# IIT JAM 2019 Mathematical Statistics (MS) Question Paper

| **Time Allowed :**3 Hours | **Maximum Marks :**100 | **Total questions :**60 |
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**General Instructions**

**General Instructions:**

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

**Q1. Let $\{x_n\}_{n\geq 1}$ be a sequence of positive real numbers. Which one of the following statements is always TRUE?**

(A) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $\{x_n\}_{n\geq 1}$ is monotone

(B) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then the sequence $\{x_n\}_{n\geq 1}$ does not converge

(C) If the sequence $\{x_{n+1} - x_n\}$ converges to 0, then the series $\sum_{m=1}^{\infty} x_m$ is convergent

(D) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $e^{x_n}$ is also a convergent sequence

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**Q2. Consider the function $f(x, y) = x^3 - 3xy^2, x, y \in \mathbb{R}$. Which one of the following statements is TRUE?**

(A) $f$ has a local minimum at $(0, 0)$

(B) $f$ has a local maximum at $(0, 0)$

(C) $f$ has global maximum at $(0, 0)$

(D) $f$ has a saddle point at $(0, 0)$

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**Q3. If $F(x) = \int_x^4 \sqrt{4 + t^2}\, dt$, for $x \in \mathbb{R}$, then $F'(1)$ equals**

(A) $-\sqrt{5}$

(B) $-2\sqrt{5}$

(C) $2\sqrt{5}$

(D) $\sqrt{5}$

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**Q4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} a \\ b \end{bmatrix}$. Then $\alpha + \beta + a + b$ equals**

(A) $\frac{2}{3}$

(B) $\frac{4}{3}$

(C) $\frac{5}{3}$

(D) $\frac{7}{3}$

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**Q5. Two biased coins $C_1$ and $C_2$ have probabilities of getting heads $\frac{2}{3}$ and $\frac{3}{4}$, respectively. When tossed. If both coins are tossed independently two times each, then the probability of getting exactly two heads out of these four tosses is**

(A) $\frac{1}{4}$

(B) $\frac{37}{144}$

(C) $\frac{41}{144}$

(D) $\frac{49}{144}$

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**Q6. Let $X$ be a discrete random variable with the probability mass function**

$$P(X = n) = \begin{cases} \frac{-2c}{n}, & n = -1, -2 \\ d, & n = 0 \\ \frac{cn}{n}, & n = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

where $c$ and $d$ are positive real numbers. If $P(|X| \leq 1) = \frac{3}{4}$, then $E(X)$ equals

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

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**Q7. Let $X$ be a Poisson random variable and $P(X = 1) + 2P(X = 0) = 12P(X = 2)$. Which one of the following statements is TRUE?**

(A) $0.40 < P(X = 0) \leq 0.45$

(B) $0.45 < P(X = 0) \leq 0.50$

(C) $0.50 < P(X = 0) \leq 0.55$

(D) $0.55 < P(X = 0) \leq 0.60$

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**Q8. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. discrete random variables with the probability mass function**

$$P(X_1 = m) = \frac{(\log 2)^m}{2(m!)} \quad \text{for} \quad m = 0, 1, 2, \ldots$$

**If $S_n = X_1 + X_2 + \cdots + X_n$, then which one of the following sequences of random variables converges to 0 in probability?**

(A) $\frac{S_n}{n \log 2}$

(B) $\frac{S_n}{n} \log 2$

(C) $S_n - \log 2$

(D) $\frac{S_n - n}{\log 2}$

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**Q9. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function**

$$f(x) = \frac{1}{2\sqrt{2\pi}} \left[ e^{-\frac{1}{2}(x-2)^2} + e^{-\frac{1}{2}(x-4)^2} \right], \quad -\infty < x < \infty$$

**If $T_n = X_1 + X_2 + \cdots + X_n$, then which one of the following is an unbiased estimator of $\mu$?**

(A) $\frac{T_n}{n}$

(B) $\frac{T_n}{2n}$

(C) $\frac{T_n}{3n}$

(D) $\frac{T_n}{4n}$

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**Q10. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. Instead of observing $X_1, X_2, \ldots, X_n$, we observe $Y_i = e^{X_i}, i = 1, 2, \ldots, n$. To test the hypothesis**

$$H_0 : \theta = 1 \quad \text{against} \quad H_1 : \theta \neq 1$$

**based on the random sample $Y_1, Y_2, \ldots, Y_n$, the rejection region of the likelihood ratio test is of the form, for some $c_1 < c_2$,**

(A) $\sum_{i=1}^{n} Y_i \leq c_1$ or $\sum_{i=1}^{n} Y_i \geq c_2$

(B) $c_1 \leq \sum_{i=1}^{n} Y_i \leq c_2$

(C) $c_1 \leq \sum_{i=1}^{n} \log Y_i \leq c_2$

(D) $\sum_{i=1}^{n} \log Y_i \leq c_1$ or $\sum_{i=1}^{n} \log Y_i \geq c_2$

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**Q11. The sum**

$$\sum_{n=4}^{\infty} \frac{6}{n^2 - 4n + 3} \quad \text{equals}$$

(A) $\frac{5}{2}$

(B) 3

(C) $\frac{7}{2}$

(D) $\frac{9}{2}$

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**Q12. Evaluate the limit**

$$\lim_{n \to \infty} \frac{1 + \frac{1}{2} + \cdots + \frac{1}{n}}{(n + e^n)^{1/n} \log_e n}$$

(A) $\frac{1}{\pi}$

(B) $\frac{1}{e}$

(C) $\frac{e}{\pi}$

(D) $\frac{\pi}{e}$

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**Q13. A possible value of $b \in \mathbb{R}$ for which the equation**

$$x^4 + bx^3 + 1 = 0$$

**has no real root is**

(A) $-\frac{11}{5}$

(B) $-\frac{3}{2}$

(C) 2

(D) $\frac{5}{2}$

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**Q14. Let the Taylor polynomial of degree 20 for $\frac{1}{(1-x)^3}$ at $x = 0$ be**

$$\sum_{n=0}^{20} a_n x^n$$

**Then $a_{15}$ equals**

(A) 136

(B) 120

(C) 60

(D) 272

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**Q15. The length of the curve**

$$y = \frac{3}{4}x^{4/3} - \frac{3}{8}x^{2/3} + 7$$

**from $x = 1$ to $x = 8$ equals**

(A) $\frac{99}{8}$

(B) $\frac{117}{8}$

(C) $\frac{99}{4}$

(D) $\frac{49}{8}$

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**Q16. The volume of the solid generated by revolving the region bounded by the parabola**

$$x = 2y^2 + 4 \quad \text{and the line} \quad x = 6 \quad \text{about the line} \quad x = 6$$

**is**

(A) $\frac{78\pi}{15}$

(B) $\frac{91\pi}{15}$

(C) $\frac{64\pi}{15}$

(D) $\frac{117\pi}{15}$

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**Q17. Let $P$ be a $3 \times 3$ non-null real matrix. If there exists a $3 \times 2$ real matrix $Q$ and a $2 \times 3$ real matrix $R$ such that $P = QR$, then**

(A) $Px = 0$ has a unique solution, where $0 \in \mathbb{R}^3$

(B) There exists $b \in \mathbb{R}^3$ such that $Px = b$ has no solution

(C) There exists a non-zero $b \in \mathbb{R}^3$ such that $Px = b$ has a unique solution

(D) There exists a non-zero $b \in \mathbb{R}^3$ such that $P^T x = b$ has a unique solution

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**Q18. If**

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \quad \text{and} \quad 6P^{-1} = aI_3 + bP - P^2, \quad \text{then the ordered pair} \quad (a, b) \quad \text{is}$$

(A) (3,2)

(B) (2,3)

(C) (4,5)

(D) (5,4)

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**Q19. Let $E, F$, and $G$ be any three events with $P(E) = 0.3$, $P(F|E) = 0.2$, $P(G|E) = 0.1$. Then $P(E - (F \cup G))$ equals**

(A) 0.155

(B) 0.175

(C) 0.225

(D) 0.255

**Q20. Let $E$ and $F$ be any two independent events with $0 < P(E) < 1$ and $0 < P(F) < 1$. Which one of the following statements is NOT TRUE?**

(A) $P(\text{Neither } E \text{ nor } F \text{ occurs}) = P(E^C) \cdot P(F^C)$

(B) $P(E) = 1 - P(F)$

(C) $P(E \text{ occurs but } F \text{ does not occur}) = P(E) - P(E \cap F)$

(D) $P(E \text{ occurs given that } F \text{ does not occur}) = P(E)$

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**Q21. Let $X$ be a continuous random variable with the probability density function**

$$f(x) = \frac{1}{3}x^2 e^{-x^2}, \quad x > 0$$

**Then the distribution of the random variable**

$$W = 2X^2 \quad \text{is}$$

(A) $\chi_2^2$

(B) $\chi_4^4$

(C) $\chi_4^2$

(D) $\chi_8^2$

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**Q22. Let $X$ be a continuous random variable with the probability density function**

$$f(x) = \frac{e^x}{(1+e^x)^2}, \quad -\infty < x < \infty$$

**Then $E(X)$ and $P(X > 1)$, respectively, are**

(A) 1 and $(1+e)^{-1}$

(B) 0 and $2(1+e)^{-2}$

(C) 2 and $(2+2e)^{-1}$

(D) 0 and $(1+e)^{-1}$

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**Q23. The lifetime (in years) of bulbs is distributed as an Exp$(1)$ random variable. Using Poisson approximation to the binomial distribution, the probability (rounded off to 2 decimal places) that out of the fifty randomly chosen bulbs at most one fails within one month equals**

(A) 0.05

(B) 0.07

(C) 0.09

(D) 0.11

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**Q24. Let $X$ follow a beta distribution with parameters $m(>0)$ and 2. If $P(X \leq \frac{1}{2}) = \frac{1}{2}$, then Var$(X)$ equals**

(A) $\frac{1}{10}$

(B) $\frac{1}{20}$

(C) $\frac{1}{25}$

(D) $\frac{1}{40}$

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**Q25. Let $X_1, X_2, X_3$ be i.i.d. $U(0,1)$ random variables. Then**

$$P(X_1 > X_2 + X_3)$$

(A) $\frac{1}{6}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

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**Q26. Let $X$ and $Y$ be i.i.d. $U(0,1)$ random variables. Then $E(X|X > Y)$ equals**

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

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**Q27. Let -1 and 1 be the observed values of a random sample of size two from $N(\theta, \theta)$ distribution. The maximum likelihood estimate of $\theta$ is**

(A) 0

(B) 2

(C) $\frac{-\sqrt{5}-1}{2}$

(D) $\frac{\sqrt{5}-1}{2}$

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**Q28. Let $X_1$ and $X_2$ be a random sample from a continuous distribution with the probability density function**

$$f(x) = \frac{1}{\theta} e^{-\frac{x-\theta}{\theta}}, \quad x > \theta$$

**If $X_{(1)} = \min\{X_1, X_2\}$ and $\overline{X} = \frac{X_1+X_2}{2}$, then which one of the following statements is TRUE?**

(A) $(\overline{X}, X_{(1)})$ is sufficient and complete

(B) $(\overline{X}, X_{(1)})$ is sufficient but not complete

(C) $(\overline{X}, X_{(1)})$ is complete but not sufficient

(D) $(\overline{X}, X_{(1)})$ is neither sufficient nor complete

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**Q29. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function $f(x)$. To test the hypothesis $H_0 : f(x) = e^{-x^2}$ against $H_1 : f(x) = e^{-2|x|}$, the rejection region of the most powerful size $\alpha$ test is of the form, for some $c > 0$,**

(A) $\sum_{i=1}^{n}(X_i - 1)^2 \geq c$

(B) $\sum_{i=1}^{n}(X_i-1)^2 \le c$

(C) $\sum_{i=1}^{n}|X_i| \ge c$

(D) $\sum_{i=1}^{n}|X_i-1|^2 \le c$

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**Q30. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. To test $H_0 : \theta = 0$ against $H_1 : \theta = 1$, assume that the critical region is given by**

$$\frac{1}{n}\sum_{i=1}^{n} X_i \ge \frac{3}{4}.$$

**Then the minimum sample size required so that $P($Type I error$) \le 0.05$ is**

(A) 3

(B) 4

(C) 5

(D) 6

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**Q31. Let $\{x_n\}_{n\ge1}$ be a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} x_n$ converges. Which of the following statements is (are) always TRUE?**

(A) The series $\sum_{n=1}^{\infty}\sqrt{x_n x_{n+1}}$ converges

(B) $\lim_{n\to\infty} x_n = 0$

(C) The series $\sum_{n=1}^{\infty} x_n^2$ converges

(D) The series $\sum_{n=1}^{\infty}\frac{\sqrt{x_n}}{1+\sqrt{x_n}}$ converges

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**Q32. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and differentiable on $(-\infty, 0) \cup (0, \infty)$. Which of the following statements is (are) always TRUE?**

(A) If $f$ is differentiable at 0 and $f'(0) = 0$, then $f$ has a local maximum or a local minimum at 0.

(B) If $f$ has a local minimum at 0, then $f$ is differentiable at 0 and $f'(0) = 0$.

(C) If $f'(x) < 0$ for all $x < 0$ and $f'(x) > 0$ for all $x > 0$, then $f$ has a global maximum at 0.

(D) If $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$, then $f$ has a global maximum at 0.

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**Q33. Let $P$ be a $2 \times 2$ real matrix such that every non-zero vector in $\mathbb{R}^2$ is an eigenvector of $P$. Suppose that $\lambda_1$ and $\lambda_2$ denote the eigenvalues of $P$ and $P \begin{bmatrix} \sqrt{2} \\ \sqrt{3} \end{bmatrix} = \begin{bmatrix} 2 \\ t \end{bmatrix}$ for some $t \in \mathbb{R}$. Which of the following statements is (are) TRUE?**

(A) $\lambda_1 \neq \lambda_2$

(B) $\lambda_1 \lambda_2 = 2$

(C) $\sqrt{2}$ is an eigenvalue of $P$

(D) $\sqrt{3}$ is an eigenvalue of $P$

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**Q34. Let $P$ be an $n \times n$ non-null real skew-symmetric matrix, where $n$ is even. Which of the following statements is (are) always TRUE?**

(A) $Px = 0$ has infinitely many solutions, where $0 \in \mathbb{R}^n$

(B) $Px = \lambda x$ has a unique solution for every non-zero $\lambda \in \mathbb{R}$

(C) If $Q = (I_n + P)(I_n - P)^{-1}$, then $Q^T Q = I_n$

(D) The sum of all the eigenvalues of $P$ is zero

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**Q35. Let $X$ be a random variable with the cumulative distribution function**

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 + x^2, & 0 \leq x < 1 \\ \frac{10}{3} + x^2, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

**Which of the following statements is (are) TRUE?**

(A) $P(1 < X < 2) = \frac{3}{10}$

(B) $P(1 < X \leq 2) = \frac{3}{5}$

(C) $P(1 \leq X < 2) = \frac{1}{2}$

(D) $P(1 \leq X \leq 2) = \frac{4}{5}$

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**Q36. Let $X$ and $Y$ be i.i.d. Exp$(\lambda)$ random variables. If $Z = \max\{X - Y, 0\}$, then which of the following statements is (are) TRUE?**

(A) $P(Z = 0) = \frac{1}{2}$

(B) The cumulative distribution function of $Z$ is $F(z) = \begin{cases} 0, & z < 0 \\ 1 - \frac{1}{2}e^{-\lambda z}, & z \geq 0 \end{cases}$

(C) $P(Z = 0) = 0$

(D) The cumulative distribution function of $Z$ is $F(z) = \begin{cases} 0, & z < 0 \\ 1 - e^{-\lambda z}, & z \geq 0 \end{cases}$

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**Q37. Let the discrete random variables $X$ and $Y$ have the joint probability mass function**

$$P(X = m, Y = n) = \begin{cases} \frac{e^{-2}}{m!n!}, & m = 0, 1, 2, \ldots, n = 0, 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

**Which of the following statements is (are) TRUE?**

(A) The marginal distribution of $X$ is Poisson with parameter 2

(B) The marginal distribution of $X$ and $Y$ are independent

(C) The joint distribution of $X$ and $X + \sqrt{Y}$ is independent

(D) The random variables $X$ and $Y$ are independent

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**Q38. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. continuous random variables with the probability density function**

$$f(x) = \begin{cases} 2e^{-2(x-\frac{1}{2})^2}, & x \geq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

**If $S_n = X_1 + X_2 + \cdots + X_n$ and $\bar{X}_n = \frac{S_n}{n}$, then the distributions of which of the following sequences of random variables converge(s) to a normal distribution with mean 0 and a finite variance?**

(A) $\frac{S_n - n}{\sqrt{n}}$

(B) $\frac{S_n}{\sqrt{n}}$

(C) $\sqrt{n}\left(\bar{X}_n - 1\right)$

(D) $\sqrt{n}\left(\bar{X}_n - 1\right)/2$

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**Q39. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U(\theta, 0)$ distribution, where $\theta < 0$. If $T_n = \min(X_1, X_2, \ldots, X_n)$, then which of the following sequences of estimators is (are) consistent for $\theta$?**

(A) $T_n$

(B) $T_n - 1$

(C) $T_n + \frac{1}{n}$

(D) $T_n - \frac{1}{n^2}$

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**Q40. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function for $\lambda > 0$**

$$f(x) = \begin{cases} 2\lambda x e^{-\lambda x^2}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

**To test the hypothesis $H_0 : \lambda = \frac{1}{2}$ against $H_1 : \lambda = \frac{3}{4}$ at the level $\alpha$ (with $0 < \alpha < 1$), which of the following statements is (are) TRUE?**

(A) The most powerful test exists for each value of $\alpha$

(B) The most powerful test does not exist for some values of $\alpha$

(C) If the most powerful test exists, it is of the form: Reject $H_0$ if $X_1^2 + X_2^2 + \cdots + X_n^2 > c$ for some $c > 0$

(D) If the most powerful test exists, it is of the form: Reject $H_0$ if $X_1^2 + X_2^2 + \cdots + X_n^2 \geq c$ for some $c \geq 0$

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**Q41. Evaluate the following limit (round off to 2 decimal places):**

$$\lim_{n\to\infty} \frac{\sqrt{n+1} + \sqrt{n+2} + \cdots + \sqrt{n+n}}{\sqrt{n}}$$

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**Q42. Let $f : [0,2] \to \mathbb{R}$ be such that $|f(x) - f(y)| \leq |x-y|^{4/3}$ for all $x, y \in [0,2]$. If $\int_0^2 f(x)\, dx = \frac{2}{3}$, then**

$$\sum_{k=1}^{2019} f\left(\frac{1}{k}\right) \text{ equals } \_\_\_\_\_\_\_\_\_\_\_\_$$

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**Q43. The value (round off to 2 decimal places) of the double integral**

$$\int_0^9 \int_{\sqrt{x}}^3 \frac{1}{1+y^3}\, dy\, dx$$

**equals .............**

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**Q44. If**

$$\begin{pmatrix} \frac{\sqrt{5}}{3} & -\frac{2}{3} & c \\ \frac{2}{3} & \frac{\sqrt{5}}{3} & d \\ a & b & 1 \end{pmatrix}$$

**is a real orthogonal matrix, then $a^2 + b^2 + c^2 + d^2$ equals ...............**

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**Q45. Two fair dice are tossed independently and it is found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals .............**

**Q46. Let $X$ be a random variable with the moment generating function**

$$M_X(t) = \left(\frac{e^{t/2} + e^{-t/2}}{2}\right)^2, \quad -\infty < t < \infty.$$

**Using Chebyshev's inequality, the upper bound for $P\left(|X| > \frac{2}{\sqrt{3}}\right)$ equals ...............**

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**Q47. In a production line of a factory, each packet contains four items. Past record shows that 20% of the produced items are defective. A quality manager inspects each item in a packet and approves the packet for shipment if at most one item in the packet is found to be defective. Then the probability (round off to 2 decimal places) that out of the three randomly inspected packets at least two are approved for shipment equals ............**

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**Q48. Let $X$ be the number of heads obtained in a sequence of 10 independent tosses of a fair coin. The fair coin is tossed again $X$ number of times independently, and let $Y$ be the number of heads obtained in these $X$ number of tosses. Then $E(X + 2Y)$ equals ............**

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**Q49. Let $0, 1, 0, 0, 1$ be the observed values of a random sample of size five from a discrete distribution with the probability mass function $P(X = 1) = 1 - P(X = 0) = 1 - e^{-\lambda}$, where $\lambda > 0$. The method of moments estimate (round off to 2 decimal places) of $\lambda$ equals .............**

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**Q50. Let $X_1, X_2, X_3$ be a random sample from $N(\mu_1, \sigma_1^2)$ distribution and $Y_1, Y_2, Y_3$ be a random sample from $N(\mu_2, \sigma_2^2)$ distribution. Also, assume that $(X_1, X_2, X_3)$ and $(Y_1, Y_2, Y_3)$ are independent. Let the observed values of $\sum_{i=1}^{3}\left[X_i - \frac{1}{3}(X_1 + X_2 + X_3)\right]^2$ and $\sum_{i=1}^{3}\left[Y_i - \frac{1}{3}(Y_1 + Y_2 + Y_3)\right]^2$ be 1 and 5, respectively. Then the solution for the 90% confidence interval for $\mu_1 - \mu_2$ equals ................**

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**Q51. Evaluate the limit**

$$\lim_{n\to\infty}\left[n-\frac{n}{e}\left(1+\frac{1}{n}\right)^{n}\right] \text{ equals ...........}$$

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**Q52. For any real number $y$, let $\lfloor y \rfloor$ be the greatest integer less than or equal to $y$ and let $\{y\} = y - \lfloor y \rfloor$. For $n = 1, 2, \ldots$, and for $x \in \mathbb{R}$, let**

$$f_{2n}(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \quad \text{and} \quad f_{2n-1}(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

**Then**

$$\lim_{x\to 0}\sum_{k=1}^{100} f_k(x) \text{ equals ............}$$

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**Q53. The volume (round off to 2 decimal places) of the region in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 2$ and $y + z = 4$ equals ................**

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**Q54. If $ad - bc = 2$ and $ps - qr = 1$, then the determinant of**

$$\begin{pmatrix} a & b & 0 & 0 \\ 3 & 10 & 2p & q \\ c & d & 0 & 0 \\ 2 & 7 & 2r & s \end{pmatrix}$$

**equals ...........**

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**Q55. In an ethnic group, 30% of the adult male population is known to have heart disease. A test indicates high cholesterol level in 80% of adult males with heart disease.**

**But the test also indicates high cholesterol levels in 10% of the adult males with no heart disease. Then the probability (round off to 2 decimal places) that a randomly selected adult male from this population does not have heart disease given that the test indicates high cholesterol level equals ............**

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**Q56. Let $X$ be a continuous random variable with the probability density function**

$$f(x) = \begin{cases} ax^2, & 0 < x < 1 \\ bx^{-4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

**where $a$ and $b$ are positive real numbers. If $E(X) = 1$, then $E(X^2)$ equals ...............**

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**Q57. Let $X$ and $Y$ be jointly distributed continuous random variables, where $Y$ is positive valued with $E(Y^2) = 6$. If the conditional distribution of $X$ given $Y = y$ is $U(1 - y, 1 + y)$, then Var$(X)$ equals ................**

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**Q58. Let $X_1, X_2, \ldots, X_{10}$ be i.i.d. $N(0, 1)$ random variables. If $T = X_1^2 + X_2^2 + \cdots + X_{10}^2$, then $E\left(\frac{1}{T}\right)$ equals .................**

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**Q59. Let $X_1, X_2, X_3$ be a random sample from a continuous distribution with the probability density function**

$$f(x) = \begin{cases} e^{-(x-\mu)}, & x \geq \mu \\ 0, & \text{otherwise} \end{cases}$$

**Let $X_{(1)} = \min\{X_1, X_2, X_3\}$ and $c > 0$ be a real number. Then $(X_{(1)} - c, X_{(1)})$ is a 97% confidence interval for $\mu$, if $c$ (round off to 2 decimal places) equals ...............**

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**Q60. Let $X_1, X_2, X_3, X_4$ be a random sample from a discrete distribution with the probability mass function**

$$P(X = 0) = 1 - P(X = 1) = 1 - p, \quad 0 < p < 1.$$

**To test the hypothesis**

$$H_0 : p = \frac{3}{4} \quad \text{against} \quad H_1 : p = \frac{4}{5},$$

**consider the test:** Reject $H_0$ if $X_1 + X_2 + X_3 + X_4 > 3$. **Let the size and power of the test be denoted by $\alpha$ and $\gamma$, respectively. Then $\alpha + \gamma$ (round off to 2 decimal places) equals**

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