IIT JAM 2020 Mathematical Statistics (MS) Question Paper

Time Allowed :3 Hours	Maximum Marks :100	Total questions :60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

Q1. If $\{x_n\}_{n\geq 1}$ is a sequence of real numbers such that $\lim_{n\to\infty} \frac{x_n}{n} = 0.001$, then

(A) $\{x_n\}_{n\geq 1}$ is a bounded sequence

(B) $\{x_n\}_{n\geq 1}$ is an unbounded sequence

(C) $\{x_n\}_{n\geq 1}$ is a convergent sequence

(D) $\{x_n\}_{n\geq 1}$ is a monotonically decreasing sequence

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Q2. For real constants a and b, let

$$f(x) = \begin{cases} \frac{a\sin x - 2x}{x}, & x < 0 \\ bx, & x \geq 0 \end{cases}$$

If f is a differentiable function, then the value of $a+b$ is

(A) 0

(B) 1

(C) 2

(D) 3

Q3. The area of the region bounded by the curves $y_1(x) = x^4 - 2x^2$ and $y_2(x) = 2x^2$, $x \in \mathbb{R}$, is

(A) $\frac{128}{15}$

(B) $\frac{129}{15}$

(C) $\frac{133}{15}$

(D) $\frac{134}{15}$

Q4. Consider the following system of linear equations:

$$\begin{cases} ax + 2y + z = 0 \\ y + 5z = 1 \\ by - 5z = -1 \end{cases}$$

Which one of the following statements is TRUE?

(A) The system has unique solution for $a = 1, b = -1$

(B) The system has unique solution for $a = -1, b = 1$

(C) The system has no solution for $a = 1, b = 0$

(D) The system has infinitely many solutions for $a = 0, b = 0$

Q5. Let E and F be two events. Then which one of the following statements is NOT always TRUE?

(A) $P(E \cap F) \leq \max\{1 - P(E^C) - P(F^C), 0\}$

(B) $P(E \cup F) \geq \max\{P(E), P(F)\}$

(C) $P(E \cup F) \leq \min\{P(E) + P(F), 1\}$

(D) $P(E \cap F) \leq \min\{P(E), P(F)\}$

Q6. Let X be a random variable having Poisson(2) distribution. Then $E\left(\frac{1}{1+X}\right)$ equals

(A) $1 - e^{-2}$

(B) e^{-2}

(C) $\frac{1}{2}(1 - e^{-2})$

(D) $\frac{1}{2}e^{-1}$

Q7. The mean and the standard deviation of weights of ponies in a large animal shelter are 20 kg and 3 kg, respectively. A pony is selected at random. Using Chebyshev's inequality, find the lower bound of the probability that its weight lies between 14 kg and 26 kg.

(A) $\frac{3}{4}$

(B) $\frac{1}{4}$

(C) 0

(D) 1

Q8. Let $X_1, X_2, \ldots, X_{10}$ be a random sample from $N(1, 2)$ distribution. If $\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$ and $S^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2$, then Var$(S^2)$ equals

(A) $\frac{2}{5}$

(B) $\frac{4}{9}$

(C) $\frac{11}{9}$

(D) $\frac{8}{9}$

Q9. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that $E(X_i) = 1$ and Var$(X_i) = 1$. Then the approximate distribution of $\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_{2i} - X_{2i-1})$, for large n, is

(A) $N(0, 1)$

(B) $N(0, 2)$

(C) $N(0, 0.5)$

(D) $N(0, 0.25)$

Q10. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables having $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$. Define

$$W = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - X_j)^2.$$

Then W, as an estimator of σ^2, is

(A) Biased and consistent

(B) Unbiased and consistent

(C) Biased and inconsistent

(D) Unbiased and inconsistent

Q11. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 1, a_2 = 7$, and $a_{n+1} = \dfrac{a_n + a_{n-1}}{2}$, $n \geq 2$. Assuming that $\lim_{n\to\infty} a_n$ exists, the value of $\lim_{n\to\infty} a_n$ is

(A) $\dfrac{19}{4}$
(B) $\dfrac{9}{2}$
(C) 5
(D) $\dfrac{21}{4}$

Q12. Which one of the following series is convergent?

(A) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5n+1}{4n+1}\right)^n$
(B) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$
(C) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^{1/n}}$
(D) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}\left(1 - \cos\left(\frac{1}{n}\right)\right)$

Q13. Let α and β be real numbers. If

$$\lim_{x\to 0} \frac{\tan 2x - 2\sin \alpha x}{x(1-\cos 2x)} = \beta,$$

then $\alpha + \beta$ equals

(A) $\dfrac{1}{2}$
(B) 1
(C) $\dfrac{3}{2}$
(D) $\dfrac{5}{2}$

Q14. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{2x^3 + 3y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Let $f_x(0,0)$ and $f_y(0,0)$ denote first order partial derivatives of $f(x,y)$ at $(0,0)$. Which one of the following statements is TRUE?

(A) f is continuous at $(0,0)$ but $f_x(0,0)$ and $f_y(0,0)$ do not exist

(B) f is differentiable at $(0,0)$

(C) f is not differentiable at $(0,0)$ but $f_x(0,0)$ and $f_y(0,0)$ exist

(D) f is not continuous at $(0,0)$ but $f_x(0,0)$ and $f_y(0,0)$ exist

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Q15. If the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$ is given by

$$\int_0^{\alpha} \int_{\beta(y)}^{2y} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} dx\, dz\, dy$$

then

(A) $\alpha = 2$ and $\beta(y) = y$, $y \in [0,2]$

(B) $\alpha = 1$ and $\beta(y) = y^2$, $y \in [0,1]$

(C) $\alpha = 2$ and $\beta(y) = y^2$, $y \in [0,2]$

(D) $\alpha = 1$ and $\beta(y) = y$, $y \in [0,1]$

Q16. The value of the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2}\, dy\, dx$$

is

(A) $\dfrac{17}{9}$

(B) $\dfrac{16}{9}$

(C) $\dfrac{14}{9}$

(D) $\frac{13}{9}$

Q17. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation. If $T(1,1,0) = (2,0,0,0)$, $T(1,0,1) = (2,4,0,0)$, and $T(0,1,1) = (0,0,2,0)$, then $T(1,1,1)$ equals

(A) $(1,1,0,0)$

(B) $(0,1,1,1)$

(C) $(2,2,1,0)$

(D) $(0,0,0,0)$

Q18. Let M be an $n \times n$ non-zero skew symmetric matrix. Then the matrix $(I_n - M)(I_n + M)^{-1}$ is always

(A) Singular

(B) Symmetric

(C) Orthogonal

(D) Idempotent

Q19. A packet contains 10 distinguishable firecrackers out of which 4 are defective. If three firecrackers are drawn at random (without replacement) from the packet, then the probability that all three firecrackers are defective equals

(A) $\frac{1}{10}$

(B) $\frac{1}{20}$

(C) $\frac{1}{30}$

(D) $\frac{1}{40}$

Q20. Let X_1, X_2, X_3, X_4 be i.i.d. random variables having a continuous distribution. Then $P(X_3 < X_2 < \max(X_1, X_4))$ equals

(A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) $\frac{1}{4}$
(D) $\frac{1}{6}$

Q21. Consider the simple linear regression model $y_i = \alpha + \beta x_i + \varepsilon_i$, where ε_i are i.i.d. random variables with mean 0 and variance σ^2. Given that $n = 20$, $\sum x_i = 100$, $\sum y_i = 50$, $\sum x_i^2 = 600$, $\sum y_i^2 = 500$, and $\sum x_i y_i = 400$, find the least squares estimates of α and β.

(A) 5 and $\frac{3}{2}$
(B) -5 and $\frac{3}{2}$
(C) 5 and $-\frac{3}{2}$
(D) -5 and $-\frac{3}{2}$

Q22. Let Z_1 and Z_2 be i.i.d. $N(0,1)$ random variables. If $Y = Z_1^2 + Z_2^2$, then $P(Y > 4)$ equals

(A) e^{-2}
(B) $1 - e^{-2}$
(C) $\frac{1}{2}e^{-2}$
(D) e^{-4}

Q23. Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{3}$. Let X denote the number of trials required to get the second success. Then $P(X \geq 5)$ equals

(A) $\frac{3}{7}$

(B) $\frac{16}{27}$

(C) $\frac{16}{21}$

(D) $\frac{9}{13}$

Q24. Let the joint probability density function of (X, Y) **be**

$$f(x,y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P\left(X < \frac{Y}{2}\right)$ **equals**

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{1}{2}$

Q25. Let X_1, X_2, X_3, X_4, X_5 **be a random sample from** $N(0,1)$ **distribution and let**

$$W = \frac{X_1^2}{X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2}.$$

Then $E(W)$ **equals**

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

Q26. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a random sample from $N(\mu_2, \sigma^2)$ distribution, where $\mu_i \in \mathbb{R}, i = 1, 2$ and $\sigma > 0$. Suppose that the two random samples are independent. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad W = \frac{\sqrt{mn}(\bar{X} - \mu_1)}{\sqrt{\sum_{i=1}^{m}(Y_i - \mu_2)^2}}.$$

Then which one of the following statements is TRUE for all positive integers m and n?

(A) $W \sim t_m$

(B) $W \sim t_n$

(C) $W^2 \sim F_{1,m}$

(D) $W^2 \sim F_{m,n}$

Q27. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta - 0.5, \theta + 0.5)$ distribution, where $\theta \in \mathbb{R}$. If $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$ and $X_{(n)} = \max(X_1, X_2, \ldots, X_n)$, then which one of the following estimators is NOT a maximum likelihood estimator (MLE) of θ?

(A) $\frac{1}{2}(X_{(1)} + X_{(n)})$

(B) $\frac{1}{4}(3X_{(1)} + X_{(n)} + 1)$

(C) $\frac{1}{4}(X_{(1)} + 3X_{(n)} - 1)$

(D) $\frac{1}{2}(3X_{(n)} - X_{(1)} - 2)$

Q28. Let $X_1, X_2, \ldots, X_n$ be a random sample from Exp(θ) distribution, where $\theta \in (0, \infty)$. If $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then a 95% confidence interval for θ is

(A) $\left(0, \frac{\chi^2_{2n,0.95}}{n\bar{X}}\right)$

(B) $\left(\frac{\chi^2_{2n,0.95}}{n\bar{X}}, \infty\right)$

(C) $\left(0, \frac{\chi^2_{2n,0.05}}{2n\bar{X}}\right)$

(D) $\left(\frac{\chi^2_{2n,0.05}}{2n\bar{X}}, \infty\right)$

Q29. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(1,2)$ and $Y_1, Y_2, \ldots, Y_n$ be a random sample from $U(0,1)$. Suppose the two samples are independent. Define

$$Z_i = \begin{cases} 1, & \text{if } X_i Y_i < 1, \\ 0, & \text{otherwise}, \end{cases} \qquad i = 1, 2, \ldots, n.$$

If $\lim_{n\to\infty} P\left(|\frac{1}{n}\sum_{i=1}^{n} Z_i - \theta| < \epsilon\right) = 1$ for all $\epsilon > 0$, **then** θ **equals**

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\log_e \frac{3}{2}$

(D) $\log_e 2$

Q30. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function

$$f_\theta(x) = \begin{cases} \theta(1-x)^{\theta-1}, & 0 < x < 1, \\ 0, & \text{otherwise}, \end{cases} \qquad \theta > 0.$$

To test $H_0 : \theta = 1$ **against** $H_1 : \theta > 1$**, the uniformly most powerful (UMP) test of size** α **would reject** H_0 **if**

(A) $-\sum_{i=1}^{n} \log_e(1 - X_i)^2 < \chi^2_{2n,1-\alpha}$

(B) $-\sum_{i=1}^{n} \log_e(1 - X_i)^2 < \chi^2_{n,1-\alpha}$

(C) $-\sum_{i=1}^{n} \log_e(1 - X_i)^2 < \chi^2_{2n,\alpha}$

(D) $\sum_{i=1}^{n} \log_e(1 - X_i)^2 < \chi^2_{n,\alpha}$

Q31. Let the sequence $\{x_n\}_{n\geq 1}$ be given by $x_n = \sin\frac{n\pi}{6}$, $n = 1, 2, \ldots$. Then which of the following statements is/are TRUE?

(A) The sequence $\{x_n\}$ has a subsequence that converges to $\frac{1}{2}$

(B) $\limsup_{n\to\infty} x_n = 1$

(C) $\liminf_{n\to\infty} x_n = -1$

(D) The sequence $\{x_n\}$ has a subsequence that converges to $\dfrac{1}{\sqrt{2}}$

Q32. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2(2 - y) - y^3 + 3y^2 + 9y$, where $(x, y) \in \mathbb{R}^2$. Which of the following is/are saddle point(s) of f?

(A) $(0, -1)$

(B) $(0, 3)$

(C) $(3, 2)$

(D) $(-3, 2)$

Q33. The arc length of the parabola $y^2 = 2x$ intercepted between the points of intersection of the parabola $y^2 = 2x$ and the straight line $y = 2x$ equals

(A) $\int_0^1 \sqrt{1 + 4y^2}\, dy$

(B) $\int_0^1 \sqrt{1 + 4y^2}\, dy$

(C) $\int_0^{1/2} \dfrac{\sqrt{1 + 4x}}{\sqrt{2x}}\, dx$

(D) $\int_0^{1/2} \dfrac{\sqrt{1 + 4x}}{\sqrt{2x}}\, dx$

Q34. For real constants a and b, let

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ a & b \end{bmatrix}$$

be an orthogonal matrix. Then which of the following statements is/are always TRUE?

(A) $a + b = 0$

(B) $b = \sqrt{1 - a^2}$

(C) $ab = -\dfrac{1}{2}$

(D) $M^2 = I_2$

Q35. Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{5}$. Then which of the following statements is/are TRUE?

(A) Expected number of trials required to get the first success is 5

(B) Expected number of successes in first 50 trials is 10

(C) Expected number of failures preceding the first success is 4

(D) Expected number of trials required to get the second success is 10

Q36. Let (X, Y) have the joint probability mass function

$$f(x,y) = \begin{cases} \binom{x+1}{y}\binom{16}{x}\left(\frac{1}{6}\right)^y\left(\frac{5}{6}\right)^{x+1-y}\left(\frac{1}{2}\right)^{16}, & y = 0, 1, \ldots, x+1;\ x = 0, 1, \ldots, 16, \\ 0, & \text{otherwise.} \end{cases}$$

Then which of the following statements is/are TRUE?

(A) $E(Y) = \frac{3}{2}$

(B) $\text{Var}(Y) = \frac{49}{36}$

(C) $E(XY) = \frac{37}{3}$

(D) $\text{Var}(X) = 3$

Q37. Let X_1, X_2, X_3 be i.i.d. $N(0, 1)$ random variables. Then which of the following statements is/are TRUE?

(A) $\dfrac{\sqrt{2}X_1}{\sqrt{X_2^2 + X_3^2}} \sim t_2$

(B) $\dfrac{\sqrt{2}X_1}{|X_2 + X_3|} \sim t_1$

(C) $\dfrac{(X_1 - X_2)^2}{(X_1 + X_2)^2} \sim F_{1,1}$

(D) $\sum_{i=1}^{3} X_i^2 \sim \chi_2^2$

Q38. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that

$$P(X_1 = 0) = \frac{1}{4}, \quad P(X_1 = 1) = \frac{3}{4}.$$

Define

$$U_n = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad V_n = \frac{1}{n}\sum_{i=1}^{n}(1 - X_i)^2, \quad n = 1, 2, \ldots$$

Then which of the following statements is/are TRUE?

(A) $\lim_{n\to\infty} P\left(|U_n - \frac{3}{4}| < \frac{1}{100}\right) = 1$

(B) $\lim_{n\to\infty} P\left(|U_n - \frac{3}{4}| > \frac{1}{100}\right) = 0$

(C) $\lim_{n\to\infty} P\left(\sqrt{n}\left(U_n - \frac{3}{4}\right) \leq 1\right) = \Phi(2)$

(D) $\lim_{n\to\infty} P\left(\sqrt{n}\left(V_n - \frac{1}{4}\right) \leq 1\right) = \Phi\left(\frac{4}{\sqrt{3}}\right)$

Q39. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Poisson(λ) random variables, where $\lambda > 0$. Define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

Then which of the following statements is/are TRUE?

(A) $\mathrm{Var}(\bar{X}) < \mathrm{Var}(S^2)$

(B) $\mathrm{Var}(\bar{X}) = \mathrm{Var}(S^2)$

(C) $\mathrm{Var}(\bar{X})$ attains the Cramer–Rao lower bound

(D) $E(\bar{X}) = E(S^2)$

Q40. Consider the two probability density functions (pdfs):

$$f_0(x) = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise,} \end{cases} \qquad f_1(x) = \begin{cases} 1, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let X be a random variable with pdf $f \in \{f_0, f_1\}$. Consider testing $H_0 : f = f_0(x)$ against $H_1 : f = f_1(x)$ at $\alpha = 0.05$ level of significance. For which observed value of X, the most powerful test would reject H_0?

(A) 0.19

(B) 0.22

(C) 0.25

(D) 0.28

Q41. Evaluate:

$$\lim_{n\to\infty}\left(\frac{\sqrt{n^2+1}+n}{\sqrt[3]{n^6+1}}\right)^2$$

Q42. The maximum value of the function

$$y = \frac{x^2}{x^4+4}, \quad x \in \mathbb{R},$$

is ....

Q43. The value of the integral

$$\int_0^1 \int_{y^2}^1 \frac{e^x}{\sqrt{x}}\, dx\, dy$$

equals .... (round off to two decimal places):

Q44. Find the rank of the matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 5 & 6 & 4 \\ 2 & 6 & 8 & 5 \end{bmatrix}$$

Rank = ?

Q45. Let (X, Y) have the joint pdf

$$f(x,y) = \begin{cases} \frac{3}{4}(y-x), & 0 < x < y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then the conditional expectation $E(X|Y=1)$ equals .... (round off to two decimal places).

Q46. Let X be a random variable having the Poisson(4) distribution and let E be an event such that $P(E|X=i) = 1-2^{-i}$, $i = 0, 1, 2, \ldots$. Then $P(E)$ equals .... (round off to two decimal places).

Q47. Let $X_1, X_2,$ and X_3 be independent random variables such that $X_1 \sim N(47, 10)$, $X_2 \sim N(55, 15)$, and $X_3 \sim N(60, 14)$. Then $P(X_1 + X_2 \geq 2X_3)$ equals .... (round off to two decimal places).

Q48. Let $U \sim F_{5,8}$ and $V \sim F_{8,5}$. If $P[U > 3.69] = 0.05$, then the value of c such that $P[V > c] = 0.95$ equals .... (round off to two decimal places).

Q49. Let the sample mean based on a random sample from Exp(λ) distribution be 3.7. Then the maximum likelihood estimate of $1 - e^{-\lambda}$ equals .... (round off to two decimal places).

Q50. Let X be a single observation drawn from $U(0, \theta)$ distribution, where $\theta \in \{1, 2\}$. To test $H_0 : \theta = 1$ against $H_1 : \theta = 2$, consider the test procedure that rejects H_0 if and only if $X > 0.75$. If the probabilities of Type-I and Type-II errors are α and β, respectively, then $\alpha + \beta$ equals .... (round off to two decimal places).

Q51. Let $f : [-1, 3] \to \mathbb{R}$ be a continuous function such that f is differentiable on $(-1, 3)$, $|f'(x)| \leq \dfrac{3}{2}$ for all $x \in (-1, 3)$, $f(-1) = 1$ and $f(3) = 7$. Then $f(1)$ equals ....

Q52. Let α be the real number such that the coefficient of x^{125} in Maclaurin's series of $(x+\alpha^3)^3 e^x$ is $\frac{28}{124!}$. Then α equals ....

Q53. Consider the matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 4 \end{bmatrix}$. Let P be a nonsingular matrix such that $P^{-1}MP$ is a diagonal matrix. Then the trace of the matrix $P^{-1}M^3P$ equals ....

Q54. Let P be a 3×3 matrix having characteristic roots $\lambda_1 = -\frac{2}{3}$, $\lambda_2 = 0$ and $\lambda_3 = 1$. Define $Q = 3P^3 - P^2 - P + I_3$ and $R = 3P^3 - 2P$. If $\alpha = \det(Q)$ and $\beta =$ trace(R), then $\alpha + \beta$ equals .... (round off to two decimal places).

Q55. Let X and Y be independent random variables with respective moment generating functions $M_X(t) = \frac{(8+e^t)^2}{81}$ and $M_Y(t) = \frac{(1+3e^t)^3}{64}$, $-\infty < t < \infty$. Then $P(X+Y=1)$ equals .... (round off to two decimal places).

Q56. Let X be a random variable having $U(0,10)$ distribution and $Y = X - [X]$, where $[X]$ denotes the greatest integer less than or equal to X. Then $P(Y > 0.25)$ equals ....

Q57. A computer lab has two printers handling certain types of printing jobs. Printer-I and Printer-II handle 40% and 60% of the jobs, respectively. For a typical printing job, printing time (in minutes) of Printer-I follows $N(10,4)$ and that of Printer-II follows $U(1,21)$. If a randomly selected printing job is found to have been completed in less than 10 minutes, then the conditional probability that it was handled by the Printer-II equals .... (round off to two decimal places).

Q58. Let $x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1, x_5 = 0, x_6 = 1$ be the data on a random sample of size 6 from Bin(1, θ) distribution, where $\theta \in (0, 1)$. Then the uniformly minimum variance unbiased estimate of $\theta(1 + \theta)$ equals ....

Q59. Let $x_1 = 1, x_2 = 4$ be the data on a random sample of size 2 from a Poisson(θ) distribution, where $\theta \in (0, \infty)$. Let $\hat{\psi}$ be the uniformly minimum variance unbiased estimate of $\psi(\theta) = \sum_{k=4}^{\infty} e^{-\theta} \frac{\theta^k}{k!}$ based on the given data. Then $\hat{\psi}$ equals .... (round off to two decimal places).

Q60. Let X be a random variable having $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$. Consider testing $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$ at $\alpha = 0.617$ level of significance. The power of the likelihood ratio test at $\theta = 1$ equals .... (round off to two decimal places).
