IIT JAM 2018 Mathematics (MA) Question Paper

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

1. Which one of the following is TRUE?

(A) $\mathbb{Z}_n$ is cyclic if and only if n is prime

(B) Every proper subgroup of $\mathbb{Z}_n$ is cyclic

(C) Every proper subgroup of S_4 is cyclic

(D) If every proper subgroup of a group is cyclic, then the group is cyclic

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2. Let $a_n = \frac{b_{n+1}}{b_n}$, where $b_1 = 1$, $b_2 = 1$, and $b_{n+2} = b_n + b_{n+1}$, $n \in \mathbb{N}$. Then $\lim_{n\to\infty} a_n$ is

(A) $\frac{1-\sqrt{5}}{2}$

(B) $\frac{1-\sqrt{3}}{2}$

(C) $\frac{1+\sqrt{3}}{2}$

(D) $\frac{1+\sqrt{5}}{2}$

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3. If $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors in a vector space over $\mathbb{R}$, then which one of the following sets is also linearly independent?

(A) $\{v_1 + v_2 - v_3, 2v_1 + 2v_2 + 3v_3, 5v_1 + 4v_2\}$

(B) $\{v_1 - v_2, v_2 - v_3, v_3 - v_1\}$

(C) $\{v_1 + v_2 - v_3, v_2 + v_3 - v_1, v_3 + v_1 - v_2\}$

(D) $\{v_1 + v_2, v_2 + 2v_3, v_3 + 3v_1\}$

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4. Let a be a positive real number. If f is a continuous and even function defined on the interval $[-a, a]$, then

(A) $\int_0^a f(x)dx$

(B) $2\int_0^a f(x)dx$

(C) $\int_{-a}^a f(x)dx$

(D) $2a\int_0^a \frac{f(x)}{1+x^2}dx$

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5. The tangent plane to the surface $z = \sqrt{x^2 + 3y^2}$ at (1, 1, 2) is given by

(A) $x - 3y + z = 0$

(B) $x + 3y - 2z = 0$

(C) $2x + 4y - 3z = 0$

(D) $3x - 7y + 2z = 0$

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6. In $\mathbb{R}^3$, the cosine of the acute angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z - x^2 - y^2 + 3 = 0$ at the point (2, 1, 2) is

(A) $\frac{8}{5\sqrt{21}}$

(B) $\frac{10}{5\sqrt{21}}$

(C) $\frac{8}{3\sqrt{21}}$

(D) $\frac{10}{3\sqrt{21}}$

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7. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a scalar field, $\vec{v} : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field and let $\vec{a} \in \mathbb{R}^3$ be a constant vector. If $\vec{r}$ represents the position vector $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then which one of the following is FALSE?

(A) curl$(\vec{f}\vec{v}) = \nabla f \times \vec{v} + f\text{curl}(\vec{v})$

(B) div$(\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

(C) curl$(\vec{a} \times \vec{r}) = 2|\vec{a}|\vec{r}$

(D) div $\left(\frac{\vec{r}}{|\vec{r}|^3}\right) = 0, \quad$ for $\quad \vec{r} \neq \vec{0}$

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8. In $\mathbb{R}^2$, the family of trajectories orthogonal to the family of asteroids $x^{2/3} + y^{2/3} = a^{2/3}$ is given by

(A) $x^{4/3} + y^{4/3} = c^{4/3}$

(B) $x^{4/3} - y^{4/3} = c^{4/3}$

(C) $x^{5/3} - y^{5/3} = c^{5/3}$

(D) $x^{2/3} - y^{2/3} = c^{2/3}$

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9. Consider the vector space V over $\mathbb{R}$ of polynomial functions of degree less than or equal to 3 defined on $\mathbb{R}$. Let $T : V \to V$ be defined by $(Tf)(x) = f(x) - xf'(x)$. Then the rank of T is

(A) 1

(B) 2

(C) 3

(D) 4

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10. Let $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ for $n \in \mathbb{N}$. Then which one of the following is TRUE for the sequence $\{s_n\}$?

(A) $\{s_n\}$ converges in $\mathbb{Q}$

(B) $\{s_n\}$ is a Cauchy sequence but does not converge in $\mathbb{Q}$

(C) The subsequence $\{s_k\}_{k=1}^{\infty}$ is convergent in $\mathbb{R}$, only when k is even natural number

(D) $\{s_n\}$ is not a Cauchy sequence

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11. Let $a_n = \begin{cases} 2 + (-1)^n \frac{n}{2^n} & \textbf{if } n \textbf{ is odd} \\ 1 + \frac{n}{2^n} & \textbf{if } n \textbf{ is even}, \end{cases} \quad n \in \mathbb{N}.$

Then which one of the following is TRUE?

(A) $\sup\{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$

(B) $\liminf(a_n) = \limsup(a_n) = \frac{3}{2}$

(C) $\sup\{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$

(D) $\liminf(a_n) = 1$ and $\limsup(a_n) = 3$

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12. Let $a, b, c \in \mathbb{R}$. Which of the following values of a, b, c do NOT result in the convergence of the series

$$\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log n)^c}?$$

(A) $|a| < 1, b \in \mathbb{R}, c \in \mathbb{R}$

(B) $a = 1, b > 1, c \in \mathbb{R}$

(C) $a = 1, b \geq 0, c < 1$

(D) $a = -1, b \geq 0, c > 0$

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13. Let $a_n = n + \frac{1}{n}$, $n \in \mathbb{N}$. Then the sum of the series $\sum_{n=1}^{\infty} (-1)^n n^{n+1} \frac{a_{n+1}}{n!}$ is

(A) $e - 1$

(B) e^{-1}

(C) $1 - e^{-1}$

(D) $1 + e^{-1}$

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14. Let $a_n = \frac{(-1)^n}{\sqrt{1+n}}$ and let $c_n = \sum_{k=0}^{n} a_{n-k} a_k$, where $n \in \mathbb{N} \cup \{0\}$. Then which one of the following is TRUE?

(A) Both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent

(B) $\sum_{n=0}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} c_n$ is not convergent

(C) $\sum_{n=1}^{\infty} c_n$ is convergent but $\sum_{n=0}^{\infty} a_n$ is not convergent

(D) Neither $\sum_{n=0}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} c_n$ is convergent

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15. Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable functions such that f is strictly increasing and g is strictly decreasing. Define $p(x) = f(g(x))$ and $q(x) = g(f(x))$, $\forall x \in \mathbb{R}$. Then, for $t > 0$, the sign of $\int_0^t p'(x) q'(x)\, dx$ is

(A) positive

(B) negative

(C) dependent on t

(D) dependent on f and g

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16. For $x \in \mathbb{R}$, let $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then which one of the following is FALSE?

(A) $\lim_{x \to 0} \frac{f(x)}{x} = 0$

(B) $\lim_{x \to 0} f(x) = 0$

(C) $f(x)$ has infinitely many maxima and minima on the interval $(0, 1)$

(D) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

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17. Let $f(x, y) = \begin{cases} \frac{xy}{(x^2+y^2)^\alpha}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ Then which one of the following is TRUE for f at the point $(0, 0)$?

(A) For $\alpha = \frac{1}{2}$, f is continuous but not differentiable

(B) For $\alpha = 1$, f is continuous and differentiable

(C) For $\alpha = \frac{1}{4}$, f is continuous and differentiable

(D) For $\alpha = \frac{3}{4}$, f is neither continuous nor differentiable

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18. Let $a, b \in \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. If $z = e^u f(v)$, where $u = ax + by$ and $v = ax - by$, then which one of the following is TRUE?

(A) $b^2 z_{xx} - a^2 z_{yy} = 4a^2 b^2 e^u f'(v)$

(B) $b^2 z_{xx} - a^2 z_{yy} = -4e^u f'(v)$

(C) $bz_x + az_y = abz$

(D) $bz_{xx} + az_y = -abz$

19. Consider the region D in the yz-plane bounded by the line $y = \frac{1}{2}$ and the curve $y^2 + z^2 = 1$, where $y \geq 0$. If the region D is revolved about the z-axis in $\mathbb{R}^3$, then the volume of the resulting solid is

(A) $\frac{\pi}{\sqrt{3}}$

(B) $\frac{2\pi}{\sqrt{3}}$

(C) $\pi\sqrt{3}$

(D) $\pi\sqrt{3}$

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20. If $\hat{F}(x, y) = (3x - 8y)\hat{i} + (4y - 6xy)\hat{j}$ for $(x, y) \in \mathbb{R}^2$, then $\oint_C \hat{F} \cdot d\hat{r}$, where C is the boundary of the triangular region bounded by the lines $x = 0$, $y = 0$, and $x + y = 1$ oriented in the anti-clockwise direction, is

(A) $\frac{5}{2}$

(B) 3

(C) 4

(D) 5

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21. Let U, V and W be finite dimensional real vector spaces, $T : U \to V$, $S : V \to W$ and $P : W \to U$ be linear transformations. If range (ST) = nullspace (P), nullspace (ST) = range (P) and rank (T) = rank (S), then which one of the following is TRUE?

(A) nullity of T = nullity of S

(B) dimension of $U \neq$ dimension of W

(C) If dimension of $V = 3$, dimension of $U = 4$, then P is not identically zero

(D) If dimension of $V = 4$, dimension of $U = 3$ and T is one-one, then P is identically zero

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22. Let $y(x)$ be the solution of the differential equation $\frac{dy}{dx} + y = f(x)$, for $x \geq 0, y(0) = 0$, where

$$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases}$$

Then $y(x)$ is

(A) $2(1 - e^{-x})$ when $0 \le x < 1$ and $2(e^{-x} - 1)$ when $x \ge 1$

(B) $2(1 - e^{-x})$ when $0 \le x < 1$ and 0 when $x \ge 1$

(C) $2(1 - e^{-x})$ when $0 \le x < 1$ and $2(1 - e^{-1})e^{-x}$ when $x \ge 1$

(D) $2(1 - e^{-x})$ when $0 \le x < 1$ and $2(1 - e^{-x})$ when $x \ge 1$

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23. An integrating factor of the differential equation

$$\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)\, dx + \frac{1}{4}(x + x^2)^2\, dy = 0$$

is

(A) x^2

(B) $3 \log x$

(C) x^3

(D) $2 \log x$

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24. A particular integral of the differential equation

$$y'' + 3y' + 2y = e^{ex}$$

is

(A) $e^{ex}e^{-x}$

(B) $e^{ex}e^{-2x}$

(C) $e^{ex}e^{2x}$

(D) $e^{ex}e^{x}$

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25. Let G be a group satisfying the property that $f : G \to \mathbb{Z}_{221}$ is a homomorphism implies $f(g) = 0, \forall g \in G$. Then a possible group G is

(A) $\mathbb{Z}_{21}$

(B) $\mathbb{Z}_{51}$

(C) $\mathbb{Z}_{91}$

(D) $\mathbb{Z}_{119}$

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26. Let H be the quotient group $\mathbb{Q}/\mathbb{Z}$. Consider the following statements.
I. Every cyclic subgroup of H is finite.
II. Every finite cyclic group is isomorphic to a subgroup of H.
Which one of the following holds?

(A) I is TRUE but II is FALSE

(B) II is TRUE but I is FALSE

(C) both I and II are TRUE

(D) neither I nor II is TRUE

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27. Let I denote the 4×4 identity matrix. If the roots of the characteristic polynomial of a 4×4 matrix M are $\pm\frac{1+\sqrt{5}}{2}$, then M^8 is

(A) $I + M^2$

(B) $2I + M^2$

(C) $2I + 3M^2$

(D) $3I + 2M^2$

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28. Consider the group $\mathbb{Z}^2 = \{(a, b)|a, b \in \mathbb{Z}\}$ under component-wise addition. Then which of the following is a subgroup of $\mathbb{Z}^2$?

(A) $\{(a, b) \in \mathbb{Z}^2|ab = 0\}$

(B) $\{(a,b) \in \mathbb{Z}^2 | 3a + 2b = 15\}$

(C) $\{(a,b) \in \mathbb{Z}^2 | 7$ divides $ab\}$

(D) $\{(a,b) \in \mathbb{Z}^2 | 2$ divides a and 3 divides $b\}$

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29. Let $f : \mathbb{R} \to \mathbb{R}$ be a function and let I be a bounded open interval in $\mathbb{R}$. Define

$$W(f,I) = \sup\{f(x) | x \in I\} - \inf\{f(x) | x \in I\}$$

Which one of the following is FALSE?

(A) $W(f,J_1) \leq W(f,J_2)$ if $J_1 \subseteq J_2$

(B) If f is a bounded function in I and $J_1 \supseteq J_2 \supseteq \cdots \supseteq J_n$ such that the length of the interval J_n tends to 0 as $n \to \infty$, then $\lim_{n\to\infty} W(f,J_n) = 0$

(C) If f is discontinuous at a point $a \in I$, then $W(f,I) \neq 0$

(D) If f is continuous at a point $a \in I$, then for any given $\epsilon > 0$ there exists an interval $I \subseteq J$ such that $W(f,I) < \epsilon$

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30. For $x > -\frac{1}{2}$, let $f_1(x) = \frac{2x}{1+2x}$, $f_2(x) = \log_e(1+2x)$ and $f_3(x) = 2x$. Then which one of the following is TRUE?

(A) $f_3(x) < f_2(x) < f_1(x)$ for $0 < x < \frac{\sqrt{3}}{2}$

(B) $f_1(x) < f_3(x) < f_2(x)$ for $x > 0$

(C) $f_1(x) + f_2(x) < \frac{f_3(x)}{2}$ for $x > \frac{\sqrt{3}}{2}$

(D) $f_2(x) < f_1(x) < f_3(x)$ for $x > 0$

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31. Let $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ be defined by $f(x) = x + \frac{1}{x^3}$. On which of the following interval(s) is f one-to-one?

(A) $(-\infty, -1)$

(B) $(0, 1)$

(C) $(0, 2)$

(D) $(0, \infty)$

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32. The solution(s) of the differential equation

$$\frac{dy}{dx} = (\sin 2x)y^{1/3} \quad \text{satisfying} \quad y(0) = 0 \quad \text{is (are)}$$

(A) $y(x) = 0$

(B) $y(x) = -\frac{\sqrt{8}}{27}\sin^3 x$

(C) $y(x) = \frac{\sqrt{8}}{27}\sin^3 x$

(D) $y(x) = \frac{\sqrt{8}}{27}\cos^3 x$

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33. Suppose f, g, h are permutations of the set $\{\alpha, \beta, \gamma, \delta\}$, where

f interchanges α and β but fixes γ and δ,

g interchanges β and γ but fixes α and δ,

h interchanges γ and δ but fixes α and β.

Which of the following permutations interchange(s) α and δ but fix(es) β and γ?

(A) $f \circ g \circ h \circ g \circ f$

(B) $g \circ h \circ f \circ h \circ g$

(C) $g \circ f \circ h \circ f \circ g$

(D) $h \circ g \circ f \circ g \circ h$

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34. Let P and Q be two non-empty disjoint subsets of $\mathbb{R}$. Which of the following is (are) FALSE?

(A) If P and Q are compact, then $P \cup Q$ is also compact

(B) If P and Q are not connected, then $P \cup Q$ is also not connected

(C) If the subsequence $\{s_n\}$ converges, then $P \cup Q$ is connected

(D) If $P \cup Q$ are closed, then Q is closed

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35. Let $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ denote the group of non-zero complex numbers under multiplication. Suppose

$$Y_n = \{z \in \mathbb{C} \mid z^n = 1\}, \quad n \in \mathbb{N}.$$

Which of the following is (are) subgroup(s) of $\mathbb{C}^*$?

(A) $\bigcup_{n=1}^{100} Y_n$

(B) $\bigcup_{n=1}^{\infty} Y_n$

(C) $\bigcup_{n=100}^{\infty} Y_n$

(D) $\bigcup_{n=1}^{\infty} Y_n$

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36. Suppose $\alpha, \beta, \gamma \in \mathbb{R}$. Consider the following system of linear equations.

$$x + y + z = \alpha, \quad x + \beta y + z = \gamma, \quad x + y + \alpha z = \beta.$$

If this system has at least one solution, then which of the following statements is (are) TRUE?

(A) If $\alpha = 1$, then $y = 1$

(B) If $\beta = 1$, then $y = \alpha$

(C) If $\beta \neq 1$, then $\alpha = 1$

(D) If $y = 1$, then $\alpha = 1$

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37. Let $m, n \in \mathbb{N}, m < n, P \in M_{m\times n}(\mathbb{R}), Q \in M_{n\times n}(\mathbb{R})$. Which of the following is (are) NOT possible?

(A) rank$(PQ) = m$

(B) rank$(P) = n$

(C) rank$(P) = n$

(D) rank$(PQ) = \frac{m+n}{2}$

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38. If $\mathbf{F}(x,y,z) = (2x+3yz)\hat{\imath} + (3xz+2y)\hat{\jmath} + (3xy+2z)\hat{k}$ for $(x,y,z) \in \mathbb{R}^3$, then which among the following is (are) TRUE?

(A) $\nabla \times \mathbf{F} = \mathbf{0}$

(B) $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ along any simple closed curve C

(C) There exists a scalar function $\varphi : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \cdot \mathbf{F} = \varphi_{xx} + \varphi_{yy} + \varphi_{zz}$

(D) $\nabla \cdot \mathbf{F} = 0$

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39. Which of the following subsets of $\mathbb{R}$ is (are) connected?

(A) $\{x \in \mathbb{R} | x^2 + x > 4\}$

(B) $\{x \in \mathbb{R} | x^2 + x < 4\}$

(C) $\{x \in \mathbb{R} | |x| < |x-4|\}$

(D) $\{x \in \mathbb{R} | |x| > |x-4|\}$

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40. Let S be a subset of $\mathbb{R}$ such that 2018 is an interior point of S. Which of the following is (are) TRUE?

(A) S contains an interval

(B) There is a sequence in S which does not converge to 2018

(C) There is an element $y \in S$, $y \neq 2018$, such that y is also an interior point of S

(D) There is a point $z \in S$, such that $|z - 2018| = 0.002018$

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41. The order of the element $(1\ 2\ 3)(2\ 4\ 5)(4\ 5\ 6)$ in the group S_6 is ....

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42. Let $\varphi(x,y,z) = 3y^2 + 3xyz$ for $(x,y,z) \in \mathbb{R}^3$. Then the absolute value of the directional derivative of φ in the direction of the line

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{-2}$$

at the point $(1,-2,1)$ **is ....**

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43. Let $f(x)=\sum_{n=0}^{\infty}(-1)^n x(x-1)^n$ **for** $0<x<2$**. Then the value of** $f\left(\frac{\pi}{4}\right)$ **is ....**

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44. Let $f:\mathbb{R}^2\to\mathbb{R}$ **be given by**

$$f(x,y)=\begin{cases}\frac{x^2y(x-y)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Then

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}f\right)-\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}f\right) \text{ at the point } (0,0) \text{ is ....}$$

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45. Let $f(x,y)=\sqrt{3}x^3y\sin\left(\frac{\pi}{2}e^{(x-1)}\right)+xy\cos\left(\frac{\pi}{3}e^{(y-1)}\right)$ **for** $(x,y)\in\mathbb{R}^2, x>0, y>0$**.**
Then $f_x(1,1)+f_y(1,1)=$ ....

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46. Let $f:[0,\infty)\to[0,\infty)$ **be continuous on** $[0,\infty)$ **and differentiable on** $(0,\infty)$**. If**

$$f(x)=\int_0^x\sqrt{f(t)}\,dt, \text{ then } f(6)= \text{ ....}$$

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47. Let

$$a_n=\frac{(1+(-1)^n)}{2n}+\frac{(1+(-1)^{n-1})}{3n}.$$

Then the radius of convergence of the power series

$$\sum_{n=1}^{\infty}a_nx^n \text{ about } x=0 \text{ is ....}$$

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48. Let A_6 be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in A_6 is ....

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49. Let W_1 be the real vector space of all 5×2 matrices such that the sum of the entries in each row is zero. Let W_2 be the real vector space of all 5×2 matrices such that the sum of the entries in each column is zero. Then the dimension of the space $W_1 \cap W_2$ is ....

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50. The coefficient of x^4 in the power series expansion of $e^{\sin x}$ about $x = 0$ is .... (correct up to three decimal places).

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51. Let $a_k = (-1)^{k-1}$, $s_n = a_1 + a_2 + \cdots + a_n$, and $\sigma_n = \frac{1}{n}(s_1 + s_2 + \cdots + s_n)$, where $k, n \in \mathbb{N}$. Then

$$\lim_{n\to\infty} \sigma_n = \text{................. (correct up to one decimal place).}$$

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52. Let $f : \mathbb{R} \to \mathbb{R}$ be such that f'' is continuous on $\mathbb{R}$ and $f(0) = 1$, $f'(0) = 0$, and $f''(0) = -1$. Then

$$\lim_{x\to\infty} \left(f\left(\sqrt{\frac{2}{x}} \right) \right)^x \text{ is .... (correct up to three decimal places).}$$

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53. Suppose x, y, z are positive real numbers such that $x + 2y + 3z = 1$. If M is the maximum value of xyz^2, then the value of $\frac{1}{M}$ is ....

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54. If the volume of the solid in $\mathbb{R}^3$ bounded by the surfaces

$$x=-1,\ x=1,\ y=-1,\ y=1,\ z=2,\ y^2+z^2=2$$

is $\alpha-\pi$, then

$$\alpha = \text{..........}$$

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55. If

$$a=\int_{\pi/3}^{\pi/6}\frac{\sin t+\cos t}{\sqrt{\sin 2t}}\,dt,$$

then the value of

$$\left(2\sin\frac{\alpha}{2}+1\right)^2 \text{ is ....}$$

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56. The value of the integral

$$\int_0^1\int_0^1 ye^{xy^2}\,dy\,dx \text{ is .... (correct up to three decimal places).}$$

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57. Suppose $Q\in M_{3\times3}(\mathbb{R})$ is a matrix of rank 2. Let $T: M_{3\times3}(\mathbb{R})\to M_{3\times3}(\mathbb{R})$ be the linear transformation defined by

$$T(P)=QP.$$

Then the rank of T is ....

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58. The area of the parametrized surface

$$S=\left\{((2+\cos u)\cos v,(2+\cos u)\sin v,\sin u)\in\mathbb{R}^3 \mid 0\le u\le\frac{\pi}{2}, 0\le v\le\frac{\pi}{2}\right\}$$

is .... (correct up to two decimal places).

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59. If $x(t)$ is the solution to the differential equation

$$\frac{dx}{dt} = xt^3 + xt, \text{ for } t > 0, \text{ satisfying } x(0) = 1,$$

then the value of $x(\sqrt{2})$ is .... (correct up to two decimal places).

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60. If $y(x) = v(x)\sec x$ is the solution of

$$y'' - (2\tan x)y' + 5y = 0, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}, \text{ satisfying } y(0) = 0 \text{ and } y'(0) = \sqrt{6},$$

then $v\left(\frac{\pi}{6\sqrt{6}}\right)$ is .... (correct up to two decimal places).
