IIT JAM 2020 Mathematics (MA) Question Paper

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

1. Let $s_n = 1 + \frac{(-1)^n}{n}$, $n \in \mathbb{N}$. **Then the sequence** $\{s_n\}$ **is**

(A) monotonically increasing and is convergent to 1

(B) monotonically decreasing and is convergent to 1

(C) neither monotonically increasing nor monotonically decreasing but is convergent to 1

(D) divergent

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2. Let $f(x) = 2x^3 - 9x^2 + 7$. **Which of the following is true?**

(A) f is one-one in the interval $[-1, 1]$

(B) f is one-one in the interval $[2, 4]$

(C) f is NOT one-one in the interval $[-4, 0]$

(D) f is NOT one-one in the interval $[0, 4]$

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3. Which of the following is FALSE?

(A) $\lim_{x\to\infty} \frac{x}{e^x} = 0$

(B) $\lim_{x\to 0^+} \frac{1}{xe^{1/x}} = 0$

(C) $\lim_{x\to 0^+} \frac{\sin x}{1+2x} = 0$

(D) $\lim_{x\to 0^+} \frac{\cos x}{1+2x} = 0$

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4. Let $g : \mathbb{R} \to \mathbb{R}$ **be a twice differentiable function. If** $f(x, y) = g(y) + xg'(y)$**, then**

(A) $\frac{\partial f}{\partial x} + y\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y}$

(B) $\frac{\partial f}{\partial y} + y\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x}$

(C) $\frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y}$

(D) $\frac{\partial f}{\partial y} + x\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x}$

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5. If the equation of the tangent plane to the surface $z = 16 - x^2 - y^2$ at the point $P(1, 3, 6)$ is $ax + by + cz + d = 0$, then the value of $|d|$ is

(A) 16

(B) 26

(C) 36

(D) 46

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6. If the directional derivative of the function $z = y^2 e^{2x}$ at $(2, -1)$ along the unit vector $\vec{b} = \alpha\hat{i} + \beta\hat{j}$ is zero, then $|\alpha + \beta|$ equals

(A) $\dfrac{1}{2\sqrt{2}}$

(B) $\dfrac{1}{\sqrt{2}}$

(C) $\sqrt{2}$

(D) $2\sqrt{2}$

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7. If $u = x^3$ and $v = y^2$ transform the differential equation $3x^5 dx - y(y^2 - x^3)dy = 0$ to $\dfrac{dv}{du} = \dfrac{\alpha u}{2(u - v)}$, then α is

(A) 4

(B) 2

(C) -2

(D) -4

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8. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(x, y) = (-x, y)$. Then

(A) $T^{2k} = T$ for all $k \geq 1$

(B) $T^{2k+1} = -T$ for all $k \geq 1$

(C) The range of T^2 is a proper subspace of the range of T

(D) The range of T^2 is equal to the range of T

9. The radius of convergence of the power series $\sum_{n=1}^{\infty} \left(\frac{n+2}{n}\right)^{n^2} x^n$ is

(A) e^2
(B) $\frac{1}{\sqrt{e}}$
(C) $\frac{1}{e}$
(D) $\frac{1}{e^2}$

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10. Consider the following group under matrix multiplication:

$$H = \left\{ \begin{bmatrix} 1 & p & q \\ 0 & 1 & r \\ 0 & 0 & 1 \end{bmatrix} : p, q, r \in \mathbb{R} \right\}.$$

Then the center of the group is isomorphic to

(A) $(\mathbb{R} \setminus \{0\}, \times)$

(B) $(\mathbb{R}, +)$

(C) $(\mathbb{R}^2, +)$

(D) $(\mathbb{R}, +) \times (\mathbb{R} \setminus \{0\}, \times)$

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11. Let $\{a_n\}$ be a sequence of positive real numbers. Suppose that $l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Which of the following is true?

(A) If $l = 1$, then $\lim_{n\to\infty} a_n = 1$

(B) If $l = 1$, then $\lim_{n\to\infty} a_n = 0$

(C) If $l < 1$, then $\lim_{n\to\infty} a_n = 1$

(D) If $l < 1$, then $\lim_{n\to\infty} a_n = 0$

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12. Define $s_1 = \alpha > 0$ **and** $s_{n+1} = \sqrt{\dfrac{1+s_n^2}{1+\alpha}}$, $n \geq 1$. **Which of the following is true?**

(A) If $s_n^2 < \dfrac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim\limits_{n\to\infty} s_n = \dfrac{1}{\sqrt{\alpha}}$

(B) If $s_n^2 < \dfrac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim\limits_{n\to\infty} s_n = \dfrac{1}{\alpha}$

(C) If $s_n^2 > \dfrac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim\limits_{n\to\infty} s_n = \dfrac{1}{\sqrt{\alpha}}$

(D) If $s_n^2 > \dfrac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim\limits_{n\to\infty} s_n = \dfrac{1}{\sqrt{\alpha}}$

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13. Suppose that S **is the sum of a convergent series** $\sum\limits_{n=1}^{\infty} a_n$. **Define** $t_n = a_n + a_{n+1} + a_{n+2}$.

Then the series $\sum\limits_{n=1}^{\infty} t_n$

(A) diverges

(B) converges to $3S - a_1 - a_2$

(C) converges to $3S - a_1 - 2a_2$

(D) converges to $3S - 2a_1 - a_2$

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14. Let $a \in \mathbb{R}$. **If** $f(x) = \begin{cases} (x+a)^2, & x \leq 0 \\ (x+a)^3, & x > 0 \end{cases}$**, then**

(A) $\dfrac{d^2f}{dx^2}$ does not exist at $x = 0$ for any value of a

(B) $\dfrac{d^2f}{dx^2}$ exists at $x = 0$ for exactly one value of a

(C) $\dfrac{d^2f}{dx^2}$ exists at $x = 0$ for exactly two values of a

(D) $\dfrac{d^2f}{dx^2}$ exists at $x = 0$ for infinitely many values of a

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15. Let $f(x,y) = \begin{cases} x^2 \sin\frac{1}{x} + y^2 \sin\frac{1}{y}, & xy \neq 0 \\ x^2 \sin\frac{1}{x}, & x \neq 0, y = 0 \\ y^2 \sin\frac{1}{y}, & y \neq 0, x = 0 \\ 0, & x = y = 0 \end{cases}$**. Which of the following is true at** $(0,0)$?

(A) f is not continuous
(B) $\dfrac{\partial f}{\partial x}$ is continuous but $\dfrac{\partial f}{\partial y}$ is not continuous
(C) f is not differentiable
(D) f is differentiable but both $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ are not continuous

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16. Let S **be the surface of the portion of the sphere with centre at the origin and radius 4, above the** xy**-plane. Let** $\vec{F} = y\hat{i} - x\hat{j} + yxz^3\hat{k}$. **If** $\hat{n}$ **is the unit outward normal to** S**, then**

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$$

equals

(A) -32π

(B) -16π

(C) 16π

(D) 32π

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17. Let $f(x,y,z) = x^3 + y^3 + z^3 - 3xyz$. **A point at which the gradient of** f **is equal to zero is**

(A) $(-1, 1, -1)$

(B) $(-1, -1, -1)$

(C) $(-1, 1, 1)$

(D) $(1, -1, 1)$

18. The area bounded by the curves $x^2 + y^2 = 2x$ and $x^2 + y^2 = 4x$, and the straight lines $y = x$ and $y = 0$ is

(A) $3\left(\frac{\pi}{2} + \frac{1}{4}\right)$
(B) $3\left(\frac{\pi}{4} + \frac{1}{2}\right)$
(C) $2\left(\frac{\pi}{4} + \frac{1}{3}\right)$
(D) $2\left(\frac{\pi}{3} + \frac{1}{4}\right)$

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19. Let M be a real 6×6 matrix. Let 2 and -1 be two eigenvalues of M. If $M^5 = aI + bM$, where $a, b \in \mathbb{R}$, then

(A) $a = 10, b = 11$

(B) $a = -11, b = 10$

(C) $a = -10, b = 11$

(D) $a = 10, b = -11$

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20. Let M be an $n \times n$ ($n \geq 2$) non-zero real matrix with $M^2 = 0$ and let $\alpha \in \mathbb{R} \setminus \{0\}$. Then

(A) α is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(B) α is the only eigenvalue of $(M + \alpha I)$ and $(\alpha I - M)$

(C) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(D) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(I - \alpha M)$

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21. Consider the differential equation $L[y] = (y - y^2)dx + xdy = 0$. The function $f(x, y)$ is said to be an integrating factor of the equation if $f(x, y)L[y] = 0$ becomes exact. If $f(x, y) = \frac{1}{x^2 y^2}$, then

(A) f is an integrating factor and $y = 1 - kxy,\ k \in \mathbb{R}$ is NOT its general solution

(B) f is an integrating factor and $y = -1 + kxy,\ k \in \mathbb{R}$ is its general solution

(C) f is an integrating factor and $y = -1 + kxy,\ k \in \mathbb{R}$ is NOT its general solution

(D) f is NOT an integrating factor and $y = 1 + kxy,\ k \in \mathbb{R}$ is its general solution

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22. A solution of the differential equation $2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - y = 0,\ x > 0$ **that passes through the point (1, 1) is**

(A) $y = \frac{1}{x}$

(B) $y = \frac{1}{x^2}$

(C) $y = \frac{1}{\sqrt{x}}$

(D) $y = \frac{1}{x^{3/2}}$

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23. Let M **be a** 4×3 **real matrix and let** $\{e_1, e_2, e_3\}$ **be the standard basis of** $\mathbb{R}^3$. **Which of the following is true?**

(A) If rank(M) = 1, then $\{Me_1, Me_2\}$ is a linearly independent set

(B) If rank(M) = 2, then $\{Me_1, Me_2\}$ is a linearly independent set

(C) If rank(M) = 2, then $\{Me_1, Me_3\}$ is a linearly independent set

(D) If rank(M) = 3, then $\{Me_1, Me_3\}$ is a linearly independent set

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24. The value of the triple integral $\iiint_V (x^2y + 1)\, dx\, dy\, dz$**, where** V **is the region given by** $x^2 + y^2 \leq 1,\ 0 \leq z \leq 2$, **is**

(A) π

(B) 2π

(C) 3π

(D) 4π

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25. Let S be the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 0$ and $z = 1$. Then the value of the surface integral $\iint_S (x^2 + y^2)\, dS$ is

(A) π

(B) $\dfrac{\pi}{\sqrt{2}}$

(C) $\dfrac{\pi}{\sqrt{3}}$

(D) $\dfrac{\pi}{2}$

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26. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},\ x, y, z \in \mathbb{R}$. Which of the following is FALSE?

(A) $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$

(B) $\nabla \cdot (\vec{a} \times \vec{r}) = 0$

(C) $\nabla \times (\vec{a} \times \vec{r}) = \vec{a}$

(D) $\nabla \cdot ((\vec{a} \cdot \vec{r})\vec{r}) = 4(\vec{a} \cdot \vec{r})$

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27. Let $D = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$ and $f : D \to \mathbb{R}$ be a non-constant continuous function. Which of the following is TRUE?

(A) The range of f is unbounded

(B) The range of f is a union of open intervals

(C) The range of f is a closed interval

(D) The range of f is a union of at least two disjoint closed intervals

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28. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that $f\left(\frac{1}{2}\right) = -\frac{1}{2}$ and

$$|f(x) - f(y) - (x - y)| \leq \sin(|x - y|^2)$$

for all $x, y \in [0, 1]$. Then $\int_0^1 f(x)\, dx$ is

(A) $-\dfrac{1}{2}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

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29. Let $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ **be the circle group under multiplication and** $i = \sqrt{-1}$. **Then the set** $\{\theta \in \mathbb{R} : (e^{i2\pi\theta})$ **is infinite**$\}$ **is**

(A) empty

(B) non-empty and finite

(C) countably infinite

(D) uncountable

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30. Let $F = \{\omega \in \mathbb{C} : \omega^{2020} = 1\}$. **Consider the groups**

$$G = \left\{ \begin{pmatrix} \omega & z \\ 0 & 1 \end{pmatrix} : \omega \in F, z \in \mathbb{C} \right\} \quad \text{and} \quad H = \left\{ \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} : z \in \mathbb{C} \right\}$$

under matrix multiplication. Then the number of cosets of H **in** G **is**

(A) 1010

(B) 2019

(C) 2020

(D) infinite

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31. Let $a, b, c \in \mathbb{R}$ **such that** $a < b < c$. **Which of the following is/are true for any continuous function** $f : \mathbb{R} \to \mathbb{R}$ **satisfying** $f(a) = b, f(b) = c$ **and** $f(c) = a$?

(A) There exists $\alpha \in (a, c)$ such that $f(\alpha) = \alpha$

(B) There exists $\beta \in (a, b)$ such that $f(\beta) = \beta$

(C) There exists $\gamma \in (a, b)$ such that $(f \circ f)(\gamma) = \gamma$

(D) There exists $\delta \in (a, c)$ such that $(f \circ f \circ f)(\delta) = \delta$

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32. If $s_n = \dfrac{(-1)^n}{2^n + 3}$ and $t_n = \dfrac{(-1)^n}{4n - 1}$, $n = 0, 1, 2, \ldots,$ then

(A) $\sum_{n=0}^{\infty} s_n$ is absolutely convergent

(B) $\sum_{n=0}^{\infty} t_n$ is absolutely convergent

(C) $\sum_{n=0}^{\infty} s_n$ is conditionally convergent

(D) $\sum_{n=0}^{\infty} t_n$ is conditionally convergent

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33. Let $a, b \in \mathbb{R}$ and $a < b$. Which of the following statement(s) is/are true?

(A) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is one-one

(B) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is onto

(C) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is one-one

(D) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is onto

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34. Let V be a non-zero vector space over a field F. Let $S \subset V$ be a non-empty set. Consider the following properties of S:

(I) For any vector space W over F, any map $f : S \to W$ extends to a linear map from V to W.

(II) For any vector space W over F and any two linear maps $f, g : V \to W$ satisfying $f(s) = g(s)$ for all $s \in S$, we have $f(v) = g(v)$ for all $v \in V$.

(III) S is linearly independent.

(IV) The span of S is V.

Which of the following statement(s) is/are true?

(A) (I) implies (IV)

(B) (I) implies (III)

(C) (II) implies (III)

(D) (II) implies (IV)

35. Let $L[y] = x^2\dfrac{d^2y}{dx^2} + px\dfrac{dy}{dx} + qy,$ where p, q are real constants. Let $y_1(x)$ and $y_2(x)$ be two solutions of $L[y] = 0,\ x > 0,$ that satisfy $y_1(x_0) = 1, y_1'(x_0) = 0, y_2(x_0) = 0, y_2'(x_0) = 1$ for some $x_0 > 0$. Then,

(A) $y_1(x)$ is not a constant multiple of $y_2(x)$

(B) $y_1(x)$ is a constant multiple of $y_2(x)$

(C) $1, \ln x$ are solutions of $L[y] = 0$ when $p = 1, q = 0$

(D) $x, \ln x$ are solutions of $L[y] = 0$ when $p + q \neq 0$

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36. Consider the following system of linear equations:

$$\begin{cases} x + y + 5z = 3, \\ x + 2y + mz = 5, \\ x + 2y + 4z = k. \end{cases}$$

The system is consistent if

(A) $m \neq 4$

(B) $k \neq 5$

(C) $m = 4$

(D) $k = 5$

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37. Let $a = \lim_{n\to\infty}\left(\frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{n-1}{n^2}\right)$ and $b = \lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}\right)$.
Which of the following is/are true?

(A) $a > b$

(B) $a < b$

(C) $ab = \ln\sqrt{2}$

(D) $\dfrac{a}{b} = \ln\sqrt{2}$

38. Let S be that part of the surface of the paraboloid $z = 16 - x^2 - y^2$ which is above the plane $z = 0$ and D be its projection on the xy-plane. Then the area of S equals

(A) $\iint_D \sqrt{1 + 4(x^2 + y^2)}\, dx\, dy$

(B) $\iint_D \sqrt{1 + 2(x^2 + y^2)}\, dx\, dy$

(C) $\int_0^{2\pi} \int_0^4 \sqrt{1 + 4r^2}\, dr\, d\theta$

(D) $\int_0^{2\pi} \int_0^4 \sqrt{1 + 4r^2}\, r\, dr\, d\theta$

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39. Let f be a real-valued function of a real variable, such that $|f^{(n)}(0)| \leq K$ for all $n \in \mathbb{N}$, where $K > 0$. Which of the following is/are true?

(A) $\dfrac{|f^{(n)}(0)|^{1/n}}{n!} \to 0$ as $n \to \infty$

(B) $\dfrac{|f^{(n)}(0)|^{1/n}}{n!} \to \infty$ as $n \to \infty$

(C) $f^{(n)}(x)$ exists for all $x \in \mathbb{R}$ and all $n \in \mathbb{N}$

(D) $\sum_{n=1}^{\infty} \dfrac{f^{(n)}(0)}{(n-1)!}$ is absolutely convergent

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40. Let G be a group with identity e. Let H be an abelian non-trivial proper subgroup of G with the property that $H \cap gHg^{-1} = \{e\}$ for all $g \notin H$. If $K = \{g \in G : gh = hg$ for all $h \in H\}$, then

(A) K is a proper subgroup of H

(B) H is a proper subgroup of K

(C) $K = H$

(D) There exists no abelian subgroup $L \subseteq G$ such that K is a proper subgroup of L

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41. Let $x_n = n^{1/n}$ and $y_n = e^{1-x_n}$, $n \in \mathbb{N}$. Then the value of $\lim_{n\to\infty} y_n$ is ....

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42. Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S be the sphere given by $(x-2)^2 + (y-2)^2 + (z-2)^2 = 4$. If $\hat{n}$ is the unit outward normal to S, then $\frac{1}{\pi}\iint_S \vec{F} \cdot \hat{n}\, dS$ is ....

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43. Let $f : \mathbb{R} \to \mathbb{R}$ be such that f, f', f'' are continuous with $f > 0, f' > 0, f'' > 0$. Then $\lim_{x \to -\infty} \dfrac{f(x) + f'(x)}{2}$ is ....

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44. Let $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ and $f : S \to \mathbb{R}$ be defined by $f(x) = \frac{1}{x}$. Then

$$\max\left\{\delta : |x - \tfrac{1}{3}| < \delta \Rightarrow |f(x) - f(\tfrac{1}{3})| < 1\right\}$$

is .... (rounded off to two decimal places).

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45. Let $f(x, y) = e^x \sin y,\ x = t^3 + 1,\ y = t^4 + t$. Then $\dfrac{df}{dt}$ at $t = 0$ is .... (rounded off to two decimal places).

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46. Consider the differential equation

$$\frac{dy}{dx} + 10y = f(x), \quad x > 0,$$

where $f(x)$ is a continuous function such that $\lim_{x\to\infty} f(x) = 1$. Then the value of $\lim_{x\to\infty} y(x)$ is ....

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47. If $\displaystyle\int_0^1 \int_{2y}^2 e^{x^2}\, dx\, dy = k(e^4 - 1)$, then k equals ....

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48. Let $f(x, y) = 0$ be a solution of the homogeneous differential equation

$(2x + 5y)dx - (x + 3y)dy = 0.$

If $f(x+\alpha, y-3)=0$ **is a solution of** $(2x+5y-1)dx+(2-x-3y)dy=0$, **then the value of** α **is ....**

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49. Consider the real vector space $P_{2020}=\left\{\sum_{i=0}^{n} a_i x^i : a_i \in \mathbb{R},\ 0 \leq n \leq 2020\right\}$. **Let** W **be the subspace given by**

$$W=\left\{\sum_{i=0}^{n} a_i x^i \in P_{2020} : a_i = 0 \text{ for all odd } i\right\}.$$

Then the dimension of W **is ....**

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50. Let $\phi : S_3 \to S_1$ **be a non-trivial non-injective group homomorphism. Then the number of elements in the kernel of** ϕ **is ....**

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51. The sum of the series

$$\frac{1}{2(2^2-1)}+\frac{1}{3(3^2-1)}+\frac{1}{4(4^2-1)}+\cdots$$

is ....

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52. Consider the expansion of the function $f(x)=\dfrac{3}{(1-x)(1+2x)}$ **in powers of** x**, valid in** $|x|<\dfrac{1}{2}$**. Then the coefficient of** x^4 **is ....**

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53. The minimum value of the function $f(x,y)=x^2+xy+y^2-3x-6y+11$ **is ....**

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54. Let $f(x)=\sqrt{x}+\alpha x,\ x>0$ **and**

$$g(x)=a_0+a_1(x-1)+a_2(x-1)^2$$

be the sum of the first three terms of the Taylor series of $f(x)$ around $x = 1$. If $g(3) = 3$, then α is ....

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55. Let C be the boundary of the square with vertices $(0,0), (1,0), (1,1), (0,1)$ oriented counterclockwise. Then the value of the line integral

$$\oint_C x^2y^2dx + (x^2 - y^2)dy$$

is .... (rounded off to two decimal places).

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56. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f'(x) = f(x)$ for all x. Suppose that $f(\alpha x)$ and $f(\beta x)$ are two non-zero solutions of the differential equation

$$4\frac{d^2y}{dx^2} - p\frac{dy}{dx} + 3y = 0$$

satisfying $f(\alpha x)f(\beta x) = f(2x)$ and $f(\alpha x)f(-\beta x) = f(x)$. Then, the value of p is ....

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57. If $x^2 + xy^2 = c$, where $c \in \mathbb{R}$, is the general solution of the exact differential equation

$$M(x,y)\,dx + 2xy\,dy = 0,$$

then $M(1,1)$ is ....

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58. Let

$$M = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}.$$

Then, the value of $\det((8I - M)^3)$ is ....

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59. Let $T : \mathbb{R}^7 \to \mathbb{R}^7$ be a linear transformation with Nullity$(T) = 2.$ Then, the minimum possible value for Rank(T^2) is ....

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60. Suppose that G is a group of order 57 which is not cyclic. If G contains a unique subgroup H of order 19, then for any $g \notin H$, the order of g is ....
