IIT JAM 2023 EN Question Paper with Answer Key PDF

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**60

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Please check that this question paper contains 60 questions.
2. This Question Paper has 60 questions. All questions are compulsory.
3. Adhere to the prescribed word limit while answering the questions.

1. A competitive firm can sell any output at price $P = 1$. Production depends on capital alone, and the production function $y = f(K)$ is twice continuously differentiable, with

$$f(0) = 0,\ f' > 0,\ f'' < 0,\ \lim_{K\to 0} f'(K) = \infty,\ \lim_{K\to\infty} f'(K) = 0.$$

The firm has positive capital stock K to start with, and can buy and sell capital at price r per unit of capital. If the firm is maximizing profit then which of the following statements is NOT CORRECT?

(1) If K is large enough, profit maximizing $y = 0$ and the profit is rK

(2) If $f'(K) > r$, the firm will buy additional capital

(3) If $f'(K) < r$, the firm will sell some of its capital

(4) If $f'(K) = r$, the firm will neither buy nor sell any capital

Correct Answer: (1) If K is large enough, profit maximizing $y = 0$ and the profit is rK

Solution:

Step 1: The firm's objective is to maximize profit. The profit is given by:

$$\pi = P \cdot f(K) - rK$$

The profit maximizing condition requires that marginal revenue (MR) equals marginal cost (MC), i.e., $f'(K) = r$. If $f'(K) > r$, the firm should buy more capital to increase production; if $f'(K) < r$, it should sell some capital. For large K, the optimal production level may lead to $y = 0$, resulting in a profit of rK.

Quick Tip

For profit maximization, equate the marginal product of capital with the cost of capital. If the marginal product is greater than the cost, the firm will buy more capital; if it's less, the firm will sell some capital.

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2. Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x + 2, & x \le 1 \\ 2x + 1, & x > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2x, & x \le 2 \\ x + 2, & x > 2 \end{cases}$$

Then:

(1) f is convex and g is concave

(2) f is concave and g is convex

(3) both f and g are concave

(4) both f and g are convex

Correct Answer: (4) both f and g are convex

Solution:

Step 1: Check the properties of the functions. $f(x)$ is piecewise linear and both pieces are convex. Similarly, $g(x)$ is also piecewise linear, with both pieces being convex.

Quick Tip

A function is convex if its second derivative is non-negative, or if it is piecewise linear with each segment being convex.

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3. Let S be a feasible set of a linear programming problem (P). If the dual problem of (P) is unbounded then:

(1) (P) is unbounded

(2) S is empty

(3) S is unbounded

(4) (P) has multiple optimal solutions

Correct Answer: (3) S is unbounded

Solution:

Step 1: According to the strong duality theorem in linear programming, if the dual problem is unbounded, it implies that the primal problem's feasible region is unbounded, i.e., S is unbounded.

Quick Tip

In linear programming, if the dual problem is unbounded, the feasible set of the primal problem must be unbounded.

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4. Which of the following is NOT CORRECT?

(1) A quasiconcave function is necessarily a concave function

(2) A concave function is necessarily a quasiconcave function

(3) A quasiconcave function can also be a quasiconvex function

(4) A quasiconcave function can also be a convex function

Correct Answer: (2) A concave function is necessarily a quasiconcave function

Solution:

Step 1: A concave function is a stronger condition than quasiconcavity. A concave function is always quasiconcave, but a quasiconcave function does not necessarily have to be concave.

> Quick Tip
>
> A concave function always satisfies the condition of quasiconcavity, but the reverse is not true.

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5. Among the following statements which one is CORRECT?

S1: $x^2 + y^2 = 6$ is a level curve of

$$f(x,y) = \sqrt{x^2 + y^2 - x^2 - y^2 + 2}$$

S2: $x^2 - y^2 = -3$ is a level curve of

$$g(x,y) = e^{-x^2} e^{y^2} + x^4 - 2 - 2x^2y^2 + y^4$$

(1) both S1 and S2

(2) only S1

(3) only S2

(4) neither S1 nor S2

Correct Answer: (2) only S1

Solution:

Step 1: A level curve is a curve where a function takes a constant value. The equation $x^2 + y^2 = 6$ is a level curve of the function $f(x,y)$. On the other hand, the equation $x^2 - y^2 = -3$ does not correspond to a level curve of $g(x,y)$, as substituting the values does not satisfy the equation.

Quick Tip

To verify if an equation represents a level curve of a function, substitute the curve's equation into the function and check if the equation holds true.

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6. Which of the following is NOT a component of Gross Domestic Product?

(1) Investment

(2) Rental Income

(3) Transfer Payments

(4) Wages and Salaries

Correct Answer: (3) Transfer Payments

Solution:

Step 1: Gross Domestic Product (GDP) includes all goods and services produced within a country. Transfer payments, such as social security payments or welfare, do not represent production and hence are not included in GDP.

Quick Tip

Transfer payments are not part of GDP since they do not reflect current production or value-added activities.

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7. Which of the following are the direct instruments exercised by the Reserve Bank of India to control the money supply?

(1) (i) Cash Reserve Ratio, (ii) Open Market Operations, (iii) Foreign Exchange Rate, (iv) Statutory Liquidity Ratio

(2) (i) Cash Reserve Ratio, (ii) Open Market Operations, (iv) Statutory Liquidity Ratio

(3) (ii) Open Market Operations, (iii) Foreign Exchange Rate, (iv) Statutory Liquidity Ratio

(4) (i) Cash Reserve Ratio, (ii) Open Market Operations, (iii) Foreign Exchange Rate

Correct Answer: (2) (i) Cash Reserve Ratio, (ii) Open Market Operations, (iv) Statutory Liquidity Ratio

Solution:

Step 1: The Reserve Bank of India uses direct instruments such as the Cash Reserve Ratio (CRR), Open Market Operations (OMOs), and the Statutory Liquidity Ratio (SLR) to control the money supply. The Foreign Exchange Rate is typically influenced through indirect instruments.

Quick Tip

Direct instruments like CRR, OMOs, and SLR are actively used by the RBI to regulate money supply in the economy.

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8. Which of the following committees for the first time recommended for India:

(1) Y K Alagh Committee

(2) D T Lakdawala Committee

(3) S D Tendulkar Committee

(4) C Rangarajan Committee

Correct Answer: (1) Y K Alagh Committee

Solution:

Step 1: The Y K Alagh Committee was the first to recommend the use of implicit prices derived from quantity and value data collected in household consumer expenditure surveys to compute and update poverty lines.

Quick Tip

The Y K Alagh Committee's recommendations laid the foundation for updating and computing poverty lines using implicit prices in India.

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9. Which of the following Five Year Plans focused on rapid industrialization-heavy and basic industries, and advocated for a socialistic pattern of society as the goal of economic policy?

(1) 1st Five Year Plan (1951-56)

(2) 2nd Five Year Plan (1956-61)

(3) 3rd Five Year Plan (1961-66)

(4) 4th Five Year Plan (1969-74)

Correct Answer: (2) 2nd Five Year Plan (1956-61)

Solution:

Step 1: The 2nd Five Year Plan (1956-61) focused on rapid industrialization, with a special emphasis on heavy and basic industries, aligning with a socialist pattern of society.

Quick Tip

The 2nd Five Year Plan in India focused on setting up the industrial base of the country, including the establishment of key public sector industries.

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10. Let M and N be events defined on the sample space S. If $P(M) = \frac{1}{3}$ and $P(N^c) = \frac{1}{4}$, then which one of the following is necessarily CORRECT?

(1) M and N are disjoint

(2) M and N are not disjoint

(3) M and N are independent

(4) M and N are not independent

Correct Answer: (2) M and N are not disjoint

Solution:

Step 1: The probability of $P(M) = \frac{1}{3}$ and $P(N^c) = \frac{1}{4}$, so $P(N) = 1 - P(N^c) = \frac{3}{4}$. For events to be disjoint, their intersection must be empty, but here we cannot confirm if $M \cap N = \emptyset$, so they are not necessarily disjoint.

Quick Tip

Two events are disjoint if $P(M \cap N) = 0$. If this is not true, the events are not disjoint.

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11. Consider a 2-agent, 2-good exchange economy where agent i has utility function $u_i(x_i, y_i) = \max\{x_i, y_i\}, i = 1, 2$. The initial endowments of goods X and Y that the agents have are $(x_1, y_1), (x_2, y_2) = (25, 5, 5, 5)$. Then select the CORRECT choice below where the price vector (p_x, p_y) specified is part of a competitive equilibrium.

(1) $(p_x, p_y) = (2, 1)$

(2) $(p_x, p_y) = (2, 2)$

(3) $(p_x, p_y) = (1, 2)$

(4) $(p_x, p_y) = (4, 2)$

Correct Answer: (1) $(p_x, p_y) = (2, 1)$

Solution:

Step 1: In a competitive equilibrium, the price vector must balance the total demand and supply for both goods. The utility functions and endowments are used to determine the equilibrium prices.

> **Quick Tip**
>
> For competitive equilibrium, the demand and supply for each good must match at the equilibrium prices.

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12. For a firm operating in a perfectly competitive market which of the following statements is CORRECT?

(1) Profit function is convex and homogeneous of degree 1 in prices

(2) Profit function is concave and homogeneous of degree 1 in prices

(3) Profit function is convex but not homogeneous in prices

(4) Profit function is neither concave nor convex in prices

Correct Answer: (1) Profit function is convex and homogeneous of degree 1 in prices

Solution:

Step 1: The profit function in a perfectly competitive market is typically convex due to diminishing returns and homogeneous of degree 1 in prices due to the scaling property of production functions.

> **Quick Tip**
>
> Profit functions in perfectly competitive markets are convex and homogeneous of degree 1 in prices.

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13. A firm is operating in a perfectly competitive environment. A change in the market condition leads to an increase in the firm's profit by an amount K. Which of the following describes the change in the Producer's Surplus due to the above change in the market condition?

(1) The Producer's Surplus increases by K

(2) The Producer's Surplus increases by less than K but greater than 0

(3) The Producer's Surplus changes but it is not possible to know the direction of the change

(4) The Producer's Surplus doesn't change

Correct Answer: (1) The Producer's Surplus increases by K

Solution:

Step 1: In perfect competition, an increase in the firm's profit will directly reflect as an increase in the producer's surplus, since producer's surplus is the area between the price line and the supply curve.

Quick Tip

An increase in profit directly increases the producer's surplus in a perfectly competitive market.

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14. Two people, 1 and 2, are engaged in a joint project. Person $i \in \{1, 2\}$ puts in effort x_i ($0 \leq x_i \leq 1$), and incurs cost $C_i(x_i) = x_i$. The monetary outcome of the project is $4x_1x_2$ which is split equally between them. Considering the situation as a strategic game, the set of all Nash Equilibria in pure strategies is:

(1) $\{(0, 0), (1, 1)\}$

(2) $\{(0, 0), (\frac{1}{4}, \frac{3}{4}), (\frac{3}{4}, \frac{1}{4}), (1, 1)\}$

(3) $\{(0, 0), (\frac{1}{2}, \frac{1}{2}), (1, 1)\}$

(4) A null set

Correct Answer: (3) $\{(0, 0), (\frac{1}{2}, \frac{1}{2}), (1, 1)\}$

Solution:

Step 1: The Nash Equilibria are found by solving the optimization problem for both players.

In this case, both players maximize their individual payoffs considering the effort exerted by the other player.

Quick Tip

Nash Equilibria are found by solving the best response functions for both players in a strategic game.

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15. Two firms, X and Y, are operating in a perfectly competitive market. The price elasticity of supply of X and Y are respectively 0.5 and 1.5. Then:

(1) If the market price increases by 1%, X supplies 0.5% less quantity

(2) Y experiences a slower increase in marginal cost in comparison to X

(3) If market price increases by 0.5%, X supplies 1% more quantity

(4) Y experiences a rapid increase in marginal cost in comparison to X

Correct Answer: (3) If market price increases by 0.5%, X supplies 1% more quantity

Solution:

Step 1: The price elasticity of supply indicates how much the supply changes with a change in price. Since X's elasticity is 0.5, a 0.5% price increase leads to a 1% increase in X's quantity supplied.

Quick Tip

Price elasticity of supply shows the responsiveness of quantity supplied to price changes.

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16. Let $y - y(x)$ be a solution curve of the differential equation

$$x\frac{dy}{dx} = y\ln\left(\frac{y}{x}\right), \; y > x > 0.$$

If $y(1) = e^2$ and $y(2) = \alpha$, then the value of $\frac{dy}{dx}$ at $(2, \alpha)$ is equal to:

(1) α

(2) $\frac{\alpha}{2}$

(3) 2α

(4) $\frac{3\alpha}{2}$

Correct Answer: (2) $\frac{\alpha}{2}$

Solution:

Step 1: Solve the given differential equation using the initial conditions. After solving, substitute the values $y(1) = e^2$ and $y(2) = \alpha$ into the equation to get the value of $\frac{dy}{dx}$.

Quick Tip

Differential equations involving logarithms can often be solved by separating variables and integrating both sides.

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17. Let $2z = -3 + \sqrt{3}i$, $i = \sqrt{-1}$. Then $2z^8$ is equal to:

(1) $-81(1 + \sqrt{3}i)$

(2) $81(-1 + \sqrt{3}i)$

(3) $81(\sqrt{3} + i)$

(4) $9(-\sqrt{3} + i)$

Correct Answer: (3) $81(\sqrt{3} + i)$

Solution:

Step 1: First, compute the value of $2z$, and then raise it to the power of 8. Use the properties of complex numbers to simplify the result.

Quick Tip

To compute powers of complex numbers, use De Moivre's Theorem or directly multiply the number in polar form.

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18. Let $a_n = \left(1 + \frac{1}{n}\right)^{\frac{n}{2}}$ be the n-th term of the sequence $\{a_n\}, n = 1, 2, 3, \ldots$. Then which one of the following is NOT CORRECT?

(1) $\{a_n\}$ is bounded

(2) $\{a_n\}$ is increasing

(3) $\sum_{n=1}^{\infty} \ln(a_n)$ is a convergent series

(4) $\lim_{n\to\infty} \left(\frac{1}{n}\sum_{k=1}^{n} a_k\right) = \sqrt{e}$

Correct Answer: (3) $\sum_{n=1}^{\infty} \ln(a_n)$ is a convergent series

Solution:

Step 1: The sequence a_n is increasing and bounded. The series $\sum_{n=1}^{\infty} \ln(a_n)$ is divergent because the terms do not decay quickly enough.

> **Quick Tip**
>
> To test for convergence of a series, check the behavior of the terms. If the terms do not tend to 0 rapidly enough, the series will diverge.

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19. Consider a linear programming problem (P)

$$\min\ z = 4x_1 + 6x_2 + 6x_3$$

subject to

$$x_1 + 3x_2 \geq 3, \quad x_1 + 2x_3 \geq 5, \quad x_1, x_2, x_3 \geq 0.$$

If $x^* = (x_1^*, x_2^*, x_3^*)$ **is an optimal solution and** z^* **is an optimal value of** (P)**, and** $w^* = (w_1^*, w_2^*)$ **is an optimal solution of the dual of** (P)**, then:**

(1) $x_2^* + x_3^* = w_1^* + w_2^*$

(2) $z^* = 4(x_1^* + w_2^*)$

(3) $z^* = 6(w_1^* + x_3^*)$

(4) $x_1^* + x_3^* = w_1^* + w_2^*$

Correct Answer: (1) $x_2^* + x_3^* = w_1^* + w_2^*$

Solution:

Step 1: The relationship between the primal and dual solutions involves complementary slackness. The optimal values x_2^* and x_3^* should add up to the sum of w_1^* and w_2^*, according to duality theory.

Quick Tip

In linear programming, the optimal solution to the primal and dual problems are related by complementary slackness.

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20. For $\alpha, \beta \in \mathbb{R}$, consider the system of linear equations

$$x + y + z = 1, \quad 3x + y + 2z = 2, \quad 5x + \alpha y + \beta z = 3.$$

Then:

(1) for every (α, β), $\alpha = \beta$, the system is consistent

(2) there exists (α, β), satisfying $\alpha - 2\beta + 5 = 0$, for which the system has a unique solution

(3) there exists a unique pair (α, β) for which the system has infinitely many solutions

(4) for every (α, β), $\alpha \neq \beta$, satisfying $\alpha - 2\beta + 5 = 0$, the system has infinitely many solutions

Correct Answer: (2) there exists (α, β), satisfying $\alpha - 2\beta + 5 = 0$, for which the system has a unique solution

Solution:

Step 1: The system will have a unique solution when the determinant of the coefficient matrix is non-zero. This is true when $\alpha - 2\beta + 5 = 0$.

Quick Tip

Check the determinant of the coefficient matrix to determine if a system has a unique solution.

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21. For a positively sloped LM curve, which of the following statements is CORRECT?

(1) A decrease in the price level will shift the LM curve to the left

(2) A lower nominal money supply will shift the LM curve to the right

(3) An increase in the price level will shift the LM curve to the right

(4) A higher nominal money supply will shift the LM curve to the right

Correct Answer: (4) A higher nominal money supply will shift the LM curve to the right

Solution:

Step 1: The LM curve shows the combinations of interest rates and levels of income that equate money demand and money supply. A higher nominal money supply increases the amount of money available, shifting the LM curve to the right.

Quick Tip

The LM curve shifts to the right when the money supply increases and shifts to the left when the money supply decreases.

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22. Consider an Economy that produces only Apples and Bananas. The following Table contains per unit price (in INR) and quantity (in kg) of these goods. Assuming 2010 as the Base Year and using GDP deflator to calculate the annual inflation rate, which of the following options is CORRECT?

(1) GDP deflator for the year 2011 is 100 and the inflation rate for the year 2011 is 0(2) GDP deflator for the year 2012 is 50 and the inflation rate for the year 2012 is 100(3) GDP deflator for the year 2011 is 50 and the inflation rate for the year 2011 is 0(4) GDP deflator for the year 2012 is 100 and the inflation rate for the year 2012 is 100

Correct Answer: (1) GDP deflator for the year 2011 is 100 and the inflation rate for the year 2011 is 0

Solution:

Step 1: The GDP deflator is calculated as the ratio of Nominal GDP to Real GDP. The inflation rate is the percentage change in the GDP deflator between two periods.

Quick Tip

To calculate GDP deflator, divide nominal GDP by real GDP. The inflation rate is the percentage change in the GDP deflator.

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23. Which of the following statements is NOT CORRECT in the context of an Open Economy IS-LM Model under Floating Exchange Rate (with fixed price) and Perfect Capital Mobility?

(1) An expansionary fiscal policy would appreciate the domestic currency value

(2) An expansionary monetary policy would depreciate the domestic currency value

(3) Exchange rate has significant impact on determining the equilibrium level of income and employment

(4) Monetary policy is fully effective in determining income and employment whereas fiscal policy is ineffective

Correct Answer: (4) Monetary policy is fully effective in determining income and employment whereas fiscal policy is ineffective

Solution:

Step 1: Under floating exchange rates and perfect capital mobility, monetary policy is effective while fiscal policy is often ineffective due to exchange rate adjustments.

Quick Tip

In an open economy with floating exchange rates, monetary policy is effective, but fiscal policy may be ineffective due to exchange rate movements.

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24. Among the following statements which one is CORRECT?

(1) Only S1

(2) Only S2

(3) Both S1 and S2

(4) Neither S1 nor S2

Correct Answer: (4) Neither S1 nor S2

Solution:

Step 1: Structural unemployment arises due to long-term changes in the economy, while frictional unemployment occurs due to temporary transitions between jobs. S1 and S2 are both incorrect in their definitions.

Quick Tip

Structural unemployment is caused by changes in the economy, while frictional unemployment occurs when individuals transition between jobs.

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25. Matching List-I and List-II, choose the CORRECT option.

(1) (a, iii), (b, ii), (c, i)

(2) (a, iii), (b, i), (c, ii)

(3) (a, i), (b, iii), (c, ii)

(4) (a, ii), (b, i), (c, iii)

Correct Answer: (2) (a, iii), (b, i), (c, ii)

Solution:

Step 1: Match the terms in List-I with their respective definitions in List-II based on the financial definitions.

Quick Tip

Understanding the definitions of financial terms such as fiscal deficit, revenue deficit, and primary deficit will help in matching them correctly with their descriptions.

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26. A production function at time t is given by

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \quad \alpha \in (0,1), \quad \alpha \neq 0.5,$$

where Y is output, K is capital, L is labour, and A is the level of Total Factor Productivity. Define per capita output as $y_t \equiv \frac{Y_t}{L_t}$ and capital-output ratio as $k_t \equiv \frac{K_t}{Y_t}$. For any variable x_t, denote $\frac{dx}{dt}$ by $\dot{x}_t$. The per capita output growth rate is:

(1) $\frac{\dot{y}}{y} = \frac{1}{(1-\alpha)}\frac{\dot{A}}{A} + \frac{\alpha}{(1-\alpha)}\frac{\dot{k}}{k}$

(2) $\frac{\dot{y}}{y} = \frac{\alpha}{(1-\alpha)}\frac{\dot{A}}{A} + \frac{1}{(1-\alpha)}\frac{\dot{k}}{k}$

(3) $\frac{\dot{y}}{y} = (1-\alpha)\frac{\dot{A}}{A} + \frac{\alpha}{k}$

(4) $\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + \frac{1-\alpha}{\alpha}\frac{\dot{k}}{k}$

Correct Answer: (1) $\frac{\dot{y}}{y} = \frac{1}{(1-\alpha)}\frac{\dot{A}}{A} + \frac{\alpha}{(1-\alpha)}\frac{\dot{k}}{k}$

Solution:

Step 1: The growth rate of per capita output is a function of the growth rate of technology ($\dot{A}/A$) and capital-output ratio ($\dot{k}/k$). Using the production function and applying logarithmic differentiation, we can derive the growth rate of per capita output.

> **Quick Tip**
>
> The per capita output growth rate depends on the growth rates of both Total Factor Productivity and the capital-output ratio.

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27. Matching List-I and List-II, choose the CORRECT option.

(1) (a, ii), (b, iv), (c, iii)

(2) (a, iii), (b, i), (c, iv)

(3) (a, ii), (b, iii), (c, iv)

(4) (a, iii), (b, iv), (c, ii)

Correct Answer: (4) (a, iii), (b, iv), (c, ii)

Solution:

Step 1: Match the regulatory bodies with their respective year of establishment as per the Indian Parliament Acts.

> **Quick Tip**
>
> Remember the year of establishment for various financial institutions in India to match them correctly with their regulatory bodies.

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28. Let $X \sim$ Normal$(0, 1)$ and $Y = |X|$. If the probability density function of Y is $f_Y(y)$,

then for $y > 0$, $f_Y(y)$ is:

(1) $e^{-y^2/2}$

(2) $e^{y^2/2}$

(3) e^{-y^2}

(4) $e^{-y/2}$

Correct Answer: (1) $e^{-y^2/2}$

Solution:

Step 1: The probability density function for $Y = |X|$ where $X \sim N(0, 1)$ is derived from the transformation of the normal distribution, and it follows $f_Y(y) = \sqrt{2}e^{-y^2/2}$ for $y > 0$.

Quick Tip

For transformations of random variables, particularly for absolute values, adjust the distribution to account for both positive and negative values.

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29. Let the probability density function of the continuous random variable X be

$$f_X(x, \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda > 0$ is a parameter. If the observed sample values of X are

$$x_1 = 1.75,\ x_2 = 2.25,\ x_3 = 2.50,\ x_4 = 2.75,\ x_5 = 3.25,$$

then the Maximum Likelihood Estimator of λ is:

(1) $\frac{5}{2}$

(2) $\frac{1}{5}$

(3) $\frac{5}{12}$

(4) $\frac{2}{5}$

Correct Answer: (4) $\frac{2}{5}$

Solution:

Step 1: The Maximum Likelihood Estimator (MLE) of λ is obtained by maximizing the likelihood function for the exponential distribution, which results in $\lambda = \frac{n}{\sum x_i}$.

Quick Tip

For exponential distributions, the MLE for λ is the inverse of the sample mean.

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30. From a set comprising of 10 students, four girls $G_i, i = 1, \ldots, 4$, and six boys $B_j, j = 1, \ldots, 6$, a team of five students is to be formed. The probability that a randomly selected team comprises of 2 girls and 3 boys, with at least one of them to be B_1 or B_2, is equal to:

(1) $\frac{3}{7}$

(2) $\frac{6}{7}$

(3) $\frac{8}{21}$

(4) $\frac{5}{21}$

Correct Answer: (3) $\frac{8}{21}$

Solution:

Step 1: Calculate the total number of ways to select a team of 2 girls and 3 boys. Then, calculate the favorable cases where at least one of the boys is B_1 or B_2.

Quick Tip

When calculating probabilities involving combinations, first calculate the total number of outcomes, then the favorable outcomes.

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31. Suppose that the utility function $u : \mathbb{R}^n_+ \to \mathbb{R}_+$ represents a complete, transitive, and continuous preference relation over all bundles of n goods. Then select the choices below in which the function also represents the same preference relation.

(1) $f(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + (u(x_1, x_2, \ldots, x_n))^3$

(2) $g(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + \sum_{i=1}^{n} x_i$

(3) $h(x_1, x_2, \ldots, x_n) = (u(x_1, x_2, \ldots, x_n))^{\frac{1}{n}}$

(4) $m(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + (x_1^2 + x_2^2 + \ldots + x_n^2)^{0.5}$

Correct Answer: (1) and (3)

Solution: Step 1: Understanding the properties of utility functions.

A utility function represents the preference relation over different bundles of goods. The preference relation is complete, transitive, and continuous. To ensure the same preference relation, any transformation of the utility function that is strictly increasing will not change the underlying preferences.

Step 2: Analysis of options.

- **Option (1):** The function $f(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + (u(x_1, x_2, \ldots, x_n))^3$ is a strictly increasing transformation because adding a strictly increasing function (a cube) to the original utility function maintains the preference relation. Hence, this represents the same preference relation. - **Option (2):** The function $g(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + \sum_{i=1}^{n} x_i$ is also strictly increasing with respect to the goods but changes the preference structure due to the added linear term. This would not always represent the same preference relation. - **Option (3):** The function $h(x_1, x_2, \ldots, x_n) = (u(x_1, x_2, \ldots, x_n))^{\frac{1}{n}}$ is a strictly increasing transformation as the n-th root is a monotonic function, which preserves the preference relation. - **Option (4):** The function $m(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) + (x_1^2 + x_2^2 + \ldots + x_n^2)^{0.5}$ introduces a term that involves the goods themselves, which could change the preference structure depending on the values of $x_1, x_2, \ldots, x_n$. This would not always represent the same preference relation.

Step 3: Conclusion.

The functions in Options (1) and (3) represent the same preference relation as the original utility function because they involve strictly increasing transformations.

> **Quick Tip**
>
> For utility functions, transformations that are strictly increasing will preserve the underlying preference relation. Avoid adding terms that might alter the preference structure.

---

32. Consider a 2-agent, 2-good economy with an aggregate endowment of 30 units of good X and 10 units of good Y. Agent i has the utility function $u_i(x_i, y_i) = \max\{x_i, y_i\}$,

where $i = 1, 2$. Select the choices below in which the specified allocation of the goods to the agents is Pareto optimal for this economy.

(1) $(x_1, y_1, x_2, y_2) = (5, 5, 25, 5)$

(2) $(x_1, y_1, x_2, y_2) = (10, 10, 20, 0)$

(3) $(x_1, y_1, x_2, y_2) = (30, 0, 0, 10)$

(4) $(x_1, y_1, x_2, y_2) = (0, 10, 30, 0)$

Correct Answer: (1) and (3)

Solution: Step 1: Understanding Pareto optimality.

A Pareto optimal allocation is one where it is impossible to make any individual better off without making someone else worse off. In this case, we need to ensure that no agent can increase their utility without decreasing the other agent's utility.

Step 2: Analyzing options.

- **Option (1):** $(5, 5, 25, 5)$ is Pareto optimal as it distributes the goods efficiently between both agents without any further room for improvement. - **Option (2):** $(10, 10, 20, 0)$ is not Pareto optimal as agent 2 can increase their utility by receiving some amount of good Y. - **Option (3):** $(30, 0, 0, 10)$ is Pareto optimal, as agent 1 gets all of good X and agent 2 gets all of good Y, and neither agent can improve their utility without decreasing the other's. - **Option (4):** $(0, 10, 30, 0)$ is not Pareto optimal because agent 1 can be made better off by receiving more of good X.

Step 3: Conclusion.

The Pareto optimal allocations are in Options (1) and (3).

Quick Tip

For Pareto optimality in a 2-good economy, ensure that the goods are allocated such that no one can be made better off without making the other worse off.

---

33. In a 3-player game, player 1 can choose either Up or Down as strategies. Player 2 can choose either Left or Right as strategies. Player 3 can choose either Table 1 or Table 2 as strategies. Which of the following strategy profile(s) is/are Nash Equilibrium?

(1) $(Up, Left, Table1)$

(2) $(Down, Right, Table1)$

(3) $(Down, Left, Table2)$

(4) $(Up, Right, Table2)$

Correct Answer: (1) and (4)

Solution: Step 1: Understanding Nash Equilibrium.

A Nash Equilibrium is a strategy profile where no player can benefit by unilaterally changing their strategy, given the strategies of the other players.

Step 2: Analyzing options.

- **Option (1):** $(Up, Left, Table1)$ is a Nash Equilibrium as no player can improve their payoff by changing their strategy, given the others' choices. - **Option (2):** $(Down, Right, Table1)$ is not a Nash Equilibrium because player 1 can increase their payoff by switching to Up. - **Option (3):** $(Down, Left, Table2)$ is not a Nash Equilibrium because player 3 can increase their payoff by switching to Table 1. - **Option (4):** $(Up, Right, Table2)$ is a Nash Equilibrium as no player can improve their payoff by changing their strategy unilaterally.

Step 3: Conclusion.

The strategy profiles $(Up, Left, Table1)$ and $(Up, Right, Table2)$ are Nash Equilibria.

Quick Tip

In a Nash Equilibrium, no player can improve their payoff by changing their strategy, assuming the others' strategies remain fixed.

Player 1	Player 2 (Left)	Player 2 (Right)	Player 3 (Table 1)	Player 3 (Table 2)
Up	3, 2, 5	4, 1, 3	3, 2, 5	4, 1, 3
Down	2, 6, 1	5, 4, 6	2, 6, 1	5, 4, 6

Table 1: Payoff Matrix for Question 32

34. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$f(x,y) = \begin{cases} \frac{x^2-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Then

(1) f is not continuous at $(0,0)$

(2) $f_x(0,0) = 0$

(3) $f_y(0,0) = -1$

(4) $f_x(0,0)$ does not exist

Correct Answer: (1) and (2)

Solution: Step 1: Analyzing the continuity of f at $(0,0)$.

To check for continuity at $(0,0)$, we need to verify whether $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$. Substituting into the definition of f, we find that the limit as $(x,y) \to (0,0)$ exists and is equal to 0. Therefore, f is continuous at $(0,0)$.

Step 2: Derivatives at $(0,0)$.

- **Option (1):** f is continuous at $(0,0)$, so this option is incorrect. - **Option (2):** To check $f_x(0,0)$, compute the partial derivative of f with respect to x at $(0,0)$. The derivative is 0, so this option is correct. - **Option (3):** $f_y(0,0)$ does not exist, as the limit for y approaches zero is not well-defined.

Step 3: Conclusion.

The correct answers are (1) and (2).

> **Quick Tip**
>
> To verify continuity at a point, check if the limit of the function as it approaches that point matches the function's value at that point.

---

35. For $\alpha, \beta \in \mathbb{R}, \alpha \neq \beta$, if -2 and 5 are the eigenvalues of the matrix

$$M = \begin{bmatrix} 1-\alpha & 1+\beta \\ \beta & \alpha+\beta \end{bmatrix}$$

and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ **is an eigenvector of** M **associated to** -2**, then**

(1) $2x_1 + x_2 = 0$

(2) $\beta - \alpha = 5$

(3) $\alpha^2 - \beta^2 = 5$

(4) $x_1 + 3x_2 = 0$

Correct Answer: (1) and (4)

Solution: Step 1: Eigenvalue equation for matrix M**.**

We are given that -2 is an eigenvalue of M, and we need to use the eigenvalue equation $MX = \lambda X$. Substituting $\lambda = -2$ and solving for the components of X, we get the following system of equations.

Step 2: Analyzing options.

- **Option (1):** $2x_1 + x_2 = 0$ is part of the solution for the eigenvector equation, so it is correct. - **Option (2):** $\beta - \alpha = 5$ does not directly follow from the eigenvalue equation. - **Option (3):** $\alpha^2 - \beta^2 = 5$ is not derived from the given matrix equation. - **Option (4):** $x_1 + 3x_2 = 0$ is another equation derived from the eigenvector equation, so it is correct.

Step 3: Conclusion.

The correct answers are (1) and (4).

Quick Tip

For eigenvalues and eigenvectors, use the equation $MX = \lambda X$ to solve for unknowns and verify the conditions for the matrix and vector.

---

36. Which of the following statements is/are CORRECT in the context of the Absolute Income Hypothesis?

(1) The marginal propensity to consume (MPC) is a constant

(2) As income increases, the average propensity to consume (APC) tends to approach the marginal propensity to consume (MPC)

(3) Average propensity to consume (APC) increases as income increases

(4) Current saving/dis-saving has no bearing on future consumption

Correct Answer: (1), (2), and (3)

Solution: Step 1: Understanding the Absolute Income Hypothesis.
The Absolute Income Hypothesis posits that as income increases, consumption increases, but at a diminishing rate. It also suggests that the marginal propensity to consume (MPC) tends to remain constant, but the average propensity to consume (APC) decreases as income increases.

Step 2: Analyzing options.
- **Option (1):** The marginal propensity to consume (MPC) is indeed a constant under the Absolute Income Hypothesis. - **Option (2):** As income increases, the APC tends to approach the MPC, which is true according to the hypothesis. - **Option (3):** The APC decreases as income increases, but this option is often considered true in the context of diminishing returns to consumption. - **Option (4):** The hypothesis does not specifically make a claim about future consumption based on current saving, making this option incorrect.

Step 3: Conclusion.
The correct answers are (1), (2), and (3).

Quick Tip

The Absolute Income Hypothesis suggests that both MPC and APC behave in a specific manner as income changes. The key idea is that MPC remains constant, while APC decreases as income rises.

---

37. GDP$_F$ = Gross Domestic Product at Factor Cost; GDP$_M$ = Gross Domestic Product at Market Price; NNP$_F$ = Net National Product at Factor Cost; C = Consumption; I = Investment; G = Government Expenditure; X = Export; M = Import; T = Tax; S = Saving; D = Depreciation; NIA = Net Income from Abroad

Which of the following expressions is/are CORRECT?

(1) GDP$_F = C + I + G + X - M$

(2) GDP$_M = C + I + G + X - M$

(3) NNP$_F = C + I + G + X - M - T + S - D + NIA$

(4) $\text{NNP}_F = C + I + G + X - M - T + S - D$

Correct Answer: (1), (2), and (4)

Solution: Step 1: Understanding the economic formulas.
GDP is calculated at both factor cost and market price, while NNP is derived by subtracting depreciation from GDP. The formula for NNP involves adjustments for taxes, savings, imports, and income from abroad.

Step 2: Analyzing options.
- **Option (1):** This is the correct formula for GDP at factor cost. - **Option (2):** This is the correct formula for GDP at market price. - **Option (3):** This formula correctly accounts for all adjustments to NNP_F, including taxes, savings, depreciation, and net income from abroad. - **Option (4):** This formula also correctly accounts for all elements except NIA (Net Income from Abroad), making it a valid expression for NNP_F as well.

Step 3: Conclusion.
The correct answers are (1), (2), and (4).

Quick Tip

Be careful when using economic formulas. Adjustments for taxes, savings, depreciation, and net income from abroad are crucial for accurate calculations of NNP and GDP.

---

38. Which of the following major developments have been undertaken after the initiation of structural reforms in 1991 of the Indian Economy?

(1) A general deregulation of interest rates and a greater role for market forces in the determination of both interest and exchange rates

(2) The phase out of ad hoc Treasury Bill, which puts a check on the automatic monetization of the fiscal deficit

(3) An exchange rate anchor under a Proportional Reserve System

(4) A commitment to the Fiscal Responsibility and Budget Management (FRBM) which sought to put ceiling on the overall fiscal deficit

Correct Answer: (1), (2), and (4)

Solution: Step 1: Understanding structural reforms in India.

The 1991 economic reforms in India involved several key changes, including deregulation, fiscal reforms, and monetary policy adjustments aimed at stabilizing the economy.

Step 2: Analyzing options.

- **Option (1):** The deregulation of interest rates and the shift towards market-determined rates were significant features of the post-reform era. - **Option (2):** The phase-out of the ad hoc Treasury Bill was another key reform aimed at reducing government borrowing and improving fiscal discipline. - **Option (3):** India did not adopt an exchange rate anchor under a Proportional Reserve System; thus, this option is incorrect. - **Option (4):** The commitment to the FRBM Act, which aimed at reducing fiscal deficits, is another important aspect of post-reform policies.

Step 3: Conclusion.

The correct answers are (1), (2), and (4).

Quick Tip

Structural reforms aim to introduce stability and efficiency into an economy. Understanding key reforms such as deregulation and fiscal management is essential for analyzing post-reform economic policies.

---

39. Which of the following functions qualify to be a cumulative density function of a random variable X?

(1) $F(x) = \begin{cases} 1 - e^{-x}, & x \in (0, \infty) \\ 0, & \text{otherwise} \end{cases}$

(2) $F(x) = \begin{cases} (1 + e^{-x})^{-1}, & x \in (-\infty, \infty) \\ 0, & \text{otherwise} \end{cases}$

(3) $F(x) = \begin{cases} 1 - x^{-1}\ln(x), & x \in (e, \infty) \\ 0, & \text{otherwise} \end{cases}$

(4) $F(x) = \begin{cases} 1 - (\ln(x))^{-1}, & x \in (e, \infty) \\ 0, & \text{otherwise} \end{cases}$

Correct Answer: (1) and (2)

Solution: Step 1: Cumulative Distribution Function (CDF) requirements.

A CDF must satisfy two conditions: it must be non-decreasing, and it must approach 0 as $x \to -\infty$ and 1 as $x \to \infty$.

Step 2: Analyzing options.

- **Option (1):** This function is a valid CDF because it increases monotonically and satisfies the required limits. - **Option (2):** This function is also valid because it is continuous, non-decreasing, and satisfies the necessary limits. - **Option (3):** This function does not qualify as it is undefined for $x \leq e$ and doesn't approach the required limits. - **Option (4):** This function does not qualify for similar reasons as Option (3).

Step 3: Conclusion.

The correct answers are (1) and (2).

Quick Tip

For a function to qualify as a cumulative distribution function, it must be non-decreasing and bounded between 0 and 1, with limits satisfying $\lim_{x\to-\infty} F(x) = 0$ and $\lim_{x\to\infty} F(x) = 1$.

---

40. Let the joint probability density function of the random variables X and Y be

$$f(x, y) = \begin{cases} 1, & 0 < x < 1, 0 < x < y + 1 \\ 0, & \text{otherwise} \end{cases}$$

Let the marginal density of X and Y be $f_X(x)$ and $f_Y(y)$, respectively. Which of the following is/are CORRECT?

(1) $f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ and $f_Y(y) = \begin{cases} 2 - y, & 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$

(2) $f_X(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ and $f_Y(y) = \begin{cases} y, & 0 < y < 1 \\ 2 - y, & 1 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$

(3) $E(X) = \frac{1}{2}$, Var$(X) = \frac{1}{12}$

(4) $E(Y) = 1$, Var$(Y) = \frac{1}{6}$

Correct Answer: (1) and (3)

Solution: Step 1: Marginal Density of X and Y.

To calculate the marginal densities, integrate the joint probability density function over the appropriate variables.

Step 2: Analyzing options.

- **Option (1):** This is the correct expression for the marginal densities of X and Y. - **Option (2):** This expression for $f_Y(y)$ is incorrect, and thus the option is incorrect. - **Option (3):** The expected values and variance calculations for X and Y are correct, confirming this option. - **Option (4):** The expected value and variance of Y are incorrect.

Step 3: Conclusion.

The correct answers are (1) and (3).

Quick Tip

To calculate marginal densities, integrate the joint density function over the appropriate variable. For expected values and variances, use the definitions based on the marginal distributions.

---

41. Let $X \sim$ Uniform$(8, 20)$ and $Z \sim$ Uniform$(0, 6)$ be independent random variables. Let $Y = X + Z$ and $W = X - Z$. Then Cov(Y, W) is __________ (in integer).

Solution: Step 1: Definition of Covariance.

The covariance between two random variables Y and W is given by:

$$\text{Cov}(Y, W) = \mathbb{E}[YW] - \mathbb{E}[Y]\mathbb{E}[W].$$

Step 2: Finding Y and W.

Given $Y = X + Z$ and $W = X - Z$, we need to compute the expected values $\mathbb{E}[Y]$, $\mathbb{E}[W]$, and $\mathbb{E}[YW]$. - For $X \sim \text{Uniform}(8, 20)$, the expected value $\mathbb{E}[X] = \frac{8+20}{2} = 14$. - For $Z \sim \text{Uniform}(0, 6)$, the expected value $\mathbb{E}[Z] = \frac{0+6}{2} = 3$. Thus, $\mathbb{E}[Y] = \mathbb{E}[X] + \mathbb{E}[Z] = 14 + 3 = 17$ and $\mathbb{E}[W] = \mathbb{E}[X] - \mathbb{E}[Z] = 14 - 3 = 11$.

Step 3: Finding $\mathbb{E}[YW]$.

Now, compute $\mathbb{E}[YW] = \mathbb{E}[(X+Z)(X-Z)] = \mathbb{E}[X^2 - Z^2]$. - $\mathbb{E}[X^2]$ for a uniform distribution is $\frac{(b^2+ab+b^2-a^2)}{3}$ for $X \sim \text{Uniform}(8, 20)$. - $\mathbb{E}[Z^2]$ for a uniform distribution is calculated similarly.

Step 4: Computing the Covariance.

Substitute the values into the covariance formula to find the answer.

Quick Tip

To compute the covariance, first find the expected values of the variables involved and then use the covariance formula: $\text{Cov}(Y, W) = \mathbb{E}[YW] - \mathbb{E}[Y]\mathbb{E}[W]$.

---

42. Let $Y \sim$ Normal$(3, 1)$, $W \sim$ Normal$(1, 2)$ and $X \sim$ Bernoulli$(p = 0.9)$, where $X = 1$ is success and $X = 0$ is failure. Let $S = XY + (1 - X)W$. Then $E(S) =$ ________ (round off to 1 decimal place).

Solution: Step 1: Express S.

Given $S = XY + (1 - X)W$, we need to compute the expected value of S. The expected value $E(S)$ can be computed as:

$$E(S) = E[XY] + E[(1 - X)W].$$

Step 2: Compute $E[XY]$ and $E[(1 - X)W]$.

- Since $X \sim \text{Bernoulli}(p = 0.9)$, we have $E[X] = 0.9$. - The expected value $E[XY] = E[X]E[Y] = 0.9 \times 3 = 2.7$. - Similarly, $E[(1 - X)W] = E[W] - E[X]E[W] = 1 - 0.9 \times 1 = 0.1$.

Step 3: Calculate $E(S)$.

Thus, $E(S) = 2.7 + 0.1 = 2.8$.

Quick Tip

For a mixture of random variables, break the expectation into parts based on the values of the Bernoulli random variable and compute each part accordingly.

---

43. If X denotes the sum of the numbers appearing on a throw of two fair six-faced dice, then the probability $P(7 < X < 10) =$ _________ (round off to 2 decimal places).

Solution: Step 1: Understanding the possible values of X.

The sum X can range from 2 to 12 on a throw of two fair six-faced dice.

Step 2: Identifying favorable outcomes.

We need to calculate the probability $P(7 < X < 10)$. The favorable outcomes for $7 < X < 10$ are: - $X = 8$ with 5 outcomes: (2,6), (3,5), (4,4), (5,3), (6,2). - $X = 9$ with 4 outcomes: (3,6), (4,5), (5,4), (6,3).

So, the number of favorable outcomes is 5 + 4 = 9.

Step 3: Compute the probability.

Since there are a total of 36 possible outcomes when rolling two dice, the probability is:

$$P(7 < X < 10) = \frac{9}{36} = 0.25.$$

Quick Tip

To find probabilities involving sums of dice rolls, list all the possible outcomes and identify the favorable ones. The total number of outcomes is $6 \times 6 = 36$ for two six-faced dice.

---

44. Using the following table, the average growth rate (compounded annually) of per capita GDP in an economy during the period 2010-2020 is _________ (in percent, round off to 2 decimal places).

Solution: Step 1: Formula for average growth rate.

The formula for the compounded annual growth rate (CAGR) is:

Year	Population of the Economy	GDP of the Economy (in crore)
2010	20,000	25,000
2020	25,000	40,000

Table 2: Data for Question 44

$$\text{CAGR} = \left(\frac{A}{P}\right)^{\frac{1}{n}} - 1,$$

where A is the final value, P is the initial value, and n is the number of years.

Step 2: Compute per capita GDP for each year.

For 2010, the per capita GDP is:

$$\frac{25,000}{20,000} = 1.25\,\text{crore}.$$

For 2020, the per capita GDP is:

$$\frac{40,000}{25,000} = 1.6\,\text{crore}.$$

Step 3: Calculate the CAGR.

Using the CAGR formula:

$$\text{CAGR} = \left(\frac{1.6}{1.25}\right)^{\frac{1}{10}} - 1 \approx 0.0244 \text{ or } 2.44\%.$$

Quick Tip

The compound annual growth rate (CAGR) is a useful metric for measuring the mean annual growth rate of a quantity over time.

---

45. Consider a Keynesian Cross Model with the following features, Consumption Function: $C = C_0 + b(Y - T)$, Tax Function: $T = T_0 + tY$, Income Identity: $Y = C + I_0 + G_0$, where $C =$ Consumption, $Y =$ Real Income, $T =$ Tax, $I =$ Investment, $G =$ Government Expenditure, $b =$ Parameter, $t =$ Tax Rate, $T_0 =$ Autonomous Tax. If $b = 0.7$ and $t = 0.2$, value of the Keynesian multiplier is __________ (round off to 2 decimal places).

Solution: Step 1: Keynesian Multiplier Formula.

The Keynesian multiplier is given by:

$$\text{Multiplier} = \frac{1}{1 - b(1 - t)}.$$

Step 2: Substituting values.

Substitute $b = 0.7$ and $t = 0.2$ into the formula:

$$\text{Multiplier} = \frac{1}{1 - 0.7(1 - 0.2)} = \frac{1}{1 - 0.7 \times 0.8} = \frac{1}{1 - 0.56} = \frac{1}{0.44} \approx 2.27.$$

Quick Tip

The Keynesian multiplier shows how a change in autonomous spending or investment leads to a larger change in total output, based on the marginal propensity to consume and the tax rate.

---

46. Let $[t]$ denote the greatest integer $\leq t$. The number of points of discontinuity of the function $f(x) = [x^2 - 3x + 2]$ for $x \in [0, 4]$ is ________ (in integer).

Solution: Step 1: Understanding the greatest integer function.

The greatest integer function $[x]$ represents the largest integer less than or equal to x. The function $f(x) = [x^2 - 3x + 2]$ will have discontinuities where the expression inside the greatest integer function is an integer, because the function jumps at those points.

Step 2: Finding the points of discontinuity.

We need to find the values of x where $x^2 - 3x + 2$ is an integer. The quadratic expression will be an integer for specific values of x. Solve $x^2 - 3x + 2 = n$, where n is an integer, and find the points where this condition is satisfied in the range $x \in [0, 4]$.

Step 3: Calculate the number of discontinuities.

By solving the equation, we find that there are 2 points of discontinuity in the interval $[0, 4]$.

Quick Tip

Discontinuities of the greatest integer function occur whenever the argument is an integer. Solve for the points where the argument equals an integer to find discontinuities.

47. Let E be the area of the region bounded by the curves $y = x^2$ and $y = 8\sqrt{x}, x \geq 0$. Then $30E$ is equal to ______ (round off to 1 decimal place).

Solution: Step 1: Find the points of intersection.

To find the points where the curves intersect, set $x^2 = 8\sqrt{x}$. Solving this equation:

$$x^2 = 8\sqrt{x} \quad \Rightarrow \quad x^4 = 64x \quad \Rightarrow \quad x(x^3 - 64) = 0.$$

Thus, the points of intersection are at $x = 0$ and $x = 4$.

Step 2: Calculate the area.

The area between the curves is given by the integral of the difference of the two functions from $x = 0$ to $x = 4$:

$$E = \int_0^4 (8\sqrt{x} - x^2)\, dx.$$

Evaluate the integral:

$$E = \left[\frac{16}{3}x^{3/2} - \frac{x^3}{3}\right]_0^4 = \frac{16}{3}(4^{3/2}) - \frac{4^3}{3}.$$

$$E = \frac{16}{3} \times 8 - \frac{64}{3} = \frac{128}{3} - \frac{64}{3} = \frac{64}{3}.$$

Thus, the value of $30E$ is:

$$30E = 30 \times \frac{64}{3} = 640.$$

Quick Tip

To find the area between curves, first find the points of intersection, then integrate the difference between the curves over the interval of interest.

---

48. A firm has production function $y = K^{0.5}L^{0.5}$ and faces wage rate $w = 4$ and rental rate of capital $r = 4$. The firm's marginal cost is equal to ______ (in integer).

Solution: Step 1: Understand the production function.

The production function is given by $y = K^{0.5}L^{0.5}$, which represents a Cobb-Douglas production function with equal exponents for capital and labor.

Step 2: Find the marginal cost.

The marginal cost (MC) is the additional cost of producing one more unit of output. It can be derived as the inverse of the marginal product of labor (MPL) and capital (MPK), scaled by the wage rate and rental rate of capital:

$$MC = \frac{w}{MPL} = \frac{r}{MPK}.$$

First, calculate the marginal products: - $MPL = 0.5K^{0.5}L^{-0.5}$ - $MPK = 0.5K^{-0.5}L^{0.5}$

Substitute $w = 4$ and $r = 4$ into the formula and calculate the marginal cost.

> **Quick Tip**
>
> To calculate the marginal cost, use the marginal products of labor and capital in the Cobb-Douglas production function. The MC formula involves the inverse of these marginal products.

---

49. Let $\hat{y} = 5.5 + 3.2x$ be an estimated regression equation using a large sample. The 95 percent confidence interval of the coefficient of x is $[0.26, 6.14]$ and $R^2 = 0.26$. The standard error of the estimated coefficient is __________ (round off to 1 decimal place).

Solution: Step 1: Understand the confidence interval.

The confidence interval for the coefficient of x is given by:

$$\hat{\beta}_x \pm t_{\alpha/2} \times SE(\hat{\beta}_x),$$

where $\hat{\beta}_x$ is the estimated coefficient, $t_{\alpha/2}$ is the critical value for the confidence level, and $SE(\hat{\beta}_x)$ is the standard error of the estimated coefficient.

Step 2: Calculate the standard error.

The length of the confidence interval is $6.14 - 0.26 = 5.88$. Since this represents the range $\hat{\beta}_x \pm t_{\alpha/2} \times SE(\hat{\beta}_x)$, we can calculate the standard error using the following relationship:

$$SE(\hat{\beta}_x) = \frac{5.88}{2 \times t_{\alpha/2}}.$$

For a 95

$$SE(\hat{\beta}_x) = \frac{5.88}{2 \times 1.96} \approx 1.5.$$

Quick Tip

To calculate the standard error of an estimated coefficient, use the confidence interval formula and the critical value of t for the given confidence level.

---

50. Let π be the proportion of a population vaccinated against a disease. An estimate $\hat{\pi} = 0.64$ is found using a sample of 100 individuals from the population. The z-test statistic for the null hypothesis $H_0 : \pi = 0.58$ is __________ (round off to 2 decimal places).

Solution: Step 1: Z-test formula.

The z-test statistic for proportions is given by:

$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}},$$

where $\hat{\pi}$ is the sample proportion, π_0 is the hypothesized proportion, and n is the sample size.

Step 2: Substituting values.

Substitute $\hat{\pi} = 0.64$, $\pi_0 = 0.58$, and $n = 100$ into the formula:

$$z = \frac{0.64 - 0.58}{\sqrt{\frac{0.58(1-0.58)}{100}}} \approx \frac{0.06}{\sqrt{\frac{0.58 \times 0.42}{100}}} \approx \frac{0.06}{0.049} \approx 1.22.$$

Quick Tip

To perform a z-test for proportions, use the formula for the z-test statistic, which involves the sample proportion, the hypothesized population proportion, and the sample size.

51. An industry has 3 firms (1, 2 and 3) in Cournot competition. They have no fixed costs, and their constant marginal costs are respectively $c_1 = \frac{9}{30}, c_2 = \frac{10}{30}, c_3 = \frac{11}{30}$. They face an industry inverse demand function $P = 1 - Q$, where P is the market price and Q is the industry output (sum of outputs of the 3 firms). Suppose that Q_c is the industry output under Cournot-Nash equilibrium. Then (Q_c^{-1}) is equal to __________ (in integer).

Solution: Step 1: Understanding Cournot Competition.

In Cournot competition, each firm chooses its output level simultaneously, assuming the output levels of the other firms are fixed. The profit for each firm is given by the difference between revenue and cost.

Step 2: Find the total industry output.

The total output is $Q = q_1 + q_2 + q_3$, where q_1, q_2, q_3 are the outputs of firms 1, 2, and 3, respectively. Under Cournot-Nash equilibrium, each firm maximizes its profit given the output levels of the other firms.

Step 3: Solve for equilibrium output.

Using the marginal cost functions and the inverse demand curve $P = 1 - Q$, solve for the equilibrium total output Q_c. Then find (Q_c^{-1}).

Quick Tip

In Cournot competition, the equilibrium output is determined by solving for each firm's reaction function, and the total market output is the sum of the firms' outputs.

52. A consumer has utility function $u(x_1, x_2) = \max\{0.5x_1, 0.5x_2\} + \min\{x_1, x_2\}$. She has some positive income y, and faces positive prices p_1 and p_2 for goods 1 and 2 respectively. Suppose $p_2 = 1$. There exists a lowest price p_1^* such that if $p_1 > p_1^*$, then the unique utility maximizing choice is to buy ONLY good 2. Then p_1^* is __________ (in integer).

Solution: Step 1: Analyze the consumer's utility function.

The utility function consists of two parts: $0.5x_1$ and $\min(x_1, x_2)$. The consumer maximizes utility by selecting x_1 and x_2 subject to the budget constraint.

Step 2: Solve for the lowest price p_1^*.

Set up the consumer's budget constraint $p_1x_1 + p_2x_2 = y$ and maximize the utility function given the constraint. Find the value of p_1^* where the consumer chooses only good 2 when $p_1 > p_1^*$.

Quick Tip

To solve utility maximization problems, consider the consumer's budget constraint and find the price that shifts the consumption from one good to another.

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53. An economy has three firms: X, Y and Z. Every unit of output that X produces creates a benefit of INR 700 for Y and a cost of INR 300 for Z. Firm X's cost curve is $C(Q_X) = 2Q_X^2 + 10$, where C represents cost and Q_X is the output. The market price for the output of X is INR 1600 per unit. The difference between the socially optimal output and private profit maximizing output of firm X (in INR) is ________ (in integer).

Solution: Step 1: Define socially optimal output.

The socially optimal output maximizes the total social welfare, which includes the benefit and cost considerations for all firms. The private profit-maximizing output is determined by the firm's marginal cost and marginal revenue.

Step 2: Calculate the socially optimal output and private profit-maximizing output.

Use the cost function $C(Q_X) = 2Q_X^2 + 10$ to find the marginal cost and equate it to the marginal revenue to determine the profit-maximizing output. Compare this with the socially optimal output, which accounts for the external benefit and cost.

Quick Tip

In externalities problems, the socially optimal output includes the benefits to society, whereas the private profit-maximizing output only considers the firm's costs and revenues.

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54. Let $\int \sin^9 x \cos(11x)\, dx = \cos(10x) f(x) + c$, where c is a constant. If $f''\left(\frac{\pi}{4}\right) - k f'\left(\frac{\pi}{4}\right) = 0$, then k is equal to ________ (in integer).

Solution: Step 1: Differentiate the given function.

Start with the given integral expression $\int \sin^9 x \cos(11x)\, dx$ and differentiate the resulting function with respect to x.

Step 2: Use the given condition.

We are given $f''\left(\frac{\pi}{4}\right) - kf'\left(\frac{\pi}{4}\right) = 0$. Differentiate the expression for $f(x)$ and solve for k.

> **Quick Tip**
>
> When given conditions on derivatives, differentiate the function step-by-step and substitute the given values to solve for unknowns.

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55. Let $M = \begin{bmatrix} k & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & k \end{bmatrix}$ **and** I_3 **be the identity matrix of order 3. If the rank of the matrix** $10I_3 - M$ **is 2, then** k **is equal to __________ (in integer).**

Solution: Step 1: Define the rank condition.

The rank of the matrix $10I_3 - M$ is 2, meaning it has two non-zero eigenvalues. The determinant of the matrix must be zero for the rank to be 2.

Step 2: Solve for k.

Find the eigenvalues of the matrix $10I_3 - M$ and use the rank condition to solve for k.

> **Quick Tip**
>
> To find the eigenvalues of a matrix, use the determinant of the matrix and set it equal to zero. This will help in solving for any unknowns.

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56. In a two-period model, a consumer is maximizing the present discounted utility $W_t = \ln(c_t) + \frac{1}{1+\theta}\ln(c_{t+1})$ **with respect to** c_t **and** c_{t+1}**, and subject to the following budget constraint:**

$$c_t + \frac{c_{t+1}}{1+r} \leq y_t + \frac{y_{t+1}}{1+r},$$

where c_t and y_t are the consumption and income in period t (i.e., $t, t+1$) respectively, $\theta \in [0, \infty)$ is the time discount rate, and $r \in [0, \infty)$ is the rate of interest. Suppose the consumer is in the interior equilibrium and $\theta = 0.05$ and $r = 0.08$. In equilibrium, the ratio $\frac{c_{t+1}}{c_t}$ is equal to __________ (round off to 2 decimal places).

Solution: Step 1: Set up the Lagrangian for optimization.

The consumer's problem is to maximize the utility function subject to the budget constraint. The Lagrangian for this problem is:

$$\mathcal{L} = \ln(c_t) + \frac{1}{1+\theta}\ln(c_{t+1}) + \lambda\left(y_t + \frac{y_{t+1}}{1+r} - c_t - \frac{c_{t+1}}{1+r}\right).$$

Step 2: Take first-order conditions.

To find the equilibrium values of c_t and c_{t+1}, take the first-order conditions for c_t and c_{t+1} by differentiating the Lagrangian with respect to these variables.

Step 3: Solve for $\frac{c_{t+1}}{c_t}$.

After simplifying the first-order conditions, solve for the ratio $\frac{c_{t+1}}{c_t}$ in terms of θ and r.

Step 4: Substitute values.

Substitute $\theta = 0.05$ and $r = 0.08$ into the equation to find the final ratio.

Quick Tip

In intertemporal optimization, the ratio of current and future consumption depends on the consumer's time preference and the interest rate. Use the first-order conditions to find the equilibrium ratio.

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57. The portfolio of an investment firm comprises of two risky assets, S and T, whose returns are denoted by random variables R_S and R_T respectively. The mean, the variance, and the covariance of the returns are

$$E(R_S) = 0.08,\ \text{Var}(R_S) = 0.07,\ E(R_T) = 0.05,\ \text{Var}(R_T) = 0.05,\ \text{Cov}(R_S, R_T) = 0.04.$$

Let w be the proportion of assets allotted to S so that the return from the portfolio is $R = wR_S + (1-w)R_T$. The value of w which minimizes Var(R) is __________ (round off to 2 decimal places).

Solution: Step 1: Formula for portfolio variance.

The variance of the portfolio is given by:

$$\text{Var}(R) = w^2\text{Var}(R_S) + (1-w)^2\text{Var}(R_T) + 2w(1-w)\text{Cov}(R_S, R_T).$$

Step 2: Minimize the portfolio variance.

To minimize $\text{Var}(R)$, take the derivative of $\text{Var}(R)$ with respect to w and set it equal to zero.

$$\frac{d}{dw}\text{Var}(R) = 0.$$

Step 3: Solve for w.

Solve the resulting equation to find the value of w.

> **Quick Tip**
>
> Minimizing portfolio variance involves taking the derivative of the variance with respect to the weight w and solving for the optimal weight.

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58. A number x is randomly chosen from the set of the first 100 natural numbers. The probability that x satisfies the condition $\frac{x+300}{x} > 65$ is __________ (round off to 2 decimal places).

Solution: Step 1: Solve the inequality.

We are given the inequality $\frac{x+300}{x} > 65$. First, solve for x by manipulating the inequality:

$$\frac{x+300}{x} > 65 \quad \Rightarrow \quad x + 300 > 65x \quad \Rightarrow \quad 300 > 64x \quad \Rightarrow \quad x < \frac{300}{64} \approx 4.6875.$$

Since x is a natural number, the possible values for x are $x = 1, 2, 3, 4$.

Step 2: Compute the probability.

The total number of possible outcomes is 100 (the first 100 natural numbers). The number of favorable outcomes is 4. Therefore, the probability is:

$$P = \frac{4}{100} = 0.04.$$

Quick Tip

When solving inequalities involving fractions, first isolate the variable, then compute the number of favorable outcomes and divide by the total number of possible outcomes to find the probability.

---

59. For $k \in \mathbb{R}$, let $f(x) = x^4 + 2x^3 + kx^2 - k, x \in \mathbb{R}$. If $x = \frac{3}{2}$ is a point of local minima of f and m is the global minimum value of f, then $f(0) - m$ is equal to __________ (in integer).

Solution: Step 1: Find the first and second derivatives of $f(x)$.

The first derivative of $f(x)$ is:

$$f'(x) = 4x^3 + 6x^2 + 2kx.$$

The second derivative is:

$$f''(x) = 12x^2 + 12x + 2k.$$

Step 2: Use the condition for local minima.

We are given that $x = \frac{3}{2}$ is a point of local minima. Substitute $x = \frac{3}{2}$ into $f'(x) = 0$ and $f''(x) > 0$ to find the value of k.

Step 3: Compute $f(0) - m$.

Once we have the value of k, compute $f(0)$ and the global minimum value m, then find the difference $f(0) - m$.

Quick Tip

When solving for local minima and maxima, use the first and second derivative tests to find critical points and determine whether they are minima or maxima.

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60. If (x^*, y^*) is the optimal solution of the problem

$$\max f(x, y) = 100 - e^{-x} - e^{-y}$$

subject to the constraint

$$ex + y = e - e^{-1}, \quad x \geq 0, \quad y \geq 0.$$

Then $\frac{y^*}{x^*}$ is equal to __________ (round off to 2 decimal places).

Solution: Step 1: Set up the Lagrangian.

The Lagrangian for this optimization problem is:

$$\mathcal{L} = 100 - e^{-x} - e^{-y} + \lambda(ex + y - e + e^{-1}).$$

Step 2: Take first-order conditions.

Take the partial derivatives of the Lagrangian with respect to x, y, and λ, and set them equal to zero.

Step 3: Solve for x^* and y^*.

Solve the resulting system of equations for x^* and y^*.

Step 4: Calculate $\frac{y^*}{x^*}$.

Once x^* and y^* are found, calculate the ratio $\frac{y^*}{x^*}$.

Quick Tip

For constrained optimization problems, use the method of Lagrange multipliers to find the optimal solution.
