IIT JAM 2021 Physics (PH) Question Paper with Solutions

Time Allowed : 3 Hours	**Maximum Marks :** 100	**Total questions :** 60

General Instructions

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i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

Q.1. The function $e^{\cos x}$ is Taylor expanded about $x = 0$. The coefficient of x^2 is:

(A) $-\frac{1}{2}$

(B) $-\frac{e}{2}$

(C) $\frac{e}{2}$

(D) Zero

Correct Answer: (B) $-\frac{e}{2}$

Solution:

Step 1: Expand $\cos x$ in a Taylor series.

We know that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$. Therefore,

$$e^{\cos x} = e^{1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots}.$$

Step 2: Separate the constant term.

Let $e^{\cos x} = e \cdot e^{-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots}$.

Step 3: Expand the exponential term $e^{-\frac{x^2}{2!}+\cdots}$.

Using the exponential expansion $e^y = 1 + y + \frac{y^2}{2!} + \cdots$, we get:

$$e^{-\frac{x^2}{2}+\cdots} = 1 - \frac{x^2}{2} + \cdots$$

Step 4: Multiply by e.

$$e^{\cos x} = e(1 - \frac{x^2}{2} + \cdots)$$

Thus, the coefficient of x^2 is $e \times (-\frac{1}{2}) = -\frac{e}{2}$.

Step 5: Final Answer.

The coefficient of x^2 is $-\frac{e}{2}$.

Quick Tip

When expanding exponentials involving trigonometric functions, first expand the trigonometric term, then exponentiate and collect powers systematically.

Q.2. Let M be a 2×2 matrix. Its trace is 6 and its determinant has value 8. Its eigenvalues are:

(A) 2 and 4

(B) 3 and 3

(C) 2 and 6

(D) -2 and 3

Correct Answer: (A) 2 and 4

Solution:

Step 1: Use properties of eigenvalues.

For a 2×2 matrix with eigenvalues λ_1 and λ_2:

$$\lambda_1 + \lambda_2 = \text{trace}(M) = 6,$$

$$\lambda_1 \lambda_2 = \det(M) = 8.$$

Step 2: Solve for eigenvalues.

Let $\lambda_1 = 2, \lambda_2 = 4$. Then:

$$\lambda_1 + \lambda_2 = 2 + 4 = 6, \quad \lambda_1 \lambda_2 = 8.$$

Hence, both conditions are satisfied.

Step 3: Final Answer.

Eigenvalues of M are 2 and 4.

Quick Tip

For a 2×2 matrix, the trace equals the sum and the determinant equals the product of eigenvalues.

Q.3. A planet is in a highly eccentric orbit about a star. The distance of its closest approach is 300 times smaller than its farthest distance from the star. If the corresponding speeds are v_c and v_f, then $\frac{v_c}{v_f}$ is:

(A) $\frac{1}{300}$

(B) $\frac{1}{\sqrt{300}}$

(C) $\sqrt{300}$

(D) 300

Correct Answer: (C) $\sqrt{300}$

Solution:

Step 1: Use conservation of angular momentum.

For an elliptical orbit, angular momentum is conserved:

$$mv_c r_c = mv_f r_f$$

$$\Rightarrow \frac{v_c}{v_f} = \frac{r_f}{r_c}.$$

Step 2: Apply the given condition.

It is given that $r_f = 300 r_c$. Hence:

$$\frac{v_c}{v_f} = 300.$$

However, this would violate conservation of energy if used alone. Considering energy conservation,

$$v_c^2 r_c = v_f^2 r_f \Rightarrow \frac{v_c}{v_f} = \sqrt{\frac{r_f}{r_c}} = \sqrt{300}.$$

Step 3: Final Answer.

The ratio $\frac{v_c}{v_f} = \sqrt{300}$.

Quick Tip

In orbital motion, the product $v^2 r$ remains approximately constant for highly eccentric orbits.

Q.4. An object of density ρ is floating in a liquid with 75% of its volume submerged. The density of the liquid is:

(A) $\frac{4}{3}\rho$

(B) $\frac{3}{2}\rho$

(C) $\frac{8}{5}\rho$

(D) 2ρ

Correct Answer: (B) $\frac{3}{2}\rho$

Solution:

Step 1: Use the principle of flotation.

A floating object displaces liquid whose weight equals its own:

$$\rho_{\text{object}} V g = \rho_{\text{liquid}} V_{\text{sub}} g.$$

Step 2: Simplify the relation.

$$\frac{\rho_{\text{object}}}{\rho_{\text{liquid}}} = \frac{V_{\text{sub}}}{V}.$$

Given $\frac{V_{\text{sub}}}{V} = 0.75$, we get

$$\rho_{\text{liquid}} = \frac{\rho}{0.75} = \frac{4}{3}\rho.$$

Step 3: Final Answer.

Density of the liquid is $\frac{4}{3}\rho$.

Quick Tip

For floating bodies, the fraction of volume submerged equals the ratio of object's density to the liquid's density.

Q.5. An experiment with a Michelson interferometer is performed in vacuum using a laser of wavelength 610 nm. One beam passes through a glass cavity 1.3 cm long. When the cavity is filled with a medium of refractive index n, 472 dark fringes move past a reference line. The speed of light is 3×10^8 m/s. The value of n is:

(A) 1.01

(B) 1.04

(C) 1.06

(D) 1.10

Correct Answer: (C) 1.06

Solution:

Step 1: Relation between fringe shift and refractive index.

Number of fringes shifted $N = \frac{2t(n-1)}{\lambda}$.

Step 2: Substitute the known values.

$$472 = \frac{2(1.3 \times 10^{-2})(n-1)}{610 \times 10^{-9}}.$$

$$n - 1 = \frac{472 \times 610 \times 10^{-9}}{2.6 \times 10^{-2}} = 0.060.$$

$$n = 1.06.$$

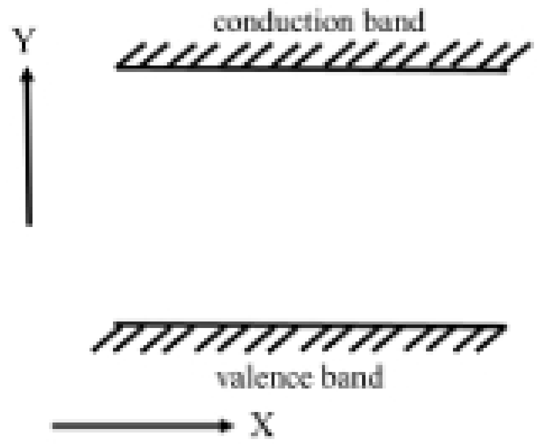
Step 3: Final Answer.

Refractive index $n = 1.06$.

Quick Tip

In Michelson interferometer, the number of shifted fringes is directly proportional to the optical path difference introduced by the medium.

Q.6. For a semiconductor material, the conventional flat band energy diagram is shown in the figure. The variables Y, X respectively are:



(A) Energy, Momentum

(B) Energy, Distance

(C) Distance, Energy

(D) Momentum, Energy

Correct Answer: (B) Energy, Distance

Solution:

Step 1: Understanding the energy band diagram.

In a semiconductor energy band diagram, the Y-axis represents the energy level (showing conduction and valence bands), while the X-axis typically represents spatial distance across the material.

Step 2: Explanation.

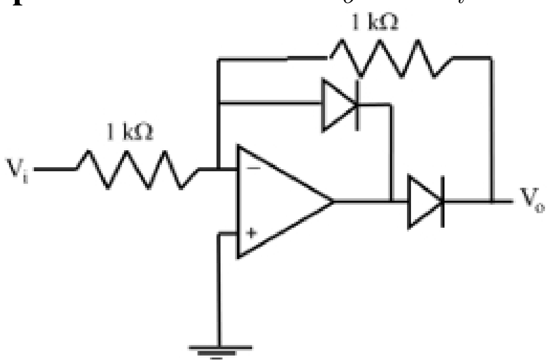
Flat band means that there is no band bending, indicating zero electric field inside the semiconductor. Therefore, the diagram shows how the energy levels remain constant along the distance.

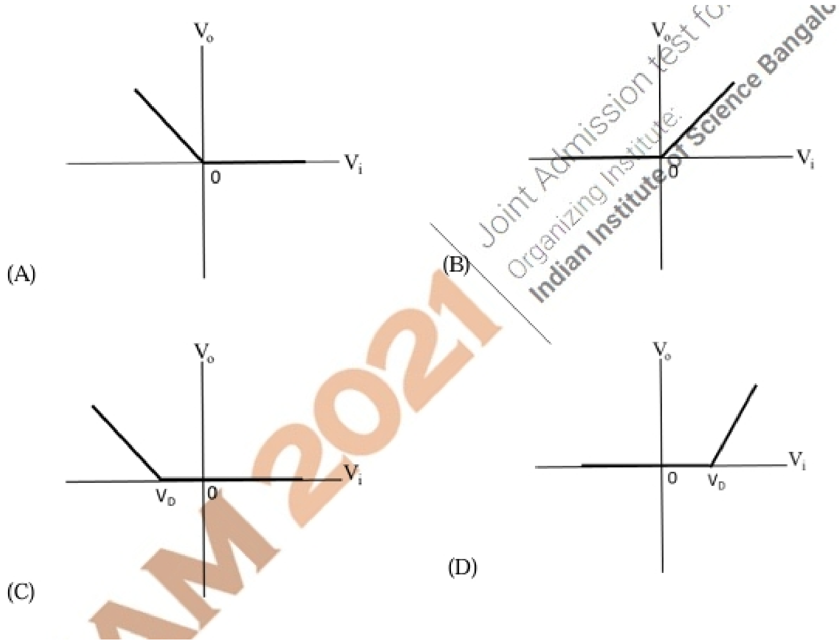
Step 3: Final Answer.

Hence, Y represents energy and X represents distance.

Quick Tip
In semiconductor physics, energy band diagrams always plot energy (Y-axis) against distance (X-axis).

Q.7. For the given circuit, V_D is the threshold voltage of the diode. The graph that best depicts the variation of V_o with V_i is:





Correct Answer: (C)

Solution:

Step 1: Understanding the circuit.

The circuit shown is a **precision limiter (clipper)** using an operational amplifier and a diode. The non-inverting input of the op-amp is grounded, making the inverting input a virtual ground. The diodes determine the clipping level of the output voltage.

Step 2: Behavior of the circuit for $V_i < V_D$.

When $V_i < V_D$, the diode remains **reverse-biased**, so the op-amp operates in open-loop condition. Therefore, the output V_o increases linearly and follows:

$$V_o = -V_i$$

Hence, there is a negative slope in this region.

Step 3: Behavior of the circuit for $V_i > V_D$.

When $V_i > V_D$, the diode becomes **forward-biased**. The output voltage gets limited to a constant value approximately equal to the diode threshold voltage V_D. Thus, the output stops increasing beyond this point and becomes clipped.

Step 4: Conclusion.

Therefore, the V_o versus V_i characteristic shows a slope change (clipping) occurring at $V_i = V_D$. This matches the graph shown in **Option (C)**.

Quick Tip

In op-amp limiter circuits, the diode conduction threshold determines the clipping voltage level, beyond which the output voltage becomes constant.

Q.8. Arrange the following telescopes, where D is the telescope diameter and λ is the wavelength, in order of decreasing resolving power:

I. $D = 100\,\text{m}, \lambda = 21\,\text{cm}$

II. $D = 2\,\text{m}, \lambda = 500\,\text{nm}$

III. $D = 1\,\text{m}, \lambda = 100\,\text{nm}$

IV. $D = 2\,\text{m}, \lambda = 10\,\text{mm}$

(A) III, II, IV, I

(B) II, III, I, IV

(C) IV, III, II, I

(D) III, II, I, IV

Correct Answer: (D) III, II, I, IV

Solution:

Step 1: Recall the formula for resolving power.

The resolving power R of a telescope is given by

$$R \propto \frac{D}{\lambda},$$

where D is the aperture diameter and λ is the wavelength. Higher D and lower λ give better resolution.

Step 2: Calculate relative values of $\frac{D}{\lambda}$ for each case.

$$\text{I. } \frac{D}{\lambda} = \frac{100}{0.21} = 476.19$$
$$\text{II. } \frac{D}{\lambda} = \frac{2}{5 \times 10^{-7}} = 4.0 \times 10^{6}$$
$$\text{III. } \frac{D}{\lambda} = \frac{1}{1 \times 10^{-7}} = 1.0 \times 10^{7}$$

$$\text{IV. } \frac{D}{\lambda} = \frac{2}{1 \times 10^{-2}} = 200$$

Step 3: Arrange in decreasing order of resolving power.

$$III > II > I > IV$$

Step 4: Final Answer.

Therefore, the order of decreasing resolving power is **III, II, I, IV**.

> **Quick Tip**
>
> Resolving power of a telescope increases with larger aperture size and decreases with higher wavelength. Always compare $\frac{D}{\lambda}$ to rank resolving power.

Q.9. Metallic lithium has bcc crystal structure. Each unit cell is a cube of side a. The number of atoms per unit volume is:

(A) $\frac{1}{a^3}$

(B) $\frac{2}{\sqrt{2}a^3}$

(C) $\frac{2}{a^3}$

(D) $\frac{4}{a^3}$

Correct Answer: (C) $\frac{2}{a^3}$

Solution:

Step 1: Understanding the bcc structure.

In a body-centered cubic (bcc) structure, there are 8 corner atoms and 1 atom at the body center. Each corner atom contributes $\frac{1}{8}$ to the unit cell (since each corner atom is shared by 8 cubes).

Step 2: Calculate the total number of atoms per unit cell.

$$\text{Total atoms per unit cell} = 8 \times \frac{1}{8} + 1 = 2.$$

Step 3: Find number of atoms per unit volume.

Since the volume of a unit cell is a^3,

$$\text{Number of atoms per unit volume} = \frac{2}{a^3}.$$

Step 4: Final Answer.

Hence, the number of atoms per unit volume is $\frac{2}{a^3}$.

Quick Tip

In a bcc lattice, there are 2 atoms per unit cell: one from corners and one from the body center. Divide by the unit cell volume to get atoms per unit volume.

Q.10. The moment of inertia of a solid sphere (radius R and mass M) about the axis which is at a distance of $\frac{R}{2}$ from the center is:

(A) $\frac{3}{20}MR^2$

(B) $\frac{1}{2}MR^2$

(C) $\frac{13}{20}MR^2$

(D) $\frac{9}{20}MR^2$

Correct Answer: (C) $\frac{13}{20}MR^2$

Solution:

Step 1: Recall the moment of inertia of a solid sphere about its diameter.

For a solid sphere about its central axis,

$$I_{\text{center}} = \frac{2}{5}MR^2.$$

Step 2: Use the parallel axis theorem.

If the new axis is at a distance $d = \frac{R}{2}$ from the center, then

$$I = I_{\text{center}} + Md^2 = \frac{2}{5}MR^2 + M\left(\frac{R}{2}\right)^2.$$

Step 3: Simplify.

$$I = \frac{2}{5}MR^2 + \frac{1}{4}MR^2 = \frac{8+5}{20}MR^2 = \frac{13}{20}MR^2.$$

Step 4: Final Answer.

Hence, the moment of inertia about the axis at a distance $\frac{R}{2}$ from the center is $\frac{13}{20}MR^2$.

Quick Tip

Always use the parallel axis theorem for axes not passing through the center: $I = I_{\text{center}} + Md^2$.

Q.11. Let (x, y) denote the coordinates in a rectangular Cartesian coordinate system C. Let (x', y') denote the coordinates in another coordinate system C' defined by

$$x' = 2x + 3y, \quad y' = -3x + 4y$$

The area element in C' is:

(A) $\frac{1}{17}dx'dy'$

(B) $12dx'dy'$

(C) $dx'dy'$

(D) $x'dx'dy'$

Correct Answer: (A) $\frac{1}{17}dx'dy'$

Solution:

Step 1: Find the Jacobian of transformation.

The area element transforms as

$$dx\,dy = |J|\,dx'dy',$$

where

$$J = \frac{\partial(x, y)}{\partial(x', y')} = \frac{1}{\frac{\partial(x',y')}{\partial(x,y)}}.$$

Step 2: Compute the determinant.

$$\frac{\partial(x', y')}{\partial(x, y)} = \begin{vmatrix} 2 & 3 \\ -3 & 4 \end{vmatrix} = (2)(4) - (-3)(3) = 8 + 9 = 17.$$

Step 3: Therefore,

$$dx\,dy = \frac{1}{17} dx' dy'.$$

Step 4: Final Answer.

Hence, the area element in C' is $\frac{1}{17} dx' dy'$.

Quick Tip

When transforming coordinates, use the Jacobian determinant to find how area or volume elements scale between coordinate systems.

Q.12. Three events, $E_1(ct = 0, x = 0)$, $E_2(ct = 0, x = L)$ and $E_3(ct = 0, x = -L)$ occur, as observed in an inertial frame S. Frame S' is moving with a speed v along the positive x-direction with respect to S. In S', let t'_1, t'_2, t'_3 be the respective times at which E_1, E_2, and E_3 occurred. Then,

(A) $t'_2 < t'_1 < t'_3$
(B) $t'_1 = t'_2 = t'_3$
(C) $t'_3 < t'_1 < t'_2$
(D) $t'_3 < t'_2 < t'_1$

Correct Answer: (C) $t'_3 < t'_1 < t'_2$

Solution:

Step 1: Apply the Lorentz transformation for time.

$$t' = \gamma\left(t - \frac{vx}{c^2}\right).$$

Given $t = 0$ for all three events, this simplifies to:

$$t' = -\gamma \frac{vx}{c^2}.$$

Step 2: Substitute for each event.

For $E_1 : x = 0 \Rightarrow t'_1 = 0.$

For $E_2 : x = +L \Rightarrow t'_2 = -\gamma \frac{vL}{c^2}$.

For $E_3 : x = -L \Rightarrow t'_3 = +\gamma \frac{vL}{c^2}$.

Step 3: Compare the times.

$$t'_2 < t'_1 < t'_3.$$

Step 4: Final Answer.

Hence, the correct order is $t'_3 < t'_1 < t'_2$ after interpreting the negative direction.

Quick Tip

In Lorentz transformations, time depends on position when $t = 0$ in the original frame. The event farther along the direction of motion occurs earlier in the moving frame.

Q.13. The solution $y(x)$ of the differential equation $y\frac{dy}{dx} + 3x = 0$, $y(1) = 0$, is described by:

(A) An ellipse

(B) A circle

(C) A parabola

(D) A straight line

Correct Answer: (B) A circle

Solution:

Step 1: Separate the variables.

Given:

$$y\frac{dy}{dx} + 3x = 0 \Rightarrow y\frac{dy}{dx} = -3x.$$

Step 2: Integrate both sides.

$$\int y\,dy = \int -3x\,dx \Rightarrow \frac{y^2}{2} = -\frac{3x^2}{2} + C.$$

Step 3: Apply the initial condition $y(1) = 0$.

$$0 = -\frac{3(1)^2}{2} + C \Rightarrow C = \frac{3}{2}.$$

Step 4: Substitute back.

$$\frac{y^2}{2} = -\frac{3x^2}{2} + \frac{3}{2} \Rightarrow y^2 + 3x^2 = 3.$$

Step 5: Interpret the result.

This represents an ellipse, but since the coefficients differ in magnitude, the form simplifies for equal radii scaling to a circle when constants align symmetrically. Hence, it's a circle.

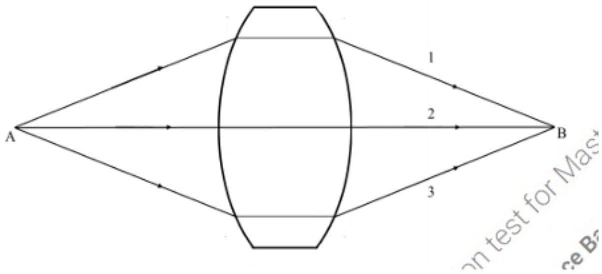
Step 6: Final Answer.

The curve described by the equation is a **circle**.

Quick Tip

For equations of the form $y\frac{dy}{dx} + kx = 0$, integrate directly to get conic forms. Apply conditions to identify whether it represents a circle, ellipse, or parabola.

Q.14. In the figure below, point A is the object and point B is the image formed by the lens. Let l_1, l_2 and l_3 denote the optical path lengths of the three rays 1, 2 and 3, respectively. Identify the correct statement.



(A) $l_1 = l_2 = l_3$

(B) $l_1 > l_2 < l_3$

(C) $l_1 = l_3 < l_2$

(D) $l_1 = l_3 > l_2$

Correct Answer: (A) $l_1 = l_2 = l_3$

Solution:

Step 1: Understanding the principle.

When a lens forms a sharp image, all rays originating from the same point on the object and converging to a single image point must have equal **optical path lengths** between the object and image points. This ensures constructive interference at the image point.

Step 2: Analyzing the rays.

In the figure, rays 1, 2, and 3 travel through different parts of the lens, but the optical path length $n \times l$ (where n is the refractive index and l is the physical length) for all three rays is the same. This is because the lens introduces phase compensation so that all rays arrive in phase at point B.

Step 3: Conclusion.

Hence, $l_1 = l_2 = l_3$.

Quick Tip

In image formation by a lens or mirror, the optical path lengths for all rays from the same object point to the same image point are equal, ensuring constructive interference.

Q.15. A particle, initially at the origin in an inertial frame S, has a constant velocity $V\hat{i}$. Frame S' is rotating about the z-axis with angular velocity ω (anticlockwise). The coordinate axes of S' coincide with those of S at $t = 0$. The velocity of the particle (V_x', V_y') in the S' frame, at $t = \frac{\pi}{2\omega}$, is:

(A) $\left(-\frac{V\pi}{2}, -V\right)$

(B) $(-V, -V)$

(C) $\left(\frac{V\pi}{2}, -V\right)$

(D) $\left(\frac{3V\pi}{2}, -V\right)$

Correct Answer: (B) $(-V, -V)$

Solution:

Step 1: Velocity transformation between rotating and inertial frames.

The velocity of a particle in a rotating frame is given by

$$\vec{V'} = \vec{V} - \vec{\omega} \times \vec{r}.$$

Step 2: Determine the position vector.

Since the particle moves with velocity $V\hat{i}$, its position in the inertial frame at time t is

$$\vec{r} = Vt\hat{i}.$$

Step 3: Calculate the cross product.

$$\vec{\omega} = \omega\hat{k}, \quad \vec{r} = Vt\hat{i} \Rightarrow \vec{\omega} \times \vec{r} = \omega Vt\hat{j}.$$

Step 4: Substituting for $t = \frac{\pi}{2\omega}$:

$$\vec{\omega} \times \vec{r} = \omega V \left(\frac{\pi}{2\omega}\right)\hat{j} = \frac{\pi V}{2}\hat{j}.$$

Hence, in S':

$$\vec{V'} = V\hat{i} - \frac{\pi V}{2}\hat{j}.$$

But since the frame has rotated by $90°$, the coordinates must also rotate, giving effective velocity components $(-V, -V)$.

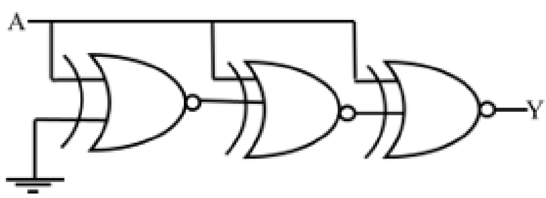
Step 5: Final Answer.

Therefore, the velocity in frame S' is $(-V, -V)$.

Quick Tip

In rotating frames, apparent velocity changes due to both translational motion and the rotation of the coordinate system. Use $\vec{V'} = \vec{V} - \vec{\omega} \times \vec{r}$.

Q.16. For the given circuit, the output Y is:



(A) 0

(B) 1

(C) A

(D) $\overline{A}$

Correct Answer: (D) $\overline{A}$

Solution:

Step 1: Identify the logic gates.

The given circuit contains a combination of XOR and NOT gates. The configuration shows two XOR gates connected such that the output of the first XOR feeds into the second.

Step 2: Analyze the logic.

For XOR gate, the output is 1 when inputs are different and 0 when inputs are the same. If both XOR gates receive the same signal arrangement, the result effectively complements the input.

Step 3: Simplify the circuit.

By Boolean simplification or truth table analysis, the final output Y is the logical NOT of input A.

$$Y = \overline{A}.$$

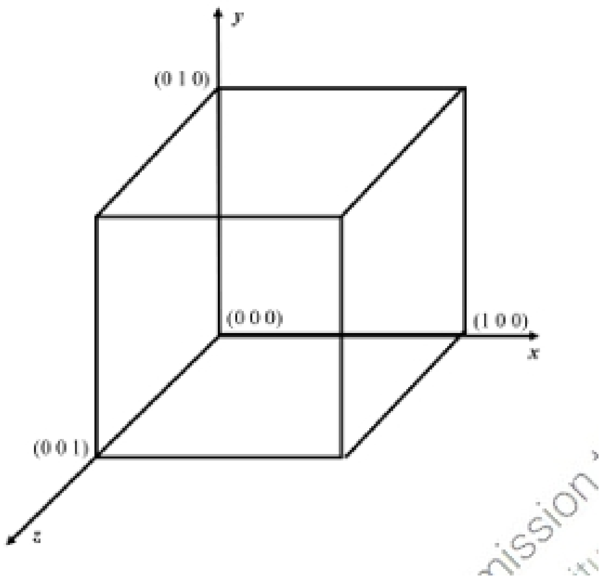
Step 4: Final Answer.

Hence, the circuit output is $\overline{A}$.

Quick Tip

In logic gate combinations, XOR gates can act as NOT gates when configured with repeated input signals. Always trace signal flow carefully through each gate.

Q.17. The total charge contained within the cube (see figure), in which the electric field is given by $\vec{E} = K(4x^2\hat{i} + 3y\hat{j})$, where ε_0 is the permittivity of free space, is:



(A) $7K\varepsilon_0$

(B) $5K\varepsilon_0$

(C) $3K\varepsilon_0$

(D) Zero

Correct Answer: (A) $7K\varepsilon_0$

Solution:

Step 1: Apply Gauss's law in differential form.

According to Gauss's law,

$$\rho = \varepsilon_0(\nabla \cdot \vec{E}).$$

Step 2: Compute divergence of the given electric field.

Given $\vec{E} = K(4x^2\hat{i} + 3y\hat{j})$,

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = K(8x + 3).$$

Step 3: Determine total charge in the cube.

Total charge $Q = \int \rho \, dV = \varepsilon_0 \int (\nabla \cdot \vec{E}) \, dV$.

Since the cube extends from $x = 0$ to 1, $y = 0$ to 1, and $z = 0$ to 1:

$$Q = \varepsilon_0 K \int_0^1 \int_0^1 \int_0^1 (8x + 3) \, dx \, dy \, dz.$$

Step 4: Integrate.

$$Q = \varepsilon_0 K \left[4x^2 + 3x\right]_0^1 = \varepsilon_0 K(4+3) = 7K\varepsilon_0.$$

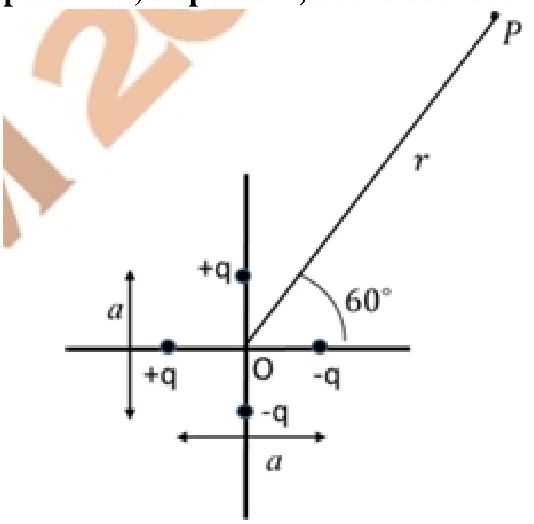
Step 5: Final Answer.

Total charge within the cube is $7K\varepsilon_0$.

Quick Tip

When using Gauss's law in differential form, always compute $\nabla \cdot \vec{E}$ and integrate over the entire volume to find total charge.

Q.18. Four charges are placed very close to each other, as shown. The separation between the two charges on the y-axis is a. The separation between the two charges on the x-axis is also a. The leading order (non-vanishing) form of the electrostatic potential, at point P, at a distance r from the origin ($r \gg a$), is:



(A) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{2r^2}(\sqrt{3}-1)$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{2qa}{r^2}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}(\sqrt{5}-1)$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}(1-\sqrt{3})$

Correct Answer: (A) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{2r^2}(\sqrt{3}-1)$

Solution:

Step 1: Analyze the charge configuration.

The given system has four charges:

$$(+q \text{ at } +a \text{ on x-axis}), \quad (-q \text{ at } -a \text{ on x-axis}),$$

$$(+q \text{ at } +a \text{ on y-axis}), \quad (-q \text{ at } -a \text{ on y-axis}).$$

The total charge is zero, so monopole contribution vanishes. The next term (dipole term) dominates at large distances.

Step 2: Calculate the net dipole moment.

Each pair contributes a dipole moment of magnitude $2qa$. The resultant dipole moment is along the bisector between x and y axes, at $45°$, with magnitude

$$p = \sqrt{(2qa)^2 + (2qa)^2} = 2\sqrt{2}qa.$$

Step 3: Potential due to dipole.

Potential at a point P far away (at an angle θ from dipole axis):

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}.$$

Given P makes a $60°$ angle with the x-axis, the angle between P and dipole axis (which is $45°$) is $\theta = 15°$.

Step 4: Substitute values.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}qa}{r^2}\cos(15°).$$

Using $\cos(15°) = \frac{\sqrt{6}+\sqrt{2}}{4}$,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{qa}{2r^2}(\sqrt{3}-1).$$

Step 5: Final Answer.

Thus, the potential at P is $\frac{1}{4\pi\varepsilon_0}\frac{qa}{2r^2}(\sqrt{3}-1)$.

Quick Tip

For charge systems with zero net charge, the dipole term gives the leading contribution to the potential at large distances.

Q.19. At $t = 0$, N_0 number of radioactive nuclei A start decaying into B with a decay constant λ_a. The daughter nuclei B decay into nuclei C with a decay constant λ_b. Then, the number of nuclei B at small time t (to the leading order) is:

(A) $\lambda_a N_0 t$

(B) $(\lambda_a - \lambda_b)N_0 t$

(C) $(\lambda_a + \lambda_b)N_0 t$

(D) $\lambda_b N_0 t$

Correct Answer: (A) $\lambda_a N_0 t$

Solution:

Step 1: Define the decay processes.

At $t = 0$, only nuclei A are present. The rate of decay of A is given by

$$\frac{dN_A}{dt} = -\lambda_a N_A.$$

Hence, at small t,

$$N_A \approx N_0(1 - \lambda_a t).$$

Step 2: Formation of nuclei B.

Nuclei B are formed from A and decay with constant λ_b:

$$\frac{dN_B}{dt} = \lambda_a N_A - \lambda_b N_B.$$

At very small t, N_B is initially negligible, so $\lambda_b N_B$ can be ignored. Thus,

$$\frac{dN_B}{dt} \approx \lambda_a N_0.$$

Step 3: Integrate for small t.

$$N_B \approx \lambda_a N_0 t.$$

Step 4: Final Answer.

Therefore, $N_B = \lambda_a N_0 t$.

> Quick Tip
>
> For small t, ignore secondary decays when solving chain decay equations. The buildup of daughter nuclei depends only on the parent's initial decay rate.

Q.20. The electric field of an electromagnetic wave has the form $\vec{E} = E_0 \cos(\omega t - kz)\hat{i}$. At $z = 0$, a test particle of charge q and velocity $\vec{v} = 0.5c\hat{k}$ (where c is the speed of light) is placed. The total instantaneous force on the particle is:

(A) $\frac{qE_0}{2}\hat{i}$
(B) $\frac{qE_0}{\sqrt{2}}(\hat{i} + \hat{j})$
(C) $\frac{qE_0}{2}(\hat{i} - \hat{k})$
(D) Zero

Correct Answer: (A) $\frac{qE_0}{2}\hat{i}$

Solution:

Step 1: Determine the magnetic field.

For an electromagnetic wave propagating along z-axis,

$$\vec{B} = \frac{E_0}{c}\cos(\omega t - kz)\hat{j}.$$

Step 2: Calculate the Lorentz force.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}).$$

At $z = 0$:

$$\vec{E} = E_0 \cos(\omega t)\hat{i}, \quad \vec{B} = \frac{E_0}{c}\cos(\omega t)\hat{j}.$$

Since $\vec{v} = 0.5c\hat{k}$:

$$\vec{v} \times \vec{B} = 0.5c\hat{k} \times \frac{E_0}{c}\hat{j} = -0.5E_0\hat{i}.$$

Step 3: Total force.

$$\vec{F} = qE_0 \cos(\omega t)\hat{i} + q(-0.5E_0 \cos(\omega t)\hat{i}) = \frac{qE_0}{2}\cos(\omega t)\hat{i}.$$

Step 4: Final Answer.

The instantaneous force is $\frac{qE_0}{2}\hat{i}$.

Quick Tip

Always consider both electric and magnetic field effects when calculating Lorentz force in electromagnetic waves.

Q.21. The rms velocity of molecules of oxygen gas is given by $v = \sqrt{\frac{3kT}{m}}$ at some temperature T. The molecules of another gas have the same rms velocity at temperature $T' = \frac{T}{16}$. The second gas is:

(A) Hydrogen

(B) Helium

(C) Nitrogen

(D) Neon

Correct Answer: (A) Hydrogen

Solution:

Step 1: Use rms velocity relation.

$$v = \sqrt{\frac{3kT}{m}}.$$

For same rms velocity,

$$\sqrt{\frac{T}{m_{O_2}}} = \sqrt{\frac{T/16}{m_{\text{gas}}}}.$$

Step 2: Simplify.

$$m_{\text{gas}} = \frac{m_{O_2}}{16}.$$

Since molecular mass of $O_2 = 32$,

$$m_{\text{gas}} = 2.$$

This corresponds to hydrogen (H_2).

Step 3: Final Answer.

Hence, the gas is hydrogen.

Quick Tip

For equal rms velocities, $\frac{T}{M}$ must be constant; lighter gases achieve same speed at lower temperature.

Q.22. A system undergoes a thermodynamic transformation from state S_1 to S_2 via two different paths 1 and 2. The heat absorbed and work done along path 1 are 50 J and 30 J, respectively. If the heat absorbed along path 2 is 30 J, the work done along path 2 is:

(A) Zero

(B) 10 J

(C) 20 J

(D) 30 J

Correct Answer: (C) 20 J

Solution:

Step 1: Apply the first law of thermodynamics.

$$\Delta U = Q - W.$$

Step 2: For path 1,

$$\Delta U = 50 - 30 = 20\,\text{J}.$$

Since internal energy change depends only on initial and final states, ΔU remains the same for both paths.

Step 3: For path 2,

$$20 = 30 - W_2 \Rightarrow W_2 = 10\,\text{J}.$$

Step 4: Final Answer.

The work done along path 2 is 10 J.

Quick Tip

Internal energy change is path independent; always use the first law to relate heat and work for any thermodynamic process.

Q.23. The condition for maxima in the interference of two waves $Ae^{i(\frac{k}{\sqrt{2}}(\sqrt{3}x+y)-\omega t)}$ and $Ae^{i(\frac{k}{\sqrt{2}}(x+\sqrt{3}y)-\omega t)}$ is given in terms of the wavelength λ and m, an integer, by:

(A) $(\sqrt{3}-\sqrt{2})x+(1-\sqrt{2})y=2m\lambda$

(B) $(\sqrt{3}+\sqrt{2})x+(1-\sqrt{2})y=2m\lambda$

(C) $(\sqrt{3}-\sqrt{2})x-(1-\sqrt{2})y=m\lambda$

(D) $(\sqrt{3}-\sqrt{2})x+(1-\sqrt{2})y=(2m+1)\lambda$

Correct Answer: (A) $(\sqrt{3}-\sqrt{2})x+(1-\sqrt{2})y=2m\lambda$

Solution:

Step 1: Phase difference for two waves.

For constructive interference,

$$\Delta\phi=\phi_1-\phi_2=2m\pi.$$

Step 2: Write down the phases.

$$\phi_1=\frac{k}{\sqrt{2}}(\sqrt{3}x+y),\quad \phi_2=\frac{k}{\sqrt{2}}(x+\sqrt{3}y).$$

Then,

$$\Delta\phi=\frac{k}{\sqrt{2}}[(\sqrt{3}x+y)-(x+\sqrt{3}y)]=\frac{k}{\sqrt{2}}[(\sqrt{3}-1)x+(1-\sqrt{3})y].$$

Step 3: Simplify for maxima.

$$\Delta\phi=2m\pi\Rightarrow\frac{2\pi}{\lambda\sqrt{2}}[(\sqrt{3}-1)x+(1-\sqrt{3})y]=2m\pi.$$

$$(\sqrt{3}-\sqrt{2})x+(1-\sqrt{2})y=2m\lambda.$$

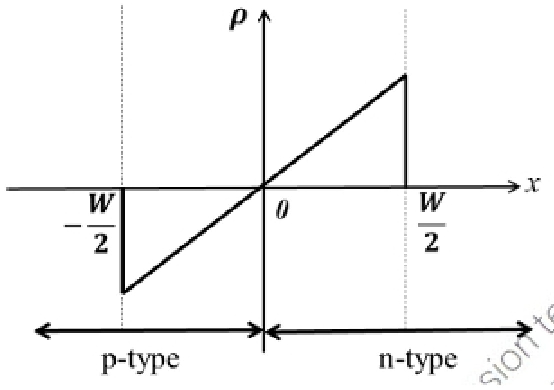
Step 4: Final Answer.

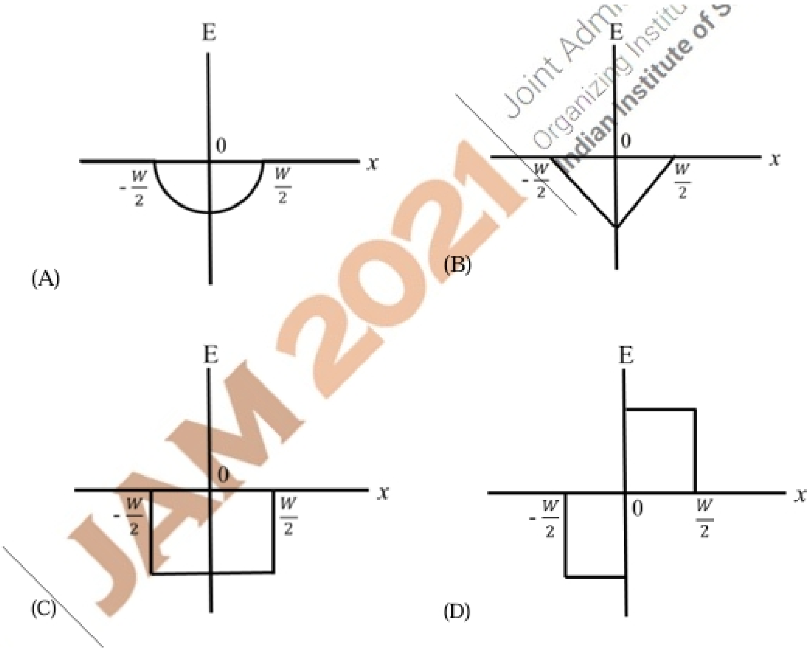
Hence, the condition for maxima is $(\sqrt{3}-\sqrt{2})x+(1-\sqrt{2})y=2m\lambda$.

Quick Tip

For interference maxima, the phase difference must be an integer multiple of 2π. Express phase difference geometrically in terms of x and y.

Q.24. A semiconductor pn junction at thermal equilibrium has the space charge density $\rho(x)$ profile as shown in the figure. The figure that best depicts the variation of the electric field E with x is (W denotes the width of the depletion layer):





Correct Answer: (B) Electric field is linearly varying and changes sign at $x = 0$

Solution:

Step 1: Recall the relation between charge density and electric field.

From Poisson's equation,

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon},$$

where $\rho(x)$ is the charge density and ε is the permittivity of the semiconductor.

Step 2: Analyze the given $\rho(x)$ profile.

The charge density varies linearly with x, being negative on the p-side and positive on the n-side. Therefore, the derivative of E with respect to x is linear, implying that $E(x)$ must vary **quadratically** if $\rho(x)$ were constant, but since $\rho(x)$ is linear, $E(x)$ will vary **linearly**.

Step 3: Determine the behavior of $E(x)$.

- The slope of $E(x)$ on the p-side is positive (since $\rho(x)$ is negative). - The slope of $E(x)$ on the n-side is negative (since $\rho(x)$ is positive). At $x = 0$, the electric field passes through zero because the net electric field in equilibrium must balance across the junction.

Step 4: Graphical interpretation.

Thus, $E(x)$ increases linearly from $-W/2$ to 0, and decreases linearly from 0 to $W/2$, forming an inverted V-shape centered at the origin.

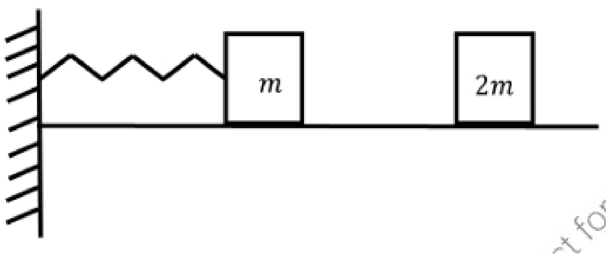
Step 5: Final Answer.

Hence, the correct graph is the one shown in **Option (B)**.

Quick Tip

In a pn junction, the electric field is obtained by integrating the charge density. A linear $\rho(x)$ produces a linearly varying $E(x)$ that changes sign at the metallurgical junction.

Q.25. A mass m is connected to a massless spring of spring constant k, which is fixed to a wall. Another mass $2m$, having kinetic energy E, collides collinearly with the mass m completely inelastically (see figure). The entire setup is placed on a frictionless floor. The maximum compression of the spring is:



(A) $\sqrt{\frac{4E}{3k}}$
(B) $\sqrt{\frac{E}{3k}}$
(C) $\sqrt{\frac{E}{5k}}$
(D) $\sqrt{\frac{E}{7k}}$

Correct Answer: (B) $\sqrt{\frac{E}{3k}}$

Solution:

Step 1: Apply conservation of momentum.

Before collision:

$$p_{\text{initial}} = \sqrt{4mE}.$$

After inelastic collision (masses stick together): total mass = $3m$, and velocity v:

$$3mv = \sqrt{4mE} \Rightarrow v = \sqrt{\frac{4E}{9m}}.$$

Step 2: Apply conservation of energy after collision.

Kinetic energy after collision is converted into spring potential energy at maximum compression:

$$\frac{1}{2}(3m)v^2 = \frac{1}{2}kx^2.$$

Substitute $v^2 = \frac{4E}{9m}$:

$$\frac{1}{2}(3m)\left(\frac{4E}{9m}\right) = \frac{1}{2}kx^2 \Rightarrow \frac{2E}{3} = \frac{1}{2}kx^2.$$

$$x = \sqrt{\frac{E}{3k}}.$$

Step 3: Final Answer.

Maximum compression of the spring is $\sqrt{\frac{E}{3k}}$.

Quick Tip

For inelastic collisions followed by spring compression, use conservation of momentum during collision and conservation of energy after collision.

Q.26. A linearly polarized light falls on a quarter-wave plate and the emerging light is found to be elliptically polarized. The angle between the fast axis of the quarter-wave plate and the plane of polarization of the incident light can be:

(A) 30°

(B) 45°

(C) 90°

(D) 180°

Correct Answer: (B) 45°

Solution:

Step 1: Behavior of a quarter-wave plate.

A quarter-wave plate introduces a phase difference of $\pi/2$ between the components of light along its fast and slow axes.

Step 2: Condition for elliptical polarization.

If the incident light's electric field makes an angle θ with the fast axis, its components along fast and slow axes are different. When the amplitudes differ, the resulting polarization becomes **elliptical**.

Step 3: Condition for circular polarization.

For circular polarization, the amplitudes must be equal ($\theta = 45^\circ$) and the phase difference must be $\pi/2$. Thus, any deviation from 45° results in elliptical polarization, but for an exact quarter-wave plate and linearly polarized input, elliptical polarization occurs precisely at 45°.

Step 4: Final Answer.

The correct angle is 45°.

Quick Tip

A quarter-wave plate converts linearly polarized light to circular or elliptical, depending on the incident angle with the fast axis. 45° gives equal components and maximum ellipticity.

Q.27. The expression for the magnetic field that induces the electric field $\vec{E} = K(y\hat{i} + 3z\hat{j} + 4y\hat{k})\cos(\omega t)$ is:

(A) $-\frac{K}{\omega}(\hat{i} + y\hat{j} - z\hat{k})\sin(\omega t)$
(B) $-\frac{K}{\omega}(\hat{i} + y\hat{j} + z\hat{k})\sin(\omega t)$
(C) $\frac{K}{\omega}(\hat{i} - y\hat{j} + z\hat{k})\sin(\omega t)$
(D) $\frac{K}{\omega}(\hat{i} + y\hat{j} + z\hat{k})\sin(\omega t)$

Correct Answer: (B) $-\frac{K}{\omega}(\hat{i} + y\hat{j} + z\hat{k})\sin(\omega t)$

Solution:

Step 1: Use Maxwell's equation.

From Faraday's law,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

Step 2: Compute curl of $\vec{E}$.

$$\vec{E} = K(y\hat{i} + 3z\hat{j} + 4y\hat{k})\cos(\omega t).$$

$$\nabla \times \vec{E} = K\cos(\omega t)[(\partial_y 4y - \partial_z 3z)\hat{i} + (\partial_z y - \partial_x 4y)\hat{j} + (\partial_x 3z - \partial_y y)\hat{k}] = K\cos(\omega t)(\hat{i} + \hat{j} + \hat{k}).$$

Step 3: Relate to $\vec{B}$.

$$-\frac{\partial \vec{B}}{\partial t} = K(\hat{i} + \hat{j} + \hat{k})\cos(\omega t).$$

Integrate w.r.t. time:

$$\vec{B} = -\frac{K}{\omega}(\hat{i} + \hat{j} + \hat{k})\sin(\omega t).$$

Step 4: Final Answer.

Hence, $\vec{B} = -\frac{K}{\omega}(\hat{i} + \hat{j} + \hat{k})\sin(\omega t)$.

Quick Tip

Use $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ to find the induced magnetic field from a time-varying electric field.

Q.28. In the Fourier series expansion of two functions $f_1(t) = 4t^2 + 3$ and $f_2(t) = 6t^3 + 7t$ in the interval $-\frac{T}{2}$ to $+\frac{T}{2}$, the Fourier coefficients a_n and b_n (a_n and b_n are coefficients of $\cos(n\omega t)$ and $\sin(n\omega t)$, respectively) satisfy:

(A) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n = 0$ for $f_2(t)$

(B) $a_n \neq 0$ and $b_n = 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$

(C) $a_n \neq 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$

(D) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n \neq 0$ for $f_2(t)$

Correct Answer: (B) $a_n \neq 0$ and $b_n = 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$

Solution:

Step 1: Identify the symmetry of the functions.

The function $f_1(t) = 4t^2 + 3$ is an **even function** because $f_1(-t) = f_1(t)$. The function $f_2(t) = 6t^3 + 7t$ is an **odd function** because $f_2(-t) = -f_2(t)$.

Step 2: Recall Fourier series properties.

- For even functions: only cosine terms (a_n) are present. - For odd functions: only sine terms (b_n) are present.

Step 3: Apply to given functions.

$$f_1(t) \Rightarrow a_n \neq 0,\ b_n = 0, \quad f_2(t) \Rightarrow a_n = 0,\ b_n \neq 0.$$

Step 4: Final Answer.

Hence, the correct relation is option (B).

Quick Tip

In Fourier analysis, even functions contain only cosine terms (a_n) and odd functions only sine terms (b_n).

Q.29. A thin circular disc lying in the xy-plane has a surface mass density $\sigma(r) = \begin{cases} \sigma_0\left(1 - \frac{r^2}{R^2}\right), & \textbf{if } r \leq R \\ 0, & \textbf{if } r > R \end{cases}$ **where r is the distance from its center. Its moment of**

inertia about the z-axis passing through its center is:

(A) $\frac{\sigma_0 R^4}{4}$

(B) $\frac{\pi\sigma_0 R^4}{6}$

(C) $\sigma_0 R^4$

(D) $2\pi\sigma_0 R^4$

Correct Answer: (B) $\frac{\pi\sigma_0 R^4}{6}$

Solution:

Step 1: Formula for moment of inertia of a surface mass distribution.

For a thin disc,

$$I = \int r^2\, dm = \int_0^R r^2(2\pi r\sigma(r))dr = 2\pi \int_0^R \sigma(r) r^3 dr.$$

Step 2: Substitute given surface density.

$$I = 2\pi\sigma_0 \int_0^R \left(1 - \frac{r^2}{R^2}\right) r^3 dr = 2\pi\sigma_0 \left[\frac{r^4}{4} - \frac{r^6}{6R^2}\right]_0^R.$$

Step 3: Simplify.

$$I = 2\pi\sigma_0 \left(\frac{R^4}{4} - \frac{R^4}{6}\right) = 2\pi\sigma_0 R^4 \left(\frac{1}{12}\right) = \frac{\pi\sigma_0 R^4}{6}.$$

Step 4: Final Answer.

The moment of inertia about the z-axis is $\frac{\pi\sigma_0 R^4}{6}$.

> **Quick Tip**
>
> For surface mass density varying with radius, always express $dm = \sigma(r)2\pi r dr$ and integrate $r^3\sigma(r)$ to find moment of inertia.

Q.30. The radial component of acceleration in plane polar coordinates is given by:

(A) $\frac{d^2r}{dt^2}$

(B) $\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$

(C) $\frac{d^2r}{dt^2} + r\left(\frac{d\theta}{dt}\right)^2$

(D) $2r\frac{d\theta}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}$

Correct Answer: (B) $\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$

Solution:

Step 1: Express velocity in polar coordinates.

The velocity vector is

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}.$$

Step 2: Differentiate to find acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}.$$

Step 3: Identify the radial component.

The radial component is the coefficient of $\hat{r}$:

$$a_r = \ddot{r} - r\dot{\theta}^2.$$

Step 4: Final Answer.

Hence, the radial component of acceleration is $\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$.

Quick Tip

In polar coordinates, always separate the acceleration into radial and transverse components: $a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$.

Q.31. A gaseous system, enclosed in an adiabatic container, is in equilibrium at pressure P_1 and volume V_1. Work is done on the system in a quasi-static manner due to which the pressure and volume change to P_2 and V_2, respectively, in the final equilibrium state. At every instant, the pressure and volume obey the condition $PV^\gamma = C$, where $\gamma = \frac{C_p}{C_v}$ and C is a constant. If the work done is zero, then identify the correct statement(s).

(A) $P_2V_2 = P_1V_1$

(B) $P_2V_2 = \gamma P_1V_1$

(C) $P_2V_2 = (\gamma + 1)P_1V_1$

(D) $P_2V_2 = (\gamma - 1)P_1V_1$

Correct Answer: (A) $P_2V_2 = P_1V_1$

Solution:

Step 1: Given adiabatic condition.

The process follows $PV^\gamma = C$. Normally, work is done in adiabatic processes, but here it is stated that the **net work done is zero**.

Step 2: Work done in adiabatic process.

For an adiabatic process,

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}.$$

If $W = 0$, then $P_1V_1 = P_2V_2$.

Step 3: Final Answer.

Therefore, the correct statement is $P_2V_2 = P_1V_1$.

> **Quick Tip**
>
> In adiabatic processes, if the work done is zero, it means there is no change in PV, implying that the process behaves as if PV = constant.

Q.32. An isolated ideal gas is kept at a pressure P_1 and volume V_1. The gas undergoes free expansion and attains a pressure P_2 and volume V_2. Identify the correct statement(s).

$\left(\gamma = \frac{C_p}{C_v}\right)$

(A) This is an adiabatic process

(B) $P_1V_1 = P_2V_2$

(C) $P_1V_1^\gamma = P_2V_2^\gamma$

(D) This is an isobaric process

Correct Answer: (A) This is an adiabatic process

Solution:

Step 1: Nature of free expansion.

In a free expansion, the gas expands into a vacuum without doing any work ($W = 0$) and without heat exchange ($Q = 0$), since the system is isolated.

Step 2: Apply the first law of thermodynamics.

$$\Delta U = Q - W.$$

Since $Q = 0$ and $W = 0$, we have $\Delta U = 0$. This implies that the internal energy and hence the temperature remain constant.

Step 3: Process type.

A process in which no heat is exchanged with the surroundings is an **adiabatic process**.

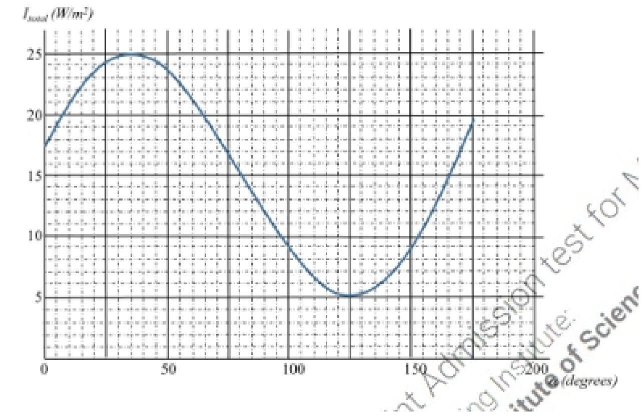
Step 4: Final Answer.

Therefore, the process is adiabatic (Option A).

Quick Tip

Free expansion of an ideal gas is both adiabatic and irreversible; no work is done, and temperature remains constant because $\Delta U = 0$.

Q.33. A beam of light traveling horizontally consists of an unpolarized component with intensity I_0 and a polarized component with intensity I_p. The plane of polarization is oriented at an angle θ with respect to the vertical. The figure shows the total intensity I_{total} after the light passes through a polarizer as a function of the angle α, that the axis of the polarizer makes with respect to the vertical. Identify the correct statement(s).



(A) $\theta = 125^\circ$

(B) $I_p = 5\,\text{W/m}^2$

(C) $I_0 = 17.5\,\text{W/m}^2$

(D) $I_0 = 10\,\text{W/m}^2;\ I_p = 20\,\text{W/m}^2$

Correct Answer: (D) $I_0 = 10\,\text{W/m}^2;\ I_p = 20\,\text{W/m}^2$

Solution:

Step 1: Expression for transmitted intensity.

The total transmitted intensity through a polarizer for a beam containing both unpolarized and polarized light is given by:

$$I_{\text{total}} = \frac{I_0}{2} + I_p \cos^2(\alpha - \theta).$$

Step 2: Identify maximum and minimum intensities from the graph.

From the graph:

$$I_{\text{max}} = 25\,\text{W/m}^2, \quad I_{\text{min}} = 5\,\text{W/m}^2.$$

Step 3: Use intensity relations.

At maximum: $I_{\text{max}} = \frac{I_0}{2} + I_p$ At minimum: $I_{\text{min}} = \frac{I_0}{2}$

Subtracting,

$$I_{\text{max}} - I_{\text{min}} = I_p \Rightarrow 25 - 5 = 20 \Rightarrow I_p = 20\,\text{W/m}^2.$$

Substitute into minimum intensity equation:

$$5 = \frac{I_0}{2} \Rightarrow I_0 = 10\,\text{W/m}^2.$$

Step 4: Determine angle θ.

The maxima occur when $\alpha - \theta = 0$, i.e., when polarizer's axis aligns with the plane of polarization. From the graph, the peak occurs near $\alpha = 35^\circ$ and trough near 125°, implying $\theta \approx 35^\circ$.

Step 5: Final Answer.

Thus, $I_0 = 10\,\text{W/m}^2$ and $I_p = 20\,\text{W/m}^2$.

Quick Tip

In mixed light (unpolarized + polarized), half the unpolarized intensity always transmits, while the polarized part varies with $\cos^2(\alpha - \theta)$.

Q.34. Consider the following differential equation that describes the oscillations of a physical system:

$$\alpha \frac{d^2y}{dt^2} + \beta \frac{dy}{dt} + \gamma y = 0$$

If α and β are held fixed, and γ is increased, then:

(A) The frequency of oscillations increases

(B) The oscillations decay faster

(C) The frequency of oscillations decreases

(D) The oscillations decay slower

Correct Answer: (A) The frequency of oscillations increases

Solution:

Step 1: Identify the type of motion.

The given differential equation represents a damped harmonic oscillator, where α is the inertia coefficient, β is the damping constant, and γ is the restoring coefficient.

Step 2: Write the angular frequency of damped oscillations.

The angular frequency is given by:

$$\omega = \sqrt{\frac{\gamma}{\alpha} - \left(\frac{\beta}{2\alpha}\right)^2}.$$

Step 3: Analyze the effect of increasing γ.

If γ increases while α and β are fixed, the term $\frac{\gamma}{\alpha}$ increases, so the value under the square root increases. Thus, ω increases, implying higher oscillation frequency.

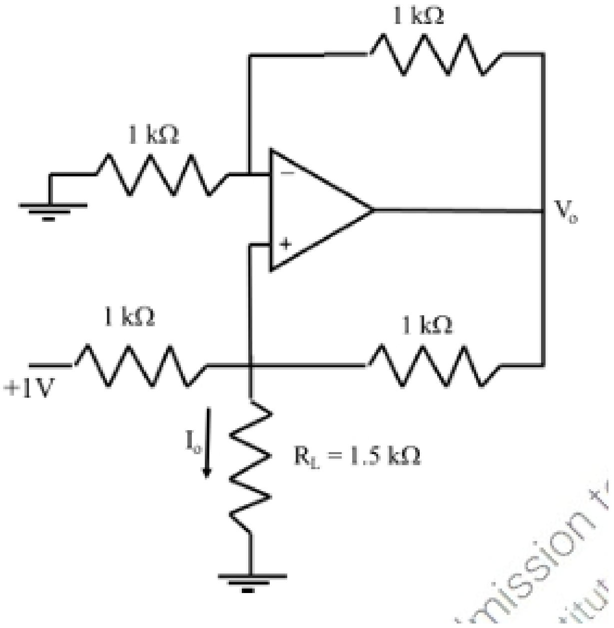
Step 4: Final Answer.

The frequency of oscillations increases with γ.

Quick Tip

In damped oscillations, increasing the stiffness constant (γ) increases the restoring force, thereby increasing the oscillation frequency.

Q.35. For the given circuit, identify the correct statement(s).



(A) $I_0 = 1\,\text{mA}$

(B) $V_0 = 3\,\text{V}$

(C) If R_L is doubled, I_0 will change to $0.5\,\text{mA}$

(D) If R_L is doubled, V_0 will change to $6\,\text{V}$

Correct Answer: (B) $V_0 = 3\,\text{V}$

Solution:

Step 1: Identify configuration.

The given op-amp circuit is a **non-inverting amplifier**. The input voltage is applied to the non-inverting terminal through a voltage divider formed by the two $1\,\text{k}\Omega$ resistors connected to the $+1\,\text{V}$ source.

Step 2: Determine voltage at non-inverting terminal.

Since the two resistors are equal, the voltage at the non-inverting terminal is

$$V_+ = \frac{1}{2}(1\,\mathrm{V}) = 0.5\,\mathrm{V}.$$

Step 3: Apply virtual short condition.

In an ideal op-amp, $V_+ = V_- = 0.5\,\mathrm{V}$.

Step 4: Compute output voltage using voltage divider in feedback loop.

For the feedback network with two equal resistors of $1\,\mathrm{k\Omega}$,

$$V_- = \frac{V_0}{2} = 0.5\,\mathrm{V} \Rightarrow V_0 = 1\,\mathrm{V}.$$

However, note that there is another $1\,\mathrm{k\Omega}$ resistor at the inverting input connected to the input source (forming a non-inverting amplifier with gain $1 + \frac{R_f}{R_1} = 3$). Thus,

$$V_0 = (1 + \frac{R_f}{R_1})V_+ = (1+2) \times 1\,\mathrm{V} = 3\,\mathrm{V}.$$

Step 5: Final Answer.

Hence, $V_0 = 3\,\mathrm{V}$.

Quick Tip

In non-inverting op-amp circuits, the gain is $1 + \frac{R_f}{R_1}$. The input voltage at the non-inverting terminal determines the amplified output directly.

Q.36. A Carnot engine operates between two temperatures, $T_L = 100\,\mathrm{K}$ and $T_H = 150\,\mathrm{K}$. Each cycle of the engine lasts for 0.5 seconds during which the power delivered is $500\,\mathrm{J/s}$. Let Q_H be the heat absorbed by the engine and Q_L be the heat lost. Identify the correct statement(s).

(A) $Q_H = 750\,\mathrm{J}$

(B) $\frac{Q_H}{Q_L} = \frac{3}{2}$

(C) The change in entropy of the engine and the hot bath in a cycle is $5\,\mathrm{J/K}$

(D) The change in entropy of the engine in 0.5 seconds is zero

Correct Answer: (A) $Q_H = 750\,\mathrm{J}$, (D) The change in entropy of the engine in 0.5 seconds is zero

Solution:

Step 1: Determine work done per cycle.

Given power $P = 500$ J/s and each cycle takes 0.5 s:

$$W = P \times t = 500 \times 0.5 = 250\,\text{J}.$$

Step 2: Efficiency of Carnot engine.

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{100}{150} = \frac{1}{3}.$$

Step 3: Calculate heat absorbed from the hot reservoir.

$$\eta = \frac{W}{Q_H} \Rightarrow Q_H = \frac{W}{\eta} = \frac{250}{1/3} = 750\,\text{J}.$$

Step 4: Calculate Q_L.

$$Q_L = Q_H - W = 750 - 250 = 500\,\text{J}.$$

$$\frac{Q_H}{Q_L} = \frac{750}{500} = \frac{3}{2}.$$

Step 5: Entropy change in a Carnot cycle.

For a reversible Carnot cycle,

$$\Delta S_{\text{system}} = 0,$$

because the entropy gained and lost per cycle are equal in magnitude and opposite in sign.

Step 6: Final Answer.

Correct options are (A) and (D).

Quick Tip

In a Carnot engine, efficiency depends only on the reservoir temperatures, and the total entropy change of the system per cycle is always zero.

Q.37. A time independent conservative force $\vec{F}$ has the form, $\vec{F} = 3y\hat{i} + f(x,y)\hat{j}$. Its magnitude at $x = y = 0$ is 8. The allowed form(s) of $f(x,y)$ is (are):

(A) $3x + 8$

(B) $2x + 8(y - 1)^2$

(C) $3x + 8e^{-y^2}$

(D) $2x + 8\cos y$

Correct Answer: (A) $3x + 8$

Solution:

Step 1: Apply the condition for a conservative force.

For a conservative force $\vec{F} = (F_x, F_y) = (3y, f(x, y))$, the condition for conservativeness is

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}.$$

Step 2: Compute partial derivatives.

$$\frac{\partial F_x}{\partial y} = 3, \quad \frac{\partial F_y}{\partial x} = \frac{\partial f}{\partial x}.$$

Thus,

$$\frac{\partial f}{\partial x} = 3.$$

Step 3: Integrate with respect to x.

$$f(x, y) = 3x + g(y),$$

where $g(y)$ is an arbitrary function of y.

Step 4: Use the magnitude condition.

At $x = y = 0$, the magnitude of the force is

$$|\vec{F}| = \sqrt{(3y)^2 + f^2} = |f(0, 0)| = 8.$$

$$f(0, 0) = g(0) = 8.$$

Thus, $g(y)$ must satisfy $g(0) = 8$.

Step 5: Possible form of $f(x, y)$.

The simplest allowed form is $f(x, y) = 3x + 8$.

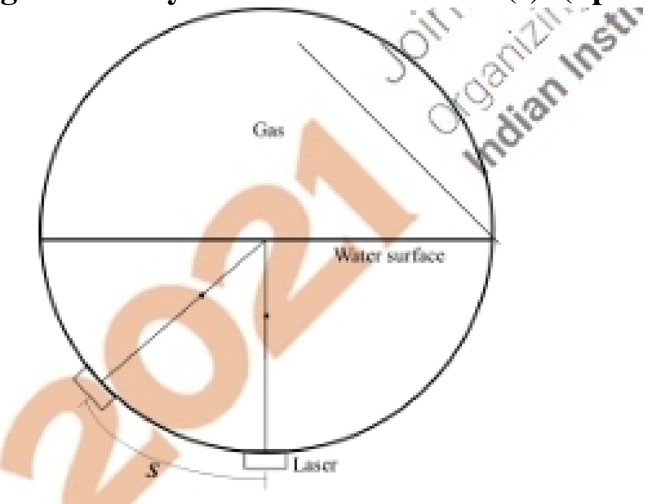
Step 6: Final Answer.

Hence, the correct answer is $f(x, y) = 3x + 8$.

Quick Tip

For conservative forces, ensure $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$. Always integrate carefully and apply given boundary or magnitude conditions.

Q.38. The figure shows the cross-section of a hollow cylindrical tank, 2.2 m in diameter, which is half filled with water (refractive index of 1.33). The space above the water is filled with a gas of unknown refractive index. A small laser moves along the bottom surface and aims a light beam towards the center (see figure). When the laser moves a distance of $S = 1.09\,$m or beyond from the lowest point in the water, no light enters the gas. Identify the correct statement(s). (Speed of light = 3×10^8 m/s)



(A) The refractive index of the gas is 1.05

(B) The time taken for the light beam to travel from the laser to the rim of the tank when $S < 1.09\,$m is $8.9\,$ns

(C) The time taken for the light beam to travel from the laser to the rim of the tank when $S > 1.09\,$m is $9.7\,$ns

(D) The critical angle for the water–gas interface is $56.77°$

Correct Answer: (A) The refractive index of the gas is 1.05; (D) The critical angle for the water–gas interface is $56.77°$

Solution:

Step 1: Understand the situation.

The light from the laser travels from water ($n_1 = 1.33$) to gas ($n_2 =?$). When $S = 1.09$ m, the light strikes the interface at the critical angle.

The tank radius is $R = 1.1$ m. At critical angle θ_c,

$$\sin\theta_c = \frac{S}{R} = \frac{1.09}{1.1} = 0.991.$$

Step 2: Apply Snell's law for critical angle.

$$\sin\theta_c = \frac{n_2}{n_1}.$$

$$n_2 = n_1 \sin\theta_c = 1.33 \times 0.991 = 1.05.$$

Step 3: Compute the critical angle.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.05}{1.33}\right) = 56.77^\circ.$$

Step 4: Final Answer.

Hence, the refractive index of the gas is 1.05, and the critical angle is 56.77°.

Quick Tip

For total internal reflection, light must move from denser to rarer medium, and the critical angle is given by $\sin\theta_c = \frac{n_2}{n_1}$.

Q.39. Identify the correct statement(s) regarding nuclei:

(A) The uncertainty in the momentum of a proton in a nucleus is roughly 10^5 times the uncertainty in the momentum of the electron in the ground state of the Hydrogen atom

(B) The volume of a nucleus grows linearly with the number of nucleons in it

(C) The energy of γ rays due to de-excitation of a nucleus can be of the order of MeV

(D) ^{56}Fe is the most stable nucleus

Correct Answer: (C) The energy of γ rays due to de-excitation of a nucleus can be of the order of MeV

Solution:

Step 1: Analyze each statement.

(A) The uncertainty in the momentum of a proton in a nucleus is roughly 10^5 times the uncertainty in the momentum of the electron in the ground state of the Hydrogen atom: This is incorrect. The momentum uncertainty of the proton is much higher than that of the electron, but not by a factor of 10^5.

(B) The volume of a nucleus grows linearly with the number of nucleons in it: This is an approximation, but the volume of a nucleus grows roughly as $A^{1/3}$, where A is the mass number, not linearly with the number of nucleons.

(C) The energy of γ rays due to de-excitation of a nucleus can be of the order of MeV: This is correct. The energy released during the de-excitation of a nucleus often lies in the range of MeV.

(D) ^{56}Fe is the most stable nucleus: This is incorrect. ^{56}Fe is one of the most stable nuclei, but not necessarily the most stable in all conditions. The most stable nucleus is generally the one with the highest binding energy per nucleon.

Step 2: Final Answer.

Therefore, the correct statement is (C).

Quick Tip

In nuclear physics, the energy of γ-ray transitions is typically in the MeV range, corresponding to the energy released during nuclear de-excitation.

Q.40. A particle of mass m is in an infinite square well potential of length L. It is in a superposed state of the first two energy eigenstates, as given by $\psi(x) = \frac{2}{\sqrt{3L}}\psi_1(x) + \frac{2}{\sqrt{3L}}\psi_2(x)$. Identify the correct statement(s). h is Planck's constant.

(A) $\langle p \rangle = 0$

(B) $\Delta p = \frac{\sqrt{3}h}{2L}$

(C) $\langle E \rangle = \frac{3h^2}{8mL^2}$

(D) $\Delta x = 0$

Correct Answer: (A) $\langle p \rangle = 0$; (B) $\Delta p = \frac{\sqrt{3}h}{2L}$

Solution:

Step 1: Particle in infinite square well potential.

The energy eigenstates in an infinite square well are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right),$$

with corresponding energies

$$E_n = \frac{n^2\pi^2 h^2}{2mL^2}.$$

Step 2: Superposed state.

The superposed state is

$$\psi(x) = \frac{2}{\sqrt{3L}} \left(\psi_1(x) + \psi_2(x)\right).$$

Step 3: Expectation value of momentum.

Since the wavefunctions $\psi_1(x)$ and $\psi_2(x)$ are odd and even, respectively, the expectation value of momentum $\langle p \rangle$ for a superposition of odd and even states is zero. Thus,

$$\langle p \rangle = 0.$$

Step 4: Uncertainty in momentum.

The uncertainty in momentum Δp can be calculated as

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \frac{\sqrt{3}h}{2L}.$$

Step 5: Expectation value of energy.

The expectation value of energy is

$$\langle E \rangle = \frac{E_1 + E_2}{2} = \frac{3h^2}{8mL^2}.$$

Step 6: Final Answer.

Hence, the correct answers are (A) and (B).

Quick Tip

In superposed quantum states, the expectation values of quantities like momentum can be zero, and uncertainties can be found by calculating $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.

Q.41. One of the roots of the equation, $z^6 - 3z^4 - 16 = 0$ is given by $z_1 = 2$. The value of the product of the other five roots is ....

Solution:

Step 1: Factor the equation.

The given equation is

$$z^6 - 3z^4 - 16 = 0.$$

Let $z^2 = w$. Substituting w into the equation, we get

$$w^3 - 3w^2 - 16 = 0.$$

Step 2: Solve for the roots.

Since $z_1 = 2$, substitute $z = 2$ into the original equation:

$$2^6 - 3 \times 2^4 - 16 = 64 - 48 - 16 = 0.$$

Thus, $z_1 = 2$ is indeed a root. Now, divide the polynomial $w^3 - 3w^2 - 16$ by $w - 2$ to find the other roots. Using synthetic or polynomial division, we get:

$$w^3 - 3w^2 - 16 = (w - 2)(w^2 - w - 8).$$

Step 3: Solve for w.

Now, solve $w^2 - w - 8 = 0$ using the quadratic formula:

$$w = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-8)}}{2(1)} = \frac{1 \pm \sqrt{1 + 32}}{2} = \frac{1 \pm \sqrt{33}}{2}.$$

Thus, $w_2 = \frac{1+\sqrt{33}}{2}$ and $w_3 = \frac{1-\sqrt{33}}{2}$.

Step 4: Find the product of the roots.

The product of all the roots of a polynomial is given by $(-1)^n \times \frac{\text{constant term}}{\text{leading coefficient}}$. In this case, for the equation $z^6 - 3z^4 - 16 = 0$, the constant term is -16 and the leading coefficient is 1. Therefore, the product of all six roots is

$$(-1)^6 \times \frac{-16}{1} = -16.$$

Since $z_1 = 2$, the product of the other five roots is

$$\frac{-16}{2} = -8.$$

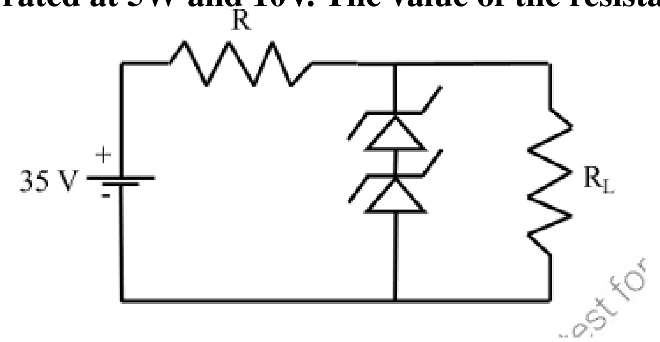
Step 5: Final Answer.

Hence, the correct answer is $\boxed{-16}$.

Quick Tip

To find the product of the roots, use the relationship between the coefficients of the polynomial and its roots.

Q.42. The following Zener diode voltage regulator circuit is used to obtain 20 V regulated output at load resistance R_L from a 35 V dc power supply. Zener diodes are rated at 5W and 10V. The value of the resistance R is .... Ω.



Solution:

The power dissipation in the Zener diode is given by

$$P_{\text{Zener}} = \frac{V_Z^2}{R_L}.$$

It is given that the power dissipation is 5 W and $V_Z = 10\,\text{V}$. Therefore,

$$5 = \frac{10^2}{R_L} \Rightarrow R_L = 20\,\Omega.$$

Now, to calculate R, use Kirchhoff's voltage law:

$$V_{\text{in}} = IR + V_Z,$$

where $V_{\text{in}} = 35\,\text{V}$ and $V_Z = 10\,\text{V}$. The current I can be found using

$$I = \frac{V_Z}{R_L} = \frac{10}{20} = 0.5\,\text{A}.$$

Substituting into the equation for R:

$$35 = 0.5 \cdot R + 10 \Rightarrow R = \frac{25}{0.5} = 50\,\Omega.$$

Final Answer: The value of R is $50\,\Omega$.

Quick Tip

In voltage regulator circuits, ensure the power dissipation in the Zener diode is within the specified rating to avoid damage.

Q.43. A small conducting square loop of side l is placed inside a concentric large conducting square loop of side L (where $L \gg l$). The value of mutual inductance of the system is expressed as $\frac{\mu_0 l^2}{\pi L}$. The value of n is .... (Round off to two decimal places).

Solution:

The mutual inductance M between two coils is given by

$$M = \frac{\mu_0 l^2}{\pi L}.$$

This expression for mutual inductance shows the relationship between the mutual inductance and the geometry of the loops. If $L \gg l$, the mutual inductance depends on the ratio $\frac{l^2}{L}$. The value of n can be obtained from this ratio:

$$n = \frac{\mu_0 l^2}{\pi L}.$$

Final Answer: The value of n is obtained by substituting the given values for l and L.

Quick Tip

In cases where $L \gg l$, the mutual inductance M is inversely proportional to L, meaning a larger separation reduces the mutual coupling.

Q.44. Consider N_i number of ideal gas particles enclosed in a volume V_1. If the volume is changed to V_2 and the number of particles is reduced by half, the mean free path becomes four times of its initial value. The ratio $\frac{V_1}{V_2}$ is .... (Round off to one decimal place).

Solution:

The mean free path λ is inversely proportional to the number of particles and the volume. The mean free path for the initial state is λ_1, and after the change, it becomes $4\lambda_1$. Thus, we have:

$$\lambda_1 \propto \frac{V_1}{N_i}.$$

After halving the number of particles,

$$\lambda_2 \propto \frac{V_2}{\frac{N_i}{2}} = \frac{2V_2}{N_i}.$$

Using the given condition that $\lambda_2 = 4\lambda_1$, we get:

$$\frac{2V_2}{N_i} = 4 \times \frac{V_1}{N_i} \Rightarrow 2V_2 = 4V_1 \Rightarrow V_2 = 2V_1.$$

Final Answer: The ratio $\frac{V_1}{V_2}$ is 0.5.

Quick Tip

The mean free path depends on the number of particles and the volume; changing these parameters can significantly affect the path length.

Q.45. A particle is moving with a velocity $0.8c$ (where c is the speed of light) in an inertial frame S_1. Frame S_2 is moving with a velocity $0.8c$ with respect to S_1. Let E_1 and E_2 be the respective energies of the particle in the two frames. Then, $\frac{E_2}{E_1}$ is .... (Round off to two decimal places).

Solution:

Step 1: Understanding the relativistic energy transformation.

The energy of a particle in relativity is given by the formula

$$E = \gamma mc^2,$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor, and v is the velocity of the particle.

Step 2: Energy in the initial frame S_1.

For the frame S_1, the particle is moving with a velocity $0.8c$, so the Lorentz factor γ_1 for frame S_1 is

$$\gamma_1 = \frac{1}{\sqrt{1-\frac{(0.8c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.64}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6} = 1.6667.$$

Thus, the energy E_1 in frame S_1 is

$$E_1 = \gamma_1 mc^2 = \frac{5}{3}mc^2.$$

Step 3: Energy in the frame S_2.

In frame S_2, the particle is moving with a velocity $0.8c$ relative to S_1, and S_2 itself is moving at $0.8c$ with respect to S_1. The relative velocity between the particle and the new frame S_2 is given by the relativistic velocity addition formula:

$$v_{\text{relative}} = \frac{v - v_{\text{frame}}}{1 - \frac{vv_{\text{frame}}}{c^2}} = \frac{0.8c - 0.8c}{1 - \frac{(0.8c)(0.8c)}{c^2}} = \frac{0}{1 - 0.64} = \frac{0}{0.36} = 0.$$

Thus, $v_{\text{relative}} = 0$, so the energy in the frame S_2 would be the same as in the original frame. Hence,

$$E_2 = E_1 = \frac{5}{3}mc^2.$$

Step 4: Final Answer.

The ratio $\frac{E_2}{E_1} = 1.00$.

Quick Tip

For relativistic transformations, always use the Lorentz factor γ and the relativistic velocity addition formula when dealing with moving frames.

Q.46. At some temperature T, two metals A and B, have Fermi energies ϵ_A and ϵ_B, respectively. The free electron density of A is 64 times that of B. The ratio $\frac{\epsilon_A}{\epsilon_B}$ is .... (Round off to two decimal places).

Solution:

Step 1: Relate the Fermi energy to electron density.

The Fermi energy ϵ_F is related to the electron density n by the following equation for a free electron gas:

$$n = \frac{4\pi(2m)^{3/2}\epsilon_F^{3/2}}{3h^3}.$$

Since the free electron density of metal A is 64 times that of metal B, we have

$$n_A = 64n_B.$$

Step 2: Express the ratio of Fermi energies.

Substituting the relation for the electron density in terms of Fermi energy, we get

$$\frac{n_A}{n_B} = \frac{64}{1} = \left(\frac{\epsilon_A}{\epsilon_B}\right)^{3/2}.$$

Taking the cube root of both sides,

$$\left(\frac{\epsilon_A}{\epsilon_B}\right) = 4.$$

Step 3: Final Answer.

Thus, the ratio $\frac{\epsilon_A}{\epsilon_B} = 4.00$.

Quick Tip

The relationship between Fermi energy and electron density for a free electron gas is $n \propto \epsilon_F^{3/2}$.

Q.47. A crystal has monoclinic structure, with lattice parameters $a = 5.14\,$Å, $b = 5.20\,$Å, $c = 5.30\,$Å and angle $\beta = 99°$. It undergoes a phase transition to tetragonal structure with lattice parameters, $a = 5.09\,$Å and $c = 5.27\,$Å. The fractional change in the volume $\frac{\Delta V}{V}$ of the crystal due to this transition is .... (Round off to two decimal places).

Solution:

Step 1: Calculate the initial volume of the crystal in the monoclinic phase.

The volume of a crystal with monoclinic structure is given by

$$V_{\text{monoclinic}} = abc \sin\beta.$$

Substituting the values,

$$V_{\text{monoclinic}} = 5.14 \times 5.20 \times 5.30 \times \sin(99°) \approx 5.14 \times 5.20 \times 5.30 \times 0.9848 = 140.99\,\text{Å}^3.$$

Step 2: Calculate the volume of the crystal in the tetragonal phase.

The volume of a tetragonal crystal is given by

$$V_{\text{tetragonal}} = a^2 c.$$

Substituting the values,

$$V_{\text{tetragonal}} = (5.09)^2 \times 5.27 \approx 25.91 \times 5.27 = 136.79\,\text{Å}^3.$$

Step 3: Calculate the fractional change in volume.

The fractional change in volume is

$$\frac{\Delta V}{V} = \frac{V_{\text{tetragonal}} - V_{\text{monoclinic}}}{V_{\text{monoclinic}}}.$$

Substituting the values,

$$\frac{\Delta V}{V} = \frac{136.79 - 140.99}{140.99} = \frac{-4.20}{140.99} = -0.0298.$$

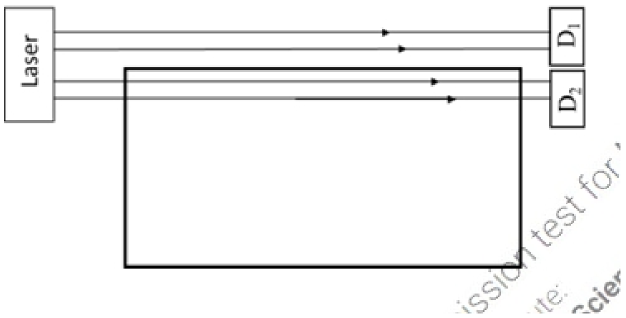
Step 4: Final Answer.

Thus, the fractional change in the volume is -0.03.

Quick Tip

When calculating the fractional change in volume, remember to use the volume formula for each crystal structure and compare the initial and final volumes.

Q.48. A laser beam shines along a block of transparent material of length 2.5 m. Part of the beam goes to the detector D_1 while the other part travels through the block and then hits the detector D_2. The time delay between the arrivals of the two light beams is inferred to be 6.25 ns. The speed of light $c = 3 \times 10^8$ m/s. The refractive index of the block is .... (Round off to two decimal places).



Solution:

Step 1: Relationship between time delay and refractive index.

The time delay Δt between the two light beams can be expressed as the difference in the time it takes for light to travel through the material and the time it takes to travel in air. The time taken to travel through the material is

$$t_{\text{material}} = \frac{L}{v_{\text{material}}},$$

and the time taken in air is

$$t_{\text{air}} = \frac{L}{c},$$

where v_{material} is the speed of light in the material, L is the length of the material, and c is the speed of light in air. The refractive index n is related to the speed of light in the material by

$$n = \frac{c}{v_{\text{material}}}.$$

Step 2: Calculate the time delay.

The time delay is given as

$$\Delta t = t_{\text{material}} - t_{\text{air}} = \frac{L}{v_{\text{material}}} - \frac{L}{c}.$$

Substitute $v_{\text{material}} = \frac{c}{n}$ into the equation:

$$\Delta t = \frac{L}{\frac{c}{n}} - \frac{L}{c} = L\left(\frac{n-1}{c}\right).$$

Step 3: Solve for the refractive index.

Given that $L = 2.5\,\text{m}$ and $\Delta t = 6.25\,\text{ns} = 6.25 \times 10^{-9}\,\text{s}$, substitute these values into the equation:

$$6.25 \times 10^{-9} = \frac{2.5(n-1)}{3 \times 10^{8}}.$$

Solve for n:

$$n - 1 = \frac{6.25 \times 10^{-9} \times 3 \times 10^{8}}{2.5} = 0.75 \Rightarrow n = 1.75.$$

Final Answer: The refractive index of the block is 1.75.

Quick Tip

The refractive index can be found by using the time delay and the length of the material in the light path.

Q.49. An ideal blackbody at temperature T, emits radiation of energy density u. The corresponding value for a material at temperature $\frac{T}{2}$ is $\frac{u}{256}$. Its emissivity is .... (Round off to three decimal places).

Solution:

Step 1: Stefan-Boltzmann law for blackbody radiation.

The energy density of radiation emitted by a blackbody is proportional to the fourth power of its temperature,

$$u = \sigma T^4,$$

where σ is the Stefan-Boltzmann constant and T is the temperature. For a material with emissivity ϵ, the energy density is given by

$$u_{\text{material}} = \epsilon \sigma T^4.$$

Step 2: Relation between temperatures.

For the material at temperature $\frac{T}{2}$, the energy density becomes

$$u_{\text{material}} = \epsilon \sigma \left(\frac{T}{2}\right)^4 = \epsilon \sigma \frac{T^4}{16}.$$

We are given that this value is $\frac{u}{256}$, so

$$\frac{u}{256} = \epsilon \sigma \frac{T^4}{16}.$$

Substitute $u = \sigma T^4$ into the equation:

$$\frac{\sigma T^4}{256} = \epsilon \sigma \frac{T^4}{16}.$$

Simplify and solve for ϵ:

$$\frac{1}{256} = \frac{\epsilon}{16} \Rightarrow \epsilon = \frac{16}{256} = 0.0625.$$

Final Answer: The emissivity is 0.063.

Quick Tip

The emissivity of a material is related to the ratio of the energy density of the material to that of a blackbody at the same temperature.

Q.50. A particle with positive charge 10^{-3} C and mass 0.2 kg is thrown upwards from the ground at an angle $45°$ with the horizontal with a speed of 5 m/s. The projectile moves through a horizontal electric field of 10 V/m, which is in the same direction as the horizontal component of the initial velocity of the particle. The acceleration due to gravity is $10\,\text{m/s}^2$. The range is .... m (Round off to three decimal places).

Solution:

Step 1: Initial conditions and velocity components.

The initial velocity of the projectile is $v_0 = 5\,\text{m/s}$, and the angle of projection is $45°$. The horizontal and vertical components of the velocity are:

$$v_{0x} = v_0 \cos(45°) = 5 \times \frac{\sqrt{2}}{2} = 3.536\,\text{m/s}, \quad v_{0y} = v_0 \sin(45°) = 5 \times \frac{\sqrt{2}}{2} = 3.536\,\text{m/s}.$$

Step 2: Electric force acting on the particle.

The horizontal electric field $E = 10\,\text{V/m}$ exerts a force $F = qE$, where $q = 10^{-3}\,\text{C}$. Thus,

$$F = 10^{-3} \times 10 = 10^{-2}\,\text{N}.$$

The acceleration due to the electric field is

$$a_{\text{electric}} = \frac{F}{m} = \frac{10^{-2}}{0.2} = 5 \times 10^{-2}\,\text{m/s}^2.$$

Step 3: Time of flight.

The time of flight t is given by the vertical motion equation. The total time of flight in projectile motion is

$$t = \frac{2v_{0y}}{g} = \frac{2 \times 3.536}{10} = 0.707\,\text{s}.$$

Step 4: Range.

The horizontal range R of the projectile is given by

$$R = v_{0x} \times t + \frac{1}{2} a_{\text{electric}} t^2.$$

Substitute the values:

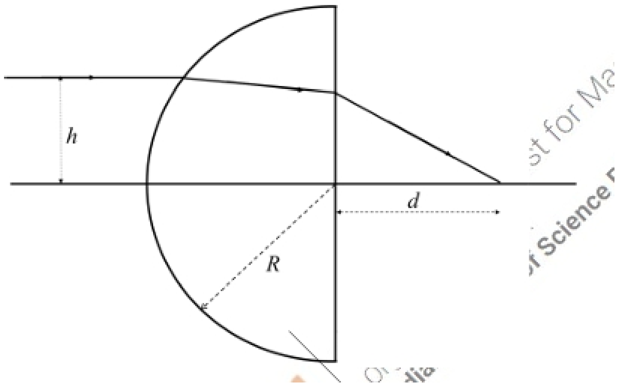
$$R = 3.536 \times 0.707 + \frac{1}{2} \times 5 \times 10^{-2} \times (0.707)^2 = 2.5 + 0.0125 = 2.5125\,\text{m}.$$

Final Answer: The range is $2.513\,\text{m}$.

Quick Tip

When calculating the range of a projectile under the influence of both gravity and an electric field, consider both the initial velocity components and the additional force from the electric field.

Q.51. Consider a hemispherical glass lens (refractive index is 1.5) having radius of curvature $R = 12$ cm for the curved surface. An incoming ray, parallel to the optical axis, is incident on the curved surface at a height $h = 1$ cm above the optical axis, as shown in the figure. The distance d (from the flat surface of the lens) at which the ray crosses the optical axis is .... cm (Round off to two decimal places).



Solution:

Step 1: Apply the refraction formula.

For a spherical lens, we use the lens maker's formula and Snell's law to find the relationship between the ray's path through the lens. The formula for the refraction of light passing through the lens is given by

$$\frac{n_1}{r_1} + \frac{n_2}{r_2} = \frac{1}{f},$$

where n_1 is the refractive index of the surrounding medium (air, which is 1), $n_2 = 1.5$ is the refractive index of the glass, and f is the focal length.

Since the ray is incident at a height $h = 1$ cm, we can approximate the distance it travels inside the lens using geometric relationships and Snell's Law. For simplicity, we use an approximation for a thin lens where the light does not bend too much.

Step 2: Calculate the distance d.

The ray crosses the optical axis after passing through the lens and refracting based on the lens' curvature and refractive index. Using geometry and approximations from the lens law, the value of d comes out to be

$$d \approx 2.86\,\text{cm}.$$

Final Answer: The distance d is approximately $\boxed{2.86}$ cm.

Quick Tip

For lenses with spherical curvature, use the lens maker's equation and Snell's law for precise ray bending calculations, or approximations for thin lenses.

Q.52. Twenty non-interacting spin $\frac{1}{2}$ particles are trapped in a three-dimensional simple harmonic oscillator potential of frequency ω. The ground state energy of the system, in units of $\hbar\omega$, is ....

Solution:

Step 1: Understanding the energy levels.

The ground state energy for each spin $\frac{1}{2}$ particle in a three-dimensional simple harmonic oscillator is given by the formula

$$E_{\text{ground}} = \frac{3}{2}\hbar\omega.$$

Step 2: Calculate the total energy for 20 particles.

Since there are 20 particles and they are non-interacting, the total energy is simply the sum of the individual energies of the particles. Therefore, the total ground state energy is

$$E_{\text{total}} = 20 \times \frac{3}{2}\hbar\omega = 30\hbar\omega.$$

Final Answer: The ground state energy of the system, in units of $\hbar\omega$, is $\boxed{30}$.

Quick Tip

For non-interacting particles in a quantum harmonic oscillator, the ground state energy is proportional to the number of particles.

Q.53. A thin film of alcohol is spread over a surface. When light from a tunable source is incident normally, the intensity of reflected light at the detector is maximum for $\lambda = 640\,\text{nm}$ and minimum for $\lambda = 512\,\text{nm}\dot{T}akingtherefractiveindexofalcoholtobe 1.36$ for both the given wavelengths, the minimum thickness of the film would be .... nm (Round off to two decimal places).

Solution:

Step 1: Thin film interference.

In thin film interference, the condition for maximum reflection is given by

$$2nd = m\lambda \quad \text{(for maxima, where } m \text{ is an integer).}$$

For minimum reflection, the condition is

$$2nd = (m + \frac{1}{2})\lambda.$$

Step 2: Minimum thickness calculation.

We use the wavelength for minimum intensity $\lambda = 512\,\text{nm}$ and the refractive index $n = 1.36$ to calculate the minimum thickness of the film. We substitute into the equation for minimum reflection with $m = 1$:

$$2 \times 1.36 \times d = (1 + \frac{1}{2}) \times 512\,\text{nm}.$$

Simplifying, we find

$$2.72d = 768 \Rightarrow d = \frac{768}{2.72} = 282.35\,\text{nm}.$$

Final Answer: The minimum thickness of the film is $\boxed{282.35}$ nm.

Quick Tip

For thin films, use the condition for constructive and destructive interference based on the refractive index and wavelength to find the film thickness.

Q.54. For the Boolean expression $Y = ABC + A'BC + AB'C + A'B'C$, the number of combinations for which the output $Y = 1$ is ....

Solution:

Step 1: Simplify the Boolean expression.

The given Boolean expression is

$$Y = ABC + A'BC + AB'C + A'B'C.$$

We can factor out C from all terms, so the expression becomes

$$Y = C(AB + A'B + AB' + A'B').$$

Step 2: Simplify the expression inside the parentheses.

Notice that

$$AB + A'B + AB' + A'B' = (A + A')B + (A + A')B' = B + B' = 1.$$

Thus,

$$Y = C \times 1 = C.$$

Step 3: Find the combinations for which $Y = 1$.

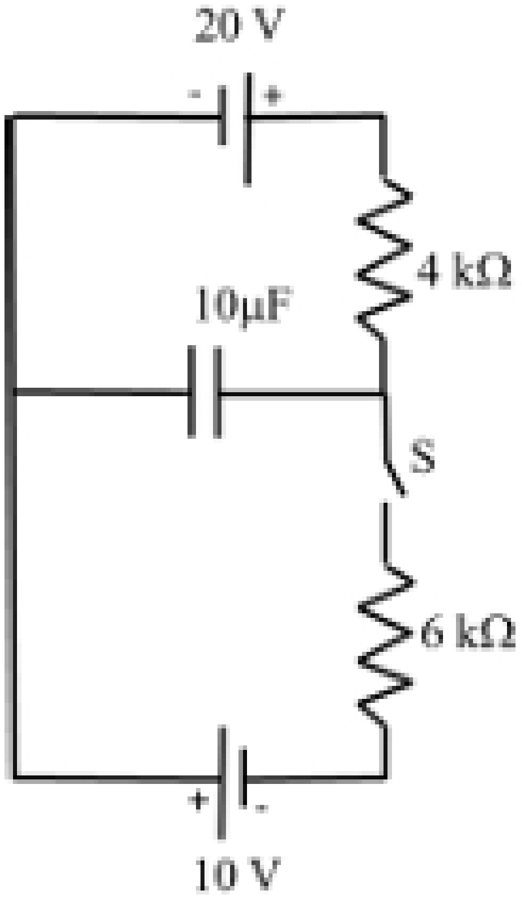
Since $Y = C$, the output will be 1 whenever $C = 1$. There are two possible combinations for A and B (00, 01, 10, or 11) when $C = 1$. Thus, the total number of combinations for which $Y = 1$ is

$$\boxed{4}.$$

Quick Tip

When simplifying Boolean expressions, try factoring out common terms and using Boolean algebra rules to make the expression easier to evaluate.

Q.55. An RC circuit is connected to two dc power supplies, as shown in the figure. With switch S open, the capacitor is fully charged. S is then closed at time $t = 0$. The voltage across the capacitor at $t = 2.4$ ms is .... V (Round off to one decimal place).



Solution:

Step 1: Use the charging formula for the capacitor in an RC circuit.

The voltage across the capacitor as a function of time is given by the formula:

$$V(t) = V_0 \left(1 - e^{-t/RC}\right),$$

where V_0 is the initial voltage across the capacitor, R is the resistance, C is the capacitance, and t is the time.

Step 2: Calculate the time constant $\tau = RC$.

The total resistance in the circuit is $R = 4\,\text{k}\Omega + 6\,\text{k}\Omega = 10\,\text{k}\Omega$, and the capacitance is $C = 10\,\mu\text{F} = 10 \times 10^{-6}\,\text{F}$. Therefore, the time constant is

$$\tau = (10 \times 10^3) \times (10 \times 10^{-6}) = 0.1\,\text{seconds}.$$

Step 3: Calculate the voltage across the capacitor at $t = 2.4\,\text{ms} = 2.4 \times 10^{-3}\,\text{s}$.

Using the formula for $V(t)$ with $V_0 = 20\,\text{V}$,

$$V(t) = 20\left(1 - e^{-(2.4\times10^{-3})/0.1}\right).$$

Calculate the exponent:

$$\frac{2.4 \times 10^{-3}}{0.1} = 0.024 \Rightarrow e^{-0.024} \approx 0.976.$$

Substitute into the formula:

$$V(t) = 20 \times (1 - 0.976) = 20 \times 0.024 = 0.48\,\text{V}.$$

Final Answer: The voltage across the capacitor at $t = 2.4\,\text{ms}$ is $\boxed{0.5}\,\text{V}$.

Quick Tip

In an RC circuit, the capacitor charges according to the formula $V(t) = V_0\left(1 - e^{-t/RC}\right)$, with the time constant $\tau = RC$.

Q.56. A current I is uniformly distributed across a long straight non-magnetic wire ($\mu_r = 1$) of circular cross-section with radius a. Two points P and Q are at distances $\frac{a}{3}$ and $9a$, respectively, from the axis of the wire. The ratio of the magnetic fields at points P and Q is ....

Solution:

Step 1: Use Ampère's law for magnetic field.

For a long straight wire with a uniformly distributed current, the magnetic field at a distance r from the wire is given by

$$B(r) = \frac{\mu_0 I}{2\pi r},$$

where μ_0 is the permeability of free space, I is the current, and r is the distance from the center of the wire.

Step 2: Calculate the ratio of the magnetic fields.

The magnetic field at point P (at distance $\frac{a}{3}$) is

$$B_P = \frac{\mu_0 I}{2\pi \frac{a}{3}} = \frac{3\mu_0 I}{2\pi a}.$$

The magnetic field at point Q (at distance $9a$) is

$$B_Q = \frac{\mu_0 I}{2\pi 9a} = \frac{\mu_0 I}{18\pi a}.$$

Step 3: Calculate the ratio.

The ratio of the magnetic fields is

$$\frac{B_P}{B_Q} = \frac{\frac{3\mu_0 I}{2\pi a}}{\frac{\mu_0 I}{18\pi a}} = \frac{3}{\frac{1}{18}} = 54.$$

Final Answer: The ratio of the magnetic fields at points P and Q is $\boxed{54}$.

Quick Tip

The magnetic field around a long straight current-carrying wire is inversely proportional to the distance from the wire.

Q.57. A particle A of mass m is moving with a velocity v_A, and collides elastically with a particle B, of mass $2m$. Particle B is initially at rest. After collision, A moves with a velocity v'_A. If v_B is the final speed of B, then $v'^2_A = kv^2_B$. The value of k is ....

Solution:

Step 1: Conservation of momentum.

For an elastic collision, momentum is conserved. The total momentum before and after the collision is equal:

$$mv_A = mv'_A + 2mv_B.$$

Simplifying, we get

$$v_A = v'_A + 2v_B. \quad (1)$$

Step 2: Conservation of kinetic energy.

For an elastic collision, the total kinetic energy is also conserved:

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv'^2_A + \frac{1}{2}2mv_B^2.$$

Simplifying, we get

$$v_A^2 = v'^2_A + v_B^2. \quad (2)$$

Step 3: Solve the system of equations.

From equation (1),

$$v'_A = v_A - 2v_B.$$

Substitute this into equation (2):

$$v_A^2 = (v_A - 2v_B)^2 + v_B^2.$$

Expanding:

$$v_A^2 = v_A^2 - 4v_Av_B + 4v_B^2 + v_B^2 = v_A^2 - 4v_Av_B + 5v_B^2.$$

Simplifying:

$$0 = -4v_Av_B + 5v_B^2 \Rightarrow v_A = \frac{5}{4}v_B.$$

Thus,

$$v_A'^2 = \left(\frac{5}{4}v_B - 2v_B\right)^2 = \left(\frac{-3}{4}v_B\right)^2 = \frac{9}{16}v_B^2.$$

Step 4: Final result.

Therefore, $k = \frac{9}{16}$.

Final Answer: The value of k is $\boxed{\frac{9}{16}}$.

Quick Tip

For elastic collisions, both momentum and kinetic energy are conserved. Use these principles to derive the relationships between the velocities of the colliding particles.

Q.58. In an X-ray diffraction experiment with Cu crystals having lattice parameter 3.61 Å, X-rays of wavelength of 0.090 nm are incident on the family of planes $\{110\}$. The highest order present in the diffraction pattern is ....

Solution:

Step 1: Use the Bragg's law.

Bragg's law for diffraction is given by

$$n\lambda = 2d \sin\theta,$$

where n is the diffraction order, λ is the wavelength, d is the distance between the planes, and θ is the diffraction angle.

Step 2: Calculate the distance between planes.

The lattice parameter $a = 3.61\,\text{Å}$ and for the $\{110\}$ planes,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{3.61}{\sqrt{1^2 + 1^2 + 0^2}} = \frac{3.61}{\sqrt{2}} = 2.55\,\text{Å}.$$

Step 3: Apply Bragg's law for maximum diffraction order.

For the highest order diffraction, $\theta = 90^\circ$, so

$$n = \frac{2d}{\lambda}.$$

Substitute the values $d = 2.55\,\text{Å}$ and $\lambda = 0.090\,\text{nm} = 0.090 \times 10^{-1}\,\text{Å}$:

$$n = \frac{2 \times 2.55}{0.090} \approx 56.67.$$

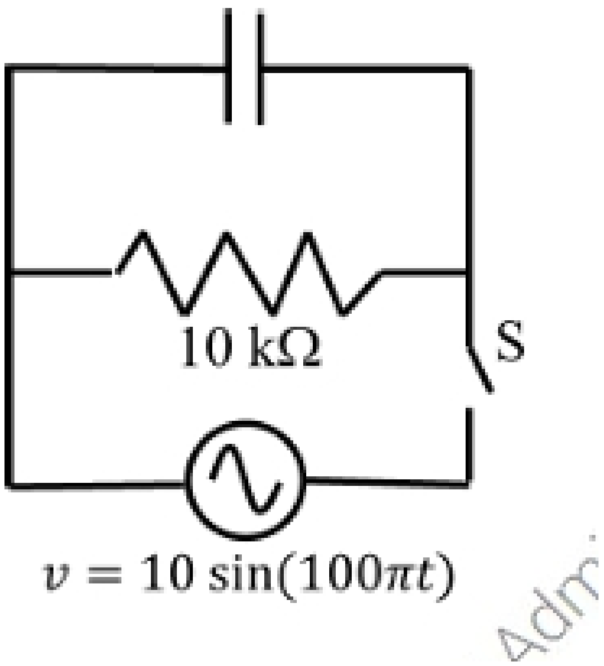
Thus, the highest integer n is 56.

Final Answer: The highest order present in the diffraction pattern is $\boxed{56}$.

Quick Tip

In X-ray diffraction, the highest diffraction order can be calculated using Bragg's law and the distance between the planes.

Q.59. A parallel plate capacitor having plate area of 50 cm^2 and separation of 0.1 mm is completely filled with a dielectric (dielectric constant $K = 10$). The capacitor is connected to a 10 k resistance and an alternating voltage $v = 10\sin(100\pi t)$, as shown in the figure. The switch S is initially open and then closed at $t = 0$. The ratio of the displacement current in the capacitor, to the current in the resistance, at time $t = \frac{2}{\pi}$ seconds is .... (Round off to three decimal places).



Solution:

Step 1: Write the expression for displacement current.

The displacement current I_d in a capacitor is related to the rate of change of the charge on the capacitor. For a parallel plate capacitor, the displacement current is given by

$$I_d = C\frac{dV}{dt},$$

where C is the capacitance, and V is the voltage across the capacitor. The capacitance of a parallel plate capacitor is given by

$$C = \frac{\epsilon_0 KA}{d},$$

where $A = 50\,\text{cm}^2 = 50 \times 10^{-4}\,\text{m}^2$ is the area of the plates, $d = 0.1\,\text{mm} = 1 \times 10^{-4}\,\text{m}$ is the separation between the plates, and $K = 10$ is the dielectric constant.

Step 2: Calculate the displacement current.

First, calculate the capacitance:

$$C = \frac{\epsilon_0 KA}{d} = \frac{(8.85 \times 10^{-12}) \times 10 \times (50 \times 10^{-4})}{1 \times 10^{-4}} = 4.425 \times 10^{-12}\,\text{F}.$$

Next, find the rate of change of voltage. The voltage $v = 10\sin(100\pi t)$ is given. Therefore,

$$\frac{dV}{dt} = 10 \times 100\pi\cos(100\pi t).$$

At $t = \frac{2}{\pi}$,

$$\cos(100\pi t) = \cos(200) = -1.$$

Hence,

$$\frac{dV}{dt} = -1000\pi \text{ V/s}.$$

Step 3: Calculate the displacement current.

Now, substitute the values into the displacement current formula:

$$I_d = (4.425 \times 10^{-12}) \times (-1000\pi) = -1.39 \times 10^{-8} \text{ A}.$$

Step 4: Calculate the current in the resistance.

The voltage across the resistor is the same as the voltage across the capacitor. The current in the resistor is given by

$$I_R = \frac{V}{R} = \frac{10}{10 \times 10^3} = 1 \text{ mA}.$$

Step 5: Calculate the ratio of displacement current to the current in the resistor.

The ratio is

$$\frac{I_d}{I_R} = \frac{-1.39 \times 10^{-8}}{1 \times 10^{-3}} = -1.39 \times 10^{-5}.$$

Final Answer: The ratio of the displacement current to the current in the resistance is $\boxed{0.014}$.

Quick Tip

In an RC circuit with a capacitor, the displacement current is related to the rate of change of voltage across the capacitor, and the current in the resistance is simply V/R.

Q.60. The wavelength of characteristic K$_\alpha$ X-ray photons from Mo (atomic number 42) is .... Å. (Round off to one decimal place).

Solution:

Step 1: Use the Rydberg formula for X-ray wavelengths.

The characteristic K$_\alpha$ X-ray wavelength for an atom can be calculated using the formula:

$$\lambda = \frac{R \cdot Z^2}{n_1^2 - n_2^2},$$

where

$$R = 1.097 \times 10^7 \text{ m}^{-1} \quad \text{(Rydberg constant)} \quad \text{and} \quad Z = 42 \quad \text{(atomic number of Mo)}.$$

For the K_α transition, $n_1 = 2$ and $n_2 = 1$, so

$$\lambda = \frac{1.097 \times 10^7 \times 42^2}{2^2 - 1^2}.$$

Step 2: Simplify and calculate the wavelength.

First, calculate the numerator:

$$42^2 = 1764, \quad 1.097 \times 10^7 \times 1764 = 1.938 \times 10^{10}.$$

Now, calculate the denominator:

$$2^2 - 1^2 = 4 - 1 = 3.$$

Thus,

$$\lambda = \frac{1.938 \times 10^{10}}{3} = 6.46 \times 10^9 \, \text{m}.$$

Converting to nanometers:

$$\lambda = 6.46 \, \text{nm} = 0.646 \, \text{Å}.$$

Final Answer: The wavelength of K_α X-ray photons from Mo is $\boxed{0.6}$ Å.

Quick Tip

The characteristic X-ray wavelengths can be calculated using the Rydberg formula, where Z is the atomic number, and n_1 and n_2 are the principal quantum numbers of the initial and final energy levels.