IIT JAM 2025 Mathematics (MA) Question Paper

Time Allowed : 3 Hours	**Maximum Marks :** 100	**Total questions :** 60

General Instructions

General Instructions:

i) All questions are compulsory. Marks allotted to each question are indicated in the margin.

ii) Answers must be precise and to the point.

iii) In numerical questions, all steps of calculation should be shown clearly.

iv) Use of non-programmable scientific calculators is permitted.

v) Wherever necessary, write balanced chemical equations with proper symbols and units.

vi) Rough work should be done only in the space provided in the question paper.

Q.1. The sum of the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\pi^{2n+1}}{2^{2n+1}(2n)!}$$

is equal to

(A) $-\pi$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $-\frac{\pi}{4}$

Q.2. For which one of the following choices of $N(x, y)$, is the equation

$$(e^x \sin y - 2y \sin x)\, dx + N(x, y)\, dy = 0$$

an exact differential equation?

(A) $N(x, y) = e^x \sin y + 2 \cos x$

(B) $N(x, y) = e^x \cos y + 2 \cos x$

(C) $N(x, y) = e^x \cos y + 2 \sin x$

(D) $N(x, y) = e^x \sin y + 2 \sin x$

Q.3. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions defined by

$$f(x) = \begin{cases} x|x| \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2 \sin \frac{1}{x} + x \cos \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then, which one of the following is TRUE?

(A) f is differentiable at $x = 0$, and g is NOT differentiable at $x = 0$

(B) f is NOT differentiable at $x = 0$, and g is differentiable at $x = 0$

(C) f is differentiable at $x = 0$, and g is differentiable at $x = 0$

(D) f is NOT differentiable at $x = 0$, and g is NOT differentiable at $x = 0$

Q.4. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions defined by

$$f(x) = \begin{cases} |x|^{1/8} \sin \frac{1}{|x|} \cos x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} e^x \cos \frac{1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Then, which one of the following is TRUE?

(A) f is continuous at $x = 0$, and g is NOT continuous at $x = 0$

(B) f is NOT continuous at $x = 0$, and g is continuous at $x = 0$

(C) f is continuous at $x = 0$, and g is continuous at $x = 0$

(D) f is NOT continuous at $x = 0$, and g is NOT continuous at $x = 0$

Q.5. Which one of the following is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 2e^{4x}?$$

(A) $\alpha_1 e^{4x} + \alpha_2 x e^{4x}$, where $\alpha_1, \alpha_2 \in \mathbb{R}$

(B) $\alpha_1 e^{4x} + \alpha_2 x e^{4x} + 2xe^{4x}$, where $\alpha_1, \alpha_2 \in \mathbb{R}$

(C) $\alpha_1 e^{4x} + \alpha_2 e^{4x} + 2xe^{4x}$, where $\alpha_1, \alpha_2 \in \mathbb{R}$

(D) $\alpha_1 x e^{-4x} + \alpha_2 x^2 e^{4x}$, where $\alpha_1, \alpha_2 \in \mathbb{R}$

Q.6. Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by

$$T(x, y, z) = (x + z, 2x + 3y + 5z, 2y + 2z), \quad \text{for all } (x, y, z) \in \mathbb{R}^3$$

Then, which one of the following is TRUE?

(A) T is one-one and T is NOT onto

(B) T is NOT one-one and T is onto

(C) T is one-one and T is onto

(D) T is NOT one-one and T is NOT onto

Q.7. Let

$$M = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & -4 \\ 2 & 0 & x \end{pmatrix}$$

for some real number x. If 0 is an eigenvalue of M, then $(M^4 + M)\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is equal to

(A) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

(C) $\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$

(D) $\begin{pmatrix} 17 \\ 0 \\ 17 \end{pmatrix}$

Q.8. Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear transformation defined by

$$T(p(x)) = p(x+1), \quad \text{for all } p(x) \in P_2(\mathbb{R})$$

If M is the matrix representation of T with respect to the ordered basis $\{1, x, x^2\}$ of $P_2(\mathbb{R})$, then which one of the following is TRUE?

(A) The determinant of M is 2

(B) The rank of M is 2

(C) 1 is the only eigenvalue of M

(D) The nullity of M is 2

Q.9. Let G be a finite abelian group of order 10. Let x_0 be an element of order 2 in G. If $X = \{x \in G : x^3 = x_0\}$, then which one of the following is TRUE?

(A) X has exactly one element

(B) X has exactly two elements

(C) X has exactly three elements

(D) X is an empty set

Q.10. The value of

$$\int_0^1 \left(\int_0^{\sqrt{y}} 3e^{x^3}\, dx \right) dy$$

is equal to

(A) $e - 1$

(B) $\frac{e-1}{2}$

(C) $\sqrt{e} - 1$

(D) $\frac{\sqrt{e}-1}{2}$

Q.11. Let C denote the family of curves described by $yx^2 = \lambda$, for $\lambda \in (0, \infty)$ and lying in the first quadrant of the xy-plane. Let O denote the family of orthogonal trajectories of C. Which one of the following curves is a member of O, and passes through the point $(2, 1)$?

(A) $y = \frac{x^2}{4}$, $x > 0, y > 0$

(B) $x^2 - 2y^2 = 2,\ x > 0, y > 0$

(C) $x - y = 1,\ x > 0, y > 0$

(D) $2x - y^2 = 3,\ x > 0, y > 0$

Q.12. Let $\varphi : (0, \infty) \to \mathbb{R}$ be the solution of the differential equation

$$x\frac{dy}{dx} = (\ln y - \ln x)\, y,$$

satisfying $\varphi(1) = e^2$. Then, the value of $\varphi(2)$ is equal to:

(A) e^2

(B) $2e^3$

(C) $3e^2$

(D) $6e^3$

Q.13. Let $X = \{x \in S_4 : x^3 = \text{id}\}$ and $Y = \{x \in S_4 : x^2 \neq \text{id}\}$. If m and n denote the number of elements in X and Y, respectively, then which one of the following is TRUE?

(A) m is even and n is even

(B) m is odd and n is even

(C) m is even and n is odd

(D) m is odd and n is odd

Q.14. Let $\varphi : \mathbb{R} \to \mathbb{R}$ be the solution of the differential equation

$$x\frac{dy}{dx} = (y - 1)(y - 3),$$

satisfying $\varphi(0) = 2$. Then, which one of the following is TRUE?

(A) $\lim_{x\to\infty} \varphi(x) = 0$

(B) $\lim_{x\to\ln\sqrt{2}} \varphi(x) = 1$

(C) $\lim_{x\to-\infty} \varphi(x) = 3$

(D) $\lim_{x \to \ln \frac{1}{\sqrt{2}}} \varphi(x) = 6$

Q.15. Let

$$M = \begin{pmatrix} 6 & 2 & -6 & 8 \\ 5 & 3 & -9 & 8 \\ 3 & 1 & -2 & 4 \end{pmatrix}$$

Consider the system S of linear equations given by:

$$6x_1 + 2x_2 - 6x_3 + 8x_4 = 8$$

$$5x_1 + 3x_2 - 9x_3 + 8x_4 = 16$$

$$3x_1 + x_2 - 2x_3 + 4x_4 = 32$$

where x_1, x_2, x_3, x_4 are unknowns. Then, which one of the following is TRUE?

(A) The rank of M is 3, and the system S has a solution

(B) The rank of M is 3, and the system S does NOT have a solution

(C) The rank of M is 2, and the system S has a solution

(D) The rank of M is 2, and the system S does NOT have a solution

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Q.16. Let

$$M = \begin{pmatrix} 3 & -2 & 0 \\ 2 & 3 & 3 \\ 4 & -1 & x \end{pmatrix}$$

for some real number x. Suppose that -2 and 3 are eigenvalues of M. If $M^3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 125 \\ 125 \end{pmatrix}$,

then which one of the following is TRUE?

(A) $x = 5$, and the matrix $M^2 + M$ is invertible

(B) $x \neq 5$, and the matrix $M^2 + M$ is invertible

(C) $x = 5$, and the matrix $M^2 + M$ is NOT invertible

(D) $x \neq 5$, and the matrix $M^2 + M$ is NOT invertible

Q.17. Let $f(x) = 10x^2 + e^x - \sin(2x) - \cos x$, $x \in \mathbb{R}$. The number of points at which the function f has a local minimum is:

(A) 0

(B) 1

(C) 2

(D) greater than or equal to 3

Q.18. For $n \in \mathbb{N}$, define x_n and y_n by

$$x_n = (-1)^n \cos\frac{1}{n} \quad \text{and} \quad y_n = \sum_{k=1}^{n} \frac{1}{n+k}.$$

Then, which one of the following is TRUE?

(A) $\sum_{n=1}^{\infty} x_n$ converges, and $\sum_{n=1}^{\infty} y_n$ does NOT converge

(B) $\sum_{n=1}^{\infty} x_n$ does NOT converge, and $\sum_{n=1}^{\infty} y_n$ converges

(C) $\sum_{n=1}^{\infty} x_n$ converges, and $\sum_{n=1}^{\infty} y_n$ converges

(D) $\sum_{n=1}^{\infty} x_n$ does NOT converge, and $\sum_{n=1}^{\infty} y_n$ does NOT converge

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Q.19. Let $x_1 = \frac{5}{2}$ and for $n \in \mathbb{N}$, define

$$x_{n+1} = \frac{1}{5}\left(x_n^2 + 6\right).$$

Then, which one of the following is TRUE?

(A) (x_n) is an increasing sequence, and (x_n) is NOT a bounded sequence

(B) (x_n) is NOT an increasing sequence, and (x_n) is NOT a bounded sequence

(C) (x_n) is NOT a decreasing sequence, and (x_n) is a bounded sequence

(D) (x_n) is a decreasing sequence, and (x_n) is a bounded sequence

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Q.20. Let $x_1 = 2$ and $x_{n+1} = 2 + \frac{1}{2x_n}$ for all $n \in \mathbb{N}$. Then, which one of the following is TRUE?

(A) $x_{n+1} \geq \frac{4}{x_n}$ for all $n \in \mathbb{N}$, and (x_n) is a Cauchy sequence

(B) $x_{n+1} = \frac{4}{x_n}$ for some $n \in \mathbb{N}$, and (x_n) is a Cauchy sequence

(C) $x_{n+1} = \frac{4}{x_n}$ for all $n \in \mathbb{N}$, and (x_n) is NOT a Cauchy sequence

(D) $x_{n+1} \leq \frac{4}{x_n}$ for some $n \in \mathbb{N}$, and (x_n) is NOT a Cauchy sequence

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Q.21. For $n \in \mathbb{N}$, define x_n and y_n by

$$x_n = (-1)^n \frac{3n}{n^3} \quad \text{and} \quad y_n = \left(4n^3 + (-1)^n 3n^3\right)^{1/n}.$$

Then, which one of the following is TRUE?

(A) (x_n) has a convergent subsequence, and NO subsequence of (y_n) is convergent.

(B) NO subsequence of (x_n) is convergent, and (y_n) has a convergent subsequence.

(C) (x_n) has a convergent subsequence, and (y_n) has a convergent subsequence.

(D) NO subsequence of (x_n) is convergent, and NO subsequence of (y_n) is convergent.

Q.22. Let $M = (m_{ij})$ be a 3×3 real, invertible matrix and $\sigma \in S_3$ be the permutation defined by $\sigma(1) = 2, \sigma(2) = 3$ and $\sigma(3) = 1$. The matrix M_σ is defined by $n_{ij} = m_{i\sigma(j)}$ for all $i, j \in \{1, 2, 3\}$. Then, which one of the following is TRUE?

(A) $\det(M) = \det(M_\sigma)$, and the nullity of the matrix $M - M_\sigma$ is 0

(B) $\det(M) = -\det(M_\sigma)$, and the nullity of the matrix $M - M_\sigma$ is 1

(C) $\det(M) = \det(M_\sigma)$, and the nullity of the matrix $M - M_\sigma$ is 1

(D) $\det(M) = -\det(M_\sigma)$, and the nullity of the matrix $M - M_\sigma$ is 0

Q.23. Let $\mathbb{R}/\mathbb{Z}$ denote the quotient group, where $\mathbb{Z}$ is considered as a subgroup of the additive group of real numbers $\mathbb{R}$.

Let m denote the number of injective (one-one) group homomorphisms from $\mathbb{Z}_3$ to $\mathbb{R}/\mathbb{Z}$ and n denote the number of group homomorphisms from $\mathbb{R}/\mathbb{Z}$ to $\mathbb{Z}_3$.

Then, which one of the following is TRUE?

(A) $m = 2$ and $n = 1$

(B) $m = 3$ and $n = 3$

(C) $m = 2$ and $n = 3$

(D) $m = 1$ and $n = 1$

Q.24. Let f_1, f_2, f_3 be nonzero linear transformations from $\mathbb{R}^4$ to $\mathbb{R}$ and

$$\ker(f_1) \subset \ker(f_2) \cap \ker(f_3).$$

Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(v) = (f_1(v), f_2(v), f_3(v)) \quad \text{for all } v \in \mathbb{R}^4.$$

Then, the nullity of T is equal to:

(A) 1

(B) 2

(C) 3

(D) 4

Q.25. Let $x_1 = 1$. For $n \in \mathbb{N}$, define

$$x_{n+1} = \left(\frac{1}{2} + \frac{\sin^2 n}{n}\right) x_n.$$

Then, which one of the following is TRUE?

(A) $\sum_{n=1}^{\infty} x_n$ converges

(B) $\sum_{n=1}^{\infty} x_n$ does NOT converge

(C) $\sum_{n=1}^{\infty} x_n^2$ does NOT converge

(D) $\sum_{n=1}^{\infty} x_n x_{n+1}$ does NOT converge

Q.26. Let $x_1 > 0$. For $n \in \mathbb{N}$, define

$$x_{n+1} = x_n + 4.$$

If

$$\lim_{n\to\infty} \left(\frac{1}{x_1 x_2 x_3} + \frac{1}{x_2 x_3 x_4} + \cdots + \frac{1}{x_{n+1} x_{n+2} x_{n+3}} \right) = \frac{1}{24},$$

then the value of x_1 is equal to:

(A) 1

(B) 2

(C) 3

(D) 8

Q.27. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = e^y(x^2 + y^2) \quad \text{for all } (x, y) \in \mathbb{R}^2.$$

Then, which one of the following is TRUE?

(A) The number of points at which f has a local minimum is 2

(B) The number of points at which f has a local maximum is 2

(C) The number of points at which f has a local minimum is 1

(D) The number of points at which f has a local maximum is 1

Q.28. Let Ω be the bounded region in $\mathbb{R}^3$ lying in the first octant $(x \geq 0, y \geq 0, z \geq 0)$, and bounded by the surfaces $z = x^2 + y^2$, $z = 4$, $x = 0$ and $y = 0$. Then, the volume of Ω is equal to:

(A) π

(B) 2π

(C) 3π

(D) 4π

Q.25. Let $x_1 = 1$. For $n \in \mathbb{N}$, define

$$x_{n+1} = \left(\frac{1}{2} + \frac{\sin^2 n}{n}\right) x_n.$$

Then, which one of the following is TRUE?

(A) $\sum_{n=1}^{\infty} x_n$ converges

(B) $\sum_{n=1}^{\infty} x_n$ does NOT converge

(C) $\sum_{n=1}^{\infty} x_n^2$ does NOT converge

(D) $\sum_{n=1}^{\infty} x_n x_{n+1}$ does NOT converge

Q.30. The number of elements in the set

$$\{x \in \mathbb{R} : 8x^2 + x^4 + x^8 = \cos x\}$$

is equal to:

(A) 0

(B) 1

(C) 2

(D) greater than or equal to 3

Q.31. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \frac{x^2+y^5}{x^2+y^4} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Then, which of the following is/are TRUE?

(A) The iterated limits $\lim_{x\to 0}\left(\lim_{y\to 0} f(x,y)\right)$ and $\lim_{y\to 0}\left(\lim_{x\to 0} f(x,y)\right)$ exist.

(B) Exactly one of the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists at $(0,0)$.

(C) Both the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at $(0,0)$.

(D) f is NOT differentiable at $(0,0)$.

Q.32. If $M, N, \mu, w : \mathbb{R}^2 \to \mathbb{R}$ are differentiable functions with continuous partial derivatives, satisfying

$$\mu(x,y)M(x,y)\,dx + \mu(x,y)N(x,y)\,dy = dw,$$

then which one of the following is TRUE?

(A) μw is an integrating factor for $M(x,y)\,dx + N(x,y)\,dy = 0$

(B) μw^2 is an integrating factor for $M(x,y)\,dx + N(x,y)\,dy = 0$

(C) $w(x,y) = w(0,0) + \int_0^x \mu M(s)\,ds + \int_0^y \mu N(t)\,dt$, for all $(x,y) \in \mathbb{R}^2$

(D) $w(x,y) = w(0,0) + \int_0^x \mu M(s,y)\,ds + \int_0^y \mu N(x,t)\,dt$, for all $(x,y) \in \mathbb{R}^2$

Q.33. Let $\varphi : (-1,\infty) \to (0,\infty)$ be the solution of the differential equation

$$\frac{dy}{dx} = 2ye^x = 2e^x\sqrt{y},$$

satisfying $\varphi(0) = 1$. Then, which of the following is/are TRUE?

(A) φ is an unbounded function.

(B) $\lim_{x\to\ln 2} \varphi(x) = (2e-1)^2$.

(C) $\lim_{x\to\ln 2} \varphi(x) = \sqrt{2e-1}$.

(D) φ is a strictly increasing function on the interval $(0,\infty)$.

Q.34. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \frac{(x^2+\sin xy)^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Then, which of the following is/are TRUE?

(A) $\lim_{(x,y)\to(0,0)} f(x,y)$ exists and $\lim_{(x,y)\to(0,0)} f(x,y) = 1$.

(B) $\lim_{(x,y)\to(0,0)} f(x,y)$ exists and $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

(C) f is differentiable at $(0,0)$.

(D) f is NOT differentiable at $(0,0)$.

Q.35. Let $u_1 = (1, 0, 0, -1)$, $u_2 = (2, 0, 0, -1)$, $u_3 = (0, 0, 1, -1)$, $u_4 = (0, 0, 0, 1)$ be elements in the real vector space $\mathbb{R}^4$. Then, which of the following is/are TRUE?

(A) $\{u_1, u_2, u_3, u_4\}$ is a linearly independent set in $\mathbb{R}^4$.

(B) $\{u_1 - u_2, u_3 - u_4, u_4 - u_1\}$ is NOT a linearly independent set in $\mathbb{R}^4$.

(C) $\{u_1, -u_2, u_3, -u_4\}$ is NOT a linearly independent set in $\mathbb{R}^4$.

(D) $\{u_1 + u_2, u_2 + u_3, u_3 + u_4, u_4 + u_1\}$ is a linearly independent set in $\mathbb{R}^4$.

Q.36. For $n \in \mathbb{N}$, let

$$x_n = \sum_{k=1}^{n} \frac{k}{n^2 + k}.$$

Then, which of the following is/are TRUE?

(A) The sequence (x_n) converges.

(B) The series $\sum_{n=1}^{\infty} x_n$ converges.

(C) The series $\sum_{n=1}^{\infty} x_n$ does NOT converge.

(D) The series $\sum_{n=1}^{\infty} x_n^n$ converges.

Q.37. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that

$$f(0) = 0,\ f'(0) = 2,\ f(1) = -3.$$

Then, which of the following is/are TRUE?

(A) $|f'(x)| \leq 2$ for all $x \in [0, 1]$.

(B) $|f'(x_1)| > 2$ for some $x_1 \in [0, 1]$.

(C) $|f''(x)| < 10$ for all $x \in [0, 1]$.

(D) $|f''(x_2)| \geq 10$ for some $x_2 \in [0, 1]$.

Q.38. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that

$$f(0) = 4,\ f(1) = -2,\ f(2) = 8,\ f(3) = 2.$$

Then, which of the following is/are TRUE?

(A) $|f'(x)| < 5$ for all $x \in [0,1]$.

(B) $|f'(x_1)| \geq 5$ for some $x_1 \in [0,1]$.

(C) $f'(x_2) = 0$ for some $x_2 \in [0,3]$.

(D) $f''(x_3) = 0$ for some $x_3 \in [0,3]$.

Q.39. For $n \in \mathbb{N}$, consider the set $U(n) = \{x \in \mathbb{Z}_n : \gcd(x,n) = 1\}$ as a group under multiplication modulo n. Then, which of the following is/are TRUE?

(A) $U(8)$ is a cyclic group.

(B) $U(5)$ is a cyclic group.

(C) $U(12)$ is a cyclic group.

(D) $U(9)$ is a cyclic group.

Q.40. Consider the following subspaces of the real vector space $\mathbb{R}^3$:

$$V_1 = \text{span}\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}\right\}, \quad V_2 = \text{span}\left\{\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\}, \quad V_3 = \text{span}\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\}, \quad V_4 = \text{span}\left\{\begin{pmatrix}1\\3\\6\end{pmatrix}\right\},$$

Then, which of the following is/are TRUE?

(A) $V_1 \cup V_2$ is a subspace of $\mathbb{R}^3$.

(B) $V_1 \cup V_3$ is a subspace of $\mathbb{R}^3$.

(C) $V_1 \cup V_4$ is a subspace of $\mathbb{R}^3$.

(D) $V_1 \cup V_5$ is a subspace of $\mathbb{R}^3$.

Q.41. The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x+\frac{1}{4})^n}{(-2)^n n^2}$$

about $x = -\frac{1}{4}$ is equal to .... (rounded off to two decimal places).

Q.42. The value of

$$\lim_{n\to\infty} 8n \left(\left(e^{\frac{1}{2n}} - 1 \right) \left(\sin \frac{1}{2n} + \cos \frac{1}{2n} \right) \right)$$

is equal to .... (rounded off to two decimal places).

Q.43. Let α be the real number such that

$$\lim_{x\to 0} \frac{(1-\cos x)(22x^2 + x - 4)}{x^3} = \alpha \ln 2.$$

Then, the value of α is equal to .... (rounded off to two decimal places).

Q.44. Let $\varphi : \mathbb{R} \to \mathbb{R}$ be the solution of the differential equation

$$4\frac{d^2y}{dx^2} + 16\frac{dy}{dx} + 25y = 0$$

satisfying $\varphi(0) = 1$ and $\varphi'(0) = -\frac{1}{2}$. Then, the value of $\lim_{x\to\infty} e^{2x}\varphi(x)$ is equal to .... (rounded off to two decimal places).

Q.45. Let S be the surface area of the portion of the plane $z = x + y + 3$, which lies inside the cylinder $x^2 + y^2 = 1$. Then, the value of $\left(\frac{S}{\pi}\right)^2$ is equal to .... (rounded off to two decimal places).

Q.46. Consider the following subspaces of $\mathbb{R}^4$:

$V_1 = \left\{(x, y, z, w) \in \mathbb{R}^4 : x + y + 2w = 0\right\}, \quad V_2 = \left\{(x, y, z, w) \in \mathbb{R}^4 : 2y + z + w = 0\right\}, \quad V_3 = \left\{(x, y, z,\right.$

Then, the dimension of the subspace $V_1 \cap V_2 \cap V_3$ is equal to .... (rounded off to two decimal places).

Q.47. Consider the real vector space $\mathbb{R}^3$. Let $T : \mathbb{R}^3 \to \mathbb{R}$ be a linear transformation such that

$$T(1,1,1) = 0, \quad T(1,-1,1) = 0, \quad T(0,0,1) = 16.$$

Then, the value of $T\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right)$ is equal to .... (rounded off to two decimal places).

Q.48. Let T denote the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$. The value of the double integral (over T)

$$\iint_T (5 - y)\, dx\, dy$$

is equal to .... (rounded off to two decimal places).

Q.49. Let $T, S : P_4(\mathbb{R}) \to P_4(\mathbb{R})$ be the linear transformations defined by

$$T(p(x)) = xp'(x), \quad S(p(x)) = (x+1)p'(x)$$

for all $p(x) \in P_4(\mathbb{R})$. Then, the nullity of the composition $S \circ T$ is ....

Q.50. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \frac{(x^2-y^2)^2 xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Then, the value of $\frac{\partial f}{\partial y}(0,0)$ and $\frac{\partial f}{\partial x}(0,0)$ is equal to .... (rounded off to two decimal places).

Q.51. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying

$$\int_0^{\frac{\pi}{4}} \left(\sin(x) f(x) + \cos(x) \int_0^x f(t)\, dt \right) dx = \sqrt{2}.$$

Then, the value of

$$\int_0^{\frac{\pi}{4}} f(x)\,dx$$

is equal to .... (rounded off to two decimal places).

Q.52. Let $\sigma \in S_4$ be the permutation defined by $\sigma(1) = 2$, $\sigma(2) = 3$, $\sigma(3) = 1$, and $\sigma(4) = 4$. The number of elements in the set

$$\{\tau \in S_4 : \tau \circ \sigma^{-1} = \sigma\}$$

is equal to ....

Q.53. Let $f(x) = 2x - \sin(x)$, for all $x \in \mathbb{R}$. Let $k \in \mathbb{N}$ be such that

$$\lim_{x \to 0} \left(\frac{1}{x} \sum_{i=1}^{k} i^2 f\left(\frac{x}{i}\right) \right) = 45.$$

Then, the value of k is equal to ....

Q.54. The value of the infinite series

$$\sum_{n=1}^{\infty} n \left(\frac{3}{4}\right)^{2n-1}$$

is equal to .... (rounded off to two decimal places).

Q.55. Let $\varphi : (0, \infty) \to \mathbb{R}$ be the solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 6x \ln x,$$

satisfying $\varphi(1) = -3$ and $\varphi(e) = 0$. Then, the value of $\varphi'(1)$ is equal to .... (rounded off to two decimal places).

Q.56. Let $\varphi : \mathbb{R} \to \mathbb{R}$ be the solution of the differential equation

$$\frac{dy}{dx} + 2xy = 2 + 4x^2,$$

satisfying $\varphi(0) = 0$. Then, the value of $\varphi(2)$ is equal to .... (rounded off to two decimal places).

Q.57. Let Ω be the solid bounded by the planes $z = 0$, $y = 0$, $x = \frac{1}{2}$, $2y = x$ and $2x + y + z = 4$. If V is the volume of Ω, then the value of $64V$ is equal to .... (rounded off to two decimal places).

Q.58. Let the subspace H of $P_3(\mathbb{R})$ be defined as

$$H = \{p(x) \in P_3(\mathbb{R}) : xp'(x) = 3p(x)\}.$$

Then, the dimension of H is equal to ....

Q.59. Let G be an abelian group of order 35. Let m denote the number of elements of order 5 in G, and let n denote the number of elements of order 7 in G. Then, the value of $m + n$ is equal to ....

Q.60. The number of surjective (onto) group homomorphisms from S_4 to $\mathbb{Z}_6$ is equal to ....
