IIT JAM 2019 Mathematics (MA) Question Paper

Time Allowed : 3 Hours	**Maximum Marks :** 100	**Total questions :** 60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

Q1. Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by

$$a_n = \sum_{m=2}^{n} (-1)^m m(\log(m))^m$$

$$b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m}.$$

Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?

(A) Both $\{a_n\}$ and $\{b_n\}$ are divergent

(B) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent

(C) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent

(D) Both $\{a_n\}$ and $\{b_n\}$ are convergent

Q2. Let $M_{n\times p}(\mathbb{R})$ be the subspace of $M_{n\times p}(\mathbb{R})$ defined by

$$V = \{X \in M_{n\times p}(\mathbb{R}) : TX = 0\}.$$

Then the dimension of V is

(A) pn – rank(T)

(B) mn – $p$rank(T)

(C) $p(m$ – rank$(T))$

(D) $p($n – rank$(T))$

Q3. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by

$$f(x, y, z) = g(x^2 + y^2 - 2z^2).$$

Then

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

is equal to

(A) $(4x^2 + 4y^2 - 42z^2)g''(x^2 + y^2 - 2z^2)$

(B) $(4x^2 + y^2 + 42z^2)g''(x^2 + y^2 - 2z^2)$

(C) $(4x^2 + y^2 - 22z^2)g''(x^2 + y^2 - 2z^2)$

(D) $(4x^2 + y^2 + 42z^2)g''(x^2 + y^2 - 2z^2) + 8g'(x^2 + y^2 - 2z^2)$

Q4. Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real numbers such that $na_n < b_n < n^2 a_n$, for all $n \geq 2$. If the radius of convergence of the power series

$$\sum_{n=0}^{\infty} a_n x^n$$

is 4, then the power series

$$\sum_{n=0}^{\infty} b_n x^n$$

is

(A) converges for all x with $|x| < 2$

(B) converges for all x with $|x| > 2$

(C) does not converge for any x with $|x| > 2$

(D) does not converge for any x with $|x| < 2$

Q5. Let S be the set of all limit points of the set

$$\left\{ \frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N} \right\}.$$

Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then

(A) $\mathbb{Q}_+ \subset S$

(B) $S \subset \mathbb{Q}_+$

(C) $S \cap (\mathbb{R} \setminus \mathbb{Q}_+) \neq \emptyset$

(D) $S \cap \mathbb{Q}_+ \neq \emptyset$

Q6. If $x^h y^k$ is an integrating factor of the differential equation

$$y(1+xy)\,dx + x(1-xy)\,dy = 0,$$

then the ordered pair (h,k) is equal to

(A) $(-2,-2)$

(B) $(-2,-1)$

(C) $(-1,-2)$

(D) $(-1,-1)$

Q7. If $y(x) = 2e^{2x} + e^{\beta x}, \beta \neq 2$, is a solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0,$$

satisfying $\frac{dy}{dx}(0) = 5$, then $y(0)$ is equal to

(A) 1

(B) 4

(C) 5

(D) 9

Q8. The equation of the tangent plane to the surface

$$x^2 z + \sqrt{8 - x^2 - y^4} = 6 \text{ at the point } (2,0,1)$$

is

(A) $2x + z = 5$

(B) $3x + 4z = 10$

(C) $3x - z = 10$

(D) $7x - 4z = 10$

Q9. The value of the integral

$$\int_0^1 \int_0^{1-y^2} y \sin(\pi(1-x^2)^2)\, dx\, dy$$

is

(A) $\frac{1}{2\pi}$

(B) 2π

(C) $\frac{\pi}{2}$

(D) $\frac{2}{\pi}$

Q10. The area of the surface generated by rotating the curve $y = x^3$, $0 \leq x \leq 1$, about the y-axis, is

(A) $\frac{\pi}{27} 10^{3/2}$

(B) $\frac{4\pi}{3}(10^{3/2} - 1)$

(C) $\frac{\pi}{27} 10^{3/2}$

(D) $\frac{4\pi}{3} 10^{3/2}$

Q11. Let H and K be subgroups of $\mathbb{Z}_{144}$. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H \cap K$ is

(A) 3

(B) 4

(C) 6

(D) 12

Q12. Let P be a 4×4 matrix with entries from the set of rational numbers. If $\sqrt{2} + i$, with $i = \sqrt{-1}$, is a root of the characteristic polynomial of P and I is the 4×4 identity matrix, then

(A) $P^4 = 4P^2 + 9I$

(B) $P^4 = 4P^2 - 9I$

(C) $P^4 = 2P^2 - 9I$

(D) $P^4 = P^2 + 9I$

Q13. The set

$$\left\{\frac{x}{1+x^2} : -1 < x < 1\right\}, \text{ as a subset of } \mathbb{R}, \text{ is}$$

(A) connected and compact

(B) connected but not compact

(C) not connected but compact

(D) neither connected nor compact

Q14. The set

$$\left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\} \cup \{0\}, \text{ as a subset of } \mathbb{R}, \text{ is}$$

(A) compact and open

(B) compact but not open

(C) not compact but open

(D) neither compact nor open

Q15. For $-1 < x < 1$, the sum of the power series

$$1 + \sum_{n=2}^{\infty} (-1)^{n-1} n^2 x^{n-1} \text{ is}$$

(A) $\frac{1-x}{(1+x^3)}$

(B) $\frac{1+x^2}{(1+x^4)}$

(C) $\frac{1-x}{(1+x^2)}$

(D) $\frac{(1+x^2)^3}{(1+x^3)}$

Q16. Let $f(x) = (\ln x)^2, x > 0$. Then

(A) $\lim_{x\to\infty} f(x)$ does not exist

(B) $\lim_{x\to\infty} f(x) = 2$

(C) $\lim_{x\to\infty} (f(x+1) - f(x)) = 0$

(D) $\lim_{x\to\infty} (f(x+1) - f(x))$ does not exist

Q17. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) > f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$. Then $f(1)$ lies in the interval

(A) $(0, e^{-1})$

(B) $(e^{-1}, \sqrt{e})$

(C) $(\sqrt{e}, e)$

(D) (e, ∞)

Q18. For which one of the following values of k, the equation

$$2x^3 + 3x^2 - 12x - k = 0$$

has three distinct real roots?

(A) 16

(B) 20

(C) 26

(D) 31

Q19. Which one of the following series is divergent?

(A) $\sum_{n=1}^{\infty} \frac{\sin^2\left(\frac{1}{n}\right)}{n^2}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n \log n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n \tan^{-1} n}$

Q20. Let S be the family of orthogonal trajectories of the family of curves

$$2x^2 + y^2 = k, \text{ for } k \in \mathbb{R} \text{ and } k > 0.$$

If $\ell \in S$ and C passes through the point $(1, 2)$, then C also passes through

(A) $(4, -\sqrt{2})$

(B) $(2, -4)$

(C) $(2, 2\sqrt{2})$

(D) $(4, 2\sqrt{2})$

Q21. Let $x + e^x$ and $1 + x + e^x$ be solutions of a linear second-order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0) = 3$ and $y'(0) = 4$, then $y(1)$ is equal to

(A) 1

(B) 2e + 3

(C) 3e + 2

(D) 3e + 1

Q22. The function

$$f(x, y) = x^3 + 2xy + y^3$$

has a saddle point at

(A) $(0, 0)$

(B) $\left(-\frac{2}{3}, -\frac{2}{3}\right)$

(C) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

(D) $(-1, -1)$

Q23. The area of the part of the surface of the paraboloid

$$x^2 + y^2 + z = 8$$

lying inside the cylinder

$$x^2 + y^2 = 4$$

is

(A) $\frac{\pi}{2}(17^{3/2} - 1)$

(B) $\pi(17^{3/2} - 1)$

(C) $\frac{\pi}{6}(17^{3/2} - 1)$

(D) $\frac{\pi}{3}(17^{3/2} - 1)$

Q24. Let C be the circle $(x-1)^2 + y^2 = 1$, oriented counterclockwise. Then the value of the line integral

$$\int_C \left(\frac{4}{3}xy^3\, dx + x^4\, dy\right)$$

(A) 6π

(B) 8π

(C) 12π

(D) 14π

Q25. Let $\mathbf{F}(x, y, z) = 2y\hat{\imath} + x^2\hat{\jmath} + xy\hat{k}$ and let C be the curve of intersection of the plane

$$x + y + z = 1$$

and the cylinder

$$x^2 + y^2 = 1.$$

Then the value of

$$\left| \int_C \mathbf{F} \cdot d\mathbf{r} \right|$$

(A) π

(B) $\frac{3\pi}{2}$

(C) 2π

(D) 3π

Q26. The tangent line to the curve of intersection of the surface $x^2 + y^2 - z = 0$ and the plane $x + z = 3$ at the point $(1, 1, 2)$ passes through

(A) $(-1, -2, 4)$

(B) $(-1, 4, 4)$

(C) $(3, 4, 4)$

(D) $(-1, 4, 0)$

Q27. The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}?$$

(A) $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$

(B) $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Q28. Let (a_n) be a sequence of positive real numbers. The series

$$\sum_{n=1}^{\infty} a_n^2 \quad \text{converges if the series}$$

(A) $\sum_{n=1}^{\infty} a_n$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n} a_n$ converges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} a_n$ converges

(D) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

Q29. For $\beta \in \mathbb{R}$, define

$$f(x,y) = \begin{cases} x^2|x|^{\beta} y & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Then, at $(0,0)$, the function f is

(A) continuous for $\beta = 0$

(B) continuous for $\beta > 0$

(C) not differentiable for any β

(D) continuous for $\beta < 0$

Q30. Let (a_n) be a sequence of positive real numbers such that

$$a_1 = 1, \quad a_{n+1} = 2a_n a_{n+1} - a_n = 0 \text{ for all } n \geq 1.$$

Then the sum of the series

$$\sum_{n=1}^{\infty} a_n$$

lies in the interval

(A) $(1,2)$

(B) $(2,3)$

(C) $(3, 4)$

(D) $(4, 5)$

Q31. Let G be a noncyclic group of order 4. Consider the statements I and II:

I. There is NO injective (one-one) homomorphism from G to $\mathbb{Z}_4$

II. There is NO surjective (onto) homomorphism from $\mathbb{Z}_4$ to G

(A) I is true

(B) I is false

(C) II is true

(D) II is false

Q32. Let G be a nonabelian group, $y \in G$, and let the maps f, g, h from G to itself be defined by

$$f(x) = yxy^{-1}, \quad g(x) = x^{-1} \quad \text{and} \quad h = g \circ f \circ g.$$

Then

(A) g and h are homomorphisms and f is not a homomorphism

(B) h is a homomorphism and g is not a homomorphism

(C) f is a homomorphism and g is not a homomorphism

(D) f, g and h are homomorphisms

Q33. Let S and T be linear transformations from a finite dimensional vector space V to itself such that $S(T(v)) = 0$ for all $v \in V$. Then

(A) rank$(T) \geq$ nullity(S)

(B) rank$(S) \geq$ nullity(T)

(C) rank$(T) \leq$ nullity(S)

(D) rank$(S) \leq$ nullity(T)

Q34. Let F and G be differentiable vector fields and let g be a differentiable scalar function. Then

(A) $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

(B) $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} + \mathbf{F} \cdot \nabla \times \mathbf{G}$

(C) $\nabla \cdot (\mathbf{G} \times \mathbf{F}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

(D) $\nabla \cdot (g\mathbf{F}) = g\nabla \cdot \mathbf{F} + \mathbf{F} \cdot \nabla g$

Q35. Consider the intervals $S = (0, 2]$ and $T = [1, 3]$. Let S° and T° be the sets of interior points of S and T, respectively. Then the set of interior points of $S \setminus T$ is equal to

(A) $S \setminus T^\circ$

(B) $S \setminus T$

(C) $S^\circ \setminus T^\circ$

(D) $S^\circ \setminus T$

Q36. Let a_n be the sequence given by

$$a_n = \max\left(\sin\left(\frac{n\pi}{3}\right), \cos\left(\frac{n\pi}{3}\right)\right), \quad n \geq 1.$$

Then which of the following statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+1}\}$?

(A) Both the subsequences are convergent

(B) Only one of the subsequences is convergent

(C) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$ and $\{a_{6n+1}\}$ converges to $\frac{1}{2}$

(D) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

Q37. Let

$$f(x) = \cos(|\pi - x|) + (x - \pi)\sin|x| \quad \text{and} \quad g(x) = x^2 \text{ for } x \in \mathbb{R}.$$

If $h(x) = f(g(x))$, then

(A) h is not differentiable at $x = 0$

(B) $h'(\sqrt{2}) = 0$

(C) $h'(x) = 0$ has a solution in $(-\pi, \pi)$

(D) There exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

Q38. Let

$$f(x) = (\sin x)^\pi - \pi \sin x + \pi.$$

Then which of the following statements is/are TRUE?

(A) f is an increasing function

(B) f is a decreasing function

(C) $f(x) > 0$ for all $x \in (0, \pi)$

(D) $f(x) < 0$ for some $x \in (0, \frac{\pi}{2})$

Q39. Let

$$f(x,y) = \begin{cases} \frac{|x|}{|x|+|y|}\sqrt{x^4 + y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Then at $(0,0)$**,**

(A) f is continuous

(B) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y}$ does not exist

(C) $\frac{\partial f}{\partial x}$ does not exist and $\frac{\partial f}{\partial y} = 0$

(D) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

Q40. Let $\{a_n\}$ be the sequence of real numbers such that

$$a_1 = 1 \quad \text{and} \quad a_{n+1} = a_n + a_n^2 \quad \text{for all } n \geq 1.$$

Then

(A) $a_4 = a_1(1 + a_1)(1 + a_2)(1 + a_3)$

(B) $\lim_{n\to\infty} \frac{1}{a_n} = 0$

(C) $\lim_{n\to\infty} \frac{1}{a_n} = 1$

(D) $\lim_{n\to\infty} a_n = 0$

Q41. Let x be the 100-cycle $(1\,2\,3\ \ldots\ 100)$ and let y be the transposition $(49\,50)$ in the permutation group S_{100}. Then the order of xy is

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Q42. Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$$W_1 = \{(x_1, x_2, \ldots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 4\},$$

$$W_2 = \{(x_1, x_2, \ldots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 5\}.$$

Then the dimension of $W_1 \cap W_2$ is

Q43. Consider the following system of three linear equations in four unknowns x_1, x_2, x_3, x_4:

$$x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 5$$

$$x_1 + 3x_2 + 5x_3 + kx_4 = 5$$

If the system has no solutions, then $k =$ ....

Q44. Let $\vec{F}(x,y) = -y\hat{i} + x\hat{j}$ and let C be the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

with counterclockwise orientation. Then the value of $\oint_C \vec{F} \cdot d\vec{r}$ (rounded to 2 decimal places) is ....

Q45. The coefficient of $(x - \frac{\pi}{2})$ in the Taylor series expansion of the function

$$f(x) = \begin{cases} \frac{4(1-\sin(x))}{2x-\pi}, & x \neq \frac{\pi}{2} \\ 0, & x = \frac{\pi}{2} \end{cases}$$

about $x = \frac{\pi}{2}$ is ....

Q46. Let $f : [0,1] \to \mathbb{R}$ be given by

$$f(x) = \frac{\left(\frac{1}{1+x^3}\right)^3 + \left(\frac{1}{1-x^3}\right)^3}{8(1+x)}.$$

Then $\max\{f(x) : x \in [0,1]\} - \min\{f(x) : x \in [0,1]\}$ is ....

Q47. If

$$g(x) = \int_{x(x-2)}^{4x-5} f(t)\,dt, \quad \text{where} \quad f(x) = \sqrt{1+3x^4} \quad \text{for} \quad x \in \mathbb{R}$$

then $g'(1) =$ ....

Q48. Let

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2-y^2}, & x^2 - y^2 \neq 0 \\ 0, & x^2 - y^2 = 0 \end{cases}$$

Then the directional derivative of f at $(0,0)$ in the direction of $\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$ is ....

Q49. The value of the integral

$$\int_{-1}^{1}\int_{-1}^{1} |x+y|\,dx\,dy$$

(round off to 2 decimal places) is ....

Q50. The volume of the solid bounded by the surfaces

$$x = 1 - y^2, \quad x = y^2 - 1, \quad \text{and the planes} \quad z = 0 \quad \text{and} \quad z = 2$$

(round off to 2 decimal places) is ....

Q51. The volume of the solid of revolution of the loop of the curve

$$y^2 = x^4(x+2)$$

about the x-axis (round off to 2 decimal places) is ....

Q52. The greatest lower bound of the set

$$\left\{(e^n + 2^n)^{\frac{1}{n}} : n \in \mathbb{N}\right\},$$

(round off to 2 decimal places) is ....

Q53. Let

$$G = \{n \in \mathbb{N} : n \leq 55,\, \gcd(n, 55) = 1\}$$

be the group under multiplication modulo 55. Let $x \in G$ **be such that** $x^2 = 26$ **and** $x > 30$. **Then** x **is equal to** ....

Q54. The number of critical points of the function

$$f(x,y) = (x^2 + 3y^2)^2 e^{-(x^2+y^2)}$$

is ....

Q55. The number of elements in the set

$\{x \in S_3 : x^4 = e\},$ where e is the identity element of the permutation group $S_3,$ is..............

Q56. If

$$\begin{pmatrix} z \\ y \end{pmatrix}$$

is an eigenvector corresponding to a real eigenvalue of the matrix

$$\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$$

then $z - y$ **is equal to** ....

Q57. Let M and N be any two 4 $\times 4$ matrices with integer entries satisfying $MN = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = 2$

Then the maximum value of $\det(M) + \det(N)$ is..............

Q58. Let M be a 3×3 matrix with real entries such that $M^2 =$ $M + 2I,$ where I denotes the 3×3 identity matrix. **If** α, β, γ are eigenvalues of M such that $\alpha\beta\gamma = -4,$ then $\alpha + \beta + \gamma$ **is equal to** ....

Q59. Let $y(x) = xv(x)$ be a solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0.$$

If $v(0) = 0$ and $v(1) = 1$, **then** $v(-2)$ **is equal to** ....

Q60. If $y(x)$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, \quad y(0) = 2, \quad \frac{dy}{dx}(0) = 0,$$

then $y(\ln 2)$ **is (round off to 2 decimal places) equal to** ....
