

## IIT JAM 2021 Economics (EN) Question Paper

<b>Time Allowed :3 Hours</b>	<b>Maximum Marks :100</b>	<b>Total questions :60</b>
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### General Instructions

#### General Instructions:

- i) All questions are compulsory. Marks allotted to each question are indicated in the margin.
- ii) Answers must be precise and to the point.
- iii) In numerical questions, all steps of calculation should be shown clearly.
- iv) Use of non-programmable scientific calculators is permitted.
- v) Wherever necessary, write balanced chemical equations with proper symbols and units.
- vi) Rough work should be done only in the space provided in the question paper.

**1. When the expected future marginal product of capital increases, then the IS curve**

- (A) shifts up and to the right
  - (B) shifts down and to the left
  - (C) becomes steeper
  - (D) becomes flatter
- 

**2. An unanticipated inflation would cause**

- (A) redistribution of wealth from lenders to borrowers
  - (B) redistribution of wealth from borrowers to lenders
  - (C) gains for both borrowers and lenders
  - (D) losses for both borrowers and lenders
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**3. Let  $\{x_1, x_2, \dots, x_n\}$  be the realization of a randomly drawn sample of size  $n$  with sample mean  $\bar{x}$ , and let  $k$  be a real number other than  $\bar{x}$ . Let  $S_1$  and  $S_2$  be the sums of squared deviations defined as**

$$S_1 = \sum_{i=1}^n (x_i - \bar{x})^2 \quad \text{and} \quad S_2 = \sum_{i=1}^n (x_i - k)^2$$

**Then,**

- (A)  $S_1 > S_2$
  - (B)  $S_1 > S_2$  only if  $\bar{x} < k$
  - (C)  $S_1 < S_2$
  - (D)  $S_1 > S_2$  only if  $\bar{x} > k$
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**4. You have a budget of Rs. 4000 and would like to purchase LPG cylinders from a local seller who charges Rs. 50 per cylinder. The seller has a subsidy scheme by which if you return the empty cylinder purchased from him, you will get a refund of Rs. 20 per**

**cylinder. You cannot borrow money from anyone. The maximum number of cylinders you can purchase is**

- (A) 131
- (B) 132
- (C) 133
- (D) 134

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**5. Which one of the following is NOT a feature of the New Industrial Policy, 1991?**

- (A) Abolition of industrial licensing
- (B) Privatisation of public industries
- (C) Removal of restrictions on foreign trade
- (D) Restrictions on foreign technology agreements

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**6. Which one of the following is a possible reason for underestimation of the official poverty ratio in India?**

- (A) Changes in the World Bank's definition of poverty
- (B) Price indices used in the official poverty estimation may not be adequately capturing the actual increase in the cost of living over the years
- (C) Existence of identical poverty lines for all the states and union territories
- (D) Existence of identical poverty lines for rural and urban areas

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**7. Which one of the following committees is NOT associated with financial sector reforms in India?**

- (A) Raghuram Rajan Committee (2013)
- (B) Narasimham Committee (1991)
- (C) Tarapore Committee (1997)

**8. The differential equation**

$$(3x^2y + y^3) dx + (x^3 + 3xy^2) dy = 0$$

**is**

- (A) homogeneous and exact
  - (B) neither separable nor exact
  - (C) exact and not homogeneous
  - (D) homogeneous and not exact
- 

**9. Which one of the following statements is correct?**

- (A) If  $\langle a_n \rangle$  is a bounded sequence, then it is convergent
  - (B) If  $\langle a_n \rangle$  is a convergent sequence, then it is monotonic
  - (C) If  $\langle a_n \rangle$  is a convergent sequence and converges to zero, then the series  $\sum_{n=1}^{\infty} a_n$  is convergent
  - (D) If a series  $\sum_{n=1}^{\infty} a_n$  is convergent, then the sequence  $\langle a_n \rangle$  is convergent and converges to zero
- 

**10. Let  $\|\cdot\|$  and  $\langle \cdot, \cdot \rangle$  denote the standard norm and inner product in  $\mathbb{R}^n$ , respectively. If  $u, v \in \mathbb{R}^3$  such that  $\|u\| = \|v\| = 2$  and the angle between  $u$  and  $v$  is  $\pi/3$ , then**

- (A)  $\|u - v\| = 2\sqrt{2}$
  - (B)  $\langle u, v \rangle = 2\sqrt{3}$
  - (C)  $\|u - v\| = 2\sqrt{3}$
  - (D)  $\|u + v\| = 2\sqrt{3}$
-

**11. A monopoly canteen serves packed meals to two groups of consumers, group  $X$  and group  $Y$ . The demand for packed meals for  $X$  and  $Y$  are given by,**

$$Q_X = 200 - 4P \quad \text{and} \quad Q_Y = 300 - P,$$

**where  $P$  is the uniform price per unit. The unit cost of producing each meal is Rs. 50. The value of  $P$  (in Rs.) that maximizes the canteen's profit is**

- (A) 75
- (B) 50
- (C) 125
- (D) 175

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**12. Consider a Solow growth model without technological progress. The production function is**

$$Y_t = K_t^\alpha N_t^{1-\alpha},$$

**where  $Y_t$ ,  $K_t$ , and  $N_t$  are aggregate output, capital, and population at time  $t$ , respectively. The population grows at a constant rate  $g_N > 0$ , savings rate is constant at  $s \in (0, 1)$ , and capital depreciates at a constant rate  $\delta \geq 0$ . Denote per capita capital as**

$$k_t = \frac{K_t}{N_t},$$

**and define the steady state as a situation where  $k_{t+1} = k_t = k^*$ , where  $k^*$  is a positive constant. Suppose the population growth rate exogenously increases to  $g'_N$ . At the new steady state, the aggregate output will grow at a rate**

- (A)  $g_N$
- (B)  $g'_N$
- (C)  $(1 - \alpha)g_N$
- (D)  $(1 - \alpha)g'_N$

**13. The utility from wealth ( $w$ ) for an individual is given by  $u(w) = \sqrt{w}$ . The individual owns a risky asset that is equally likely to yield either Rs. 400 or Rs. 900. The risk premium of the asset (in Rs.) is**

- (A) 5
- (B) 25
- (C) 625
- (D) 650

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**14. Let  $\bar{\alpha}_1$  and  $\bar{\alpha}_2$  be two independent unbiased estimators of the parameter  $\alpha$  with standard errors  $\sigma_1$  and  $\sigma_2$ , respectively, with  $\sigma_1 \neq \sigma_2$ . The linear combination of  $\bar{\alpha}_1$  and  $\bar{\alpha}_2$  that yields an unbiased estimator of  $\alpha$  with the minimum variance is**

- (A)  $\left(\frac{\sigma_1}{\sigma_1 + \sigma_2}\right) \bar{\alpha}_1 + \left(\frac{\sigma_2}{\sigma_1 + \sigma_2}\right) \bar{\alpha}_2$
- (B)  $\left(\frac{\sigma_2}{\sigma_1 + \sigma_2}\right) \bar{\alpha}_1 + \left(\frac{\sigma_1}{\sigma_1 + \sigma_2}\right) \bar{\alpha}_2$
- (C)  $\left(\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\right) \bar{\alpha}_1 + \left(\frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\right) \bar{\alpha}_2$
- (D)  $\left(\frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\right) \bar{\alpha}_1 + \left(\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\right) \bar{\alpha}_2$

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**15. Let  $X$  be a uniformly distributed random variable in  $[0, b]$ . If the critical region for testing the null hypothesis  $H_0 : b = 2$  against the alternative  $H_A : b \neq 2$  is  $\{x \leq 0.1 \text{ or } x \geq 1.9\}$ , where  $x$  is the value of a single draw of  $X$ , then the probability of Type-I error is**

- (A) 0.2
- (B) 0.1
- (C) 0.05
- (D) 0.01

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**16. Let  $X$  be a uniformly distributed random variable in  $[a, b]$ . The values of an independently drawn sample of size five from  $X$  are given by  $\{1.3, 0.8, 9.5, 20.2, 8.2\}$ . Let**

$\hat{a}$  and  $\hat{b}$  denote the Maximum Likelihood Estimates for the parameters  $a$  and  $b$ , respectively. Then,

- (A)  $\hat{a} = 0.8; \hat{b} = 20.2$
- (B)  $\hat{a} = 1.3; \hat{b} = 9.5$
- (C)  $\hat{a} = 1.3; \hat{b} = 8.2$
- (D)  $\hat{a} = 0; \hat{b} = 21$

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**17. There are only two firms in an industry producing a homogeneous product and having identical production technology. The cost function of firm  $i$  is**

$$C_i(q_i) = q_i^2, \quad \text{for } i = 1, 2;$$

**where  $q_i$  is the quantity produced by firm  $i$ . The market demand for the product is  $p = 100 - q$ , where  $p$  is the unit price and  $q = q_1 + q_2$  is the aggregate quantity. Assuming the firms are price takers, the competitive equilibrium solution of  $p$  and  $q$  in this market is**

- (A)  $p = 80, q = 20$
- (B)  $p = 20, q = 80$
- (C)  $p = \frac{200}{3}, q = \frac{100}{3}$
- (D)  $p = 50, q = 50$

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**18. An upstream paper mill dumps effluents in a river. The total benefit and total cost to the mill are  $TB = 120Q - Q^2$  and  $TC = 20Q$ , respectively, where  $Q$  is the amount of output it produces. The environmental cost due to the negative externality is  $EC = Q^2$ . The government wants to impose a production tax of  $t$  per unit of output on the mill. The value of  $t$  to achieve the socially optimal level of production is**

- (A) 6
- (B) 25
- (C) 50

(D) 70

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**19. Which one of the following statements is NOT correct regarding changes in the occupational structure of the workforce between 1951 and 1991 in India?**

- (A) Proportion of cultivators has increased
  - (B) Proportion of agricultural labour has increased
  - (C) Proportion of those employed in the tertiary sector has increased
  - (D) Proportion of those employed in the primary sector has decreased
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**20. Let  $W$  be a subspace of the vector space  $\mathbb{R}^3$  over the field  $\mathbb{R}$  spanned by**

$$\begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}.$$

**Which one of the following vectors lies in  $W$ ?**

- (A)  $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
  - (B)  $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$
  - (C)  $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
  - (D)  $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$
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**21. Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be defined by**

$$f(x) = xe^{-x} \quad \text{and} \quad g(x) = x|x|.$$

**Then, on  $\mathbb{R}$ ,**

- (A) both  $f$  and  $g$  are convex
  - (B)  $f$  is convex and  $g$  is not convex
  - (C)  $f$  is not quasiconvex and  $g$  is quasiconvex
  - (D) neither  $f$  nor  $g$  is quasiconvex
- 

**22. Let  $(x_1^* = 1, x_2^* = 0, x_3^* = 2)$  be an optimal solution of the linear programming problem**

$$\text{Minimize} \quad x_1 + 5x_2 + 2x_3$$

**subject to**

$$\begin{cases} x_1 - x_2 \leq 1, \\ x_1 + x_2 + x_3 \geq 3, \\ x_1, x_2, x_3 \geq 0. \end{cases}$$

**If  $(\lambda_1^*, \lambda_2^*)$  is an optimal solution of its dual, then**

- (A)  $2\lambda_1^* = 3\lambda_2^*$
  - (B)  $2\lambda_1^* = \lambda_2^*$
  - (C)  $\lambda_1^* = 2\lambda_2^*$
  - (D)  $\lambda_1^* = \lambda_2^*$
- 

**23. Let  $X$  and  $Y$  be two independent random variables with the cumulative distribution functions**

$$F_X(x) = 1 - \left(\frac{3}{4}\right)^x, \quad x = 1, 2, 3, \dots$$

$$F_Y(y) = 1 - \left(\frac{2}{3}\right)^y, \quad y = 1, 2, 3, \dots$$

**respectively. Let  $Z = \min\{X, Y\}$ . Then, the probability  $P(Z \geq 6)$  is**

- (A)  $\frac{1}{64}$   
 (B)  $\frac{1}{32}$   
 (C)  $\frac{64}{63}$   
 (D)  $\frac{31}{32}$

**24. Let  $X$  and  $Y$  be two random variables with the joint probability density function**

$$f_{X,Y}(x, y) = \begin{cases} 6xy, & 0 < y \leq \sqrt{x} \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

**Then, the conditional probability  $P(Y \geq \frac{1}{3} \mid X = \frac{2}{3})$  is**

- (A)  $\frac{1}{2}$   
 (B)  $\frac{5}{9}$   
 (C)  $\frac{5}{6}$   
 (D)  $\frac{3}{4}$

**25. Which one of the following statements is NOT correct in the context of economic planning in India?**

- (A) In the investment strategy for the Second Five Year Plan, a high priority was accorded to the development of heavy capital goods industries over light industries  
 (B) The sectoral allocation to industry was the highest in the First Five Year Plan  
 (C) Plan Holiday for three years was declared after the Third Five Year Plan  
 (D) In each of the first ten Five Year Plan periods, the average incremental capital-output ratio (ICOR) did not exceed 10%

**26. For any two sets  $S_1, S_2 \subseteq \mathbb{R}$ , define the set  $S_1 - S_2 = \{x \in S_1, x \notin S_2\}$ . Let**

$$P = \{x \in \mathbb{R} : x^2 - 2x - 3 \leq 0\} \quad \text{and} \quad Q = \{x \in \mathbb{R} : \log_5(1 + x^2) \leq 1\}.$$

**Then,**

- (A)  $P - Q = [2, 3]$
- (B)  $Q - P = (1, 2]$
- (C)  $P - Q = [-3, -2)$
- (D)  $Q - P = [-2, -1)$

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**27. The workforce participation rate of a country is 60%. This country has a population of 100 million, of which 6 million are unemployed. The unemployment rate for this country is**

- (A)  $\frac{2}{11}$
- (B)  $\frac{1}{11}$
- (C)  $\frac{3}{50}$
- (D)  $\frac{1}{10}$

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**28. According to John Maynard Keynes, which one of the following statements is correct for a closed economy operating at less than the full employment level of output?**

- (A) Savings determines investment
- (B) Investment determines savings
- (C) Changes in the money supply have no impact on output
- (D) Speculative demand for money is determined by the output level

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**29. A monopolist is facing a downward sloping linear market demand. His variable cost of production is zero. The profit maximizing price will**

- (A) lie in the strictly inelastic region of the demand curve
- (B) lie in the strictly elastic region of the demand curve
- (C) be at the unitary elastic point of the demand curve

(D) be equal to the marginal cost of production

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**30. X pays Rs. 5 lakhs to a person to transport fake currency worth Rs. 50 lakhs. The Police department pays Rs. 5 lakhs to a detective to investigate the crime. The detective's income is taxed at 10%. If the above transactions happen in the same year and within the boundary of a country, the contribution of these transactions to GDP (in Rs. lakhs) is**

- (A) 5.5
  - (B) 5
  - (C) 10
  - (D) 4.5
- 

**31. An amateur singer has just recorded his first music album with a recording company. The demand for his album is given by  $Q = 40000 - 800P$ , where  $Q$  is the number of albums sold and  $P$  is the price of each album. Furthermore, per unit cost of producing each album is given by Rs. 8. A profit maximizing recording company has offered the following contract options to the singer**

- (i)** 20% of the revenue from the sales of the album;
- (ii)** Rs. 2 per album sold;
- (iii)** A fixed fee of Rs. 32,000

**Which of the following is/are correct?**

- (A) Contract (i) yields the highest payment to the singer
  - (B) Contract (ii) yields the highest payment to the singer
  - (C) Contract (iii) yields the highest payment to the singer
  - (D) Contract (ii) and (iii) yield the same payment to the singer
-

**32. There are two firms in an oligopolistic industry competing in prices and selling a homogeneous product. Total cost of production for firm  $i$  is**

$$C_i(q_i) = 10q_i, \quad i = 1, 2;$$

**where  $q_i$  is the quantity produced by firm  $i$ . Suppose firm  $i$  sets price  $p_i$  and firm  $j$  sets price  $p_j$ . The market demand faced by firm  $i$  is given by**

$$q_i(p_i, p_j) = \begin{cases} 100 - p_i, & \text{if } p_i < p_j, \\ 0, & \text{if } p_i > p_j, \\ \frac{100 - p_i}{2}, & \text{if } p_i = p_j, \end{cases}$$

**for all  $i, j = 1, 2$  and  $i \neq j$ . Price can only take integer values in this market. Nash equilibrium/equilibria is/are given by**

- (A)  $p_1 = 10, p_2 = 10$
- (B)  $p_1 = 12, p_2 = 12$
- (C)  $p_1 = 40, p_2 = 40$
- (D)  $p_1 = 11, p_2 = 11$

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**33. Which of the following statements is/are correct about the Indian economy during the colonial period?**

- (A) The average annual growth of per capita income was lower during the period 1920-25 to 1947 than the period 1865 to 1920-25.
  - (B) The colonial administration generated a large amount of revenue from peasants by raising the land revenue.
  - (C) The British brought capital from England for the construction of Railways and passed on the burden of interest on it to the Indian taxpayers.
  - (D) Dadabhai Naoroji's estimates of the drain of wealth from India to England included, among other things, the home charges.
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**34. In the context of Expectations Augmented Phillips Curve (EAPC), which of the following statements is/are correct?**

- (A) An increase in the natural rate of unemployment shifts EAPC to the left.
  - (B) An increase in the expected inflation shifts EAPC up and to the right.
  - (C) If actual unemployment rate equals the natural rate of unemployment, the unanticipated inflation equals zero.
  - (D) As long as actual unemployment rate exceeds the natural rate of unemployment, the actual inflation rate exceeds the expected inflation.
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**35. Let  $f$  be a function defined on  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  as**

$$f(x) = \frac{\cos\left(\frac{\pi}{2} + |x|\right)}{\sin\left(\frac{\pi}{2} - |x|\right)}.$$

**Then,**

- (A)  $f$  is not continuous at  $x = 0$
  - (B)  $f$  is continuous but not differentiable at  $x = 0$
  - (C)  $\lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x} = -1$
  - (D)  $f'(0) = -1$
- 

**36. Let  $x, y \in \mathbb{R}$  and the matrix**

$$M = \begin{bmatrix} x + y & x - y \\ x - y & x + y \end{bmatrix}.$$

**Also, let  $\text{adj}(M)$  be the adjoint and  $\det(M)$  be the determinant of the matrix  $M$ . If**

$$M \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix},$$

**then**

- (A)  $x + y = -\frac{3}{4}$

- (B)  $x - y = \frac{3}{4}$   
(C)  $\det(M) = -1$   
(D)  $\det(\text{adj}(M)) = 1$
- 

**37. The net inflow of foreign currency into a country on current account and capital account combined is negative in a particular year. The country could be following a fixed or a flexible exchange rate regime. Which of the following scenarios is/are possible for the country's economy in that year?**

- (A) The country's foreign exchange reserves may increase  
(B) The country's exchange rate may appreciate  
(C) The country's foreign exchange reserves may decrease  
(D) The country's exchange rate may depreciate
- 

**38. Let  $k \in \mathbb{R}$ . Which of the following statements is/are correct for the roots of the quadratic equation**

$$x^2 + 2(k + 1)x + 9k - 5 = 0$$

**?**

- (A) If  $k \leq 1$ , then the roots are real and positive  
(B) If  $2 \leq k \leq 4$ , then the roots are complex  
(C) If  $4 < k < 6$ , then the roots are real and opposite in sign  
(D) If  $k \geq 6$ , then the roots are real and negative
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**39. If the number of employed workers in a country increases while its population does not change, then the unemployment rate in the country**

- (A) will always increase  
(B) will always decrease

- (C) may increase  
(D) may decrease
- 

**40. There are two sellers,  $H$  and  $L$ , in a second-hand goods market where product quality varies. The sellers know the quality of their own product but the buyers cannot distinguish the product quality without further information. Sellers' valuation of their own product is based on the quality.  $H$  is willing to sell his product with quality  $Q_H$  at a price  $P_H$  per unit and  $L$  is willing to sell the product with quality  $Q_L$  at a price  $P_L$  per unit such that**

$$Q_H > Q_L \quad \text{and} \quad P_H > P_L.$$

**This market will suffer from**

- (A) adverse selection  
(B) moral hazard  
(C) market failure  
(D) excess supply
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**41. The amount of money a gambler can win in a casino is determined by three independent rolls of a six-faced fair dice. The gambler wins Rs. 800 if he gets three sixes, Rs. 400 if he gets two sixes, and Rs. 100 in the event of getting only one six. The gambler does not win or lose any money in all other possible outcomes. The probability that a gambler will win at least Rs. 400 is \_\_\_\_\_. (round off to 2 decimal places)**

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**42. Consider an economy where the full employment output is 1 trillion Rupees and the natural rate of unemployment is 6%. If actual unemployment rate is 8%, then according to Okun's law, the absolute gap between the full employment output and actual output (in billion Rupees) will be \_\_\_\_\_. (in integer)**

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**43. The values of normalized indices for a country are as follows.**

Dimension	Value of normalized index
Standard of living	0.4
Education	0.2
Health	0.8

**Following the current UNDP methodology, the value of Human Development Index (HDI) for the country is \_\_\_\_\_. (round off to 1 decimal place)**

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**44. The value of the integral**

$$\int_0^9 \frac{x-1}{1+\sqrt{x}} dx$$

**is \_\_\_\_\_. (in integer)**

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**45. Consider the first order difference equation**

$$x_n = \left(\frac{n+1}{n}\right) x_{n-1}, \quad n = 1, 2, 3, \dots$$

**If  $x_0 = 2$ , then  $x_{100} - x_{50}$  equals \_\_\_\_\_. (in integer)**

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**46. In a small open economy, the desired domestic savings ( $S^d$ ) and the desired domestic investment ( $I^d$ ) are as follows, where  $r^w$  is the world real interest rate.**

$$S^d = 10 + 100r^w, \quad I^d = 15 - 100r^w$$

**If  $r^w = 3\%$ , the current account balance in the equilibrium would be \_\_\_\_\_. (in integer)**

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**47. Let  $X_1 \sim N(\mu_1, \sigma_1^2)$  and  $X_2 \sim N(\mu_2, \sigma_2^2)$  be two normally distributed random variables, where  $\mu_1 = 2, \mu_2 = 3$  and  $\sigma_1^2 = 4, \sigma_2^2 = 9$ . The correlation coefficient between them is 0.5. The variance of the random variable  $(X_1 + X_2)$  is \_\_\_\_\_. (in integer)**

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**48. A consumer always spends 50% of his monthly income on food. Introduction of value added tax on food items has led to a 20% increase in food prices while his monthly income remained unchanged. The consumer's price elasticity of demand for food is ..... (in integer)**

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**49. The utility function of a consumer from consumption of  $x_1$  and  $x_2$  is given by**

$$u(x_1, x_2) = x_1 + 2\sqrt{x_2}.$$

**At the current prices and income, the consumer's optimal consumption bundle is given by  $(x_1 = 10, x_2 = 10)$ . The consumer's optimal choice of  $x_2$ , if his income increases by 50% but prices remain unchanged, is ..... (in integer)**

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**50. The following data relate to a country's GDP in 2012–13 (in local currency).**

Item	Value
GDP	59,816
Private sector investment	17,811
Exports	14,498
Investment expenditure by the government	7,087
Net Factor Income from Abroad	-265
Consumption expenditure by the government	6,620
Private sector consumption	35,695

**The value of this country's imports (in local currency) in 2012–13 is ..... (in integer)**

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**51. Amar has an endowment of food  $F_A = 2$  and water  $W_A = 5$ . Barun has an endowment of food  $F_B = 8$  and water  $W_B = 5$ . Amar's utility function is given by**

$$U_A(f_A, w_A) = f_A^2 w_A;$$

where  $f_A$  and  $w_A$  are his consumption of food and water, respectively.

Barun's utility function is given by

$$U_B(f_B, w_B) = \min\{f_B, w_B\};$$

where  $f_B$  and  $w_B$  are his consumption of food and water, respectively. They exchange food and water at prices  $p_f$  and  $p_w$ , respectively, to maximize their utilities. In the competitive equilibrium,  $\frac{p_f}{p_w}$  equals \_\_\_\_\_. (in integer)

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52. The supply and demand curves of a vaccine are

$$q = 14 + 5p \quad \text{and} \quad q = 329 - 5p,$$

respectively, where  $p$  is the price per unit of vaccine and  $q$  is quantity of vaccine. The government decides that the maximum price of the vaccine would be Rs. 25 per unit. To avoid any shortage in supply at the ceiling price, the government also decides to subsidize the sellers so that the market clears. Subsidy is given on per unit basis. The total expenditure of the government in providing the subsidy is Rs. \_\_\_\_\_. (in integer)

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53. A firm has two manufacturing plants, 1 and 2 to produce the same product. The total costs of production are given by

$$TC_1 = 500 + 30Q_1 \quad \text{and} \quad TC_2 = 1500 + 20Q_2$$

in plants 1 and 2, respectively, where  $Q_1$  and  $Q_2$  are the respective quantities. The demand for the product is given by  $Q^d = 150 - \frac{P}{3}$ , where  $P$  is the price per unit. The value of  $Q_1$  that maximizes the profit of the firm is \_\_\_\_\_. (in integer)

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54. Let  $y(x) > 0$  be a solution of the differential equation

$$\frac{dy}{dx} + y = y^2.$$

If  $y(\ln 2) = \frac{1}{3}$ , where  $\ln$  denotes the natural logarithmic function, then  $y(\ln 3)$  equals ..... (round off to 2 decimal places)

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**55. The optimal value of the constrained optimization problem**

$$\text{minimize } 2xy \quad \text{subject to } 9x^2 + 4y^2 \leq 36$$

is ..... (in integer)

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**56. For  $\beta > 0$ , let the variables  $x_1$  and  $x_3$  be the optimal basic feasible solution of the linear programming problem**

$$\begin{aligned} &\text{maximize } z = x_1 + 2x_2 + 3x_3 \\ &\text{subject to } \begin{cases} 2x_1 - x_2 + x_3 = 9, \\ x_1 + 2x_2 - \beta x_3 = 1, \\ x_1, x_2, x_3 \geq 0. \end{cases} \end{aligned}$$

If the optimal value is 7, then  $\beta$  equals ..... (in integer)

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**57. Let  $X_1, X_2, X_3, X_4$  be independent random variables following the standard normal distribution. Let  $Y$  be defined as**

$$Y = (X_1 + X_2)^2 + (X_3 + X_4)^2.$$

Then the variance of  $Y$  equals ..... (in integer)

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**58. The aggregate production function for a country is  $Y = 10N - 0.005N^2$ , where  $N$  is the quantity of labor input. The aggregate labor supply function is  $N = 55 + 5w$ , where  $w$  is the real wage rate. Assuming perfectly competitive labor and product markets, the equilibrium real wage is ..... (in integer)**

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**59. Individuals in a country start earning and consuming at the age of 18 years, retire at the age of 60 years and die at the age of 90 years, without leaving any debt and bequests. The income of an individual at age  $t$  (in years) is given by the expression  $100t - t^2$ . The price level is constant and the interest rate is zero. According to the life cycle theory of consumption, the average annual consumption of an individual is \_\_\_\_\_. (in integer)**

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**60. The IS–LM model for a closed economy is given below, where  $Y$  is output,  $C$  is consumption,  $I$  is investment,  $T$  is income tax,  $\frac{M^d}{P}$  is money demand,  $P$  is price level,  $r$  is real interest rate,  $\pi^e$  is expected inflation rate and  $G$  is government expenditure:**

$$C = 200 + 0.8(Y - T) - 500r,$$

$$I = 200 - 500r,$$

$$T = 20 + 0.25Y,$$

$$\frac{M^d}{P} = 0.5Y - 250(r + \pi^e).$$

**If  $G = 196$ ,  $\pi^e = 0.1$ , the nominal money supply equals 9890 and the full employment output equals 1000, the full employment equilibrium price level in the economy is \_\_\_\_\_. (in integer)**

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