IIT JAM 2023 MA Question Paper PDF

Time Allowed :1 Hour	**Maximum Marks :**100	**Total Questions :**60

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Please check that this question paper contains 60 questions.
2. Please write down the Serial Number of the question in the answer- book at the given place before attempting it.
3. This Question Paper has 60 questions. All questions are compulsory.
4. Adhere to the prescribed word limit while answering the questions.

1. Let G be a finite group. Then G is necessarily a cyclic group if the order of G is

(1) 4
(2) 7
(3) 6
(4) 10

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2. Let $v_1, \dots, v_9$ be the column vectors of a non-zero 9×9 real matrix A. Let $a_1, \dots, a_9 \in \mathbb{R}$, not all zero, such that $\sum_{i=1}^{9} a_i v_i = 0$. Then the system $Ax = \sum_{i=1}^{9} v_i$ has
(1) no solution
(2) a unique solution
(3) more than one but only finitely many solutions
(4) infinitely many solutions

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3. Which of the following is a subspace of the real vector space $\mathbb{R}^3$?
(A) $\{(x, y, z) \in \mathbb{R}^3 : (y + z)^2 + (2x - 3y)^2 = 0\}$
(B) $\{(x, y, z) \in \mathbb{R}^3 : y \in \mathbb{Q}\}$
(C) $\{(x, y, z) \in \mathbb{R}^3 : yz = 0\}$
(D) $\{(x, y, z) \in \mathbb{R}^3 : x + 2y - 3z + 1 = 0\}$

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4. Consider the initial value problem $\frac{dy}{dx} + \alpha y = 0,\ y(0) = 1$, where $\alpha \in \mathbb{R}$. Then
(A) there is an α such that $y(1) = 0$
(B) there is a unique α such that $\lim_{x\to\infty} y(x) = 0$
(C) there is no α such that $y(2) = 1$
(D) there is a unique α such that $y(1) = 2$

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5. Let $p(x) = x^{57} + 3x^{10} - 21x^3 + x^2 + 21$ and $q(x) = p(x) + \sum_{j=1}^{57} p^{(j)}(x)$, where $p^{(j)}(x)$ is the j^{th} derivative of $p(x)$. Then the function $q(x)$ admits

(A) neither a global maximum nor a global minimum on $\mathbb{R}$
(B) a global maximum but not a global minimum on $\mathbb{R}$
(C) a global minimum but not a global maximum on $\mathbb{R}$
(D) a global minimum and a global maximum on $\mathbb{R}$

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6. Evaluate the limit

$$\lim_{a\to 0}\left(\frac{\int_0^a \sin(x^2)\,dx}{\int_0^a (\ln(x+1))^2\,dx}\right)$$

(A) 0
(B) 1
(C) $\frac{\pi}{e}$
(D) non-existent

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7. The value of

$$\int_0^1 \int_0^{1-x} \cos(x^3 + y^2)\,dy\,dx - \int_0^1 \int_0^{1-y} \cos(x^3 + y^2)\,dx\,dy$$

is

(A) 0
(B) $\frac{\cos(1)}{2}$
(C) $\frac{\sin(1)}{2}$
(D) $\cos\left(\frac{1}{2}\right) - \sin\left(\frac{1}{2}\right)$

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8. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (e^x \cos y, e^x \sin y)$. Then the number of points in $\mathbb{R}^2$ that do NOT lie in the range of f is

(A) 0
(B) 1
(C) 2
(D) infinite

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9. Let $a_n = \left(1 + \frac{1}{n}\right)^n$ and $b_n = n\cos\left(\frac{n!\pi}{2^{10}}\right)$ for $n \in \mathbb{N}$. Then

(A) (a_n) is convergent and (b_n) is bounded
(B) (a_n) is not convergent and (b_n) is bounded
(C) (a_n) is convergent and (b_n) is unbounded
(D) (a_n) is not convergent and (b_n) is unbounded

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10. Let (a_n) be a sequence defined by

$$a_n = \begin{cases} 1 & \text{if } n \text{ is prime} \\ -1 & \text{if } n \text{ is not prime} \end{cases}$$

and let $b_n = \frac{a_n}{n}$. Then

(A) both (a_n) and (b_n) are convergent
(B) (a_n) is convergent but (b_n) is not convergent
(C) (a_n) is not convergent but (b_n) is convergent
(D) both (a_n) and (b_n) are not convergent

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11. Let $a_n = \sin\left(\frac{1}{n^3}\right)$ **and** $b_n = \sin\left(\frac{1}{n}\right)$ **for** $n \in \mathbb{N}$. **Then**

(1) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

(2) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is NOT convergent

(3) $\sum_{n=1}^{\infty} a_n$ is NOT convergent but $\sum_{n=1}^{\infty} b_n$ is convergent

(4) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are NOT convergent

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12. Consider the following statements:
I. There exists a linear transformation from $\mathbb{R}^3$ to itself such that its range space and null space are the same.
II. There exists a linear transformation from $\mathbb{R}^2$ to itself such that its range space and null space are the same.
Then

(A) both I and II are TRUE
(B) I is TRUE but II is FALSE
(C) II is TRUE but I is FALSE
(D) both I and II are FALSE

---

13. Let

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -2 & 2 & 2 \end{pmatrix}, \quad B = A^5 + A^4 + I_3.$$

Which of the following is NOT an eigenvalue of B**?**

(A) 1
(B) 2
(C) 49
(D) 3

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14. The system of linear equations in x_1, x_2, x_3

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & \alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ \beta \end{pmatrix},$$

where $\alpha, \beta \in \mathbb{R}$, has:

(A) at least one solution for any α, β
(B) a unique solution for any β when $\alpha \neq 1$
(C) no solution for any α when $\beta \neq 5$
(D) infinitely many solutions for any α when $\beta = 5$

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15. Let S and T be non-empty subsets of $\mathbb{R}^2$, and W be a non-zero proper subspace of $\mathbb{R}^2$. Consider the following statements:

I. If $\text{span}(S) = \mathbb{R}^2$, then $\text{span}(S \cap W) = W$.
II. $\text{span}(S \cup T) = \text{span}(S) \cup \text{span}(T)$.
Then

(A) both I and II are TRUE
(B) I is TRUE but II is FALSE
(C) II is TRUE but I is FALSE
(D) both I and II are FALSE

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16. Let $f(x,y) = e^{x^2+y^2}$ for $(x,y) \in \mathbb{R}^2$, and a_n be the determinant of the matrix

$$\begin{pmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial x \partial y} \\ \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial y^2} \end{pmatrix}$$

evaluated at $(\cos n, \sin n)$. Then the limit $\lim_{n\to\infty} a_n$ is

(A) non-existent
(B) 0
(C) $6e^2$
(D) $12e^2$

---

17. Let $f(x,y) = \ln(1 + x^2 + y^2)$. Define

$$P = \frac{\partial^2 f}{\partial x^2}\bigg|_{(0,0)}, \quad Q = \frac{\partial^2 f}{\partial x \partial y}\bigg|_{(0,0)}, \quad R = \frac{\partial^2 f}{\partial y \partial x}\bigg|_{(0,0)}, \quad S = \frac{\partial^2 f}{\partial y^2}\bigg|_{(0,0)}.$$

Then

(A) $PS - QR > 0$ and $P < 0$
(B) $PS - QR > 0$ and $P > 0$
(C) $PS - QR < 0$ and $P > 0$
(D) $PS - QR < 0$ and $P < 0$

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18. The area of the curved surface $S = \{(x,y,z) \in \mathbb{R}^3 : z^2 = (x-1)^2 + (y-2)^2\}$ lying between $z = 2$ and $z = 3$ is

(A) $4\pi\sqrt{2}$
(B) $5\pi\sqrt{2}$
(C) 9π
(D) $9\pi\sqrt{2}$

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19. Let $a_n = \frac{1+2^{-2}+\cdots+n^{-2}}{n}$ for $n \in \mathbb{N}$. Then

(A) both (a_n) and $\sum a_n$ are convergent
(B) (a_n) is convergent but $\sum a_n$ is NOT convergent
(C) both (a_n) and $\sum a_n$ are NOT convergent
(D) (a_n) is NOT convergent but $\sum a_n$ is convergent

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20. Let (a_n) be a sequence of real numbers such that the series $\sum_{n=0}^{\infty} a_n(x-2)^n$ converges at $x = -5$. Then this series also converges at

(A) $x = 9$
(B) $x = 12$
(C) $x = 5$
(D) $x = -6$

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21. Let (a_n) and (b_n) be sequences of real numbers such that $|a_n - a_{n+1}| = \frac{1}{2^n}$ and $|b_n - b_{n+1}| = \frac{1}{\sqrt{n}}$ for $n \in \mathbb{N}$. Then

(A) both (a_n) and (b_n) are Cauchy sequences
(B) (a_n) is a Cauchy sequence but (b_n) need NOT be a Cauchy sequence
(C) (a_n) need NOT be a Cauchy sequence but (b_n) is a Cauchy sequence
(D) both (a_n) and (b_n) need NOT be Cauchy sequences

22. Consider the family of curves $x^2 + y^2 = 2x + 4y + k$ with real parameter $k > -5$. Then the orthogonal trajectory to this family passing through (2, 3) also passes through

(A) (3, 4)
(B) (-1, 1)
(C) (1, 0)
(D) (3, 5)

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23. Consider the following statements:
I. Every infinite group has infinitely many subgroups.
II. There are only finitely many non-isomorphic groups of a given finite order.
Then

(A) both I and II are TRUE
(B) I is TRUE but II is FALSE
(C) I is FALSE but II is TRUE
(D) both I and II are FALSE

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24. Suppose $f : (-1, 1) \to \mathbb{R}$ is an infinitely differentiable function such that

$$\sum_{j=0}^{\infty} a_j \frac{x^j}{j!} = f(x),$$

where

$$a_j = \int_0^{\pi/2} \theta^j \cos^j(\tan\theta) d\theta + \int_{\pi/2}^{\pi} (\theta - \pi)^j \cos^j(\tan\theta) d\theta.$$

Then

(A) $f(x) = 0$ for all $x \in (-1, 1)$
(B) f is a non-constant even function on $(-1, 1)$
(C) f is a non-constant odd function on $(-1, 1)$
(D) f is neither odd nor even on $(-1, 1)$

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25. Let $f(x) = \cos x$ and $g(x) = 1 - \frac{x^2}{2}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then

(A) $f(x) \geq g(x)$ for all x
(B) $f(x) \leq g(x)$ for all x
(C) $f(x) - g(x)$ changes sign exactly once
(D) $f(x) - g(x)$ changes sign more than once

26. Let

$$f(x,y) = \iint_{(u-x)^2+(v-y)^2\leq 1} e^{-\sqrt{(u-x)^2+(v-y)^2}} du\, dv.$$

Then $\lim_{n\to\infty} f(n, n^2)$ is

(A) non-existent
(B) 0
(C) $\pi(1-e^{-1})$
(D) $2\pi(1-2e^{-1})$

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27. How many group homomorphisms are there from $\mathbb{Z}_2$ to S_5?

(A) 40
(B) 41
(C) 26
(D) 25

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28. Let $y : \mathbb{R} \to \mathbb{R}$ be twice differentiable such that $y(0) = y(1) = 0$ and $y''(x) + x^2 < 0$ on $[0,1]$. Then

(A) $y(x) > 0$ for all $x \in (0,1)$
(B) $y(x) < 0$ for all $x \in (0,1)$
(C) $y(x) = 0$ has exactly one solution in $(0,1)$
(D) $y(x) = 0$ has more than one solution in $(0,1)$

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29. From the additive group $\mathbb{Q}$, to which of the following groups does there exist a non-trivial group homomorphism?

(A) $\mathbb{R}^\times$
(B) $\mathbb{Z}$
(C) $\mathbb{Z}_2$
(D) $\mathbb{Q}^\times$

---

30. Let $f : \mathbb{R} \to \mathbb{R}$ be infinitely differentiable such that f'' has exactly two distinct zeros. Then

(A) f' has at most three distinct zeros
(B) f' has at least one zero

(C) f has at most three distinct zeros
(D) f has at least two distinct zeros

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31. For each $t \in (0,1)$, the surface $P_t \subset \mathbb{R}^3$ is defined by $P_t = \{(x,y,z) : (x^2+y^2)z = 1,\ t^2 \le x^2+y^2 \le 1\}$. Let $a_t \in \mathbb{R}$ be the surface area of P_t. Then

(A) $a_t = \iint_{t^2 \le x^2+y^2 \le 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^4} + \frac{4y^2}{(x^2+y^2)^4}}\, dx\, dy$

(B) $a_t = \iint_{t^2 \le x^2+y^2 \le 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^2} + \frac{4y^2}{(x^2+y^2)^2}}\, dx\, dy$

(C) The limit $\lim_{t \to 0^+} a_t$ does NOT exist

(D) The limit $\lim_{t \to 0^+} a_t$ exists

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32. Let $A \subseteq \mathbb{Z}$ with $0 \in A$. For $r, s \in \mathbb{Z}$, define $rA = \{ra : a \in A\}$ and $rA + sA = \{ra + sb : a, b \in A\}$. Which of the following conditions imply that A is a subgroup of the additive group $\mathbb{Z}$?

(A) $-2A \subseteq A$, $A + A = A$
(B) $A = -A$, $A + 2A = A$
(C) $A = -A$, $A + A = A$
(D) $2A \subseteq A$, $A + A = A$

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33. Let $y : (\sqrt{2/3}, \infty) \to \mathbb{R}$ be the solution of $(2x - y)y' + (2y - x) = 0$, with $y(1) = 3$. Then

(A) $y(3) = 1$
(B) $y(2) = 4 + \sqrt{10}$
(C) y' is bounded on $(\sqrt{2/3}, 1)$
(D) y' is bounded on $(1, \infty)$

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34. Let $f : (-1,1) \to \mathbb{R}$ be differentiable with $f(0) = 0$, and suppose $|f'(x)| \le M|x|$ for all $x \in (-1,1)$. Then

(A) f' is continuous at $x = 0$
(B) f' is differentiable at $x = 0$
(C) ff' is differentiable at $x = 0$
(D) $(f')^2$ is differentiable at $x = 0$

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35. Which of the following functions is/are Riemann integrable on $[0,1]$?

(A) $f(x) = \int_0^x \left|\frac{1}{2} - t\right| dt$

(B) $f(x) = \begin{cases} x\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

(C) $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \cap [0,1] \\ -1, & \text{otherwise} \end{cases}$

(D) $f(x) = \begin{cases} x, & x \in [0,1) \\ 0, & x = 1 \end{cases}$

---

36. A subset $S \subseteq \mathbb{R}^2$ is said to be bounded if there exists $M > 0$ such that $|x| \leq M$ and $|y| \leq M$ for all $(x,y) \in S$. Which of the following subsets of $\mathbb{R}^2$ is/are bounded?

(A) $\{(x,y) \in \mathbb{R}^2 : e^{x^2} + y^2 \leq 4\}$
(B) $\{(x,y) \in \mathbb{R}^2 : x^4 + y^2 \leq 4\}$
(C) $\{(x,y) \in \mathbb{R}^2 : |x| + |y| \leq 4\}$
(D) $\{(x,y) \in \mathbb{R}^2 : e^{x^3} + y^2 \leq 4\}$

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37. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \dfrac{x^4y^3}{x^6+y^6}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Then

(A) $\lim_{t\to 0} \frac{f(t,t)-f(0,0)}{t} = \frac{1}{2}$
(B) $\frac{\partial f}{\partial x}(0,0) = 0$
(C) $\frac{\partial f}{\partial y}(0,0) = 0$
(D) $\lim_{t\to 0} \frac{f(t,2t)-f(0,0)}{t} = \frac{1}{3}$

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38. Which of the following statements are true about linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$?

(A) Every linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ maps lines onto points or lines
(B) Every surjective linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ maps lines onto lines
(C) Every bijective linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ maps pairs of parallel lines to pairs of parallel lines
(D) Every bijective linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ maps pairs of perpendicular lines to pairs of perpendicular lines

39. Which of the following mappings are linear transformations?

(A) $T : \mathbb{R} \to \mathbb{R}$, $T(x) = \sin(x)$
(B) $T : M_2(\mathbb{R}) \to \mathbb{R}$, $T(A) = \text{trace}(A)$
(C) $T : \mathbb{R}^2 \to \mathbb{R}$, $T(x, y) = x + y + 1$
(D) $T : P_2(\mathbb{R}) \to \mathbb{R}$, $T(p(x)) = p(1)$

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40. Let R_1 and R_2 be the radii of convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^{n+1}}{n(n+1)} \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^n x^{n-1},$$

respectively. Then

(A) $R_1 = R_2$
(B) $R_2 > 1$
(C) $\sum_{n=1}^{\infty} (-1)^n x^{n-1}$ converges for all $x \in [-1, 1]$
(D) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{n+1}}{n(n+1)}$ converges for all $x \in [-1, 1]$

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41. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as follows:

$$f(x, y) = \begin{cases} (x^2 - 1)^2 \cos^2\left(\frac{y^2}{(x^2-1)^2}\right), & x \neq \pm 1, \\ 0, & x = \pm 1. \end{cases}$$

The number of points of discontinuity of $f(x, y)$ is equal to _________.

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42. Let $T : P_2(\mathbb{R}) \to P_4(\mathbb{R})$ be a linear transformation defined by $T(p(x)) = p(x^2)$. Find the rank of T.

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43. If y is the solution of the differential equation $y'' - 2y' + y = e^x$ with $y(0) = 0$ and $y'(0) = -\frac{1}{2}$, then $y(1)$ is equal to ______ (rounded to two decimal places).

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44. The value of

$$\lim_{n \to \infty} \left(n \int_0^1 \frac{x^n}{x + 1} \, dx \right)$$

is equal to _______ (rounded to two decimal places).

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45. For $\sigma \in S_8$, let $o(\sigma)$ denote the order of σ. Then $\max\{o(\sigma) : \sigma \in S_8\}$ is equal to ______.

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46. For $g \in \mathbb{Z}$, let $\bar{g} \in \mathbb{Z}_8$ denote the residue class of g modulo 8. Consider the group $\mathbb{Z}_8^\times = \{\bar{x} \in \mathbb{Z}_8 : 1 \leq x \leq 7, \gcd(x, 8) = 1\}$ under multiplication mod 8. The number of group isomorphisms from $\mathbb{Z}_8^\times$ onto itself is equal to ____.

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47. Let $f(x) = \sqrt[3]{x}$ for $x \in (0, \infty)$, and $\theta(h)$ be defined by

$$f(3+h) - f(3) = hf'(3 + \theta(h)h), \quad \text{for all } h \in (-1, 1).$$

Then $\lim_{h \to 0} \theta(h) =$ _____ (rounded off to two decimal places).

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48. Let V be the volume of the region $S \subseteq \mathbb{R}^3$ defined by

$$S = \{(x, y, z) \in \mathbb{R}^3 : xy \leq z \leq 4,\ 0 \leq x^2 + y^2 \leq 1\}.$$

Then $\dfrac{V}{\pi} =$ _____ (rounded off to two decimal places).

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49. The sum of the series $\sum_{n=1}^{\infty} \dfrac{2n+1}{(n^2+1)(n^2+2n+2)}$ is equal to _____ (rounded off to two decimal places).

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50. Evaluate

$$\lim_{n \to \infty} \left(1 + \frac{1}{2^n} + \frac{1}{3^n} + \cdots + \frac{1}{(2023)^n}\right)^{1/n}.$$

(Rounded off to two decimal places.)

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51. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined as $f(x, y, z) = x^3 + y^3 + z^3$, and let $L : \mathbb{R}^3 \to \mathbb{R}$ be the linear map satisfying

$$\lim_{(x,y,z) \to (0,0,0)} \frac{f(1+x, 1+y, 1+z) - f(1, 1, 1) - L(x, y, z)}{\sqrt{x^2 + y^2 + z^2}} = 0.$$

Then $L(1, 2, 4)$ is equal to ______. (rounded off to two decimal places.)

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52. The global minimum value of $f(x) = |x - 1| + |x - 2|^2$ on $\mathbb{R}$ is equal to ______. (rounded off to two decimal places.)

53. Let $y : (1, \infty) \to \mathbb{R}$ satisfy $y'' - \dfrac{2y}{(1-x)^2} = 0$, with $y(2) = 1$ and $\lim_{x\to\infty} y(x) = 0$. Find $y(3)$ (rounded to two decimal places).

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54. The number of permutations in S_4 having exactly two cycles in their cycle decomposition is equal to _____.

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55. Let S be the triangular region with vertices $(0,0)$, $\left(0, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, 0\right)$. Then the value of $\iint_S \sin(x)\cos(y)\,dx\,dy$ is equal to _____. (rounded to two decimal places.)

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56. Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 1 & 4 & 4 & 4 \end{pmatrix}$$

and let B be a 5×5 real matrix such that $AB = 0$. Then the maximum possible rank of B is equal to _____.

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57. Let $W \subseteq M_3(\mathbb{R})$ consist of all matrices where each row and each column sums to zero. Then the dimension of W is equal to _____.

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58. The maximum number of linearly independent eigenvectors of

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

is equal to _____.

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59. Let S be the set of all real numbers α such that the solution of $\dfrac{dy}{dx} = y(2-y)$, $y(0) = \alpha$ exists on $[0, \infty)$. Find the minimum of S.

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60. Let $f : \mathbb{R} \to \mathbb{R}$ be bijective with

$$f(x) = \sum_{n=1}^{\infty} a_n x^n, \quad f^{-1}(x) = \sum_{n=1}^{\infty} b_n x^n,$$

and f^{-1} **is the inverse of** f**. If** $a_1 = 2, a_2 = 4$**, find** b_1**.**
