IIT JAM 2018 Mathematical Statistics (MS) Question Paper

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

1. Let {a_n} be a sequence of real numbers such that $a_1 = 2$, and for $n \geq 1$, $a_{n+1} = \frac{2a_n+1}{a_n+1}$.

(A) $1.5 \leq a_n \leq 2$, for all natural numbers $n \geq 1$

(B) There exists a natural number $n \geq 1$ such that $a_n > 2$

(C) There exists a natural number $n \geq 1$ such that $a_n < 1.5$

(D) There exists a natural number $n \geq 1$ such that $a_n = \frac{1+\sqrt{5}}{2}$

---

2. The value of

$$\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^{n^2} e^{-2n} \text{ is}$$

(A) e^{-2}

(B) e^{-1}

(C) e

(D) e^2

---

3. Let {a_n} and {b_n} be two convergent sequences of real numbers. For $n \geq 1$, define $u_n = \max\{a_n, b_n\}$ and $v_n = \min\{a_n, b_n\}$. Then

(A) Neither {a_n} nor {b_n} converges

(B) $\{u_n\}$ converges but $\{v_n\}$ does not converge

(C) $\{u_n\}$ does not converge but $\{v_n\}$ converges

(D) Both $\{u_n\}$ and $\{v_n\}$ converge

---

4. Let

$$M = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}.$$

If I is the 2×2 identity matrix and 0 is the 2×2 zero matrix, then

(A) $20M^2 - 13M + 7I = 0$

(B) $20M^2 - 13M - 7I = 0$

(C) $20M^2 + 13M + 7I = 0$

(D) $20M^2 + 13M - 7I = 0$

---

5. Let X be a random variable with the probability density function

$$f(x) = \begin{cases} \frac{x^p}{\Gamma(p)} e^{-\alpha x} x^{p-1}, & x \geq 0,\, \alpha > 0,\, p > 0, \\ 0, & \text{otherwise.} \end{cases}$$

If $E(X) = 20$ and Var$(X) = 10$, then (α, p) is

(A) (2, 20)

(B) (2, 40)

(C) (4, 20)

(D) (4, 40)

---

6. Let X be a random variable with the distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{4} + \frac{4x - x^2}{8}, & 0 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$$

Then

$$P(X = 0) + P(X = 1.5) + P(X = 2) + P(X \geq 1)$$

equals

(A) $\frac{3}{8}$

(B) $\frac{5}{8}$

(C) $\frac{7}{8}$

(D) 1

---

7. Let X_1, X_2 and X_3 be i.i.d. $U(0,1)$ random variables. Then

$$E\left(\frac{X_1 + X_2}{X_1 + X_2 + X_3}\right)$$

equals

(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) $\frac{3}{4}$

---

8. Let $x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3$ and $x_5 = 0$ be the observed values of a random sample of size 5 from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \frac{\theta}{3}, & x = 0, \\ \frac{2\theta}{3}, & x = 1, \\ \frac{1-\theta}{2}, & x = 2, 3, \end{cases}$$

where $\theta \in [0, 1]$ is the unknown parameter. Then the maximum likelihood estimate of θ is

(A) $\frac{2}{5}$
(B) $\frac{3}{5}$
(C) $\frac{5}{7}$
(D) $\frac{5}{9}$

---

9. Consider four coins labelled as 1, 2, 3 and 4. Suppose that the probability of obtaining a 'head' in a single toss of the ith coin is $\frac{i}{4}$, $i = 1, 2, 3, 4$. A coin is chosen uniformly at random and flipped. Given that the flip resulted in a 'head', the conditional probability that the coin was labelled either 1 or 2 equals

(A) $\frac{1}{10}$
(B) $\frac{2}{10}$
(C) $\frac{3}{10}$
(D) $\frac{4}{10}$

10. Consider the linear regression model

$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad i = 1, 2, \ldots, n,$ where ϵ_i are i.i.d. standard normal random variables. Given that

$$\frac{1}{n}\sum_{i=1}^{n} x_i = 3.2, \quad \frac{1}{n}\sum_{i=1}^{n} y_i = 4.2, \quad \frac{1}{n}\sum_{j=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = 1.5,$$

$$\frac{1}{n}\sum_{j=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)\left(y_j - \frac{1}{n}\sum_{i=1}^{n} y_i\right) = 1.7,$$

the maximum likelihood estimates of β_0 and β_1, respectively, are

(A) $\frac{17}{15}$ and $\frac{32}{75}$

(B) $\frac{32}{75}$ and $\frac{17}{15}$

(C) $\frac{43}{75}$ and $\frac{17}{15}$

(D) $\frac{43}{75}$ and $\frac{5}{9}$

---

11. Let $f : [-1, 1] \to \mathbb{R}$ be defined by

$$f(x) = \frac{x^2 + [\sin(\pi x)]}{1 + |x|},$$ where $[y]$ denotes the greatest integer less than or equal to y.

Then

(A) f is continuous at $-1, 0, 1$

(B) f is discontinuous at $-1, 0, \frac{1}{2}$

(C) f is discontinuous at $-1, \frac{1}{2}, 0, \frac{1}{2}$

(D) f is continuous everywhere except at 0

---

12. Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x^2 - \frac{\cos(x)}{2}, \quad g(x) = \frac{x\sin(x)}{2}.$$

Then

(A) $f(x) = g(x)$ for more than two values of x

(B) $f(x) \neq g(x)$, for all $x \in \mathbb{R}$

(C) $f(x) = g(x)$ for exactly one value of x

(D) $f(x) = g(x)$ for exactly two values of x

---

13. Consider the domain $D = \{(x, y) \in \mathbb{R}^2 : x \leq y\}$ and the function $h : D \to \mathbb{R}$ defined by

$$h((x, y)) = (x - 2)^4 + (y - 1)^4, \quad (x, y) \in D.$$

Then the minimum value of h on D equals

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{1}{16}$

---

14. Let $M = [X\ Y\ Z]$ be an orthogonal matrix with $X, Y, Z \in \mathbb{R}^3$ as its column vectors. Then

$$Q = XX^T + YY^T \quad \text{and} \quad QZ = Z$$

implies

(A) M is a skew-symmetric matrix

(B) M is the 3×3 identity matrix

(C) $Q^2 = Q$

(D) Q satisfies $QZ = Z$

---

15. Let $f : [0, 3] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 0, & 0 \leq x < 1, \\ e^{x^2} - e, & 1 \leq x < 2, \\ e^{x^2} + 1, & 2 \leq x \leq 3. \end{cases}$$

Now, define $F : [0,3] \to \mathbb{R}$ by

$$F(0) = 0 \quad \text{and} \quad F(x) = \int_0^x f(t)\,dt, \text{ for } 0 < x \leq 3.$$

Then

(A) F is differentiable at $x = 1$ and $F'(1) = 0$

(B) F is differentiable at $x = 2$ and $F'(2) = 0$

(C) F is not differentiable at $x = 1$

(D) F is differentiable at $x = 2$ and $F''(2) = 1$

---

16. If x, y, z are real numbers such that

$$4x + 2y + z = 31 \quad \text{and} \quad 2x + 4y - z = 19,$$

then the value of $9x + 7y + z$ is

(A) cannot be computed from the given information

(B) equals $\frac{281}{3}$

(C) equals $\frac{182}{3}$

(D) equals $\frac{218}{3}$

---

17. Let

$$M = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}.$$

If

$$V = \{(x, y, 0) \in \mathbb{R}^3 : M \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\},$$

$$W = \{(x, y, z) \in \mathbb{R}^3 : M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\},$$

then

(A) the dimension of V equals 2

(B) the dimension of V equals 4

(C) the dimension of W equals 1

(D) $V \cap W = \{(0,0,0)\}$

---

18. Let M be a 3×3 non-zero, skew-symmetric real matrix. If I is the 3×3 identity matrix, then

(A) M is invertible

(B) The matrix $I + M$ is invertible

(C) There exists a non-zero real number α such that $\alpha I + M$ is not invertible

(D) All the eigenvalues of M are real

---

19. Let X be a random variable with the moment generating function

$$M_X(t) = \frac{6}{\pi^2} \sum_{n \geq 1} \frac{e^{t^2/n}}{n^2}, \quad t \in \mathbb{R}.$$

Then $P(X \in \mathbb{Q})$, where $\mathbb{Q}$ is the set of rational numbers, equals

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

---

20. Let X be a discrete random variable with the moment generating function

$$M_X(t) = \frac{(1+3e^t)^2(3+e^t)^3}{1024}, \quad t \in \mathbb{R}.$$

Then

(A) $E(X) = \frac{9}{4}$

(B) $\text{Var}(X) = \frac{15}{32}$

(C) $P(X \geq 1) = \frac{27}{1024}$

(D) $P(X = 5) = \frac{3}{1024}$

---

21. Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables with X_n having the probability density function as

$$f_n(x) = \begin{cases} \frac{1}{2n^{1/2}\Gamma\left(\frac{5}{2}\right)} e^{-x^2/2}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\lim_{n\to\infty} \left[P(X_n > \frac{3n}{4}) + P(X_n > n + 2\sqrt{2n})\right]$$

equals

(A) 1 + $\Phi(2)$

(B) 1 - $\Phi(2)$

(C) $\Phi(2)$

(D) 2 - $\Phi(2)$

---

22. Let X be a Poisson random variable with mean $\frac{1}{2}$. Then $E((X+1)!)$ equals

(A) $2e^{-\frac{1}{2}}$

(B) $4e^{-\frac{1}{2}}$

(C) $4e^{-1}$

(D) $2e^{-1}$

---

23. Let X be a standard normal random variable. Then $P(X^3 - 2X^2 - X + 2 > 0)$ equals

(A) $2\Phi(1) - 1$

(B) $1 - \Phi(2)$

(C) $2\Phi(1) - \Phi(2)$

(D) $\Phi(2) - \Phi(1)$

---

24. Let X and Y have the joint probability density function

$$f(x,y) = \begin{cases} 2, & 0 \le x \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $a = E(Y|X = \frac{1}{2})$ and $b = \textbf{Var}(Y|X = \frac{1}{2})$. Then (a, b) is

(A) $\left(\frac{3}{4}, \frac{7}{12}\right)$

(B) $\left(\frac{1}{4}, \frac{7}{12}\right)$

(C) $\left(\frac{3}{4}, \frac{1}{48}\right)$

(D) $\left(\frac{3}{4}, \frac{1}{48}\right)$

---

25. Let X and Y have the joint probability mass function

$$P(X = m, Y = n) = \frac{m+n}{21}, \quad m = 1,2,3; n = 1,2, \quad \text{otherwise.}$$

Then $P(X = 2|Y = 2)$ equals

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

---

26. Let X and Y be two independent standard normal random variables. Then the probability density function of $Z = \frac{|X|}{|Y|}$ is

(A) $f(z) = \frac{1}{\sqrt{\pi}} e^{-z^2/2}, \quad z > 0, \quad 0, \quad \text{otherwise}$

(B) $f(z) = \frac{2}{\sqrt{2\pi}} e^{-z^2/2}, \quad z > 0, \quad 0, \quad \text{otherwise}$

(C) $f(z) = e^{-z}, \quad z > 0, \quad 0, \quad \text{otherwise}$

(D) $f(z) = \frac{2}{\pi(1+z^2)}, \quad z > 0, \quad 0, \quad \text{otherwise}$

27. Let X and Y have the joint probability density function

$$f(x, y) = \begin{cases} e^{-y}, & 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Then the correlation coefficient between X and Y equals

(A) $\frac{1}{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{2}{\sqrt{3}}$

---

28. Let $x_1 = -2, x_2 = 1$ and $x_3 = -1$ be the observed values of a random sample of size three from a discrete distribution with the probability mass function

$$f(x; \theta) = P(X = x) = \begin{cases} \frac{1}{2\theta+1}, & x \in \{-\theta, -\theta + 1, \ldots, 0, \ldots, \theta\}, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \{1, 2, \ldots\}$ is the unknown parameter. Then the method of moment estimate of θ is

(A) 1

(B) 2

(C) 3

(D) 4

---

29. Let X be a random sample from a discrete distribution with the probability mass function

$$f(x; \theta) = P(X = x) = \begin{cases} \frac{1}{\theta}, & x = 1, 2, \ldots, \theta, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \{20, 40\}$ is the unknown parameter. Consider testing

$$H_0 : \theta = 40 \quad \text{against} \quad H_1 : \theta = 20$$

at a level of significance $\alpha = 0.1$. Then the uniformly most powerful test rejects H_0 if and only if

(A) $X \leq 4$

(B) $X > 4$

(C) $X \geq 3$

(D) $X < 3$

---

30. Let X_1 and X_2 be a random sample of size 2 from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \theta, & x = 0, \\ 1-\theta, & x = 1, \end{cases}$$

where $\theta \in \{0.2, 0.4\}$ is the unknown parameter. For testing

$$H_0 : \theta = 0.2 \quad \text{against} \quad H_1 : \theta = 0.4,$$

consider a test with the critical region

$$C = \{(x_1, x_2) \in \{0,1\}^2 : x_1 + x_2 < 2\}.$$

Let α and β denote the probability of Type I error and power of the test, respectively. Then (α, β) is

(A) $(0.36, 0.74)$

(B) $(0.64, 0.36)$

(C) $(0.05, 0.64)$

(D) $(0.05, 0.36)$

---

31. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that

$$a_n = \sum_{k=n+1}^{2n} \frac{1}{k}, \quad n \geq 1.$$

Then which of the following statement(s) is (are) true?

(A) $\{a_n\}_{n\geq 1}$ is an increasing sequence

(B) $\{a_n\}_{n\geq 1}$ is bounded below

(C) $\{a_n\}_{n\geq 1}$ is bounded above

(D) $\{a_n\}_{n\geq 1}$ is a convergent sequence

---

32. Let $\sum_{n\geq 1} a_n$ be a convergent series of positive real numbers. Then which of the following statement(s) is (are) true?

(A) $\sum_{n\geq 1}(a_n)^2$ is always convergent

(B) $\sum_{n\geq 1}\sqrt{a_n}$ is always convergent

(C) $\sum_{n\geq 1}\frac{\sqrt{a_n}}{n}$ is always convergent

(D) $\sum_{n\geq 1}\frac{\sqrt{a_n}}{n^{1/4}}$ is always convergent

---

33. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 3$ and, for $n \geq 1$,

$$a_{n+1} = \frac{a_n^2 - 2a_n + 4}{2}.$$

Then which of the following statement(s) is (are) true?

(A) $\{a_n\}_{n\geq 1}$ is a monotone sequence

(B) $\{a_n\}_{n\geq 1}$ is a bounded sequence

(C) $\{a_n\}_{n\geq 1}$ is convergent

(D) $a_n \to 2$ as $n \to \infty$

---

34. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2(2 + \sin\frac{1}{x}), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then which of the following statement(s) is (are) true?

(A) f attains its minimum at 0

(B) f is monotone

(C) f is differentiable at 0

(D) $f(x) > 2x^4 + x^3,$ for all $x > 0$

---

35. Let P be a probability function that assigns the same weight to each of the points of the sample space $\Omega = \{1, 2, 3, 4\}$. Consider the events

$$E = \{1, 2\}, \quad F = \{1, 3\}, \quad G = \{3, 4\}.$$

Then which of the following statement(s) is (are) true?

(A) E and F are independent

(B) E and G are independent

(C) F and G are independent

(D) E, F, and G are independent

---

36. Let $X_1, X_2, \ldots, X_n$, where $n \geq 5$, be a random sample from a distribution with the probability density function

$$f(x; \theta) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statement(s) is (are) true?

(A) A 95% confidence interval of θ has to be of finite length

(B) $\left(\min(X_1, X_2, \ldots, X_n) + \frac{1}{n}\ln(0.05), \min(X_1, X_2, \ldots, X_n)\right)$ is a 95% confidence interval of θ

(C) A 95% confidence interval of θ can be of length 1

(D) A 95% confidence interval of θ can be of length 2

---

37. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$, where $\theta > 0$ is the unknown parameter. Let

$$X_{(n)} = \max(X_1, X_2, \ldots, X_n).$$

Then which of the following is (are) consistent estimator(s) of θ^3?

(A) $8X_{(n)}^3$

(B) $X_{(n)}^3$

(C) $\frac{2}{n}\sum_{i=1}^{n} X_i^3$

(D) $\frac{nX_{(n)}^3+1}{n+1}$

---

38. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} c(\theta)e^{-x-\theta}, & x \geq \theta, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statement(s) is (are) true?

(A) The maximum likelihood estimator of θ is $\min(X_1, X_2, \ldots, X_n)$

(B) $c(\theta) = 1$, for all $\theta \in \mathbb{R}$

(C) The maximum likelihood estimator of θ is $\min(X_1, X_2, \ldots, X_n)$

(D) The maximum likelihood estimator of θ does not exist

---

39. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \theta^2 x e^{-\theta x}, & x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is the unknown parameter. If $Y = \sum_{i=1}^{n} X_i$, then which of the following statement(s) is (are) true?

(A) Y is a complete sufficient statistic for θ

(B) $\frac{2n}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(C) $\frac{2n-1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(D) $\frac{2n+1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

---

40. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta, \theta+1)$, where $\theta \in \mathbb{R}$ is the unknown parameter. Let

$$U = \max(X_1, X_2, \ldots, X_n) \quad \text{and} \quad V = \min(X_1, X_2, \ldots, X_n).$$

Then which of the following statement(s) is (are) true?

(A) U is a consistent estimator of θ

(B) V is a consistent estimator of θ

(C) $2U - V$ is a consistent estimator of θ

(D) $2U - V$ is a consistent estimator of $\theta + 1$

---

41. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that

$$a_n = \frac{1 + 3 + 5 + \cdots + (2n-1)}{n!}, \quad n \geq 1.$$

Then $\sum_{n\geq 1} a_n$ converges to ....

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42. Let

$$S = \{(x, y) \in \mathbb{R}^2 : x \geq 0, \sqrt{4 - (x-2)^2} \leq y \leq \sqrt{9 - (x-3)^2}\}.$$

Then the area of S equals ....

---

43. Let

$$S = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}.$$

Then the area of S equals ....

44. Let

$$J = \frac{1}{\pi} \int_0^1 t^{-\frac{1}{2}} (1-t)^{\frac{3}{2}} \, dt.$$

Then the value of J equals ....

45. A fair die is rolled three times independently. Given that 6 appeared at least once, the conditional probability that 6 appeared exactly twice equals ....

46. Let X and Y be two positive integer-valued random variables with the joint probability mass function

$$P(X = m, Y = n) = g(m)h(n), \quad m, n \geq 1,$$

where $g(m) = \left(\frac{1}{2}\right)^{m-1}$, $m \geq 1$, and $h(n) = \left(\frac{1}{3}\right)^{n}$, $n \geq 1$. Then $E(XY)$ equals ....

47. Let E, F and G be three events such that

$$P(E \cap F \cap G) = 0.1, \quad P(G \mid F) = 0.3 \quad \text{and} \quad P(E \mid F \cap G) = P(E \mid F).$$

Then $P(G \mid E \cap F)$ equals ....

48. Let A_1, A_2 and A_3 be three events such that

$$P(A_i) = \frac{1}{3}, \quad i = 1, 2, 3; \quad P(A_i \cap A_j) = \frac{1}{6}, \quad 1 \leq i \neq j \leq 3 \quad \text{and} \quad P(A_1 \cap A_2 \cap A_3) = \frac{1}{6}.$$

Then the probability that none of the events A_1, A_2, A_3 occur equals ....

49. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with the probability density function

$$f(x) = \frac{1}{4} e^{-|x-4|} + \frac{1}{4} e^{-|x-6|}, \quad x \in \mathbb{R}.$$

Then $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to ....

---

50. Let $x_1 = 1.1, x_2 = 2.2, x_3 = 3.3$ **be the observed values of a random sample of size three from a distribution with the probability density function**

$$f(x;\theta) = \begin{cases} \frac{1}{\theta}e^{-x/\theta}, & x > 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in \{1, 2, \dots\}$ **is the unknown parameter. Then the maximum likelihood estimate of** θ **equals ....**

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51. Let $f : \mathbb{R} \to \mathbb{R}$ **be a differentiable function such that** f' **is continuous on** $\mathbb{R}$ **with** $f'(3) = 18$**. Define**

$$g_n(x) = n\left(f\left(x + \frac{5}{n}\right) - f\left(x - \frac{2}{n}\right)\right).$$

Then $\lim_{n\to\infty} g_n(3)$ equals ....

---

52. Let $M = \sum_{i=1}^{4} X_i X_i^T$**, where**

$$X_1^T = [1\ \ -1\ 1\ 0], \quad X_2^T = [1\ 1\ 0\ 1], \quad X_3^T = [1\ 3\ 1\ 0] \quad \text{and} \quad X_4^T = [1\ 1\ 1\ 0].$$

Then the rank of M **equals ....**

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53. Let $f : \mathbb{R} \to \mathbb{R}$ **be a differentiable function with**

$$\lim_{x\to\infty} f(x) = \infty \quad \text{and} \quad \lim_{x\to\infty} f'(x) = 2.$$

Then

$$\lim_{x\to\infty}\left(1 + \frac{f(x)}{x^2}\right) \text{ equals ....}$$

---

54. The value of

$$\int_0^{\pi} \left(\int_0^{x} e^{\sin y} \sin x \, dy \right) dx$$

equals ....

---

55. Let X be a random variable with the probability density function

$$f(x) = \begin{cases} 4x^k, & 0 < x < 1, \\ x - \frac{x^2}{2}, & 1 \le x < 2, \\ 0, & \text{otherwise}, \end{cases}$$

where k is a positive integer. Then

$$P\left(\frac{1}{2} \le X \le \frac{3}{2}\right) \text{ equals ....}$$

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56. Let X and Y be two discrete random variables with the joint moment generating function

$$M_{X,Y}(t_1, t_2) = \left(\frac{1}{3}e^{t_1} + \frac{2}{3}\right)^2 \left(\frac{2}{3}e^{t_2} + \frac{1}{3}\right)^3, \quad t_1, t_2 \in \mathbb{R}.$$

Then $P(2X + 3Y > 1)$ equals ....

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57. Let X_1, X_2, X_3 and X_4 be i.i.d. discrete random variables with the probability mass function

$$P(X_1 = n) = \frac{3n-1}{4n}, \quad n = 1, 2, \ldots,$$

Then $P(X_1 + X_2 + X_3 + X_4 = 6)$ equals ....

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58. Let X be a random variable with the probability mass function

$$P(X = n) = \frac{1}{10}, \quad n = 1, 2, \ldots, 10.$$

Then $E(\max\{X, 5\})$ equals ....

59. Let X be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \{2, 3\}$ is the unknown parameter. For testing

$$H_0 : \theta = 2 \quad \text{against} \quad H_1 : \theta = 3,$$

let α and β be the size and power, respectively, of the test that rejects H_0 if and only if $X \geq 3.5$**. Then** $\alpha + \beta$ equals ....

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60. A fair die is rolled four times independently. For $i = 1, 2, 3, 4$, define

$$Y_i = \begin{cases} 1, & \text{if 6 appears in the } i\text{-th throw,} \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(\max\{Y_1, Y_2, Y_3, Y_4\} = 1)$ equals ....