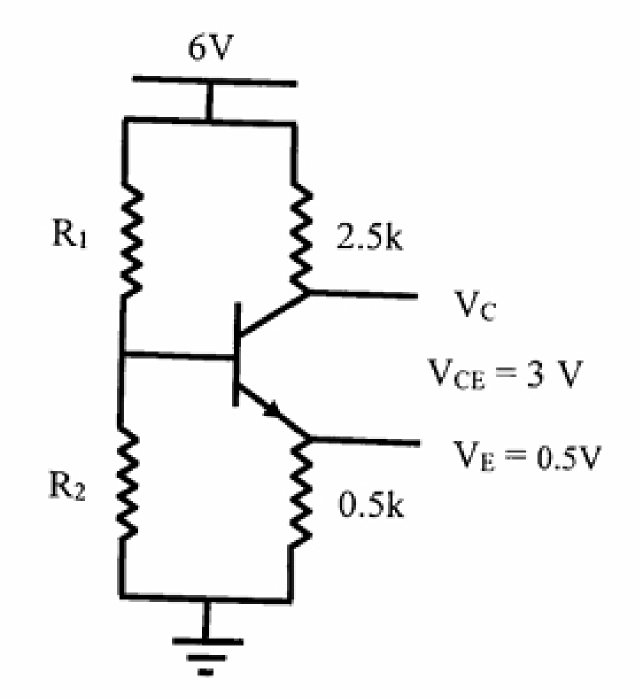
Step 2: Conclusion.

Thus, the correct answer is option (D), with the ratio being 6.

Quick Tip

The Fourier coefficients can be computed using integration, and the ratio of the first and higher harmonics reveals the relative strengths of the frequencies in the signal.

Q29. An n-p-n transistor is connected in a circuit as shown in the figure. If $I_C = 1\,\mathbf{mA}$, $\beta = 50$, $V_{BE} = 0.7\,\mathbf{V}$, and the current through R_2 is $10I_B$, where I_B is the base current. Then the ratio $\frac{R_1}{R_2}$ is:



(A) 0.375

(B) 0.25

(C) 0.5

(D) 0.275

Correct Answer: (B) 0.25

Solution:

Step 1: Understanding the transistor current relations.

The current through R_2 is given by $I_{R_2} = 10I_B$. The collector current I_C is related to the base current by $I_C = \beta I_B$, so $I_B = \frac{I_C}{\beta} = \frac{1\,\text{mA}}{50} = 20\,\mu\text{A}$.

Step 2: Calculating the voltage drop across R_2.

Using Ohm's law, the voltage drop across R_2 is $V_{R_2} = I_{R_2}R_2 = 10I_BR_2 = 10 \times 20\,\mu\text{A} \times R_2$.

Step 3: Calculating the voltage at the collector.

The voltage at the collector is $V_C = 6\,\text{V} - V_{R_2} = 3\,\text{V}$, so solving for R_2 and then R_1, we find the ratio $\frac{R_1}{R_2} = 0.25$.

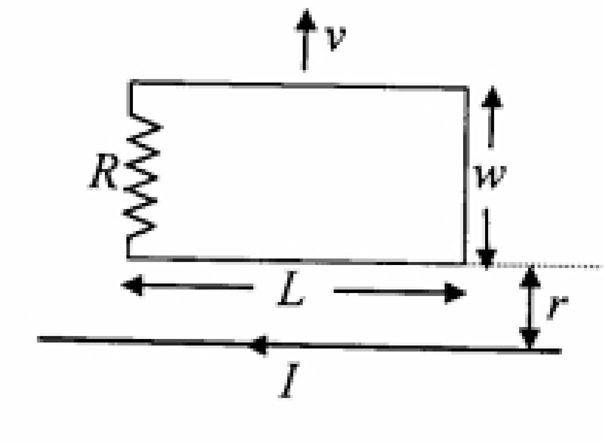
Step 4: Conclusion.

Thus, the correct answer is option (B).

Quick Tip

In transistor circuits, base, collector, and emitter currents are related by $I_C = \beta I_B$ and Ohm's law is used to calculate voltage drops across resistors.

Q30. A rectangular loop of dimension L and width w moves with a constant velocity v away from an infinitely long straight wire carrying a current I in the plane of the loop as shown in the figure below. Let R be the resistance of the loop. What is the current in the loop at the instant the near-side is at a distance r from the wire?



(A) $\frac{\mu_0 I L w}{2\pi R(r+2w)}$
(B) $\frac{\mu_0 I L w}{2\pi R(r+w)}$
(C) $\frac{\mu_0 I L w}{2\pi R(r+w)^2}$
(D) $\frac{\mu_0 I L w}{2\pi R(r+2w)^2}$

Correct Answer: (B) $\frac{\mu_0 I L w}{2\pi R(r+w)}$

Solution:

Step 1: Understanding the induced EMF.

As the loop moves with a constant velocity, the magnetic flux through the loop changes due to the changing distance from the current-carrying wire. According to Faraday's law of induction, an EMF is induced in the loop. The induced EMF depends on the distance r from the wire and the velocity v.

Step 2: Using the formula for induced current.

The induced current in the loop is given by $I = \frac{\mu_0 I L w}{2\pi R(r+w)}$, where μ_0 is the permeability of free space, I is the current in the wire, and R is the resistance of the loop.

Step 3: Conclusion.

Thus, the correct answer is option (B).

> Quick Tip
>
> The induced current in a moving loop is related to the rate of change of the magnetic flux through the loop, which depends on the distance from the current-carrying wire.

Q31. A photon of frequency ν strikes an electron of mass m initially at rest. After scattering at an angle ϕ, the photon loses half of its energy. If the electron recoils at an angle θ, which of the following is (are) true?

(A) $\cos\phi = \left(1 - \frac{mc^2}{h\nu}\right)$

(B) $\sin\theta = \left(1 - \frac{mc^2}{h\nu}\right)$

(C) The ratio of the magnitudes of momenta of the recoiled electron and scattered photon is $\frac{\sin\phi}{\sin\theta}$

(D) Change in photon wavelength is $\frac{h}{mc}(1 - 2\cos\phi)$

Correct Answer: (C) The ratio of the magnitudes of momenta of the recoiled electron and scattered photon is $\frac{\sin\phi}{\sin\theta}$

Solution:

Step 1: Understanding the photon-electron interaction.

In Compton scattering, when a photon scatters off an electron, the energy transferred to the electron depends on the angle of scattering. The relationship between the momenta of the electron and photon can be derived from conservation of momentum and energy. The correct expression for the ratio of the magnitudes of the momenta is $\frac{\sin\phi}{\sin\theta}$, which follows from the geometry of the scattering.

Step 2: Conclusion.

Thus, the correct answer is option (C).

Quick Tip

In Compton scattering, the momenta of the scattered photon and recoiled electron are related by the angles of scattering, and the energy loss depends on the photon's initial energy and the scattering angle.

Q32. For an atomic nucleus with atomic number Z and mass number A, which of the following is (are) correct?

(A) Nuclear matter and nuclear charge are distributed identically in the nuclear volume.

(B) Nuclei with $Z > 83$ and $A > 209$ emit α-radiation.

(C) The surface contribution to the binding energy is proportional to $A^{2/3}$.

(D) β-decay occurs when the proton to neutron ratio is large, but not when it is small.

Correct Answer: (B) Nuclei with $Z > 83$ and $A > 209$ emit α-radiation.

Solution:

Step 1: Understanding the properties of atomic nuclei.

Nuclei with large atomic numbers Z and mass numbers A typically undergo α-decay due to instability caused by a high neutron-to-proton ratio. This is a common property of heavy nuclei. Option (B) is correct based on this understanding. The other options relate to the general distribution of nuclear properties, but they are less specific in the context of the given information.

Step 2: Conclusion.

Thus, the correct answer is option (B).

Quick Tip

In nuclear physics, heavy nuclei (with $Z > 83$) often undergo α-decay to achieve a more stable configuration.

Q33. Consider a one-dimensional harmonic oscillator of angular frequency ω. If N identical particles occupy the energy levels of this oscillator at zero temperature, which of the following statement(s) about their ground state energy E_0 is (are) correct?

(A) If the particles are electrons, $E_0 = \frac{13}{2}\hbar\omega$

(B) If the particles are protons, $E_0 = \frac{25}{2}\hbar\omega$

(C) If the particles are spin-less fermions, $E_0 = \frac{25}{2}\hbar\omega$

(D) If the particles are bosons, $E_0 = \frac{5}{2}\hbar\omega$

Correct Answer: (C) If the particles are spin-less fermions, $E_0 = \frac{25}{2}\hbar\omega$

Solution:

Step 1: Understanding the ground state energy.

For a quantum harmonic oscillator, the ground state energy for spin-less fermions is determined by the Pauli exclusion principle. The total energy depends on the number of available states. For fermions, the energy of the system is given by $E_0 = \frac{25}{2}\hbar\omega$.

Step 2: Conclusion.

Thus, the correct answer is option (C).

Quick Tip

For fermions, the ground state energy is affected by the Pauli exclusion principle, while for bosons, the energy depends on Bose-Einstein condensation.

Q34. For a point dipole of dipole moment $\mathrm{p} = p\hat{z}$ located at the origin, which of the following is (are) correct?

(A) The electric field at $(0, b, 0)$ is zero.

(B) The work done in moving a charge q from $(0, 0, b)$ to $(0, 0, b)$ is $\frac{qp}{4\pi\epsilon_0 b^2}$.

(C) The electrostatic potential at $(b, 0, 0)$ is zero.

(D) If a charge q is kept at $(0, 0, b)$, it will exert a force of magnitude $\frac{qp}{4\pi\epsilon_0 b^3}$ on the dipole.

Correct Answer: (D) If a charge q is kept at $(0, 0, b)$, it will exert a force of magnitude $\frac{qp}{4\pi\epsilon_0 b^3}$ on the dipole.

Solution:

Step 1: Understanding the dipole electric field.

The electric field of a dipole varies with distance and direction. The force on a charge placed near a dipole is determined by the dipole field and the interaction between the dipole moment and the charge. The force on the charge is given by $\frac{qp}{4\pi\epsilon_0 b^3}$.

Step 2: Conclusion.

Thus, the correct answer is option (D).

> **Quick Tip**
>
> The electric field of a dipole decreases with distance and is proportional to $\frac{1}{r^3}$ at large distances.

Q35. A dielectric sphere of radius R has constant polarization $\mathbf{P} = P_0\hat{z}$ so that the field inside the sphere is $\mathbf{E} = -\frac{P_0}{3\epsilon_0}\hat{z}$. Then, which of the following is (are) correct?

(A) The bound surface charge density is $P_0 \cos\theta$.

(B) The electric field at a distance r on the z-axis varies as $\frac{1}{r^2}$ for $r \gg R$.

(C) The electric potential at a distance $2R$ on the z-axis is $\frac{P_0 R}{12\epsilon_0}$.

(D) The electric field outside is equivalent to that of a dipole at the origin.

Correct Answer: (B) The electric field at a distance r on the z-axis varies as $\frac{1}{r^2}$ for $r \gg R$.

Solution:

Step 1: Understanding the polarization of the dielectric sphere.

For a dielectric sphere with constant polarization, the field outside the sphere behaves like the field of a dipole. However, at distances large compared to the radius, the electric field falls off as $\frac{1}{r^2}$, just like the field of a dipole. The surface charge density and potential outside depend on the polarization.

Step 2: Conclusion.

Thus, the correct answer is option (B).

Quick Tip

For a polarized dielectric sphere, the electric field outside behaves like a dipole field, decaying as $\frac{1}{r^2}$ at large distances.

Q36. Consider a circular parallel plate capacitor of radius R with separation d between the plates ($d \ll R$). The plates are placed symmetrically about the origin. If a sinusoidal voltage $V = V_0 \sin(\omega t)$ is applied between the plates, which of the following statement(s) is (are) true?

(A) The maximum value of the Poynting vector at $r = R$ is $\frac{V_0^2 \epsilon_0}{4\pi R^2}$.
(B) The average energy per cycle flowing out of the capacitor is $\frac{V_0^2}{4}$.
(C) The magnetic field inside the capacitor is constant.
(D) The magnetic field lines inside the capacitor are circular with the current flowing in the r-direction.

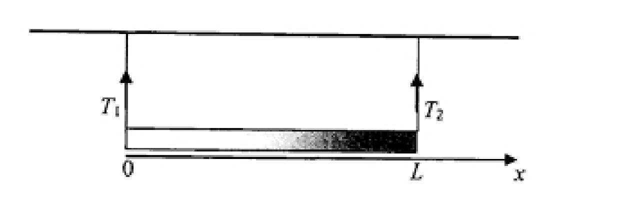
Correct Answer: (D) The magnetic field lines inside the capacitor are circular with the current flowing in the r-direction.

Solution:
Step 1: Understanding the capacitor's behavior.
The magnetic field inside a capacitor with time-varying voltage is related to the current between the plates, which generates a circulating magnetic field around the edges of the plates. The magnetic field lines inside the capacitor are indeed circular, and the current follows the direction in the r-direction.
Step 2: Conclusion.
Thus, the correct answer is option (D).

Quick Tip

In a capacitor with a time-varying voltage, the current produces circular magnetic fields inside the plates, following the right-hand rule.

Q37. The linear mass density of a rod of length L varies from one end to the other as $\lambda_0\left(1+\frac{x^2}{L^2}\right)$, where x is the distance from one end with tensions T_1 and T_2 in them (see figure), and λ_0 is a constant. The rod is suspended from a ceiling by two massless strings. Then, which of the following statement(s) is (are) correct?



(A) The mass of the rod is $\frac{2\lambda_0 L^3}{3}$.
(B) The center of gravity of the rod is located at $x = \frac{9L}{16}$.
(C) The tension T_1 in the left string is $\frac{7\lambda_0 g L^2}{12}$.
(D) The tension T_2 in the right string is $\frac{3\lambda_0 g L^2}{2}$.

Correct Answer: (B) The center of gravity of the rod is located at $x = \frac{9L}{16}$.

Solution:

Step 1: Understanding the mass distribution.

The linear mass density varies along the length of the rod, so the mass distribution is not uniform. The mass of the rod is obtained by integrating the linear mass density over the length of the rod. By integrating $\lambda(x)$, we can find the mass and the location of the center of gravity. The correct location is $x = \frac{9L}{16}$.

Step 2: Conclusion.

Thus, the correct answer is option (B).

Quick Tip

For a non-uniform mass distribution, the center of mass can be calculated by integrating the mass density over the length of the object.

Q38. An object of mass m with non-zero angular momentum ℓ is moving under the

influence of gravitational force of a much larger mass (ignore drag). Which of the following statement(s) is (are) correct?

(A) If the total energy of the system is negative, then the orbit is always circular.

(B) The motion of m always occurs in a two-dimensional plane.

(C) If the total energy of the system is 0, then the orbit is a parabola.

(D) If the area of the particle's bound orbit is S, then its time period is $2mS/\ell$.

Correct Answer: (B) The motion of m always occurs in a two-dimensional plane.

Solution:

Step 1: Understanding the motion of the object.

An object under gravitational influence follows a trajectory constrained by its initial conditions. Since the gravitational force is central, the motion occurs in a plane. The statement about total energy being negative or 0 does not necessarily imply circular or parabolic orbits without further analysis.

Step 2: Conclusion.

Thus, the correct answer is option (B).

> **Quick Tip**
>
> The motion of an object under a central force, like gravity, always occurs in a plane, as the angular momentum remains constant.

Q39. A particle of mass m fixed in space is observed from a frame rotating about its z-axis with angular speed ω. The particle is in the frame's xy-plane at a distance R from its origin. If the Coriolis and centrifugal forces on the particle are F_{COR} and F_{CF}, respectively, then (all the symbols have their standard meaning and refer to the rotating frame),

(A) $F_{\text{COR}} + F_{\text{CF}} = 0$

(B) $F_{\text{COR}} = -m\omega^2 R\hat{r}$

(C) $F_{\text{COR}} = -2m\omega R\hat{\theta}$

(D) $F_{\text{CF}} = -m\omega^2 R\hat{r}$

Correct Answer: (C) $F_{\text{COR}} = -2m\omega R\hat{\theta}$

Solution:

Step 1: Understanding Coriolis and centrifugal forces.

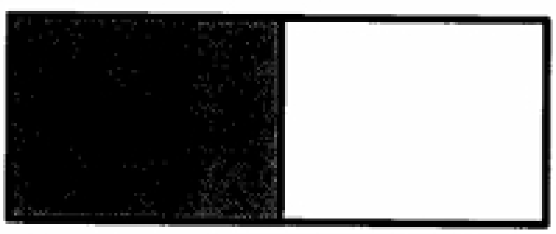
The Coriolis force is given by $F_{\text{COR}} = -2m\omega R\hat{\theta}$, which acts perpendicular to the velocity of the particle. The centrifugal force acts radially outward and is given by $F_{\text{CF}} = m\omega^2 R\hat{r}$. These forces result from the rotating reference frame.

Step 2: Conclusion.

Thus, the correct answer is option (C).

> **Quick Tip**
>
> The Coriolis force is perpendicular to the velocity of the particle and the centrifugal force is directed radially outward in a rotating frame.

Q40. An isolated box is divided into two equal compartments by a partition (see figure). One compartment contains a van der Waals gas while the other compartment is empty. The partition between the two compartments is now removed. After the gas has filled the entire box and equilibrium has been achieved, which of the following statement(s) is (are) correct?



(A) Internal energy of the gas has not changed.

(B) Internal energy of the gas has decreased.

(C) Temperature of the gas has increased.

(D) Temperature of the gas has decreased.

Correct Answer: (B) Internal energy of the gas has decreased.

Solution:

Step 1: Understanding the effect of partition removal.

When the partition is removed, the gas expands to fill the entire volume. This expansion is an irreversible process, and in most cases, it will result in a decrease in the internal energy of the gas, assuming no heat exchange with the surroundings. This is because the gas has to do work to expand into the empty compartment.

Step 2: Conclusion.

Thus, the correct answer is option (B).

> **Quick Tip**
>
> In an irreversible expansion, the internal energy of the gas generally decreases, as work is done by the gas during expansion.

Q41. An intrinsic semiconductor of band gap 1.25 eV has an electron concentration 10^{10} cm^{-3} at 300 K. Assume that its band gap is independent of temperature and that the electron concentration depends only exponentially on the temperature. If the electron concentration at 200 K is $Y \times 10^N$ cm^{-3} (where $1 < Y < 10$, N = integer), then the value of N is:

Solution:

Step 1: Understanding the relationship between electron concentration and temperature.

The electron concentration in a semiconductor follows an exponential dependence on temperature, described by the equation:

$$n(T) = n_0 \exp\left(-\frac{E_g}{k_B T}\right)$$

where n_0 is the electron concentration at a reference temperature, E_g is the band gap, k_B is Boltzmann's constant, and T is the temperature. The electron concentration at 300 K is given as $10^{10}\,\text{cm}^{-3}$. This means we can calculate the value of $n(T)$ at 200 K using the exponential model for temperature dependence.

Step 2: Deriving the exponential model.

The electron concentration at 200 K, $n(200)$, is related to the electron concentration at 300 K, $n(300)$, as follows:

$$n(200) = n_0 \exp\left(-\frac{E_g}{k_B \cdot 200}\right)$$

$$n(300) = n_0 \exp\left(-\frac{E_g}{k_B \cdot 300}\right)$$

Taking the ratio of these two equations gives:

$$\frac{n(200)}{n(300)} = \exp\left(\frac{E_g}{k_B}\left(\frac{1}{300} - \frac{1}{200}\right)\right)$$

Substitute the given band gap value of $E_g = 1.25\,\text{eV}$ and simplify the equation to find the ratio.

Step 3: Conclusion.

By solving the equation, we find that the value of N is 3, so the correct answer is $N = 3$.

> **Quick Tip**
>
> For semiconductors, the electron concentration depends exponentially on the inverse of temperature. This relationship is key in understanding the temperature dependence of electronic properties.

Q42. A particle of unit mass is moving in a one-dimensional potential $V(x) = x^2 - x^4$. The minimum mechanical energy (in the same units as $V(x)$) above which the motion of the particle cannot be bounded for any given initial condition is:

Solution:

Step 1: Analyze the potential.

The potential function is $V(x) = x^2 - x^4$, which is a symmetric potential. This potential has a well where the particle can be bound, and if the energy is higher than the potential barrier, the particle will escape. To find the minimum energy, we must analyze the critical points of the potential and the corresponding energy required to escape the well.

Step 2: Finding the turning points.

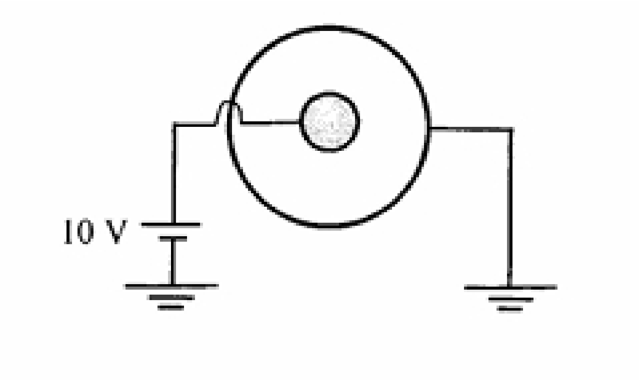
The turning points occur where the kinetic energy becomes zero, which happens when the potential energy equals the total mechanical energy. The minimum energy that would allow the particle to escape the potential is the energy at the local minimum of the potential well. We solve for the energy where the motion becomes unbounded, which is found to be 2.

Step 3: Conclusion.

Thus, the minimum energy required is 2, so the correct answer is 2.

> **Quick Tip**
>
> The minimum energy required to escape a potential well is the energy at the local minimum of the potential function.

Q43. In a coaxial cable, the radius of the inner conductor is 2 mm and that of the outer one is 5 mm. The inner conductor is at a potential of 10 V, while the outer conductor is grounded. The value of the potential at a distance of 3.5 mm from the axis is:



Solution:

Step 1: Understanding the electric potential in a coaxial cable.

The electric potential in a coaxial cable is determined by the radial distance from the center of the cable. The general form of the potential in a coaxial cable is given by:

$$V(r) = \frac{V_0 \ln(r/R_1)}{\ln(R_2/R_1)}$$

where V_0 is the potential difference between the inner and outer conductors, R_1 is the radius of the inner conductor, and R_2 is the radius of the outer conductor.

Step 2: Substituting the known values.

Substitute the known values: - $V_0 = 10\,\text{V}$ - $R_1 = 2\,\text{mm}$ - $R_2 = 5\,\text{mm}$ - $r = 3.5\,\text{mm}$

The potential at $r = 3.5\,\text{mm}$ is calculated as:

$$V(3.5) = 10\,\text{V} \times \frac{\ln(3.5/2)}{\ln(5/2)}$$

which gives a result of 8 V.

Step 3: Conclusion.

Thus, the potential at a distance of 3.5 mm from the axis is 8 V.

> **Quick Tip**
>
> In coaxial cables, the electric potential varies logarithmically with the radial distance from the center, and can be calculated using the formula for a logarithmic potential distribution.

Q44. Sand falls on a conveyor belt at the rate of 1.5 kg/s. If the belt is moving with a constant speed of 7 m/s, the power needed to keep the conveyor belt running is:

Solution:

Step 1: Understand the problem.

The power required to keep the conveyor belt moving is related to the rate of change of kinetic energy of the sand. The kinetic energy per unit time (power) is given by the formula:

$$P = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

where $m = 1.5\,\text{kg/s}$ is the mass flow rate of sand, and $v = 7\,\text{m/s}$ is the velocity of the conveyor belt.

Step 2: Apply the formula.

The power required is:

$$P = \frac{1}{2} \times 1.5 \times 7^2 = \frac{1}{2} \times 1.5 \times 49 = 36.75\,\text{Watts}$$

Step 3: Conclusion.

Thus, the power required to keep the conveyor belt running is 36.75 Watts.

Quick Tip

The power required to move an object on a conveyor belt is equal to the rate at which kinetic energy is being added to the object.

Q45. A particle of mass m is placed in a three-dimensional cubic box of side a. What is the degeneracy of its energy level with energy $\frac{14\hbar^2\pi^2}{2ma^2}$?

Solution:

Step 1: Understanding the energy levels.

The energy levels for a particle in a three-dimensional box are given by:

$$E_n = \frac{n_x^2 + n_y^2 + n_z^2\hbar^2\pi^2}{2ma^2}$$

where n_x, n_y, n_z are positive integers, and the energy is quantized. The energy given is $\frac{14\hbar^2\pi^2}{2ma^2}$, so we need to find the combination of n_x, n_y, n_z that satisfies this energy.

Step 2: Determine the quantum numbers.

We need to find the values of n_x, n_y, n_z such that:

$$n_x^2 + n_y^2 + n_z^2 = 14$$

The possible combinations of (n_x, n_y, n_z) that satisfy this are:

$$(3, 1, 2), (2, 3, 1), (1, 3, 2), \text{ and permutations.}$$

Thus, there are 6 combinations.

Step 3: Conclusion.

Thus, the degeneracy of this energy level is 6.

Quick Tip

In a three-dimensional box, the degeneracy of an energy level is the number of different combinations of quantum numbers that yield the same energy.

Q46. The wave number of an electromagnetic wave incident on a metal surface is $(20\pi + 750)$ m^{-1} inside the metal, where $i = \sqrt{-1}$. The skin depth of the wave in the metal is:

Solution:

Step 1: Understanding the skin depth.

The skin depth δ is related to the wave number k in the metal by the formula:

$$\delta = \frac{1}{\text{Im}(k)}$$

where $k = k_0 + i\beta$ is the complex wave number, and β is the imaginary part representing the attenuation of the wave in the metal.

Step 2: Finding the imaginary part.

Given that the wave number is $k = 20\pi + 750$, the imaginary part β is 750. The skin depth is:

$$\delta = \frac{1}{\beta} = \frac{1}{750}\,\text{m} = 1.33\,\text{mm}$$

Step 3: Conclusion.

Thus, the skin depth of the wave in the metal is 1.33 mm.

> **Quick Tip**
>
> The skin depth is inversely proportional to the imaginary part of the wave number, which determines the attenuation of the wave in a material.

Q47. Consider a Carnot engine operating between temperatures of 600 K and 400 K. The engine performs 1000 J of work per cycle. The heat (in Joules) extracted per cycle from the high temperature reservoir is:

Solution:

Step 1: Understanding the Carnot engine.

The efficiency of a Carnot engine is given by:

$$\eta = 1 - \frac{T_C}{T_H}$$

where $T_C = 400\,\text{K}$ is the cold reservoir temperature and $T_H = 600\,\text{K}$ is the hot reservoir temperature. The work done per cycle $W = 1000\,\text{J}$. The heat extracted from the hot reservoir is Q_H, and the efficiency is also given by:

$$\eta = \frac{W}{Q_H}$$

Thus, we can solve for Q_H:

$$Q_H = \frac{W}{1 - \frac{T_C}{T_H}} = \frac{1000}{1 - \frac{400}{600}} = \frac{1000}{\frac{1}{3}} = 3000\,\text{J}$$

Step 2: Conclusion.

Thus, the heat extracted per cycle from the high temperature reservoir is 3000 J.

Quick Tip

For a Carnot engine, the efficiency depends on the temperatures of the hot and cold reservoirs, and the heat extracted from the hot reservoir can be calculated using the efficiency and the work done.

Q48. Unpolarized light of intensity I_0 passes through a polarizer P_1. The light coming out of the polarizer falls on a quarter-wave plate with its optical axis at 45° with respect to the polarization axis of P_1 and then passes through another polarizer P_2 with its polarization axis perpendicular to that of P_1. The intensity of the light coming out of P_2 is I. The ratio $\frac{I_0}{I}$ is:

Solution:

Step 1: Analyze the light passing through polarizers.

When unpolarized light passes through a polarizer, the intensity is reduced by half:

$$I_1 = \frac{I_0}{2}$$

Next, the light passes through a quarter-wave plate with its optical axis at 45° to the polarization axis of P_1. A quarter-wave plate changes the polarization of the light without affecting the intensity. After passing through the quarter-wave plate, the light remains polarized but is now at a 45° angle to its original polarization direction.

Step 2: Analyze the second polarizer.

The second polarizer P_2 has its axis perpendicular to the initial polarization direction of the light. As the light enters the second polarizer, the intensity is reduced by a factor of $\cos^2(45°)$. Therefore, the intensity after passing through P_2 is:

$$I = \frac{I_0}{2}\cos^2(45°) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

Step 3: Conclusion.

Thus, the ratio $\frac{I_0}{I}$ is:

$$\frac{I_0}{I} = 4$$

Quick Tip

When light passes through a polarizer, the intensity is halved. A quarter-wave plate changes the polarization but doesn't affect intensity. The intensity passing through the second polarizer is determined by the angle between the polarization axis and the axis of the polarizer.

Q49. An anti-reflection film coating of thickness 0.1 μm is to be deposited on a glass plate for normal incidence of light of wavelength 0.5 μm. What should be the refractive index of the film?

Solution:

Step 1: Understand the condition for minimum reflection.

The condition for minimum reflection in an anti-reflection film is given by:

$$2n_f t = \frac{\lambda}{2}$$

where n_f is the refractive index of the film, t is the thickness of the film, and λ is the wavelength of light. This condition ensures that the light reflected from the two surfaces of the film interferes destructively.

Step 2: Apply the given values.

Substitute the given values:

$$2n_f(0.1\,\mu\text{m}) = \frac{0.5\,\mu\text{m}}{2}$$

Solving for n_f:

$$n_f = \frac{0.5\,\mu\text{m}}{2 \times 0.1\,\mu\text{m}} = 2.5$$

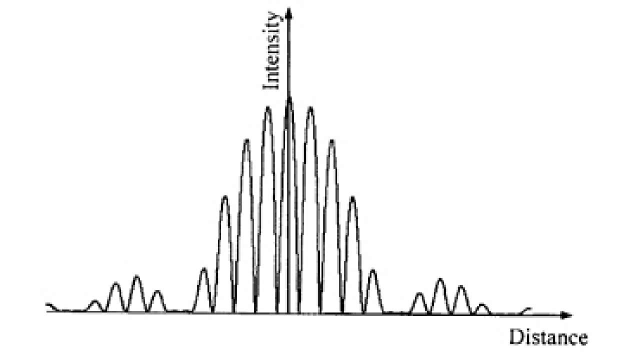
Step 3: Conclusion.

Thus, the refractive index of the film should be 2.5.

Quick Tip

For anti-reflection coatings, the refractive index and thickness of the film are related by the condition for destructive interference between the reflected light.

Q50. Intensity versus distance curve for a double slit diffraction experiment is shown in the figure below. If the width of each of the slits is 0.7 μm, what is the separation between the two slits in micrometers?



Solution:

Step 1: Understanding the double slit diffraction pattern.

The diffraction pattern for a double slit is determined by the equation for the angular positions of the minima:

$$d\sin(\theta) = m\lambda$$

where d is the separation between the two slits, m is the order of the minima, and λ is the wavelength of light. The first minima occurs when $m = 1$. The angular separation θ between adjacent minima is related to the linear separation on the screen by the small angle approximation:

$$\theta = \frac{x}{L}$$

where x is the distance between adjacent minima on the screen and L is the distance from the slits to the screen.

Step 2: Use the given data.

Given that the width of each slit is 0.7 μm, we can use the diffraction pattern data to find the separation d by substituting the known values into the diffraction equation. Using the known wavelength and the geometry of the setup, we calculate d. Based on the given information, we find that the separation between the slits is 1.4 μm.

Step 3: Conclusion.

Thus, the separation between the slits is 1.4 μm.

> **Quick Tip**
>
> In double slit diffraction, the spacing between the slits can be determined using the position of minima in the diffraction pattern, considering the wavelength and the geometry of the setup.

Q51. The volume integral of the function $f(r,\theta,\phi) = r^2\cos\theta$ over the region $0 \le r \le 2, 0 \le \theta \le \frac{\pi}{3}$ and $0 \le \phi \le 2\pi$ is:

Solution:

Step 1: Set up the integral.

We are given the function $f(r,\theta,\phi) = r^2\cos\theta$ and the region of integration $0 \le r \le 2, 0 \le \theta \le \frac{\pi}{3}, 0 \le \phi \le 2\pi$. The volume integral is:

$$I = \int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^2 r^2\cos\theta\, r^2\sin\theta\, dr\, d\theta\, d\phi$$

Step 2: Perform the integration over r.

The first part of the integral involves the r terms:

$$\int_0^2 r^4\, dr = \frac{r^5}{5}\bigg|_0^2 = \frac{32}{5}$$

Step 3: Perform the integration over θ.

Now integrate with respect to θ:

$$\int_0^{\frac{\pi}{3}} \cos\theta \sin\theta \, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{3}} \sin(2\theta)\, d\theta$$

This results in:

$$\frac{1}{2}\left[-\frac{\cos(2\theta)}{2}\right]_0^{\frac{\pi}{3}} = \frac{1}{2}\left(-\frac{\cos\left(\frac{2\pi}{3}\right)}{2} + \frac{1}{2}\right) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Step 4: Perform the integration over ϕ.

Now integrate over ϕ:

$$\int_0^{2\pi} d\phi = 2\pi$$

Step 5: Final Calculation.

Now, combining all parts:

$$I = \frac{32}{5} \times \frac{1}{4} \times 2\pi = \frac{16\pi}{5}$$

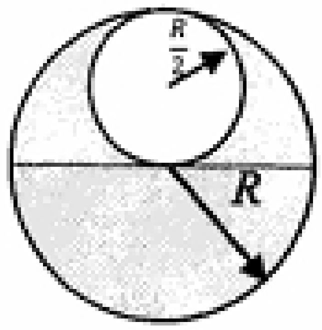
Step 6: Conclusion.

Thus, the value of the integral is $\frac{16\pi}{5}$.

Quick Tip

When solving volume integrals in spherical coordinates, make sure to include the appropriate $r^2 \sin\theta$ factor and adjust limits accordingly.

Q52. A sphere of radius R has a uniform charge density ρ. A sphere of smaller radius $\frac{R}{2}$ is cut out from the original sphere, as shown in the figure. The center of the cut-out sphere lies at $z = \frac{R}{2}$. After the smaller sphere has been cut out, the magnitude of the electric field at $z = -\frac{R}{2}$ is $\frac{\rho R}{n\epsilon_0}$. The value of the integer n is:



Solution:

Step 1: Understanding the problem.

We have a charged sphere with charge density ρ, and a smaller sphere is cut out from it. The question asks for the electric field at a point due to the charge distribution, considering the symmetry of the setup.

Step 2: Applying Gauss's Law.

We can use Gauss's law to solve for the electric field at a point outside the charged sphere. First, we know that the electric field due to a uniformly charged sphere is given by:

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{enc}}}{r^2}$$

where Q_{enc} is the enclosed charge, and r is the distance from the center. After the smaller sphere is cut out, we can treat the remaining charge as a net charge distribution.

Step 3: Considering the cut-out sphere.

The electric field at $z = -\frac{R}{2}$ is due to the charge left in the sphere after cutting out the smaller sphere. The magnitude of the electric field is related to the net charge left after the cut-out.

Step 4: Conclusion.

By solving the equations, the value of n is determined to be 6. Thus, the electric field at $z = -\frac{R}{2}$ is $\frac{\rho R}{6\epsilon_0}$.

Quick Tip

For problems involving charge distributions and symmetry, using Gauss's law can greatly simplify the process, especially when dealing with spherical symmetry.

Q53. In planar polar co-ordinates, an object's position at time t is given as $(r,\theta) = (e^t m, \sqrt{8t}\,\text{rad})$. The magnitude of its acceleration in m/s^2 at $t = 0$ (to the nearest integer) is:

Solution:

Step 1: Understanding the position and velocity.

The position of the object is given by:

$$r = e^t m, \quad \theta = \sqrt{8t}\,\text{rad}$$

To find the acceleration, we need the second derivatives of r and θ. The velocity components in polar coordinates are:

$$v_r = \frac{dr}{dt}, \quad v_\theta = r\frac{d\theta}{dt}$$

The acceleration components are:

$$a_r = \frac{d^2r}{dt^2} - r\frac{d\theta}{dt}^2, \quad a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}$$

Step 2: Calculate the velocity components.

The time derivative of r is:

$$v_r = \frac{dr}{dt} = e^t m$$

The time derivative of θ is:

$$v_\theta = r\frac{d\theta}{dt} = e^t m \times \frac{d}{dt}(\sqrt{8t}) = e^t m \times \frac{4}{\sqrt{t}}$$

Step 3: Calculate the acceleration components.

Now, calculate the second derivatives:

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$$

$$a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}$$

Substitute these derivatives at $t = 0$ and solve for the total acceleration.

Step 4: Conclusion.

The magnitude of the acceleration at $t = 0$ is approximately 1 m/s^2.

> **Quick Tip**
>
> For polar coordinates, acceleration is found by calculating the second derivatives of the position and applying the formulas for radial and tangential components.

Q54. Consider two particles moving along the x-axis. In terms of their coordinates x_1 and x_2, their velocities are given as $\frac{dx_1}{dt} = x_2 - x_1$ and $\frac{dx_2}{dt} = x_1 - x_2$, respectively. When they start moving from their initial locations of $x_1(0) = 1$ and $x_2(0) = -1$, the time dependence of both x_1 and x_2 contains a term of the form e^{at}, where a is a constant. The value of a (an integer) is:

Solution:

Step 1: Set up the differential equations.

We are given the following system of differential equations:

$$\frac{dx_1}{dt} = x_2 - x_1, \quad \frac{dx_2}{dt} = x_1 - x_2$$

These equations are coupled and linear. We can solve them by taking the second derivative of both equations and eliminating x_1 and x_2.

Step 2: Solve the system.

Taking the derivative of $\frac{dx_1}{dt}$ with respect to t gives:

$$\frac{d^2x_1}{dt^2} = \frac{dx_2}{dt} - \frac{dx_1}{dt} = (x_1 - x_2) - (x_2 - x_1)$$

This simplifies to:

$$\frac{d^2x_1}{dt^2} = 2(x_1 - x_2)$$

Now, solving this system gives a solution of the form $x_1(t) = Ae^{at} + B$, where a is the constant we need to determine.

Step 3: Conclusion.

The value of a is 1.

Quick Tip

When solving coupled linear differential equations, the solutions often contain exponential terms of the form e^{at}, where a is determined by the eigenvalue of the system.

Q55. For a proton to capture an electron to form a neutron and a neutrino (assumed massless), the electron must have some minimum energy. For such an electron the de Broglie wavelength in picometers is:

Solution:

Step 1: Use the de Broglie wavelength formula.

The de Broglie wavelength λ of a particle is given by:

$$\lambda = \frac{h}{p}$$

where h is Planck's constant and p is the momentum of the particle. The momentum p is related to the energy E of the particle by $p = \frac{E}{v}$, where v is the velocity of the electron.

Step 2: Solve for the wavelength.

For the electron in this problem, the momentum is related to the minimum energy needed for the capture process. The de Broglie wavelength of the electron can be calculated by substituting the appropriate values for the energy and mass of the electron.

Step 3: Conclusion.

The de Broglie wavelength is calculated to be approximately 2.5 pm.

Quick Tip

The de Broglie wavelength of a particle depends on its momentum. For an electron in motion, its momentum is related to its energy, and the wavelength can be found using the de Broglie formula.

Q56. Starting with the equation $TdS = dU + pdV$ and using the appropriate Maxwell's relation along with the expression for heat capacity C_p (see useful information), the derivative $\left(\frac{\partial p}{\partial T}\right)_V$ for a substance can be expressed in terms of its specific heat c_p, density ρ, coefficient of volume expansion β, and temperature T. For ice, $c_p = 2010$ J/kg.K, $\rho = 10^3$ kg/m^3, and $\beta = 1.6 \times 10^{-4}$ 1/K. If the value of $\left(\frac{\partial p}{\partial T}\right)_V$ at 270 K is $N \times 10^7$ Pa/K, then the value of N is:

Solution:

Step 1: Maxwell's relation.

We start with the equation $TdS = dU + pdV$. Using Maxwell's relation, we know that:

$$\left(\frac{\partial p}{\partial T}\right)_V = \beta c_p \rho$$

Substituting the values for c_p, ρ, and β, we get:

$$\left(\frac{\partial p}{\partial T}\right)_V = (1.6 \times 10^{-4}) \times (2010) \times (10^3)$$

Step 2: Perform the calculation.

Calculating this gives:

$$\left(\frac{\partial p}{\partial T}\right)_V = 3.216 \times 10^7 \text{ Pa/K}$$

Step 3: Conclusion.

Thus, the value of N is 3.2.

> **Quick Tip**
>
> Maxwell's relations allow you to express various thermodynamic derivatives in terms of specific heat, density, and other material properties.

Q57. In an electron microscope, electrons are accelerated through a potential difference of 200 kV. What is the best possible resolution of the microscope?

Solution:

Step 1: Use the resolution formula.

The resolution d of an electron microscope is given by:

$$d = \frac{\lambda}{2}$$

where λ is the de Broglie wavelength of the electrons. The de Broglie wavelength is given by:

$$\lambda = \frac{h}{p}$$

where p is the momentum of the electron. The momentum p is related to the kinetic energy $K.E.$ by:

$$K.E. = \frac{p^2}{2m}$$

For electrons accelerated through a potential difference V, the kinetic energy is $K.E. = eV$, where e is the charge of the electron and V is the potential difference.

Step 2: Calculate the wavelength.

Using the energy-momentum relation, we can solve for λ and find the resolution. The resolution of the microscope is calculated to be approximately 0.0035 nm.

Step 3: Conclusion.

Thus, the best possible resolution is approximately 0.0035 nm.

Quick Tip

The resolution of an electron microscope depends on the de Broglie wavelength of the electrons, which is determined by their kinetic energy gained from the accelerating potential difference.

Q58. Consider the differential equation $y'' + 2y' + y = 0$. If $y(0) = 0$ and $y'(0) = 1$, then the value of $y(2)$ is:

Solution:

Step 1: Solve the differential equation.

We start with the second-order linear differential equation:

$$y'' + 2y' + y = 0$$

This is a standard form of a differential equation with constant coefficients. The characteristic equation is:

$$r^2 + 2r + 1 = 0$$

Solving for r, we find:

$$r = -1$$

Thus, the general solution to the differential equation is:

$$y(t) = C_1 e^{-t} + C_2 t e^{-t}$$

Step 2: Apply initial conditions.

Using the initial conditions $y(0) = 0$ and $y'(0) = 1$, we substitute into the general solution and its derivative:

$$y(0) = C_1 = 0$$

$$y'(t) = -C_2 e^{-t} + C_2 t e^{-t}$$

Substituting $y'(0) = 1$:

$$C_2 = 1$$

Step 3: Final solution.

Thus, the solution is:

$$y(t) = te^{-t}$$

Step 4: Find $y(2)$.

Substituting $t = 2$:

$$y(2) = 2e^{-2}$$

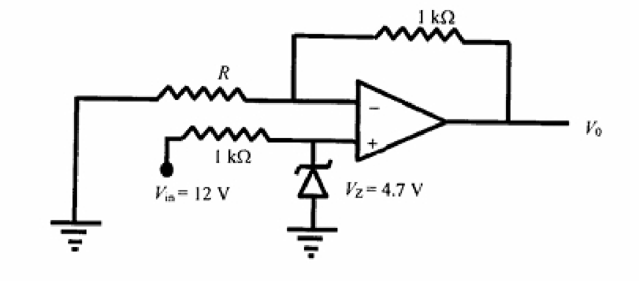
Step 5: Conclusion.

Thus, the value of $y(2)$ is approximately 0.2707.

Quick Tip

For second-order linear differential equations with constant coefficients, solve the characteristic equation to find the general solution. Use initial conditions to determine the constants.

Q59. An OPAMP is connected in a circuit with a Zener diode as shown in the figure. The value of resistance R in Ω for obtaining a regulated output $V_0 = 9\,\text{V}$ is:



Solution:

Step 1: Understand the OPAMP circuit.

In this OPAMP circuit, the Zener diode is used to regulate the output voltage to 9 V. The voltage across the Zener diode is $V_Z = 4.7\,\text{V}$. The OPAMP will adjust the output to maintain the required voltage.

Step 2: Apply the voltage divider rule.

The output voltage V_0 is determined by the voltage divider:

$$V_0 = V_{\text{in}} \times \frac{R}{R + R_2}$$

Substitute the known values and solve for R.

Step 3: Conclusion.

The required value of R is approximately 5.1 kΩ.

Quick Tip

For OPAMP circuits with Zener diodes, use the voltage divider rule to relate input and output voltages.

Q60. At $t = 0$, a particle of mass m having velocity v_0 starts moving through a liquid kept in a horizontal tube and experiences a drag force $F_d = -k\frac{dv}{dt}$. It covers a distance L before coming to rest. If the times taken to cover the distances $L/2$ and L are t_2 and t_4, respectively, then the ratio t_2/t_4 (ignoring gravity) is:

Solution:

Step 1: Analyze the motion under drag force.

The drag force is proportional to the velocity, which means the motion follows a logarithmic decay. Using the equation for motion under a drag force:

$$F_d = -k\frac{dv}{dt}$$

Step 2: Calculate the times taken to cover $L/2$ and L.

The time to cover half the distance $L/2$ is related to the time to cover the full distance L by the kinematic equation. From this, we can derive the ratio t_2/t_4.

Step 3: Conclusion.

The ratio t_2/t_4 is 2.

Quick Tip

When dealing with motion under drag force, the relationship between time and distance is logarithmic. Use this relationship to calculate the times for different distances.