## IIT JAM 2018 Physics (PH) Question Paper

**Time Allowed :**3 Hours | **Maximum Marks :**100 | **Total questions :**60

### **General Instructions**

#### **General Instructions:**

- i) All questions are compulsory. Marks allotted to each question are indicated in the margin.
- ii) Answers must be precise and to the point.
- iii) In numerical questions, all steps of calculation should be shown clearly.
- iv) Use of non-programmable scientific calculators is permitted.
- v) Wherever necessary, write balanced chemical equations with proper symbols and units.
- vi) Rough work should be done only in the space provided in the question paper.

1. Let  $f(x,y) = x^3 - 2y^3$ . The curve along which  $\nabla^2 f = 0$  is

- $(\mathbf{A}) \ x = \frac{\sqrt{2}}{2} y$
- **(B)** x = 2y
- (C)  $x = \sqrt{6}y$
- (D)  $x = -\frac{y}{2}$

**2.** A curve is given by  $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ . The unit vector of the tangent at t = 1 is

- (A)  $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$
- (B)  $\frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{6}}$ (C)  $\frac{t\hat{i} + 2t^2\hat{j} + 2t\hat{k}}{3}$ (D)  $\frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$

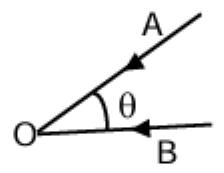
3. Three planets orbit a star at distances a, 4a, 9a. Their orbital periods are proportional to  $r^{3/2}$ . If the smallest planet has period T, after how long will all three be aligned again?

- (A) 8T
- (B) 27T
- (C) 216T
- **(D)** 512*T*

4. A current I flows through the sides of an equilateral triangle of side a. The magnetic field at the centroid is

- (A)  $\frac{9\mu_0 I}{2\pi a}$ (B)  $\frac{\mu_0 I}{\pi a}$

- 5. Two vehicles A and B are approaching an observer O at rest with equal speed as shown in the figure. Both vehicles have identical sirens blowing at a frequency  $f_s$ . The observer hears these sirens at frequency  $f_A$  and  $f_B$ , respectively. Which one of the following is correct?



- (A)  $f_A = f_B < f_s$
- (B)  $f_A = f_B > f_s$
- (C)  $f_A > f_B > f_s$
- (D)  $f_A < f_B < f_s$
- 6. Three infinite plane sheets carrying uniform charge densities  $-\sigma, \, 2\sigma, \, 3\sigma$  are placed parallel to the xz-plane at y = a, 3a, 4a, respectively. The electric field at the point (0, 2a, 0) is
- (A)  $\frac{4\sigma}{\varepsilon_0} \hat{y}$ (B)  $\frac{3\sigma}{\varepsilon_0} \hat{y}$ (C)  $\frac{2\sigma}{\varepsilon_0} \hat{y}$ (D)  $\frac{\sigma}{\varepsilon_0} \hat{y}$

7. Two boxes A and B contain an equal number of molecules of the same gas. If the volumes are  $V_A$  and  $V_B$ , and  $\lambda_A$  and  $\lambda_B$  denote respective mean free paths, then

(A) 
$$\lambda_A = \lambda_B$$

$$(B) \frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$$

(B) 
$$\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$$
  
(C)  $\frac{\lambda_A}{V_A^{1/3}} = \frac{\lambda_B}{V_B^{1/3}}$ 

(D) 
$$\lambda_A^A V_A = \lambda_B^B V_B$$

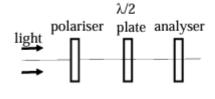
8. Let  $T_g$  and  $T_e$  be the kinetic energies of the electron in the ground and the third excited states of a hydrogen atom. According to the Bohr model, the ratio  $T_g/T_e$  is

- (A) 3
- (B)4
- (C)9
- (D) 16

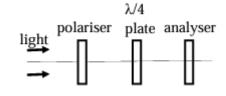
9. Which one of the following arrangements of optical components can be used to distinguish between unpolarised light and circularly polarised light?

(D)

(A)  $\lambda/2$ plate analyser

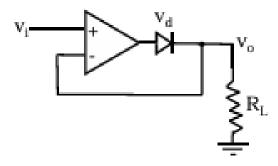


(C)  $\lambda/4$ plate analyser

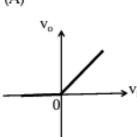


- (A) (diagram with  $\lambda/2$  plate and analyser)
- (B) (diagram with polariser,  $\lambda/2$  plate, analyser)
- (C) (diagram with  $\lambda/4$  plate and analyser)

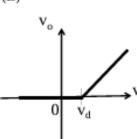
10. Which one of the following graphs shows the correct variation of  $v_o$  with  $v_i$ ? Here,  $v_d$  is the voltage drop across the diode and the Op-Amp is assumed to be ideal.



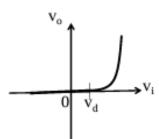
(A)



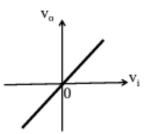
(B)



(C)



(D)



11. Which one of the figures correctly represents the T-S diagram of a Carnot engine?

(A)
T

(B) <sup>T</sup>↑



(C)



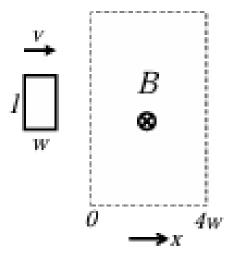
(D

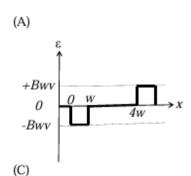


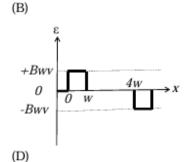
12. The plane of polarisation of a plane-polarised light rotates by  $60^{\circ}$  after passing through a wave plate. The pass-axis of the wave plate is at an angle  $\alpha$  with respect to the plane of polarisation of the incident light. The wave plate and  $\alpha$  are

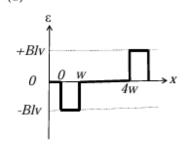
- (A)  $\lambda/4$ , 60°
- **(B)**  $\lambda/2, 30^{\circ}$
- (C)  $\lambda/2$ ,  $120^{\circ}$
- (D)  $\lambda/4$ , 30°

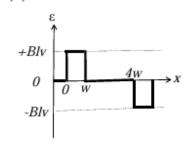
13. A rectangular loop of dimensions l and w moves with a constant speed v through a region containing a uniform magnetic field B directed into the paper and extending a distance of 4w. Which of the following figures correctly represents the variation of emf  $(\varepsilon)$  with the position (x) of the front end of the loop?











14. The equation of state for one mole of a non-ideal gas is given by  $PV = A\left(1 + \frac{B}{V}\right)$ , where the coefficients A and B are temperature dependent. If the volume changes from  $V_1$  to  $V_2$  in an isothermal process, the work done by the gas is

(A) 
$$AB\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

**(B)** 
$$AB \ln \left( \frac{V_2}{V_1} \right)$$

(C) 
$$A \ln \left(\frac{V_2}{V_1}\right) + AB \left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

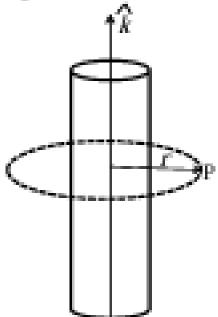
(D) 
$$A \ln \left( \frac{V_2 - V_1}{V_1} \right) + B$$

15. An ideal gas consists of three dimensional polyatomic molecules. The temperature is such that only one vibrational mode is excited. If R denotes the gas constant, then the specific heat at constant volume of one mole of the gas at this temperature is

- (A) 3R
- **(B)**  $\frac{7}{2}R$
- (C) 4R

(D)  $\frac{9}{2}R$ 

16. A long solenoid is carrying a time dependent current such that the magnetic field inside has the form  $\vec{B}(t) = B_0 t^2 \hat{k}$ , where  $\hat{k}$  is along the axis of the solenoid. The displacement current at the point P on a circle of radius r in a plane perpendicular to the axis



- (A) is inversely proportional to r and radially outward.
- (B) is inversely proportional to  $\boldsymbol{r}$  and tangential.
- (C) increases linearly with time and is tangential.
- (D) is inversely proportional to  $r^2$  and tangential.
- 17. Consider an ensemble of thermodynamic systems, each of which is characterized by the same number of particles, pressure and temperature. The thermodynamic function describing the ensemble is
- (A) Enthalpy
- (B) Helmholtz free energy

- (C) Gibbs free energy
- (D) Entropy

18. Given a spherically symmetric charge density  $\rho(r)=\begin{cases} kr^2, & r< R\\ 0, & r> R \end{cases}$  (k being a constant), the electric field for r < R is (take the total charge as Q)

- (A)  $\frac{Qr^3}{4\pi\epsilon_0 R^5} \hat{r}$ (B)  $\frac{3Qr^2}{4\pi\epsilon_0 R^3} \hat{r}$ (C)  $\frac{5Qr^3}{8\pi\epsilon_0 R^5} \hat{r}$ (D)  $\frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$

19. An infinitely long solenoid, with its axis along  $\hat{k}$ , carries a current I. In addition, there is a uniform line charge density  $\lambda$  along the axis. If  $\vec{S}$  is the energy flux, in cylindrical coordinates  $(\hat{\rho},\hat{\phi},\hat{k})$ , then

- (A)  $\vec{S}$  is along  $\hat{\rho}$
- (B)  $\vec{S}$  is along  $\hat{k}$
- (C)  $\vec{S}$  has non zero components along  $\hat{\rho}$  and  $\hat{k}$
- (D)  $\vec{S}$  is along  $\hat{\rho} \times \hat{k}$

**20.** Consider two waves  $y_1 = a\cos(\omega t - kz)$  and  $y_2 = a\cos[(\omega + \Delta\omega)t - (k + \Delta k)z]$ . The group velocity of the superposed wave will be  $(\Delta \omega \ll \omega \text{ and } \Delta k \ll k)$ 

21. Consider a convex lens of focal length f. A point object moves towards the lens along its axis between 2f and f. If the speed of the object is  $V_o$ , then its image would move with speed  $V_i$ . Which of the following is correct?

- (A)  $V_i = V_o$ ; the image moves away from the lens.
- (B)  $V_i = -V_o$ ; the image moves towards the lens.
- (C)  $V_i > V_o$ ; the image moves away from the lens.
- (D)  $V_i < V_o$ ; the image moves away from the lens.

22. A disc of radius  $R_1$  having uniform surface density has a concentric hole of radius  $R_2 < R_1$ . If its mass is M, the principal moments of inertia are

(A) 
$$\frac{M(R_1^2 - R_2^2)}{2}$$
,  $\frac{M(R_1^2 - R_2^2)}{4}$ ,  $\frac{M(R_1^2 - R_2^2)}{4}$ 

(B) 
$$\frac{M(R_1^2 + R_2^2)}{2}$$
,  $\frac{M(R_1^2 + R_2^2)}{4}$ ,  $\frac{M(R_1^2 + R_2^2)}{4}$ 

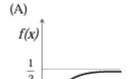
(C) 
$$\frac{M(R_1^2 + R_2^2)}{2}$$
,  $\frac{M(R_1^2 + R_2^2)}{4}$ ,  $\frac{M(R_1^2 + R_2^2)}{8}$ 

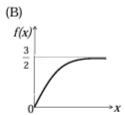
$$\begin{aligned} &\text{(A)} \ \frac{M(R_1^2-R_2^2)}{2}, \ \frac{M(R_1^2-R_2^2)}{4}, \ \frac{M(R_1^2-R_2^2)}{4} \\ &\text{(B)} \ \frac{M(R_1^2+R_2^2)}{2}, \ \frac{M(R_1^2+R_2^2)}{4}, \ \frac{M(R_1^2+R_2^2)}{4} \\ &\text{(C)} \ \frac{M(R_1^2+R_2^2)}{2}, \ \frac{M(R_1^2+R_2^2)}{4}, \ \frac{M(R_1^2+R_2^2)}{8} \\ &\text{(D)} \ \frac{M(R_1^2-R_2^2)}{2}, \ \frac{M(R_1^2-R_2^2)}{4}, \ \frac{M(R_1^2-R_2^2)}{8} \end{aligned}$$

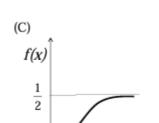
23. The function  $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$  is expanded as a Fourier series of the form  $a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$ . Which of the following is true?

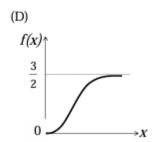
- (A)  $a_0 \neq 0, b_n = 0$
- (B)  $a_0 \neq 0, b_n \neq 0$
- (C)  $a_0 = 0, b_n = 0$
- (D)  $a_0 = 0, b_n \neq 0$

24. Which one of the following curves correctly represents (schematically) the solution for the equation  $\frac{df}{dx} + 2f = 3$ ; f(0) = 0?









25. The mean momentum  $\bar{p}$  of a nucleon in a nucleus of mass number A and atomic number Z depends on A,Z as

(A) 
$$\bar{p} \propto A^{1/3}$$

(B) 
$$\bar{p} \propto Z^{1/3}$$

(C) 
$$\bar{p} \propto A^{-1/3}$$

(D) 
$$\bar{p} \propto (AZ)^{-2/3}$$

26. The Boolean expression  $(AB)(\bar{A}+B)(A+\bar{B})$  can be simplified to

$$(A) A + B$$

(B) 
$$\bar{A}B$$

(C) 
$$A + \bar{B}$$

27. Consider the transformation to a new set of coordinates  $(\xi, \eta)$  from rectangular coordinates (x, y), where  $\xi = 2x + 3y$  and  $\eta = 3x - 2y$ . In the  $(\xi, \eta)$  coordinate system, the area element  $dx \, dy$  is

(A) 
$$\frac{1}{13} d\xi d\eta$$
(B) 
$$\frac{2}{13} d\xi d\eta$$

(B) 
$$\frac{2}{13} d\xi d\eta$$

(C) 
$$5 d\xi d\eta$$

(D) 
$$\frac{3}{5} d\xi d\eta$$

28. A particle of mass m is in a one-dimensional potential  $V(x) = \begin{cases} 0, \ 0 < x < L \\ \infty, \ \text{otherwise} \end{cases}$  . At some instant its wave function is given by  $\psi(x) = \frac{1}{\sqrt{3}}\psi_1(x) + i\sqrt{\frac{2}{3}}\psi_2(x)$ , where  $\psi_1, \psi_2$  are the ground and first excited states. Identify the correct statement.

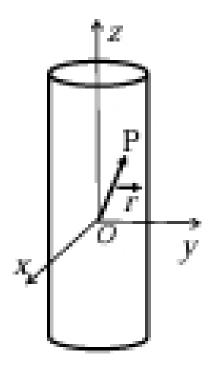
(A) 
$$\langle x \rangle = \frac{L}{2}$$
,  $\langle E \rangle = \frac{h^2}{2m} \frac{3\pi^2}{L^2}$   
(B)  $\langle x \rangle = \frac{2L}{3}$ ,  $\langle E \rangle = \frac{h^2}{2m} \frac{\pi^2}{L^2}$   
(C)  $\langle x \rangle = \frac{L}{2}$ ,  $\langle E \rangle = \frac{h^2}{2m} \frac{8\pi^2}{L^2}$   
(D)  $\langle x \rangle = \frac{2L}{3}$ ,  $\langle E \rangle = \frac{h^2}{2m} \frac{4\pi^2}{3L^2}$ 

(B) 
$$\langle x \rangle = \frac{2L}{3}, \quad \langle E \rangle = \frac{h^2}{2m} \frac{\pi^2}{L_c^2}$$

(C) 
$$\langle x \rangle = \frac{L}{2}$$
,  $\langle E \rangle = \frac{h^2}{2m} \frac{8\pi^2}{L^2}$ 

(D) 
$$\langle x \rangle = \frac{2L}{3}$$
,  $\langle E \rangle = \frac{h^2}{2m} \frac{4\pi^2}{3L^2}$ 

- 29. A raindrop falls under gravity and captures water molecules from the atmosphere. Its mass changes at the rate  $\lambda m(t)$ , where  $\lambda$  is a positive constant and m(t) is the instantaneous mass. Assume gravity is constant and captured water is at rest before capture. Which of the following statements is correct?
- (A) The speed of the raindrop increases linearly with time.
- (B) The speed of the raindrop increases exponentially with time.
- (C) The speed of the raindrop approaches a constant value when  $\lambda t \gg 1$ .
- (D) The speed of the raindrop approaches a constant value when  $\lambda t \ll 1$ .
- 30. A particle P of mass m is constrained to move on the surface of a cylinder under a force  $-k\vec{r}$  as shown in the figure (k is a positive constant). Neglect friction. Which of the following statements is correct?



- (A) Total energy of the particle is not conserved.
- (B) The motion along z direction is simple harmonic.
- (C) Angular momentum of the particle about O increases with time.
- (D) Linear momentum of the particle is conserved.

31. Let matrix 
$$M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$$
. If  $\det(M) = 0$ , then

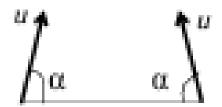
- (A) M is symmetric.
- (B) M is invertible.
- (C) One eigenvalue is 13.
- (D) Its eigenvectors are orthogonal.

# 32. Let $f(x) = 3x^6 - 2x^2 - 8$ . Which of the following statements is (are) true?

(A) The sum of all its roots is zero.

- (B) The product of its roots is  $-\frac{8}{3}$ .
- (C) The sum of all its roots is  $\frac{2}{3}$ .
- (D) Complex roots are conjugates of each other.

33. Two projectiles of identical mass are projected with same initial speed u and angle  $\alpha$  in the same plane. They collide at the highest point of their trajectories and stick to each other. Which statement is correct?



- (A) The momentum of the combined object immediately after collision is zero.
- (B) Kinetic energy is conserved.
- (C) The combined object moves vertically downward.
- (D) The combined object moves in a parabolic path.

34. Two beams of visible light (400–700 nm) interfere at a point. The optical path difference is 5000 nm. Which wavelength interferes constructively?

- (A) 416.67 nm
- (B) 555.55 nm
- (C) 625 nm
- (D) 666.66 nm

35. Which of the following relations is (are) true for thermodynamic variables?

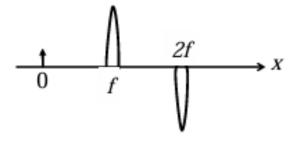
(A) 
$$TdS = C_V dT + T \left(\frac{\partial P}{\partial T}\right)_V dV$$

(B)  $TdS = C_P dT - T \left(\frac{\partial V}{\partial T}\right)_P dP$ (C) dF = -SdT + PdV

(C) 
$$dF = -SdT + PdV$$

(D) 
$$dG = -SdT + VdP$$

36. Consider a convex lens of focal length f. The lens is cut along a diameter into two parts. The two lens parts and an object are kept as shown in the figure. The images are formed at the following distances from the object:



- (A) 2f
- **(B)** 3f
- (C) 4f
- (D)  $\infty$

37. Let the electric field in some region R be given by  $\vec{E} = e^{-y^2}\hat{i} + e^{-x^2}\hat{j}$ . From this we conclude that

- (A) R has a non-uniform charge distribution.
- (B) R has no charge distribution.
- (C) R has a time-dependent magnetic field.
- (D) The energy flux in R is zero everywhere.

38. In presence of magnetic field  $\vec{B}\hat{j}$  and electric field  $(-E)\hat{k}$ , a particle moves undeflected. Which statement is correct?

(A) The particle has positive charge, velocity =  $-\frac{E}{B}\hat{i}$ 

- (B) The particle has positive charge, velocity =  $\frac{E}{B}\hat{i}$
- (C) The particle has negative charge, velocity =  $-\frac{E}{B}\hat{i}$
- (D) The particle has negative charge, velocity =  $\frac{E}{B}\hat{i}$

# 39. In a pn junction, dopant concentration on p-side is higher than n-side. Which statements are correct when the junction is unbiased?

- (A) The width of the depletion layer is larger on the n-side.
- (B) Fermi energy is higher on the p-side.
- (C) Negative charge per unit area on p-side equals positive charge per unit area on n-side.
- (D) Built-in potential depends on dopant concentration.

### 40. Which of the combinations of crystal structure and coordination number is correct?

- (A) body-centered cubic 8
- (B) face-centered cubic 6
- (C) diamond -4
- (D) hexagonal closed packed 12

41. The coefficient of  $x^3$  in the Taylor expansion of  $\sin(\sin x)$  around x=0 is ................. (Specify your answer up to two digits after the decimal point.)

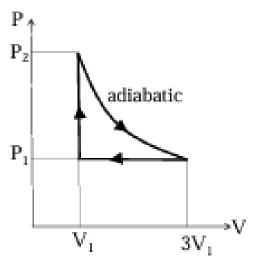
42. A particle of mass m moves in the positive x direction under the potential  $V(x)=\frac{1}{2}kx^2+\frac{\lambda}{2x^2}$ . If the particle is slightly displaced from equilibrium, the angular frequency  $\omega$  is ....... (Give answer in units of  $\sqrt{k/m}$  as an integer.)

43. A planet has the same average density as Earth but only 1/8 the mass of Earth. If  $g_p$  and  $g_e$  are the surface gravities on the planet and Earth, then  $\frac{g_p}{g_e} =$  ...... (Specify your answer up to one digit after decimal.)

44. In a grating with grating constant d=a+b, where a is the slit width and b is the separation between the slits, the diffraction pattern has the fourth order missing. The value of  $\frac{b}{a}$  is ....... (Specify your answer as an integer.)

45. Consider an electromagnetic plane wave  $\vec{E} = E_0(\hat{t} + b\hat{y})\cos\left[\frac{2\pi}{\lambda}(ct - (x - \sqrt{3}y))\right]$ , where  $\lambda$  is the wavelength and c is the speed of light. The value of b is ...... (Specify answer up to two digits after the decimal point.)

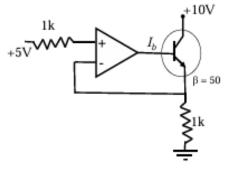
46. A monoatomic ideal gas undergoes a closed cycle shown in the P–V diagram. The ratio  $\frac{P_2}{P_1}$  is ...... (Specify your answer up to two digits after the decimal point.)



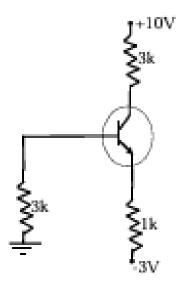
47. Using the sublimation relation  $\log_{10}(P) = \frac{C_1}{T} + C_2$  for zinc, with  $C_1 = 6790$  K,  $C_2 = 9$ , compute the latent heat of sublimation using the Clausius–Clapeyron equation. (Specify answer in kJ/mol up to one digit after decimal.)

48. A system of 8 non-interacting electrons is confined by a 3-dimensional potential  $V(r)=\frac{1}{2}m\omega^2r^2$ . The ground state energy of the system in units of  $\hbar\omega$  is ...... (Specify your answer as an integer.)

49. For the given circuit, the value of base current  $I_b$  of the npn transistor is .......... mA. ( $\beta$  is the current gain and assume the Op-Amp as ideal.) (Specify your answer in mA up to two digits after decimal.)



**50.** The lattice constant of NaCl crystal is 0.563 nm. X-rays of wavelength 0.141 nm are diffracted by this crystal. The angle at which the first order maximum occurs is ........ degrees. (Specify answer up to two digits after decimal.)



52. A body of mass 1 kg moves in an elliptical orbit with semi-major axis 1000 m and semi-minor axis 100 m. The orbital angular momentum is 100 kg  $m^2$  s<sup>-1</sup>. The time period of motion is ...... hours. (Specify answer up to two digits after the decimal point.)

53. The moon moves around the earth in a circular orbit with a period of 27 days. If  $R = 6.4 \times 10^6$  m is Earth's radius and g = 9.8 m/s<sup>2</sup>, and if D is the moon's orbital radius, find D/R. (Specify answer up to one digit after decimal point.)

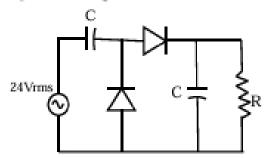
 pressure =  $10^5$  Pa, density of water =  $10^3$  kg m $^{-3}$ .) (Specify your answer in m s $^{-1}$  as an integer.)

56. A particle of mass m moves in a circular orbit with  $x=R\cos(\omega t)$  and  $y=R\sin(\omega t)$  observed in inertial frame  $S_1$ . Another frame  $S_2$  moves with velocity  $\vec{v}=\omega R\hat{i}$  with respect to  $S_1$ , and origins coincide at t=0. The angular momentum at  $t=\frac{2\pi}{\omega}$  as observed in  $S_2$  about its origin is  $(mR^2\omega)x$ . Then x is ................. (Specify answer up to two digits after decimal.)

57. Rod  $R_1$  has rest length 1 m and rod  $R_2$  has rest length 2 m.  $R_1$  and  $R_2$  move with velocities  $+v\hat{i}$  and  $-v\hat{i}$  respectively relative to the lab. If  $R_2$  has a length of 1 m in the rest frame of  $R_1$ ,  $\frac{v}{c}$  is .................... (Specify answer up to two digits after decimal.)

58. Two events occur in frame S at  $(t_1 = 0, r_1 = 0)$  and  $(t_2 = 0, x_2 = 10^8 \text{m}, y_2 = 0, z_2 = 0)$ . Another frame S' moves with v = 0.8c relative to S. The time difference  $(t'_2 - t'_1)$  in S' is ......s. (Specify answer up to two digits after decimal.)

59. In the following circuit, RC is much larger than the input period. Assume diode is ideal and R is large. The dc output voltage across R will be ............... V. (Specify answer up to one digit after the decimal point.)



| 60. For a metal, electron density is $6.4 \times 10^{28} \text{ m}^{-3}$ . The Fermi energy is eV. |
|--|
| (Specify answer up to one digit after the decimal point.)  |
|  |