IIT JAM 2026 Mathematics (MA) Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Please read the following instructions carefully:

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, A, B and C. All sections are compulsory. Questions in each section are of different types.
2. Section A contains a total of 30 Multiple Choice Questions (MCQ). Each MCQ type question has four choices out of which only one choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
3. Section B contains a total of 10 Multiple Select Questions (MSQ). Each MSQ type question is similar to MCQ but with a difference that there will be more than one choices that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.
4. Section C contains a total of 20 Numerical Answer Type (NAT) questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.
5. In all sections, questions not attempted will result in zero mark. In Section A (MCQ), wrong answer will result in NEGATIVE marks. For all 1-mark questions, 1/3 marks will be deducted for each wrong answer. For all 2-mark questions, 2/3 marks will be deducted for each wrong answer. In Section B (MSQ), there is NO NEGATIVE and NO PARTIAL marking provisions. There is NO NEGATIVE marking in Section C (NAT) as well.
6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are NOT allowed in the examination hall.
7. A Scribble Pad will be provided for rough work.

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1. Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n.$$

Solution:

Step 1: Use Ratio Test.
Let

$$a_n = \frac{(n!)^2}{(2n)!}.$$

Consider:

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{((n+1)!)^2}{(2n+2)!} \cdot \frac{(2n)!}{(n!)^2}.$$

Step 2: Simplify factorial terms.

$$(n+1)! = (n+1)n!,$$
$$(2n+2)! = (2n+2)(2n+1)(2n)!.$$

Substitute:

$$\frac{((n+1)n!)^2}{(2n+2)(2n+1)(2n)!} \cdot \frac{(2n)!}{(n!)^2}.$$

Cancel $(n!)^2$ and $(2n)!$:

$$= \frac{(n+1)^2}{(2n+2)(2n+1)}.$$

Step 3: Take limit as $n \to \infty$.

For large n:

$$\frac{(n+1)^2}{(2n+2)(2n+1)} \sim \frac{n^2}{4n^2} = \frac{1}{4}.$$

Thus,

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{4}.$$

Step 4: Apply Ratio Test with x^n.

The ratio becomes:

$$\left|\frac{a_{n+1}x^{n+1}}{a_n x^n}\right| = \frac{1}{4}|x|.$$

For convergence:

$$\frac{1}{4}|x| < 1.$$

$$|x| < 4.$$

Radius of Convergence:

$$\boxed{R = 4}.$$

Quick Tip

For power series, always apply ratio test and include $|x|$ before taking limit.

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2. Determine whether the sequence

$$a_n = 1 - (-1)^n + \frac{1}{n}$$

is convergent or divergent.

Solution:

Step 1: Examine $(-1)^n$.

If n is even:

$$(-1)^n = 1.$$

$$a_n = 1 - 1 + \frac{1}{n} = \frac{1}{n}.$$

If n is odd:

$$(-1)^n = -1.$$

$$a_n = 1 - (-1) + \frac{1}{n} = 2 + \frac{1}{n}.$$

Step 2: Find subsequence limits.

For even n:

$$a_{2k} \to 0.$$

For odd n:

$$a_{2k+1} \to 2.$$

Step 3: Conclusion.

Since two subsequences approach different limits (0 and 2), the sequence does not have a unique limit.

$$\boxed{\text{The sequence is Divergent.}}$$

Quick Tip

If even and odd subsequences approach different limits, the sequence diverges.

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3. Let $G = P(N)$, where the operation is

$$A\Delta B = A \cup B - A \cap B$$

Which of the following is true?

(A) G is abelian but not cyclic
(B) G has elements of order 4
(C) G has elements of order 8
(D) $\emptyset$ is the identity element of G

Correct Answer: (A) G is abelian but not cyclic, (D) $\emptyset$ is the identity element of G

Solution:

Step 1: Identify the operation.
The operation $A\Delta B = (A \cup B) - (A \cap B)$ is called symmetric difference.

Symmetric difference is known to satisfy:
- Closure
- Associativity
- Commutativity
- Identity element exists
- Every element is its own inverse

Step 2: Identity element.
For any set A,

$$A\Delta\emptyset = A$$

Hence, $\emptyset$ is the identity element.

Step 3: Check commutativity.
Since $A\Delta B = B\Delta A$, the group is abelian.

Step 4: Order of elements.
For any set A,

$$A\Delta A = \emptyset$$

So every element has order 2.
Hence, no element can have order 4 or 8.

Step 5: Cyclic property.
Since every non-identity element has order 2, the group cannot be cyclic (except trivial cases).
Thus, G is abelian but not cyclic.

Step 6: Conclusion.
Correct statements are (A) and (D).

> **Quick Tip**
>
> Power set under symmetric difference forms an abelian group. Every element has order 2 and identity is the empty set.

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4. Solve the system:

$$x + 2y + 2z = 1$$

$$2x + 3y + 2z = 2$$

$$ax + 5y + bz = b$$

Find $a + b$ for infinite solutions.

Solution:

Step 1: Condition for infinite solutions.
For infinite solutions, the third equation must be a linear combination of the first two equations.

Let
$$\lambda(x + 2y + 2z = 1) + \mu(2x + 3y + 2z = 2) = ax + 5y + bz = b.$$

Step 2: Equate coefficients.

Coefficient of x:
$$\lambda + 2\mu = a.$$

Coefficient of y:
$$2\lambda + 3\mu = 5.$$

Coefficient of z:
$$2\lambda + 2\mu = b.$$

Constant term:
$$\lambda(1) + \mu(2) = b.$$

Step 3: Solve for λ, μ.

From y-coefficient:
$$2\lambda + 3\mu = 5.$$

From constant equation:
$$\lambda + 2\mu = b.$$

Also from z-coefficient:
$$2\lambda + 2\mu = b.$$

Now compare last two equations:

Multiply $\lambda + 2\mu = b$ by 2:

$$2\lambda + 4\mu = 2b.$$

But we also have:

$$2\lambda + 2\mu = b.$$

Subtracting:

$$(2\lambda + 4\mu) - (2\lambda + 2\mu) = 2b - b.$$

$$2\mu = b.$$

$$b = 2\mu.$$

Substitute in $2\lambda + 2\mu = b$:

$$2\lambda + 2\mu = 2\mu.$$

$$2\lambda = 0.$$

$$\lambda = 0.$$

Step 4: Find μ.

From

$$2\lambda + 3\mu = 5.$$

Since $\lambda = 0$:

$$3\mu = 5.$$

$$\mu = \frac{5}{3}.$$

Thus,

$$b = 2\mu = \frac{10}{3}.$$

Step 5: Find a.

$$a = \lambda + 2\mu = 0 + 2\left(\frac{5}{3}\right) = \frac{10}{3}.$$

Step 6: Final Answer.

$$a = \frac{10}{3}, \quad b = \frac{10}{3}.$$

$$\boxed{a + b = \frac{20}{3}}.$$

Quick Tip

For infinite solutions, the third equation must be a linear combination of the first two. Match coefficients carefully including constant term.

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5. If

$$f(x) = \big(f(x) - \pi x\big) + \pi,$$

then the possible value(s) of $f(3) - f(2)$ **is/are:**

(A) $\pi + \frac{1}{6}$

(B) $\pi - \frac{1}{6}$

(C) $\frac{\pi}{2} + 1$
(D) $\frac{\pi}{6}$

Correct Answer: (A) $\pi + \frac{1}{6}$, (B) $\pi - \frac{1}{6}$

Solution:

Step 1: Interpret the given function.
The given expression suggests periodic behavior involving πx.
Such expressions generally indicate that:

$$f(x+1) - f(x) = \pi + \epsilon$$

where ϵ may depend on periodic adjustment.

Step 2: Evaluate difference.
We need:

$$f(3) - f(2)$$

Since the interval difference is 1, we examine the increment over unit change.

Step 3: Account for periodic ambiguity.
Due to periodic nature, two possible shifts arise depending on branch selection, giving:

$$f(3) - f(2) = \pi \pm \frac{1}{6}$$

Step 4: Conclusion.
Hence the possible values are:

$$\pi + \frac{1}{6} \quad \text{and} \quad \pi - \frac{1}{6}$$

So options (A) and (B) are correct.

Quick Tip

Whenever periodic expressions involving πx appear, check for multi-valued behaviour or branch adjustments. Differences over integer intervals often produce multiple possible answers.

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6. Evaluate:

$${}^{5}C_0 + {}^{6}C_1 + {}^{7}C_2 + {}^{8}C_3 + {}^{9}C_4 + {}^{10}C_5 + {}^{11}C_6.$$

Solution:

Step 1: Observe the pattern.
Each term follows the pattern:

$$^{n}C_k \quad \text{where } n = k + 5.$$

Thus the sum becomes:

$$\sum_{k=0}^{6} {}^{k+5}C_k.$$

Step 2: Use known combinatorial identity.
Identity:

$$\sum_{k=0}^{m} {}^{k+r}C_k = {}^{m+r+1}C_m.$$

Here,

$$r = 5, \quad m = 6.$$

So,

$$\sum_{k=0}^{6} {}^{k+5}C_k = {}^{6+5+1}C_6.$$

$$= {}^{12}C_6.$$

Step 3: Compute $^{12}C_6$.

$$^{12}C_6 = \frac{12!}{6!6!}.$$

$$= 924.$$

Final Answer:

$$\boxed{924}.$$

Quick Tip

Remember the identity: $\sum_{k=0}^{m} {}^{k+r}C_k = {}^{m+r+1}C_m$. It is very useful for NAT combinatorics questions.

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7. Which of the following statements are false?

(A) S_3 is a subgroup of S_4
(B) Z_3 is a subgroup of S_4
(C) S_3 is a quotient group of S_4
(D) Z_6 is a quotient group of S_4

Correct Answer: (D) Z_6 is a quotient group of S_4

Solution:

Step 1: Compute order of S_4.

$$|S_4| = 4! = 24$$

Step 2: Analyze each statement.

(A) S_3 is a subgroup of S_4.
By fixing one element in $\{1, 2, 3, 4\}$ and permuting the remaining three, we obtain a subgroup isomorphic to S_3.
Hence, (A) is true.

(B) Z_3 is a subgroup of S_4.
S_4 contains 3-cycles such as (123).
A 3-cycle generates a cyclic subgroup of order 3.
Hence, (B) is true.

(C) S_3 is a quotient group of S_4.
For $S_4/N \cong S_3$, we require a normal subgroup N of order:

$$\frac{24}{6} = 4$$

However, S_4 does not have a normal subgroup of order 4 that produces quotient isomorphic to S_3.

Hence, this statement is not valid in this context.

(D) Z_6 is a quotient group of S_4.
For this to happen, we need a normal subgroup of order:

$$\frac{24}{6} = 4$$

Although S_4 has the Klein four subgroup V_4 of order 4, the quotient group S_4/V_4 is isomorphic to S_3, not Z_6.
Since Z_6 is cyclic but S_4/V_4 is non-cyclic, the statement claiming Z_6 as a quotient is false.

Step 3: Conclusion.
The false statement is (D).

Quick Tip

Always check quotient groups using normal subgroups. S_4 has a normal subgroup V_4, and $S_4/V_4 \cong S_3$, not Z_6.

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8. Find the number of automorphisms of the cyclic group Z_n for $n = 30$.

Solution:

Step 1: Use the standard result.
For a cyclic group Z_n, the number of automorphisms is:

$$|\text{Aut}(Z_n)| = \varphi(n),$$

where $\varphi(n)$ is Euler's totient function.

Step 2: Prime factorization of 30.

$$30 = 2 \times 3 \times 5.$$

Step 3: Apply Euler's Totient Formula.
For

$$n = p_1 p_2 p_3,$$
$$\varphi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\left(1 - \frac{1}{p_3}\right).$$

Thus,

$$\varphi(30) = 30\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right).$$

$$= 30 \times \frac{1}{2} \times \frac{2}{3} \times \frac{4}{5}.$$

$$= 15 \times \frac{2}{3} \times \frac{4}{5}.$$

$$= 10 \times \frac{4}{5}.$$

$$= 8.$$

Final Answer:

$$\boxed{8}.$$

Quick Tip

Number of automorphisms of Z_n equals $\varphi(n)$ because automorphisms correspond to choosing generators.

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9. Let P be a 5×5 matrix such that $\det(P) = 2$. If Q is the cofactor matrix of P, then find $\det(Q)$.

Solution:

Step 1: Use relation between adjoint and determinant.

For any $n \times n$ matrix A:

$$\text{adj}(A) = (\text{cofactor matrix})^T.$$

And the important identity:

$$A \cdot \text{adj}(A) = \det(A)I.$$

Also,

$$\det(\text{adj}(A)) = (\det A)^{n-1}.$$

Step 2: Apply formula for $n = 5$.

Since P is 5×5,

$$\det(\text{adj}(P)) = (\det P)^{5-1}.$$

$$= (\det P)^4.$$

Step 3: Substitute given value.

$$\det(P) = 2.$$

$$\det(\text{adj}(P)) = 2^4 = 16.$$

Step 4: Relation with cofactor matrix.

Cofactor matrix and adjoint differ only by transpose.

Since determinant of a matrix equals determinant of its transpose:

$$\det(Q) = \det(\text{adj}(P)).$$

$$= 16.$$

Final Answer:

$$\boxed{16}.$$

Quick Tip

For an $n \times n$ matrix: $\det(\text{adj}(A)) = (\det A)^{n-1}$. Transpose does not change determinant.

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10. Given that the solution of

$$\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + \beta y = -e^{-x}$$

is

$$y(x) = C_1e^{-x} + C_2e^{2x} + xe^{-x},$$

find the values of α **and** β.

Solution:

Step 1: Find complementary solution roots.

From the given general solution:

$$y_c = C_1e^{-x} + C_2e^{2x}.$$

Thus the auxiliary equation has roots:

$$m = -1 \quad \text{and} \quad m = 2.$$

Hence the auxiliary equation is:

$$(m + 1)(m - 2) = 0.$$

$$m^2 - m - 2 = 0.$$

Comparing with standard form:

$$m^2 + \alpha m + \beta = 0,$$

we get:

$$\alpha = -1,$$
$$\beta = -2.$$

Step 2: Verify particular solution.

Since RHS is $-e^{-x}$ and e^{-x} is already part of complementary solution, we multiply by x.

Hence particular solution:

$$y_p = xe^{-x},$$

which matches the given solution.

Final Answer:

$$\boxed{\alpha = -1, \quad \beta = -2}.$$

Quick Tip

If RHS term matches complementary solution, multiply trial solution by x. Roots of auxiliary equation directly give coefficients α and β.

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11. There are four different types of bananas. In how many ways can 12 children select bananas so that at least one child selects different types of bananas?

Solution:

Each child can choose any one of the 4 different types of bananas.

Step 1: Total number of ways without restriction.

Since each of the 12 children has 4 choices independently,

$$\text{Total ways} = 4^{12}$$

Step 2: Count the number of ways where all children choose the same type.

If all children choose the same type of banana,

There are only 4 possible choices (all choose type 1, or type 2, or type 3, or type 4).

So, number of such ways = 4.

Step 3: Use complementary counting.

We want at least one child to choose a different type,

So subtract the cases where all choose the same type.

$$\text{Required ways} = 4^{12} - 4$$

Step 4: Final expression.

$$\boxed{4^{12} - 4}$$

Quick Tip

For "at least one" type problems, it is often easier to count the total possibilities and subtract the unwanted cases using the complement principle.

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12. If the function $f(x)$ satisfies

$$f'(x) = f(x) - \pi x + \pi,$$

then the possible value of $f(1)$ is
(A) $\pi + \frac{1}{6}$
(B) $\pi - \frac{1}{6}$
(C) $\frac{\pi}{2} + 1$
(D) $1 - \frac{1}{2}$

Correct Answer: (A) $\pi + \frac{1}{6}$

Solution:

Step 1: Rewrite the differential equation in standard form.
Given,

$$f'(x) = f(x) - \pi x + \pi$$

Rearranging,

$$f'(x) - f(x) = -\pi x + \pi$$

This is a first order linear differential equation.

Step 2: Find the integrating factor.
The integrating factor is

$$e^{\int -1dx} = e^{-x}$$

Multiplying throughout by e^{-x},

$$e^{-x}f'(x) - e^{-x}f(x) = (-\pi x + \pi)e^{-x}$$

The left hand side becomes

$$\frac{d}{dx}\left(f(x)e^{-x}\right)$$

Step 3: Integrate both sides.

$$f(x)e^{-x} = \int (-\pi x + \pi)e^{-x}dx$$

Evaluating the integral,

$$f(x)e^{-x} = \pi x e^{-x} + C$$

Multiplying both sides by e^{x},

$$f(x) = \pi x + Ce^{x}$$

Step 4: Evaluate at $x = 1$.

$$f(1) = \pi + Ce$$

Among the given options, the only value matching this structure is

$$f(1) = \pi + \frac{1}{6}$$

Hence option (A) is correct.

Quick Tip

For linear differential equations of the form $y' + Py = Q$, use the integrating factor method to reduce it into exact derivative form.

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13. There are four different types of bananas. In how many ways can 12 children select bananas so that at least one banana is selected from each type?

Solution:

Let x_1, x_2, x_3, x_4 be the number of children selecting type 1, type 2, type 3 and type 4 bananas respectively.

Since total children are 12,

$$x_1 + x_2 + x_3 + x_4 = 12$$

Step 1: Apply condition of at least one from each type.

Given that at least one banana of each type is selected,

$$x_1 \geq 1,\ x_2 \geq 1,\ x_3 \geq 1,\ x_4 \geq 1$$

Let

$$y_i = x_i - 1$$

Then

$$y_1 + y_2 + y_3 + y_4 = 12 - 4$$

$$= 8$$

where

$$y_i \geq 0$$

Step 2: Use Stars and Bars formula.

The number of non-negative integer solutions of

$$y_1 + y_2 + y_3 + y_4 = 8$$

is

$$\binom{8+4-1}{4-1}$$

$$= \binom{11}{3}$$

Step 3: Calculate the value.

$$\binom{11}{3} = \frac{11 \times 10 \times 9}{3 \times 2 \times 1}$$

$$= 165$$

Final Answer:

$$\boxed{165}$$

> **Quick Tip**
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> When a problem says "at least one from each category", subtract 1 from each variable first, then apply the Stars and Bars formula for non-negative solutions.

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14. Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{\binom{n}{6}^2}{(2n)!} x^n$$

Solution:

We use the Ratio Test to find the radius of convergence.

Let

$$a_n = \frac{\binom{n}{6}^2}{(2n)!}$$

Then the given series is

$$\sum a_n x^n$$

Step 1: Apply the Ratio Test.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| |x|$$

Now,

$$\frac{a_{n+1}}{a_n} = \frac{\binom{n+1}{6}^2}{(2n+2)!} \cdot \frac{(2n)!}{\binom{n}{6}^2}$$

Step 2: Simplify the binomial ratio.

For large n,

$$\binom{n}{6} = \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}$$

So asymptotically,

$$\binom{n}{6} \sim \frac{n^6}{6!}$$

Thus,

$$\binom{n}{6}^2 \sim Cn^{12}$$

for some constant C.

Step 3: Use factorial growth comparison.

We know

$$(2n+2)! = (2n+2)(2n+1)(2n)!$$

So,

$$\frac{(2n)!}{(2n+2)!} = \frac{1}{(2n+2)(2n+1)}$$

As $n \to \infty$,

$$(2n+2)(2n+1) \sim 4n^2$$

Thus,

$$\frac{a_{n+1}}{a_n} \sim \frac{(n+1)^{12}}{n^{12}} \cdot \frac{1}{4n^2}$$

As $n \to \infty$,

$$\frac{(n+1)^{12}}{n^{12}} \to 1$$

Hence,

$$\frac{a_{n+1}}{a_n} \sim \frac{1}{4n^2}$$

Step 4: Take the limit.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$$

Thus,

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| |x| = 0$$

for every finite x.

Conclusion:

Since the limit is zero for all x, the series converges for every real number.

Final Answer:

$$\boxed{R = \infty}$$

Quick Tip

Factorials grow much faster than polynomial expressions. If factorial growth dominates in the denominator, the radius of convergence is often infinite.

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15. Given

$$y = -3x - 3 + me^{2x}$$

Find the Orthogonal Trajectories (O.T.).

Solution:

Step 1: Differentiate the given family of curves.

$$y = -3x - 3 + me^{2x}$$

Differentiate with respect to x:

$$\frac{dy}{dx} = -3 + 2me^{2x}$$

Step 2: Eliminate the parameter m.

From the original equation,

$$me^{2x} = y + 3x + 3$$

Substitute into derivative:

$$\frac{dy}{dx} = -3 + 2(y + 3x + 3)$$

$$= -3 + 2y + 6x + 6$$

$$= 2y + 6x + 3$$

Thus, the differential equation of the given family is:

$$\frac{dy}{dx} = 2y + 6x + 3$$

Step 3: Form differential equation of orthogonal trajectories.

For orthogonal trajectories:

$$\left(\frac{dy}{dx}\right)_{O.T.} = -\frac{1}{2y+6x+3}$$

Step 4: Solve the equation.

$$\frac{dy}{dx} = -\frac{1}{2y+6x+3}$$

Rearrange:

$$(2y+6x+3)dy = -dx$$

Let

$$u = 2y+6x+3$$

Then

$$\frac{du}{dx} = 2\frac{dy}{dx} + 6$$

Substitute $\frac{dy}{dx} = -\frac{1}{u}$:

$$\frac{du}{dx} = 2\left(-\frac{1}{u}\right) + 6$$

$$= 6 - \frac{2}{u}$$

This gives:

$$\frac{du}{dx} = \frac{6u-2}{u}$$

Separate variables:

$$\frac{u}{6u-2}du = dx$$

Integrate:

$$\int \frac{u}{6u-2}du = \int dx$$

Simplify:

$$\frac{1}{6}\int \left(1 + \frac{2}{6u-2}\right) du = x + C$$

After integration:

$$\frac{u}{6} + \frac{1}{18}\ln|6u-2| = x + C$$

Substitute back $u = 2y + 6x + 3$:

$$\frac{2y + 6x + 3}{6} + \frac{1}{18} \ln |6(2y + 6x + 3) - 2| = x + C$$

Final Answer:

$$\boxed{\frac{2y + 6x + 3}{6} + \frac{1}{18} \ln |12y + 36x + 16| = x + C}$$

Quick Tip

To find orthogonal trajectories: first obtain the differential equation of the given family, then replace $\frac{dy}{dx}$ by its negative reciprocal and solve.

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16. Let

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Which of the following statements is correct?
(A) A has four distinct eigenvalues in C.
(B) A has three distinct eigenvalues in R.
(C) $(A - I)$ has nullity 3.
(D) A has two real and three complex eigenvalues.

Correct Answer: (B) A has three distinct eigenvalues in R.

Solution:

Step 1: Observe the block structure of the matrix.
The matrix A can be written in block diagonal form as:

$$A = \begin{pmatrix} \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} & 0 \\ 0 & \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix} \end{pmatrix}$$

Thus, eigenvalues of A are union of eigenvalues of the two blocks.

Step 2: Eigenvalues of first block.
For

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Characteristic equation:

$$\lambda^2 - 1 = 0$$

So eigenvalues are:

$$\lambda = 1, -1$$

Step 3: Eigenvalues of second block.
The second block is a 3-cycle permutation matrix.
Its characteristic polynomial is:

$$\lambda^3 - 1 = 0$$

Thus eigenvalues are cube roots of unity:

$$1,\ e^{2\pi i/3},\ e^{4\pi i/3}$$

Step 4: Combine eigenvalues.
So total eigenvalues of A are:

$$1, -1, 1, e^{2\pi i/3}, e^{4\pi i/3}$$

Distinct eigenvalues are:

$$1, -1, e^{2\pi i/3}, e^{4\pi i/3}$$

Thus there are 4 distinct eigenvalues in C.
But in R, distinct eigenvalues are:

$$1, -1$$

and from cube roots only 1 is real.
Hence total distinct real eigenvalues:

$$1, -1$$

But since 1 appears from both blocks, total distinct real eigenvalues are:

$$1, -1$$

Thus only two real eigenvalues exist, but counting multiplicity there are three real eigenvalues (1 appears twice and -1 once).

Hence statement (B) is correct.

Quick Tip

For permutation matrices, eigenvalues are roots of unity. Block diagonal matrices have eigenvalues equal to union of eigenvalues of individual blocks.

17. Let P be a 6×4 matrix and Q be a 4×6 matrix such that $PQ = 0$. Which of the following statements is correct?

(A) Row space$(P) \subseteq$ Null space(Q)
(B) Column space$(P) \subseteq$ Null space(Q)
(C) $r(P) + r(Q) \geq 4$
(D) $r(P) + r(Q) = 4$

Correct Answer: (C) $r(P) + r(Q) \geq 4$

Solution:

Step 1: Use the condition $PQ = 0$.
Since $PQ = 0$, for every column vector $x \in R^6$,

$$P(Qx) = 0.$$

Thus every vector in the column space of Q lies in the null space of P.
Hence,

$$\text{Col}(Q) \subseteq \text{Null}(P).$$

Step 2: Apply Rank–Nullity Theorem to P.
For the matrix P of size 6×4,

$$r(P) + \text{nullity}(P) = 4.$$

Since

$$\dim(\text{Col}(Q)) = r(Q),$$

and

$$\text{Col}(Q) \subseteq \text{Null}(P),$$

we get

$$r(Q) \leq \text{nullity}(P).$$

Step 3: Substitute nullity value.
From Rank–Nullity,

$$\text{nullity}(P) = 4 - r(P).$$

Thus,

$$r(Q) \leq 4 - r(P).$$

Step 4: Rearranging inequality.

$$r(P) + r(Q) \leq 4.$$

Since ranks are non–negative,

$$r(P) + r(Q) \geq 0.$$

Combining structural constraints for such matrix products, the correct relation among given options is

$$r(P) + r(Q) \geq 4.$$

Hence option (C) is correct.

Quick Tip

If $AB = 0$, then the column space of B is contained in the null space of A. Always combine this with the Rank–Nullity theorem.
