IPMAT 2025 Question Paper with Solutions

Time Allowed :2 Hours	Maximum Marks :360	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The exam is conducted in a Computer-Based Test (CBT) format.
2. There is a separate, individual time limit for each section, and candidates cannot switch between them after a section begins.
3. The total duration is 2 hours (120 minutes) for the 100-question exam.
4. Correct answers worth +4 marks and incorrect ones resulting in -1 marks.
5. Candidates will be eligible for a Personal Interview after qualifying for the aptitude test.

Quantitative Ability - SA

1. A circle of radius 13 cm touches the adjacent sides AB and BC of a square ABCD at M and N, respectively. If AB = 18 cm and the circle intersects the other two sides CD and DA at P and Q, respectively, then the area, in sq. cm, of triangle PMD is _____

Correct Answer: 153

Solution:

Step 1: Understanding the Concept:
This problem can be solved using coordinate geometry. We can place the square on a 2D Cartesian plane, determine the equation of the circle, find the coordinates of the vertices of the triangle, and then calculate its area.

Step 2: Key Formula or Approach:
1. Equation of a circle with center (h, k) and radius r: $(x-h)^2 + (y-k)^2 = r^2$.
2. Area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$: Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.
Alternatively, if one side of the triangle is parallel to an axis, the area is $\frac{1}{2} \times$ base $\times$ height.

Step 3: Detailed Explanation:
Let's set up a coordinate system with the vertex B at the origin (0, 0).
Since ABCD is a square with side length 18 cm, the coordinates of the vertices are B(0, 0), A(0, 18), C(18, 0), and D(18, 18).

The sides are AB (on the line x=0), BC (on the line y=0), CD (on the line x=18), and DA (on the line y=18).

The circle touches sides AB (x=0) and BC (y=0) and has a radius $r = 13$ cm. Therefore, the center of the circle is (r, r) = (13, 13).
The equation of the circle is:

$$(x-13)^2 + (y-13)^2 = 13^2 = 169$$

The circle touches AB at M. For point M on AB, x=0.

$$(0-13)^2 + (y-13)^2 = 169 \implies 169 + (y-13)^2 = 169 \implies y = 13$$

So, the coordinates of M are (0, 13).

The circle intersects side CD at P. For point P on CD, x=18.

$$(18-13)^2 + (y-13)^2 = 169 \implies 5^2 + (y-13)^2 = 169$$

$$25 + (y-13)^2 = 169 \implies (y-13)^2 = 144 \implies y - 13 = \pm 12$$

So, $y = 13 + 12 = 25$ or $y = 13 - 12 = 1$. Since P lies on the side CD, its y-coordinate must be between 0 and 18. Thus, y=1.
The coordinates of P are (18, 1).

We need to find the area of triangle PMD. The vertices are P(18, 1), M(0, 13), and D(18, 18). Let's consider the side PD as the base of the triangle. The points P and D have the same x-coordinate (18), so the base PD is a vertical line segment.
Length of the base PD = $|18 - 1| = 17$ cm.
The height of the triangle with respect to the base PD is the perpendicular distance from vertex M(0, 13) to the line x=18.
Height = $|18 - 0| = 18$ cm.

Area of triangle PMD = $\frac{1}{2} \times$ base $\times$ height.

$$\text{Area} = \frac{1}{2} \times 17 \times 18 = 17 \times 9 = 153$$

Step 4: Final Answer:
The area of triangle PMD is 153 sq. cm.

Quick Tip

In geometry problems involving shapes like squares and circles, establishing a coordinate system is often the most straightforward approach. Place one vertex at the origin to simplify coordinates and equations.

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2. Monica, who is 18 years old, is one-third the age of her father. The age at which she will be half the age of her father is _____

Correct Answer: 36

Solution:

Step 1: Understanding the Concept:
This is a word problem involving ages that can be solved by setting up and solving linear equations based on the given information.

Step 2: Key Formula or Approach:
1. Translate the word statements into mathematical equations.
2. Solve the equations to find the required value.

Step 3: Detailed Explanation:
Let Monica's current age be M and her father's current age be F.
Given: M = 18 years.
Also given: Monica's age is one-third her father's age.

$$M = \frac{1}{3}F$$

Substituting M = 18, we get:

$$18 = \frac{1}{3}F \implies F = 18 \times 3 = 54$$

So, the father's current age is 54 years.

Let 'x' be the number of years from now when Monica will be half her father's age.
In x years:
Monica's age will be $M' = M + x = 18 + x$.
Father's age will be $F' = F + x = 54 + x$.

According to the condition, at that time:

$$M' = \frac{1}{2}F'$$

$$18 + x = \frac{1}{2}(54 + x)$$

To solve for x, we multiply both sides by 2:

$$2(18 + x) = 54 + x$$

$$36 + 2x = 54 + x$$

$$2x - x = 54 - 36$$

$$x = 18$$

The question asks for Monica's age at that time, not the number of years from now.
Monica's age will be $18 + x = 18 + 18 = 36$ years.

To verify, the father's age at that time would be $54 + x = 54 + 18 = 72$. Is 36 half of 72? Yes, $36 = \frac{1}{2} \times 72$. The answer is correct.

Step 4: Final Answer:
Monica will be 36 years old when she is half the age of her father.

Quick Tip

For age-related problems, always define variables for the current ages first. Then, express future or past ages in terms of the current ages and the time difference. Be careful to answer what is specifically asked (e.g., Monica's age, not the number of years).

Comprehension for Q.3, Q.6, Q.8
Five teams—A, B, C, D, and E—each consisting of 15 members, are going on expeditions to five different locations. Each team includes members from three different skill sets: biologists, geologists, and explorers. However, the number of members from each skill set varies by team and each member has only one speciality. The total number of biologists, geologists, and explorers are equal.

The following additional information is available

- Every team has at least 2 members from each of the three skill sets.
- Teams C and D have 6 biologists each, and Team A has 6 geologists.
- Every team except A has more biologists than explorers.
- The number of explorers in each team is distinct and decreases in the order A, B, C, D, and E.

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3. The number of biologists in team E is _____

Correct Answer: 4

Solution:

Step 1: Understanding the Concept:
This is a logical reasoning and data interpretation problem. We need to systematically use all the given conditions to determine the exact composition of each team.

Step 2: Key Formula or Approach:
Create a table to organize the information for the five teams and three skill sets. Use the given constraints to fill in the table and solve for the unknown values using a system of equations.

Step 3: Detailed Explanation:
Total members = 5 teams $\times$ 15 members/team = 75.
The total number of biologists, geologists, and explorers are equal. Total Biologists = Total

Geologists = Total Explorers = 75 / 3 = 25.

Let B_i, G_i, E_i be the number of biologists, geologists, and explorers in team i. We know $B_i + G_i + E_i = 15$ and $B_i, G_i, E_i \geq 2$.

Let's use the explorer information first: $E_A > E_B > E_C > E_D > E_E$, they are distinct integers, and all $E_i \geq 2$. Let the numbers be $e, e+1, e+2, e+3, e+4$. The sum of explorers is 25.

$$E_A + E_B + E_C + E_D + E_E = 25$$

Let $E_E = x$, where $x \geq 2$. Then $E_D = x+1, E_C = x+2, E_B = x+3, E_A = x+4$. Sum: $(x+4) + (x+3) + (x+2) + (x+1) + x = 5x + 10$.

$$5x + 10 = 25 \implies 5x = 15 \implies x = 3$$

So, the number of explorers are: $E_E = 3, E_D = 4, E_C = 5, E_B = 6, E_A = 7$.

Now we can build a table and fill in the known values:

Team	**Biologists (B)**	**Geologists (G)**	**Explorers (E)**	**Total**
A	B_A	6	7	15
B	B_B	G_B	6	15
C	6	G_C	5	15
D	6	G_D	4	15
E	B_E	G_E	3	15
Total	**25**	**25**	**25**	**75**

From the table, using $B_i + G_i + E_i = 15$:

- Team A: $B_A + 6 + 7 = 15 \implies B_A = 2$.
- Team C: $6 + G_C + 5 = 15 \implies G_C = 4$.
- Team D: $6 + G_D + 4 = 15 \implies G_D = 5$.

Now use the total columns:

- Total Biologists: $B_A + B_B + B_C + B_D + B_E = 25 \implies 2 + B_B + 6 + 6 + B_E = 25 \implies B_B + B_E = 11$. (Eq 1)
- Total Geologists: $G_A + G_B + G_C + G_D + G_E = 25 \implies 6 + G_B + 4 + 5 + G_E = 25 \implies G_B + G_E = 10$. (Eq 2)

We also know for teams B and E:

- Team B: $B_B + G_B + 6 = 15 \implies B_B + G_B = 9$. (Eq 3)
- Team E: $B_E + G_E + 3 = 15 \implies B_E + G_E = 12$. (Eq 4)

Finally, use the condition "Every team except A has more biologists than explorers" ($B_i > E_i$ for i=B,C,D,E).

- Team B: $B_B > E_B \implies B_B > 6$.
- Team C: $B_C > E_C \implies 6 > 5$. (Satisfied)
- Team D: $B_D > E_D \implies 6 > 4$. (Satisfied)

- Team E: $B_E > E_E \implies B_E > 3$.

We have $B_B + B_E = 11$ (Eq 1) and the constraint $B_B > 6$. Since B_B is an integer, B_B can be 7, 8, 9, etc. From Eq 1, $B_E = 11 - B_B$. If $B_B = 7$, $B_E = 4$. This satisfies $B_E > 3$. If $B_B = 8$, $B_E = 3$. This does not satisfy $B_E > 3$. If $B_B > 8$, $B_E < 3$, which is also not allowed. So, the only possibility is $B_B = 7$ and $B_E = 4$.

Step 4: Final Answer:
The number of biologists in team E is 4.

Quick Tip

For logic puzzles with multiple constraints, tabulation is key. Start by filling in the most definitive information first. Then, use the process of elimination and system of equations to solve for the remaining variables.

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4. If $\log_3(x^2 - 1)$, $\log_3(2x^2 + 1)$ and $\log_3(6x^2 + 3)$ are the first three terms of an arithmetic progression, then the sum of the next three terms of the progression is _____

Correct Answer: 15

Solution:

Step 1: Understanding the Concept:
The problem involves properties of arithmetic progressions (AP) and logarithms. If three terms a, b, c are in AP, then the middle term is the arithmetic mean of the other two, i.e., $2b = a + c$.

Step 2: Key Formula or Approach:
1. AP property: $2b = a + c$.
2. Logarithm properties: $n\log_b(m) = \log_b(m^n)$ and $\log_b(m) + \log_b(n) = \log_b(mn)$.
3. Formula for the n-th term of an AP: $T_n = a + (n-1)d$.

Step 3: Detailed Explanation:
Let the three terms be $T_1 = \log_3(x^2 - 1)$, $T_2 = \log_3(2x^2 + 1)$, and $T_3 = \log_3(6x^2 + 3)$.
Since they are in AP, we have $2T_2 = T_1 + T_3$.

$$2\log_3(2x^2 + 1) = \log_3(x^2 - 1) + \log_3(6x^2 + 3)$$

Using logarithm properties:

$$\log_3((2x^2 + 1)^2) = \log_3((x^2 - 1)(6x^2 + 3))$$

Equating the arguments of the logarithm:

$$(2x^2 + 1)^2 = (x^2 - 1)(6x^2 + 3)$$

$$4x^4 + 4x^2 + 1 = 6x^4 + 3x^2 - 6x^2 - 3$$

$$4x^4 + 4x^2 + 1 = 6x^4 - 3x^2 - 3$$

Rearranging the terms to form a quadratic equation in x^2:

$$2x^4 - 7x^2 - 4 = 0$$

Let $y = x^2$. The equation becomes $2y^2 - 7y - 4 = 0$. Factoring the quadratic:

$$2y^2 - 8y + y - 4 = 0$$

$$2y(y - 4) + 1(y - 4) = 0$$

$$(2y + 1)(y - 4) = 0$$

This gives $y = -1/2$ or $y = 4$. Since $y = x^2$, it cannot be negative. So, $x^2 = 4$.

We must check that the arguments of the logarithms are positive for $x^2 = 4$:

- $x^2 - 1 = 4 - 1 = 3 > 0$ (OK)
- $2x^2 + 1 = 2(4) + 1 = 9 > 0$ (OK)
- $6x^2 + 3 = 6(4) + 3 = 27 > 0$ (OK)

Now, let's find the first three terms of the AP:

$$T_1 = \log_3(3) = 1$$

$$T_2 = \log_3(9) = \log_3(3^2) = 2$$

$$T_3 = \log_3(27) = \log_3(3^3) = 3$$

The AP is 1, 2, 3, ... with first term $a = 1$ and common difference $d = T_2 - T_1 = 2 - 1 = 1$.

We need to find the sum of the next three terms: T_4, T_5, T_6.

$$T_4 = a + 3d = 1 + 3(1) = 4$$

$$T_5 = a + 4d = 1 + 4(1) = 5$$

$$T_6 = a + 5d = 1 + 5(1) = 6$$

Sum $= T_4 + T_5 + T_6 = 4 + 5 + 6 = 15$.

Step 4: Final Answer:
The sum of the next three terms of the progression is 15.

Quick Tip

When you see logarithmic terms in an AP or GP, immediately apply the core property ($2b = a + c$ for AP, $b^2 = ac$ for GP) and then use logarithm rules to simplify the equation. Always check your final value of the variable against the domain of the logarithmic functions.

5. English exam and Math exam were conducted separately for a class of 120 students. The number of students who did not appear for the English exam is twice the number of students who did not appear for the Math exam. The number of students who passed the Math exam is twice the number of students who appeared but failed the English exam. If the number of students who passed the English exam is twice the number of students who appeared but failed the Math exam, then the number of students who appeared but failed the English exam is _____

Correct Answer: 40

Solution:

Step 1: Understanding the Concept:
This problem involves interpreting logical statements and setting up a system of linear equations. It's a word problem that can be solved using algebraic manipulation. It doesn't require Venn diagrams as there's no information about students taking both exams.

Step 2: Key Formula or Approach:
Define variables for each category of students mentioned. Translate the given sentences into equations and solve the system.

Step 3: Detailed Explanation:
Let the total number of students be $T = 120$.
Let N_M and N_E be the number of students who did not appear for Math and English exams, respectively. Given: $N_E = 2N_M$.

Let A_M and A_E be the number of students who appeared for Math and English exams. $A_M = T - N_M = 120 - N_M$. $A_E = T - N_E = 120 - 2N_M$.

For those who appeared, let P_M, F_M be the number who passed and failed Math, and P_E, F_E be the number who passed and failed English. So, $A_M = P_M + F_M$ and $A_E = P_E + F_E$.

We are given two more conditions: 1. The number of students who passed the Math exam is twice the number of students who appeared but failed the English exam: $P_M = 2F_E$. 2. The number of students who passed the English exam is twice the number of students who appeared but failed the Math exam: $P_E = 2F_M$.

Now let's write the equations for the number of students who appeared:

$$A_M = P_M + F_M \implies 120 - N_M = 2F_E + F_M \quad \text{(Equation 1)}$$

$$A_E = P_E + F_E \implies 120 - 2N_M = 2F_M + F_E \quad \text{(Equation 2)}$$

We have a system of two equations with three variables (N_M, F_M, F_E). Let's try to eliminate variables. Let's rearrange Equation 2 to express F_E:

$$F_E = 120 - 2N_M - 2F_M$$

Now, substitute this expression for F_E into Equation 1:

$$120 - N_M = 2(120 - 2N_M - 2F_M) + F_M$$

$$120 - N_M = 240 - 4N_M - 4F_M + F_M$$

$$120 - N_M = 240 - 4N_M - 3F_M$$

Now, move all terms with variables to one side and constants to the other:

$$4N_M - N_M + 3F_M = 240 - 120$$

$$3N_M + 3F_M = 120$$

Divide by 3:

$$N_M + F_M = 40$$

This gives a direct relationship between N_M and F_M.

The question asks for the number of students who appeared but failed the English exam, which is F_E. Let's use Equation 2 again:

$$120 - 2N_M = 2F_M + F_E$$

Rearrange to solve for F_E:

$$F_E = 120 - 2N_M - 2F_M$$

Factor out -2:

$$F_E = 120 - 2(N_M + F_M)$$

We found that $N_M + F_M = 40$. Substitute this into the equation for F_E:

$$F_E = 120 - 2(40)$$

$$F_E = 120 - 80 = 40$$

Step 4: Final Answer:
The number of students who appeared but failed the English exam is 40.

Quick Tip

When faced with a word problem with multiple relationships, don't be discouraged if you have more variables than equations initially. Often, algebraic substitution and simplification will reveal a direct relationship or allow you to solve for the desired quantity.

Comprehension for Q.6, Q.8
Five teams—A, B, C, D, and E—each consisting of 15 members, are going on expeditions to five different locations. Each team includes members from three different skill sets: biologists, geologists, and explorers. However, the number of members from each skill set varies by team and each member has only one speciality. The total number of biologists, geologists, and explorers are equal.

The following additional information is available

- Every team has at least 2 members from each of the three skill sets.
- Teams C and D have 6 biologists each, and Team A has 6 geologists.
- Every team except A has more biologists than explorers.
- The number of explorers in each team is distinct and decreases in the order A, B, C, D, and E.

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6. The number of teams having more geologists than biologists is _____

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:
This is a logical reasoning and data interpretation problem. We need to systematically use all the given conditions to determine the exact composition of each team and then answer the specific question.

Step 2: Key Formula or Approach:
Create a table to organize the information for the five teams and three skill sets. Use the given constraints to fill in the table and solve for the unknown values.

Step 3: Detailed Explanation:
Total members = 5 teams $\times$ 15 members/team = 75.
The total number of biologists, geologists, and explorers are equal. Total Biologists = Total Geologists = Total Explorers = 75 / 3 = 25.

Let B_i, G_i, E_i be the number of biologists, geologists, and explorers in team i. We know $B_i + G_i + E_i = 15$ and $B_i, G_i, E_i \geq 2$.

From the explorer information: $E_A > E_B > E_C > E_D > E_E$, they are distinct integers, and all $E_i \geq 2$. Let the numbers be $x, x+1, x+2, x+3, x+4$. The sum of explorers is 25.

$$E_A + E_B + E_C + E_D + E_E = (x+4) + (x+3) + (x+2) + (x+1) + x = 5x + 10 = 25$$

Solving this gives $5x = 15 \implies x = 3$. So, the number of explorers are: $E_A = 7, E_B = 6, E_C = 5, E_D = 4, E_E = 3$.

We can build a table and fill in the known values. From a full logical deduction (as required for the entire puzzle set), we find the final team compositions: Now we check which teams have more geologists than biologists (G ¿ B):

- Team A: 6 ¿ 2 (Yes)
- Team B: 2 ¡ 7 (No)
- Team C: 4 ¡ 6 (No)

Team	Biologists (B)	Geologists (G)	Explorers (E)
A	2	6	7
B	7	2	6
C	6	4	5
D	6	5	4
E	4	8	3

- Team D: 5 ¡ 6 (No)
- Team E: 8 ¿ 4 (Yes)

There are 2 such teams (A and E).

Step 4: Final Answer:
The number of teams having more geologists than biologists is 2.

Quick Tip

When multiple questions are based on the same set of data, solve the data puzzle completely first. Create a clean, final table of all values. The subsequent questions will then become simple look-up or calculation tasks based on that table.

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7. If $A = \begin{bmatrix} 2 & n \\ 1 & 4 \end{bmatrix}$ **such that** $A^3 = 27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix}$**, then** $p + q + r$ **equals** _____

Correct Answer: 109/4

Solution:

Step 1: Understanding the Concept:
This problem involves matrix multiplication and solving a system of equations derived from the equality of two matrices.

Step 2: Key Formula or Approach:
1. Calculate A^2 and then A^3 by performing matrix multiplication.
2. Equate the resulting matrix for A^3 with the given expression $27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix}$.
3. Solve for the variables n, p, q, and r by comparing the corresponding elements of the matrices.
4. Calculate the final sum $p + q + r$.

Step 3: Detailed Explanation:
First, calculate A^2:

$$A^2 = A \cdot A = \begin{bmatrix} 2 & n \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & n \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} (2)(2) + (n)(1) & (2)(n) + (n)(4) \\ (1)(2) + (4)(1) & (1)(n) + (4)(4) \end{bmatrix} = \begin{bmatrix} 4 + n & 6n \\ 6 & n + 16 \end{bmatrix}$$

Next, calculate A^3:

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4+n & 6n \\ 6 & n+16 \end{bmatrix} \begin{bmatrix} 2 & n \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} (4+n)(2)+(6n)(1) & (4+n)(n)+(6n)(4) \\ (6)(2)+(n+16)(1) & (6)(n)+(n+16)(4) \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 8+2n+6n & 4n+n^2+24n \\ 12+n+16 & 6n+4n+64 \end{bmatrix} = \begin{bmatrix} 8+8n & n^2+28n \\ n+28 & 10n+64 \end{bmatrix}$$

We are given that $A^3 = 27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix} = \begin{bmatrix} 108 & 27q \\ 27p & 27r \end{bmatrix}$.
By equating the corresponding elements of the two matrices for A^3:

1. $8+8n = 108 \implies 8n = 100 \implies n = \frac{100}{8} = \frac{25}{2}$
2. $n + 28 = 27p$
3. $n^2 + 28n = 27q$
4. $10n + 64 = 27r$

Now substitute $n = 25/2$ into equations 2, 3, and 4 to find p, q, and r.
For p:

$$27p = \frac{25}{2} + 28 = \frac{25+56}{2} = \frac{81}{2} \implies p = \frac{81}{2 \times 27} = \frac{3}{2}$$

For q:

$$27q = n(n+28) = \frac{25}{2}\left(\frac{81}{2}\right) = \frac{2025}{4} \implies q = \frac{2025}{4 \times 27} = \frac{75}{4}$$

For r:

$$27r = 10\left(\frac{25}{2}\right) + 64 = 5(25) + 64 = 125 + 64 = 189 \implies r = \frac{189}{27} = 7$$

Finally, calculate the required sum $p+q+r$:

$$p+q+r = \frac{3}{2} + \frac{75}{4} + 7 = \frac{6}{4} + \frac{75}{4} + \frac{28}{4} = \frac{6+75+28}{4} = \frac{109}{4}$$

Step 4: Final Answer:
The value of $p+q+r$ is $\frac{109}{4}$.

Quick Tip

For matrix algebra, be very careful with the multiplication process (row by column). An alternative to direct multiplication for higher powers is the Cayley-Hamilton theorem, which states that every square matrix satisfies its own characteristic equation. This can sometimes simplify calculations.

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8. The median number of biologists across five teams is _____

Correct Answer: 6

Solution:

Step 1: Understanding the Concept:
This question requires finding the median of a set of numbers. The median is the middle value of a dataset when it is arranged in ascending or descending order. The data itself comes from the solution to the logic puzzle presented before Q.6.

Step 2: Key Formula or Approach:
1. List the number of biologists for each of the five teams from the solved data puzzle. 2. Arrange these five numbers in ascending order. 3. Identify the middle value. For a set with an odd number of values (n), the median is the $(\frac{n+1}{2})^{th}$ value in the sorted list.

Step 3: Detailed Explanation:
From the detailed analysis for the puzzle, the number of biologists in each team (A, B, C, D, E) was determined as:

- Team A Biologists (B_A): 2
- Team B Biologists (B_B): 7
- Team C Biologists (B_C): 6
- Team D Biologists (B_D): 6
- Team E Biologists (B_E): 4

The dataset of the number of biologists is {2, 7, 6, 6, 4}.
To find the median, we first need to sort this data in ascending order:

$$2, 4, 6, 6, 7$$

There are n=5 data points (an odd number). The median is the middle value, which is the $(\frac{5+1}{2})^{th} = 3^{rd}$ value in the sorted list.
The third value in the list {2, 4, **6**, 6, 7} is 6.

Step 4: Final Answer:
The median number of biologists across the five teams is 6.

Quick Tip

Remember the definition of median: it's the middle value of a *sorted* dataset. A common mistake is to find the middle value of the unsorted list. For an even number of data points, the median is the average of the two middle values.

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9. If the polynomial $ax^2 + bx + 5$ leaves a remainder 3 when divided by $x - 1$, and a remainder 2 when divided by $x + 1$, then $2b - 4a$ equals _____

Correct Answer: 11

Solution:

Step 1: Understanding the Concept:
This problem uses the Remainder Theorem, which states that if a polynomial P(x) is divided by a linear factor (x - c), the remainder is P(c).

Step 2: Key Formula or Approach:
1. Let $P(x) = ax^2 + bx + 5$. 2. Apply the Remainder Theorem for the divisor $x - 1$: The remainder is $P(1) = 3$. 3. Apply the Remainder Theorem for the divisor $x + 1$: The remainder is $P(-1) = 2$. 4. Solve the resulting system of linear equations for a and b. 5. Calculate the value of the expression $2b - 4a$.

Step 3: Detailed Explanation:
According to the Remainder Theorem, when $P(x) = ax^2 + bx + 5$ is divided by $x - 1$, the remainder is $P(1)$. We are given that this remainder is 3.

$$P(1) = a(1)^2 + b(1) + 5 = 3$$

$$a + b + 5 = 3$$

$$a + b = -2 \quad \text{(Equation 1)}$$

Similarly, when $P(x)$ is divided by $x + 1$ (which is $x - (-1)$), the remainder is $P(-1)$. We are given that this remainder is 2.

$$P(-1) = a(-1)^2 + b(-1) + 5 = 2$$

$$a - b + 5 = 2$$

$$a - b = -3 \quad \text{(Equation 2)}$$

Now we solve the system of linear equations for a and b. Adding Equation 1 and Equation 2:

$$(a + b) + (a - b) = -2 + (-3)$$

$$2a = -5 \implies a = -\frac{5}{2}$$

Substituting the value of a into Equation 1:

$$-\frac{5}{2} + b = -2$$

$$b = -2 + \frac{5}{2} = \frac{-4 + 5}{2} = \frac{1}{2}$$

Now we need to find the value of $2b - 4a$.

$$2b - 4a = 2\left(\frac{1}{2}\right) - 4\left(-\frac{5}{2}\right)$$

$$= 1 - (-10) = 1 + 10 = 11$$

Step 4: Final Answer:
The value of $2b - 4a$ is 11.

Quick Tip

The Remainder Theorem is a fundamental tool for problems involving polynomial division. Always remember that dividing by $(ax - b)$ gives a remainder of $P(b/a)$. This can save time compared to performing long division.

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10. If m and n are two positive integers such that $7m+11n = 200$, then the minimum possible value of m + n is _____

Correct Answer: 20

Solution:

Step 1: Understanding the Concept:
This is a problem involving a linear Diophantine equation, where we need to find integer solutions. Since we are looking for the minimum value of a sum, we should test the possible integer solutions.

Step 2: Key Formula or Approach:
1. Isolate one variable in the equation $7m + 11n = 200$. 2. Use the constraints that m and n must be positive integers to find the range of possible values for one variable. 3. Test the possible values to find integer pairs (m, n) that satisfy the equation. 4. Calculate the sum m+n for each pair and find the minimum.

Step 3: Detailed Explanation:
We are given the equation $7m + 11n = 200$, where m and n are positive integers. Let's express m in terms of n: $m = \frac{200-11n}{7}$. Since m must be a positive integer, $200 - 11n > 0 \implies 11n < 200 \implies n \leq 18$. Also, $200 - 11n$ must be divisible by 7. We can test values of n from 1 to 18. Alternatively, using modular arithmetic: $200 - 11n \equiv 0 \pmod 7$. $200 \equiv 4 \pmod 7$ and $11 \equiv 4 \pmod 7$. So, $4 - 4n \equiv 0 \pmod 7 \implies 4n \equiv 4 \pmod 7$. Since gcd(4, 7) = 1, we can divide by 4, which gives $n \equiv 1 \pmod 7$. This means n must be of the form $7k + 1$. Possible values for n are 1, 8, 15. Let's test these values:

- If $n = 1$: $7m = 200 - 11(1) = 189 \implies m = 27$. Sum $m + n = 27 + 1 = 28$.
- If $n = 8$: $7m = 200 - 11(8) = 112 \implies m = 16$. Sum $m + n = 16 + 8 = 24$.
- If $n = 15$: $7m = 200 - 11(15) = 35 \implies m = 5$. Sum $m + n = 5 + 15 = 20$.

The possible sums are 28, 24, and 20. The minimum value is 20.

Step 4: Final Answer:
The minimum possible value of m + n is 20.

Quick Tip

For equations of the form $ax + by = c$, to minimize $x + y$ where $a < b$, you should maximize the variable with the larger coefficient (in this case, n). This reduces the value of the other variable more quickly, generally leading to a smaller sum.

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11. If the sum of the first 21 terms of the sequence $\ln \frac{a}{b}, \ln \frac{a^2}{b^3}, \ln \frac{a^3}{b^5}, \ldots$ is $\ln \frac{a^m}{b^n}$, then the value of $m + n$ is _____

Correct Answer: 672

Solution:

Step 1: Understanding the Concept:
This problem requires summing a sequence of logarithmic terms. This can be approached either by using logarithm properties ($\sum \ln(x_i) = \ln(\prod x_i)$) or by recognizing that the sequence of logs is an arithmetic progression.

Step 2: Key Formula or Approach:
The k-th term of the sequence is $T_k = \ln\left(\frac{a^k}{b^{2k-1}}\right)$. The sum of the first 21 terms is $S_{21} = \sum_{k=1}^{21} T_k = \ln\left(\prod_{k=1}^{21} \frac{a^k}{b^{2k-1}}\right)$. We need to find the sum of the exponents for 'a' and 'b'. Sum of powers of 'a': $\sum_{k=1}^{21} k = \frac{21(21+1)}{2}$. Sum of powers of 'b': $\sum_{k=1}^{21}(2k-1)$, which is the sum of the first 21 odd numbers.

Step 3: Detailed Explanation:
The sum of the sequence is given by:

$$S_{21} = \ln\left(\frac{a^1}{b^1} \cdot \frac{a^2}{b^3} \cdot \frac{a^3}{b^5} \cdots \frac{a^{21}}{b^{41}}\right)$$

$$S_{21} = \ln\left(\frac{a^{1+2+3+\cdots+21}}{b^{1+3+5+\cdots+41}}\right)$$

Let's calculate the sum of the exponents. For 'a', the sum is an arithmetic series: $m = \sum_{k=1}^{21} k = \frac{21(22)}{2} = 231$.
For 'b', the sum is the sum of the first 21 odd numbers: $n = \sum_{k=1}^{21}(2k-1) = 21^2 = 441$.
So, the sum is $S_{21} = \ln\left(\frac{a^{231}}{b^{441}}\right)$.
By comparing this with $\ln\left(\frac{a^m}{b^n}\right)$, we find $m = 231$ and $n = 441$.
The value of $m + n$ is $231 + 441 = 672$.

Step 4: Final Answer:
Based on direct calculation, the value of $m + n$ is 672.

Quick Tip

Always check if a sequence of logarithmic terms forms an arithmetic progression. $T_2 - T_1 = (\ln a^2 - \ln b^3) - (\ln a - \ln b) = \ln a - 2\ln b$. $T_3 - T_2 = \ln a - 2\ln b$. It is an AP. You can use the AP sum formula $S_N = \frac{N}{2}(2T_1 + (N-1)d)$ as an alternative way to solve.

---

12. Arpita and Nikita, working together, can complete an assigned job in 12 days. If Arpita works initially to complete 40% of the job, and the remaining job is completed by Nikita alone, then it takes 24 days to complete the job. The possible number of days that Nikita requires to complete the entire job, working alone, is

_____

Correct Answer: 20

Solution:

Step 1: Understanding the Concept:
This is a work and time problem. We can solve it by defining the rates of work for each person and setting up equations based on the information given.

Step 2: Key Formula or Approach:
1. Let Arpita's rate be A jobs/day and Nikita's rate be N jobs/day. 2. The total work is 1 job. 3. From the first statement: $A + N = \frac{1}{12}$. 4. From the second statement: Time taken by Arpita + Time taken by Nikita = 24 days. This translates to: $\frac{0.4}{A} + \frac{0.6}{N} = 24$. 5. Solve this system of two equations for N. The time Nikita takes alone is $1/N$.

Step 3: Detailed Explanation:
We have the system of equations: 1) $A + N = \frac{1}{12}$ 2) $\frac{0.4}{A} + \frac{0.6}{N} = 24$ From equation (1), we can express A as $A = \frac{1}{12} - N$. Substitute this into equation (2):

$$\frac{0.4}{\frac{1}{12} - N} + \frac{0.6}{N} = 24$$

$$\frac{0.4}{\frac{1-12N}{12}} + \frac{0.6}{N} = 24$$

$$\frac{4.8}{1-12N} + \frac{0.6}{N} = 24$$

Multiply the entire equation by $N(1-12N)$ to clear the denominators:

$$4.8N + 0.6(1-12N) = 24N(1-12N)$$

$$4.8N + 0.6 - 7.2N = 24N - 288N^2$$

$$-2.4N + 0.6 = 24N - 288N^2$$

Rearrange into a standard quadratic form $ax^2 + bx + c = 0$:

$$288N^2 - 26.4N + 0.6 = 0$$

Multiply by 10 to remove the decimal:

$$2880N^2 - 264N + 6 = 0$$

Divide by 6 to simplify:

$$480N^2 - 44N + 1 = 0$$

We solve this quadratic equation for N using the quadratic formula $N = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$:

$$N = \frac{44 \pm \sqrt{(-44)^2 - 4(480)(1)}}{2(480)} = \frac{44 \pm \sqrt{1936 - 1920}}{960} = \frac{44 \pm \sqrt{16}}{960} = \frac{44 \pm 4}{960}$$

This gives two possible values for N:
Case (i): $N = \frac{44+4}{960} = \frac{48}{960} = \frac{1}{20}$.
Case (ii): $N = \frac{44-4}{960} = \frac{40}{960} = \frac{1}{24}$.

If $N = 1/20$, Nikita takes $1/N = 20$ days. Arpita's rate is $A = 1/12 - 1/20 = (5-3)/60 = 2/60 = 1/30$, so Arpita takes 30 days. This is a valid solution.
If $N = 1/24$, Nikita takes $1/N = 24$ days. Arpita's rate is $A = 1/12 - 1/24 = (2-1)/24 = 1/24$, so Arpita also takes 24 days. In this case, their rates are equal.

Step 4: Final Answer:
Assuming Arpita and Nikita have different work rates, Nikita's rate N is 1/20. The time required for Nikita to complete the job alone is $1/N = 20$ days.

Quick Tip

In work-rate problems, always convert the information into rates (e.g., jobs per day). When solving systems of equations, be prepared for quadratic equations which may yield multiple valid solutions. Contextual clues, like different names for workers, can sometimes help in choosing the intended answer if a unique solution is expected.

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13. Eight teams take part in a tournament where each team plays against every other team exactly once. In a particular year, one team got suspended after playing 3 matches, due to a disciplinary issue. The organizers decide to proceed, nonetheless, with the remaining matches. The total number of matches that were played in the tournament that year is _____

Correct Answer: 24

Solution:

Step 1: Understanding the Concept:
This is a combinatorics problem related to round-robin tournaments. We can solve this by calculating the total matches that would have been played and subtracting the matches that

were cancelled due to the suspension.

Step 2: Key Formula or Approach:
The number of matches in a round-robin tournament with 'n' teams is given by the combination formula $\binom{n}{2} = \frac{n(n-1)}{2}$.

Step 3: Detailed Explanation:
Method 1: Subtracting Cancelled Matches
First, calculate the total number of matches that would have been played if no team was suspended. With 8 teams, this is:

$$\text{Total potential matches} = \binom{8}{2} = \frac{8 \times (8-1)}{2} = \frac{8 \times 7}{2} = 28 \text{ matches}$$

The suspended team was scheduled to play against 7 other teams, meaning it had 7 matches scheduled.
The team played 3 matches before being suspended.
The number of matches involving the suspended team that were cancelled is:

$$\text{Cancelled matches} = (\text{Scheduled matches for the team}) - (\text{Played matches}) = 7 - 3 = 4 \text{ matches}$$

The total number of matches actually played is the total potential matches minus the cancelled matches.

$$\text{Played matches} = 28 - 4 = 24$$

Method 2: Summing Played Matches
Alternatively, we can sum the matches played by the suspended team and the matches played among the other teams. The suspended team played 3 matches.
The remaining 7 teams continued the tournament among themselves. The number of matches among these 7 teams is:

$$\binom{7}{2} = \frac{7 \times (7-1)}{2} = \frac{7 \times 6}{2} = 21 \text{ matches}$$

The total number of matches played is the sum of the matches played by the suspended team and the matches played by the remaining teams.

$$\text{Total played matches} = 3 + 21 = 24$$

Step 4: Final Answer:
The total number of matches that were played in the tournament is 24.

Quick Tip

For tournament problems, there are often multiple ways to count the matches. Calculating the total and subtracting the exceptions is often as effective as adding up the different components. Choose the method that seems more straightforward to you.

14. If a, b, c are three distinct natural numbers, all less than 100, such that $|a-b|+|b-c|=|c-a|$, then the maximum possible value of b is _____

Correct Answer: 98

Solution:

Step 1: Understanding the Concept:
The given equation is a property related to the triangle inequality. The triangle inequality states that for any two numbers x and y, $|x|+|y| \geq |x+y|$. The equality holds if and only if x and y have the same sign (or one of them is zero).

Step 2: Key Formula or Approach:
1. Let $x = a-b$ and $y = b-c$. Then $x+y = (a-b)+(b-c) = a-c$. 2. The given equation is $|x|+|y| = |-(a-c)| = |a-c| = |x+y|$. 3. This implies that x and y must have the same sign. 4. Analyze the two cases: both positive or both negative. 5. Use the constraints on a, b, and c to maximize b.

Step 3: Detailed Explanation:
The condition $|a-b|+|b-c| = |a-c|$ implies that b must lie between a and c. **Case 1:** $a-b>0$ **and** $b-c>0$ This means $a>b$ and $b>c$, which can be written as $a>b>c$.
Case 2: $a-b<0$ **and** $b-c<0$ This means $a<b$ and $b<c$, which can be written as $a<b<c$.

In both cases, b is the middle value of the three distinct natural numbers.
We are given that a, b, and c are natural numbers less than 100. This means $1 \leq a, b, c \leq 99$.
We want to find the maximum possible value of b.
To maximize b, we should use the ordering $a<b<c$. Since c is a natural number and $c \leq 99$, and b must be strictly less than c, the maximum possible value for c is 99. If we set $c=99$, then the largest possible integer value for b such that $b<c$ is $b=98$.
We also need to find a value for 'a' such that $a<b$. We can choose any natural number for 'a' from 1 to 97. For instance, we can choose $a=97$. So, the set $a=97, b=98, c=99$ satisfies all conditions:

- They are distinct natural numbers less than 100.
- The condition is satisfied: $|97-98|+|98-99| = |-1|+|-1| = 1+1 = 2$. And $|99-97| = 2$. So $2=2$.

Thus, the maximum possible value for b is 98.

Step 4: Final Answer:
The maximum possible value of b is 98.

Quick Tip

The expression $|x-y|+|y-z| = |x-z|$ is a standard identity which holds if and only if y lies between x and z (inclusive). This is a useful property to remember.

15. The number of factors of $3^5 \times 5^8 \times 7^2$ that are perfect squares is _____

Correct Answer: 30

Solution:

Step 1: Understanding the Concept:
A number is a perfect square if all the exponents in its prime factorization are even. We need to find how many factors of the given number satisfy this condition.

Step 2: Key Formula or Approach:
Let the given number be $N = p_1^{e_1} \times p_2^{e_2} \times \cdots \times p_k^{e_k}$. A factor of N will be of the form $f = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_k^{a_k}$, where $0 \leq a_i \leq e_i$. For 'f' to be a perfect square, each exponent a_i must be an even number. The total number of such factors is the product of the number of possible even choices for each exponent.

Step 3: Detailed Explanation:
The given number is $N = 3^5 \times 5^8 \times 7^2$. Let a factor be $f = 3^x \times 5^y \times 7^z$. For f to be a factor of N, the exponents must satisfy: $0 \leq x \leq 5$ $0 \leq y \leq 8$ $0 \leq z \leq 2$

For f to be a perfect square, the exponents x, y, and z must be even. Let's find the number of choices for each exponent:

- Possible even values for x in the range [0, 5]: {0, 2, 4}. There are 3 choices.
- Possible even values for y in the range [0, 8]: {0, 2, 4, 6, 8}. There are 5 choices.
- Possible even values for z in the range [0, 2]: {0, 2}. There are 2 choices.

The total number of factors that are perfect squares is the product of the number of choices for each exponent.

$$\text{Number of perfect square factors} = (\text{choices for x}) \times (\text{choices for y}) \times (\text{choices for z})$$

$$= 3 \times 5 \times 2 = 30$$

Step 4: Final Answer:
The number of factors that are perfect squares is 30.

Quick Tip

To find the number of factors that are perfect cubes, you would count the number of exponents divisible by 3. For factors that are k^{th} powers, count exponents divisible by k. The principle remains the same.

Quantitative Ability - MCQ

Q.1 A circle touches the y-axis at (0, 4) and passes through the point (-2, 0). Then the radius of the circle is

(1) 5
(2) 2.4
(3) 3.7
(4) 4.6

Correct Answer: (1) 5

Solution:

Step 1: Understanding the Concept:
This problem can be solved using the standard equation of a circle and geometric properties of tangency.

Step 2: Key Formula or Approach:
1. The standard equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center and r is the radius. 2. If a circle touches a vertical line (like the y-axis, x=0), the radius is equal to the absolute difference between the x-coordinate of the center and the line's x-value. 3. The radius to the point of tangency is perpendicular to the tangent line.

Step 3: Detailed Explanation:
Let the center of the circle be (h, k) and the radius be r.
The circle touches the y-axis (the line x=0) at the point (0, 4).
Since the tangent line (y-axis) is vertical, the radius to the point of tangency (0, 4) must be horizontal. This means the y-coordinate of the center must be the same as the y-coordinate of the point of tangency. Therefore, $k=4$.
The distance from the center (h, 4) to the tangent line x=0 is the radius r. This distance is given by $r=|h-0|=|h|$. So, the equation of the circle becomes:

$$(x-h)^2+(y-4)^2=r^2=|h|^2=h^2$$

We are given that the circle passes through the point (-2, 0). We can substitute these coordinates into the equation to find h.

$$(-2-h)^2+(0-4)^2=h^2$$
$$(-(2+h))^2+(-4)^2=h^2$$
$$(2+h)^2+16=h^2$$
$$(4+4h+h^2)+16=h^2$$

Now, we solve for h:

$$4h+20+h^2=h^2$$
$$4h+20=0$$
$$4h=-20$$

$$h = -5$$

The radius of the circle is $r = |h|$.

$$r = |-5| = 5$$

Step 4: Final Answer:
The radius of the circle is 5.

Quick Tip

Visualizing the problem can be very helpful. Since the circle touches the y-axis at (0,4) and passes through (-2,0), the center must be in the second quadrant (negative h, positive k), which immediately tells you that $h < 0$.

Q.2 In triangle ABC, AB = AC = x, $\angle ABC = \theta$ and the circumradius is equal to y. Then $\frac{x}{y}$ equals

(1) $2\cos\theta$
(2) $2\sin\theta$
(3) $\sin\theta$
(4) $\cos\theta$

Correct Answer: (2) $2\sin\theta$

Solution:

Step 1: Understanding the Concept:
This problem connects the sides and angles of a triangle to its circumradius using the Sine Rule.

Step 2: Key Formula or Approach:
The Sine Rule states that for any triangle with sides a, b, c and opposite angles A, B, C, and circumradius R:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Step 3: Detailed Explanation:
In triangle ABC, we are given:

- AB = c = x
- AC = b = x
- $\angle ABC = B = \theta$
- Circumradius R = y

Since AB = AC, the triangle is isosceles. Therefore, the angles opposite to these sides are equal: $\angle ACB = C = \angle ABC = \theta$.

We can apply the Sine Rule using the side-angle pair we know. Let's use side AC (length b=x) and its opposite angle B (θ). According to the Sine Rule:

$$\frac{b}{\sin B} = 2R$$

Substituting the given values:

$$\frac{x}{\sin\theta} = 2y$$

The question asks for the value of $\frac{x}{y}$. We can rearrange the equation to find this ratio.

$$\frac{x}{y} = 2\sin\theta$$

Step 4: Final Answer:
The value of $\frac{x}{y}$ is $2\sin\theta$.

Quick Tip

The Sine Rule is the fundamental relationship between sides, angles, and the circumradius of a triangle. Whenever a problem involves the circumradius, the Sine Rule should be the first tool you consider.

Q.3 The remainder when $11^{1011} + 1011^{11}$ is divided by 9 is

(1) 0
(2) 7
(3) 9
(4) 8

Correct Answer: (4) 8

Solution:

Step 1: Understanding the Concept:
This problem requires finding the remainder of a large sum, which can be efficiently solved using modular arithmetic. We need to find the value of the expression modulo 9.

Step 2: Key Formula or Approach:
We will use the properties of congruences: If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a + c \equiv b + d \pmod{m}$ and $a^k \equiv b^k \pmod{m}$. The divisibility rule for 9 states that a number is congruent to the sum of its digits modulo 9.

Step 3: Detailed Explanation:
We need to compute $(11^{1011} + 1011^{11}) \pmod 9$. We can compute each term separately.
Term 1: $11^{1011} \pmod 9$ First, find the remainder of the base: $11 \div 9$ gives a remainder of 2. So, $11 \equiv 2 \pmod 9$. Therefore, $11^{1011} \equiv 2^{1011} \pmod 9$. Now we look for the cycle of powers

of 2 modulo 9: $2^1 \equiv 2$ $2^2 \equiv 4$ $2^3 \equiv 8$ $2^4 \equiv 16 \equiv 7$ $2^5 \equiv 14 \equiv 5$ $2^6 \equiv 10 \equiv 1$ The cycle length is 6. We need to find the exponent 1011 modulo 6. $1011 \div 6$: $1011 = 6 \times 168 + 3$. So, $1011 \equiv 3 \pmod 6$.
This means $2^{1011} \equiv 2^3 \pmod 9$.
$2^3 = 8$. So, $11^{1011} \equiv 8 \pmod 9$.

Term 2: $1011^{11} \pmod 9$ First, find the remainder of the base using the sum of digits rule: Sum of digits of 1011 is $1 + 0 + 1 + 1 = 3$. So, $1011 \equiv 3 \pmod 9$. Therefore, $1011^{11} \equiv 3^{11} \pmod 9$. Now we look at powers of 3 modulo 9:
$3^1 \equiv 3$ $3^2 = 9 \equiv 0$ For any exponent $k \geq 2$, $3^k = 3^{k-2} \cdot 3^2 \equiv 3^{k-2} \cdot 0 \equiv 0 \pmod 9$. Since the exponent is 11, which is greater than 2, we have $1011^{11} \equiv 0 \pmod 9$.

Final Sum: Now add the results for the two terms: $11^{1011} + 1011^{11} \equiv 8 + 0 \pmod 9 \equiv 8 \pmod 9$
Step 4: Final Answer:
The remainder when $11^{1011} + 1011^{11}$ is divided by 9 is 8.

Quick Tip

When finding remainders of large powers $a^b \pmod m$, first reduce the base $a \pmod m$, then use the cyclicity of the powers of the new base to reduce the exponent b. This simplifies the calculation significantly.

Q.4 A natural number n lies between 100 and 400, and the sum of its digits is 10. The probability that n is divisible by 4, is _____

(1) $\frac{7}{27}$
(2) $\frac{1}{4}$
(3) $\frac{2}{9}$
(4) $\frac{1}{3}$

Correct Answer: (1) $\frac{7}{27}$

Solution:

Step 1: Understanding the Concept:
This is a probability problem that combines number theory concepts. We need to find the total number of integers satisfying the given conditions (the sample space) and then find how many of those are divisible by 4 (the event space). The probability is the ratio of these two numbers.

Step 2: Key Formula or Approach:
1. Find all numbers n such that $100 < n < 400$ and the sum of digits is 10. 2. Count the total number of such n. This is the size of the sample space, S. 3. Among these numbers, identify

those divisible by 4. A number is divisible by 4 if the number formed by its last two digits is divisible by 4. 4. Count the number of such divisible numbers. This is the size of the event space, E. 5. Probability P = E/S.

Step 3: Detailed Explanation:
Let the number be n = htu (hundreds, tens, units digit). We are given $100 < n < 400$, so $h \in \{1, 2, 3\}$. The sum of digits is $h + t + u = 10$.
Finding the Sample Space (S):

- **Case h=1**: $t + u = 9$. The pairs (t,u) can be (0,9), (1,8), ..., (9,0). There are 10 such numbers.
- **Case h=2**: $t + u = 8$. The pairs (t,u) can be (0,8), (1,7), ..., (8,0). There are 9 such numbers.
- **Case h=3**: $t + u = 7$. The pairs (t,u) can be (0,7), (1,6), ..., (7,0). There are 8 such numbers.

Total number of possible values for n is $S = 10 + 9 + 8 = 27$.
Finding the Event Space (E): We need to find which of these 27 numbers are divisible by 4. We check the number formed by the last two digits, 'tu'.

- **Case h=1** (t+u=9): Numbers are 109, 118, 127, 136, 145, 154, 163, 172, 181, 190. The last two digits 'tu' are 09, 18, 27, 36, 45, 54, 63, 72, 81, 90. Divisible by 4 are: 36 and 72. So, the numbers are 136 and 172. (2 numbers)
- **Case h=2** (t+u=8): Numbers are 208, 217, 226, 235, 244, 253, 262, 271, 280. The last two digits 'tu' are 08, 17, 26, 35, 44, 53, 62, 71, 80. Divisible by 4 are: 08, 44, 80. So, the numbers are 208, 244, 280. (3 numbers)
- **Case h=3** (t+u=7): Numbers are 307, 316, 325, 334, 343, 352, 361, 370. The last two digits 'tu' are 07, 16, 25, 34, 43, 52, 61, 70. Divisible by 4 are: 16, 52. So, the numbers are 316 and 352. (2 numbers)

Total number of numbers divisible by 4 is $E = 2 + 3 + 2 = 7$.
Calculating the Probability: Probability $P = \frac{\text{Event Space}}{\text{Sample Space}} = \frac{E}{S} = \frac{7}{27}$.
Step 4: Final Answer:
The probability that n is divisible by 4 is $\frac{7}{27}$.

Quick Tip

For problems involving properties of digits, it's often best to break down the problem into cases based on the first digit. This makes the counting systematic and less prone to errors.

Q.5 Suppose a, b and c are three real numbers such that Max(a, b, c) + Min(a, b, c) = 15, and Median(a, b, c) - Mean(a, b, c) = 2. Then the median of a, b and c is

(1) 11
(2) 9.5
(3) 10.5
(4) 10

Correct Answer: (3) 10.5

Solution:

Step 1: Understanding the Concept:
This problem deals with the statistical measures of a set of three numbers: minimum, maximum, median, and mean. We need to use the relationships given to find the value of the median.

Step 2: Key Formula or Approach:
1. Let the three numbers be ordered as $x_1 \leq x_2 \leq x_3$. 2. By definition: Min $= x_1$, Median $= x_2$, Max $= x_3$. 3. The mean is $\frac{x_1+x_2+x_3}{3}$. 4. Set up a system of equations based on the given information and solve for the median (x_2).

Step 3: Detailed Explanation:
Let's order the numbers as $x_1 \leq x_2 \leq x_3$. The given conditions can be written as: Equation 1: Max + Min $= 15 \implies x_3 + x_1 = 15$ Equation 2: Median $-$ Mean $= 2 \implies x_2 - \frac{x_1+x_2+x_3}{3} = 2$
We need to find the value of the median, which is x_2. Let's work with Equation 2:

$$x_2 - \frac{x_1 + x_2 + x_3}{3} = 2$$

Multiply the entire equation by 3 to eliminate the fraction:

$$3x_2 - (x_1 + x_2 + x_3) = 6$$

Distribute the negative sign:

$$3x_2 - x_1 - x_2 - x_3 = 6$$

Combine the terms with x_2:

$$2x_2 - x_1 - x_3 = 6$$

Factor out the negative sign:

$$2x_2 - (x_1 + x_3) = 6$$

Now we have a simplified equation relating the three numbers. We can use Equation 1, which states $x_1 + x_3 = 15$. Substitute this value into our simplified equation:

$$2x_2 - (15) = 6$$

Now, solve for x_2:

$$2x_2 = 6 + 15$$

$$2x_2 = 21$$

$$x_2 = \frac{21}{2} = 10.5$$

Step 4: Final Answer:
The median of a, b and c is 10.5.

Quick Tip

When a problem involves Min, Median, and Max, it's almost always helpful to first assume an order for the variables (e.g., $a \leq b \leq c$). This allows you to replace the functional notation (Min, Max, etc.) with the variables themselves, making algebraic manipulation much easier.

---

6. Let A(1,3) and B(5,1) be two points. If a line with slope m intersects AB at an angle of 45°, then the possible values of m are

(1) $5, \frac{-1}{5}$
(2) $7, \frac{1}{7}$
(3) $3, \frac{1}{3}$
(4) $-3, \frac{1}{3}$

Correct Answer: (4) $-3, \frac{1}{3}$

Solution:

Step 1: Understanding the Concept:
This problem involves finding the slope of a line given the angle it makes with another line. The key is to use the formula for the angle between two lines, which relates their slopes.

Step 2: Key Formula or Approach:
The angle θ between two lines with slopes m_1 and m_2 is given by the formula:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Step 3: Detailed Explanation:
First, we need to find the slope of the line segment AB. Let's call this slope m_{AB}. Using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, with points A(1,3) and B(5,1):

$$m_{AB} = \frac{1-3}{5-1} = \frac{-2}{4} = -\frac{1}{2}$$

Let the slope of the other line be m. We are given that the angle θ between the lines is 45°. We know that $\tan(45°) = 1$. Substituting the values into the formula:

$$1 = \left|\frac{m - m_{AB}}{1 + m \cdot m_{AB}}\right| = \left|\frac{m - (-\frac{1}{2})}{1 + m(-\frac{1}{2})}\right| = \left|\frac{m + \frac{1}{2}}{1 - \frac{m}{2}}\right|$$

To simplify, multiply the numerator and denominator inside the absolute value by 2:

$$1 = \left|\frac{2m + 1}{2 - m}\right|$$

This equation gives two possibilities: **Case 1:** $\frac{2m+1}{2-m} = 1$

$$2m + 1 = 2 - m$$

$$3m = 1$$
$$m = \frac{1}{3}$$

Case 2: $\frac{2m+1}{2-m} = -1$

$$2m + 1 = -(2 - m) = -2 + m$$
$$m = -3$$

Step 4: Final Answer:
The possible values of m are $\frac{1}{3}$ and -3.

Quick Tip

When dealing with the formula for the angle between two lines, the absolute value is crucial. Remember that it leads to two separate linear equations, yielding two possible slopes for the second line (unless the lines are parallel or perpendicular).

---

7. If $y = a + b \log_e x$ then which of the following is true?

(1) $\log_e y$ is proportional to x
(2) e^y is proportional to x^b
(3) $y - a$ is proportional to x^b
(4) $\frac{1}{y-a}$ is proportional to x^b

Correct Answer: (2) e^y is proportional to x^b

Solution:

Step 1: Understanding the Concept:
This question tests the understanding of proportionality and the manipulation of logarithmic and exponential functions. A quantity Y is proportional to a quantity X if $Y = kX$ for some non-zero constant k.

Step 2: Key Formula or Approach:
We need to manipulate the given equation $y = a + b \log_e x$ algebraically to match one of the given proportionality statements. The key properties to use are:

- $k \log x = \log(x^k)$
- $e^{\log_e x} = x$
- $e^{u+v} = e^u e^v$

Step 3: Detailed Explanation:
Let's start with the given equation:

$$y = a + b \log_e x$$

We want to test the statement "e^y is proportional to x^b". This means we need to see if we can arrive at an equation of the form $e^y = (\text{constant}) \times x^b$. Let's rearrange the initial equation to isolate the logarithmic term:

$$y - a = b \log_e x$$

Using the power rule of logarithms:

$$y - a = \log_e(x^b)$$

Now, to get e^y, we should exponentiate both sides with base e:

$$e^{y-a} = e^{\log_e(x^b)}$$

Using the property that exponentiation and logarithms are inverse functions:

$$e^{y-a} = x^b$$

Using the property of exponents $e^{u-v} = e^u/e^v$:

$$\frac{e^y}{e^a} = x^b$$

Multiplying both sides by e^a:

$$e^y = e^a \cdot x^b$$

Since 'a' is a constant, e^a is also a constant. Let's call it $k = e^a$. Then we have:

$$e^y = k \cdot x^b$$

This is the definition of proportionality. Therefore, e^y is proportional to x^b.

Step 4: Final Answer:
The statement "e^y is proportional to x^b" is true.

Quick Tip

When checking proportionality statements involving logs and exponents, try to isolate the terms mentioned in the option. If an option involves e^y, your goal is to manipulate the original equation to get e^y on one side.

---

8. Consider a triangle with side lengths 4 meters, 6 meters, and 9 meters. A dog runs around the triangle in such a way that the shortest distance of the dog from the triangle is exactly 1 meter. The total distance covered (in meters) by the dog in one round is

(1) $22 - 2\pi$
(2) 22
(3) $19 + 2\pi$
(4) $22 + 2\pi$

Correct Answer: (3) $19 + 2\pi$

Solution:

Step 1: Understanding the Concept:
The path of the dog consists of straight sections parallel to the sides of the triangle and curved sections around the vertices. We need to calculate the length of both parts and add them together.

Step 2: Key Formula or Approach:
1. The total distance is the sum of the lengths of the straight paths and the curved paths. 2. The straight paths are parallel to the sides of the triangle and have the same lengths. Their total length is the perimeter of the triangle. 3. The curved paths at the vertices are circular arcs. The sum of the exterior angles of any convex polygon is 360° or 2π radians. These arcs will combine to form a full circle.

Step 3: Detailed Explanation:
The dog's path can be visualized in two parts:

1. **Straight Segments:** As the dog moves along the sides of the triangle, it stays 1 meter away. This creates three straight paths parallel to the sides of the triangle. The lengths of these paths are equal to the lengths of the sides of the triangle. Total length of straight segments = Perimeter of the triangle.

$$\text{Perimeter} = 4 + 6 + 9 = 19 \text{ meters}$$

2. **Curved Segments:** As the dog moves around each vertex of the triangle, it traces a circular arc of radius 1 meter to maintain the constant distance. The angle of each arc corresponds to the exterior angle at that vertex. The sum of the exterior angles of any triangle is always 360° or 2π radians. Therefore, the three curved paths at the vertices, when put together, form a complete circle with a radius of 1 meter. The length of this combined curved path is the circumference of this circle.

$$\text{Circumference} = 2\pi r = 2\pi(1) = 2\pi \text{ meters}$$

The total distance covered by the dog is the sum of the lengths of the straight and curved segments.

$$\text{Total Distance} = \text{Perimeter} + \text{Circumference} = 19 + 2\pi$$

Step 4: Final Answer:
The total distance covered by the dog is $19 + 2\pi$ meters.

Quick Tip

This principle applies to any convex polygon, not just a triangle. The distance covered by an object moving at a constant distance 'r' around a convex polygon with perimeter 'P' is always $P + 2\pi r$.

9. The set of all values of x satisfying the inequality $\log_{(x+\frac{1}{2})}\left[\log_2\left(\frac{x-1}{x+2}\right)\right] > 0$ **is**

(1) $(-5, -2)$
(2) $(2, 5)$
(3) Null set
(4) $(5, \infty)$

Correct Answer: (3) Null set

Solution:

Step 1: Understanding the Concept:
Solving logarithmic inequalities requires careful consideration of the domain of the functions involved. The solution must satisfy both the domain conditions and the inequality itself.

Step 2: Key Formula or Approach:
For an expression $\log_b(A)$ to be defined, we must have:

1. Argument is positive: $A > 0$
2. Base is positive: $b > 0$
3. Base is not equal to 1: $b \neq 1$

For the inequality $\log_b(A) > 0$, we have two cases:

- If $b > 1$, then $A > 1$.
- If $0 < b < 1$, then $0 < A < 1$.

Step 3: Detailed Explanation:
Let's first establish the domain for the given expression. Let the base be $b = x + \frac{1}{2}$ and the argument be $A = \log_2\left(\frac{x-1}{x+2}\right)$.
Domain Conditions:

1. **For the inner logarithm,** $\log_2(\dots)$**:** Its argument must be positive:

$$\frac{x-1}{x+2} > 0$$

   This is true when $x > 1$ or $x < -2$. (Condition I)

2. **For the outer logarithm,** $\log_{x+1/2}(\dots)$**:**

   - Its base must be positive and not 1:

$$x + \frac{1}{2} > 0 \implies x > -\frac{1}{2}$$

$$x + \frac{1}{2} \neq 1 \implies x \neq \frac{1}{2}$$

   (Condition II)

- Its argument $A = \log_2\left(\frac{x-1}{x+2}\right)$ must be positive:

$$\log_2\left(\frac{x-1}{x+2}\right) > 0$$

Since the base is 2 (which is ¿ 1), this implies:

$$\frac{x-1}{x+2} > 2^0 = 1$$

$$\frac{x-1}{x+2} - 1 > 0$$

$$\frac{(x-1)-(x+2)}{x+2} > 0$$

$$\frac{-3}{x+2} > 0$$

This inequality holds only if the denominator is negative, so:

$$x + 2 < 0 \implies x < -2$$

(Condition III)

Combining the conditions: We need to find the values of x that satisfy all the domain conditions simultaneously. From Condition II, we must have $x > -\frac{1}{2}$. From Condition III, we must have $x < -2$.
There is no real number x that can be both greater than $-\frac{1}{2}$ and less than -2 at the same time. The intersection of these conditions is an empty set. Since the domain of the inequality is the null set, there are no values of x for which the inequality is even defined.

Step 4: Final Answer:
The set of all values of x satisfying the inequality is the Null set.

Quick Tip

With complex logarithmic inequalities, always start by finding the domain. In many cases, the domain constraints might be contradictory, leading to a null set solution without needing to solve the main inequality itself. This can save a significant amount of time.

---

10. Let P(x) be a quadratic polynomial such that $\begin{vmatrix} P(0) & P(1) \\ P(0) & P(2) \end{vmatrix} = 0$. **Let P(0) = 2 and P(1) + P(2) + P(3) = 14. Then P(4) equals**

(1) -14
(2) -6
(3) 30
(4) 16

Correct Answer: (2) -6

Solution:

Step 1: Understanding the Concept:
This problem requires us to find the specific form of a quadratic polynomial using given conditions, one of which involves a determinant. Once the polynomial is determined, we can evaluate it at the desired point.

Step 2: Key Formula or Approach:
1. Let the quadratic polynomial be $P(x) = ax^2 + bx + c$. 2. Use the given conditions to form a system of equations to solve for a, b, and c. 3. The determinant condition $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC$.

Step 3: Detailed Explanation:
First, let's use the condition $P(0) = 2$. For $P(x) = ax^2 + bx + c$:

$$P(0) = a(0)^2 + b(0) + c = c$$

So, we have $c = 2$. The polynomial is $P(x) = ax^2 + bx + 2$.
Next, let's evaluate the determinant condition:

$$\begin{vmatrix} P(0) & P(1) \\ P(0) & P(2) \end{vmatrix} = P(0)P(2) - P(0)P(1) = 0$$

$$P(0)[P(2) - P(1)] = 0$$

Since we are given $P(0) = 2 \neq 0$, it must be that:

$$P(2) - P(1) = 0 \implies P(1) = P(2)$$

Now, let's use this with our polynomial form:

$$P(1) = a(1)^2 + b(1) + 2 = a + b + 2$$

$$P(2) = a(2)^2 + b(2) + 2 = 4a + 2b + 2$$

Setting them equal:

$$a + b + 2 = 4a + 2b + 2$$

$$3a + b = 0 \implies b = -3a$$

So, the polynomial can be written as $P(x) = ax^2 - 3ax + 2$.
Now, use the final condition: $P(1) + P(2) + P(3) = 14$. We already know $P(1) = P(2)$.

$$P(1) = a(1)^2 - 3a(1) + 2 = a - 3a + 2 = -2a + 2$$

$$P(3) = a(3)^2 - 3a(3) + 2 = 9a - 9a + 2 = 2$$

Substituting these into the sum equation:

$$P(1) + P(2) + P(3) = (-2a + 2) + (-2a + 2) + 2 = 14$$

$$-4a + 6 = 14$$

$$-4a = 8$$

$$a = -2$$

With $a = -2$, we can find b: $b = -3a = -3(-2) = 6$. The polynomial is $P(x) = -2x^2 + 6x + 2$. Finally, we need to find $P(4)$:

$$P(4) = -2(4)^2 + 6(4) + 2 = -2(16) + 24 + 2 = -32 + 26 = -6$$

Step 4: Final Answer:
The value of P(4) is -6.

Quick Tip

For a quadratic polynomial $P(x)$, the condition $P(x_1) = P(x_2)$ implies that the axis of symmetry is at $x = \frac{x_1+x_2}{2}$. In this case, $P(1) = P(2)$ means the axis of symmetry is at $x = 1.5$. The vertex x-coordinate is also given by $-b/2a$, so $-b/2a = 1.5$, which gives $b = -3a$, a shortcut to one of the steps.

---

11. If $8x^2 - 2kx + k = 0$ is a quadratic equation in x, such that one of its roots is p times the other, and p, k are positive real numbers, then k equals

(1) $(p + \frac{1}{p})$
(2) $(\sqrt{p} + \frac{1}{\sqrt{p}})^2$
(3) $2(p + \frac{1}{p})$
(4) $2(\sqrt{p} + \frac{1}{\sqrt{p}})^2$

Correct Answer: (4) $2(\sqrt{p} + \frac{1}{\sqrt{p}})^2$

Solution:

Step 1: Understanding the Concept:
This problem relates the coefficients of a quadratic equation to its roots using Vieta's formulas (sum and product of roots). We are given a relationship between the two roots, which allows us to solve for the coefficient k.

Step 2: Key Formula or Approach:
For a quadratic equation $ax^2 + bx + c = 0$ with roots α and β:

- Sum of roots: $\alpha + \beta = -b/a$
- Product of roots: $\alpha\beta = c/a$

Step 3: Detailed Explanation:
The given equation is $8x^2 - 2kx + k = 0$. Let the roots be α and β. We are given that $\beta = p\alpha$. Using Vieta's formulas:

1. **Sum of roots:**
$$\alpha + \beta = \alpha + p\alpha = \alpha(1+p) = -\frac{-2k}{8} = \frac{k}{4}$$
$$\alpha(1+p) = \frac{k}{4} \quad \text{(Equation 1)}$$

2. **Product of roots:**
$$\alpha\beta = \alpha(p\alpha) = p\alpha^2 = \frac{k}{8} \quad \text{(Equation 2)}$$

We have a system of two equations with two unknowns (α and k). We want to find k. From Equation 1, we can express α in terms of k:

$$\alpha = \frac{k}{4(1+p)}$$

Now substitute this expression for α into Equation 2:

$$p\left(\frac{k}{4(1+p)}\right)^2 = \frac{k}{8}$$

$$p\frac{k^2}{16(1+p)^2} = \frac{k}{8}$$

Since k is a positive real number, $k \neq 0$, so we can divide both sides by k:

$$p\frac{k}{16(1+p)^2} = \frac{1}{8}$$

Now, solve for k:

$$k = \frac{16(1+p)^2}{8p} = \frac{2(1+p)^2}{p}$$

This is a correct expression for k, but it doesn't match the form of the options. We need to manipulate it algebraically.

$$k = \frac{2(1^2+2p+p^2)}{p} = 2\left(\frac{1}{p} + \frac{2p}{p} + \frac{p^2}{p}\right) = 2\left(\frac{1}{p} + 2 + p\right)$$

Now let's expand the expression in option (4):

$$2\left(\sqrt{p} + \frac{1}{\sqrt{p}}\right)^2 = 2\left((\sqrt{p})^2 + 2(\sqrt{p})\left(\frac{1}{\sqrt{p}}\right) + \left(\frac{1}{\sqrt{p}}\right)^2\right)$$

$$= 2\left(p + 2 + \frac{1}{p}\right)$$

This matches our derived expression for k.

Step 4: Final Answer:
The value of k is $2(\sqrt{p} + \frac{1}{\sqrt{p}})^2$.

Quick Tip

When solving for a parameter in a quadratic equation based on a relationship between roots, the standard procedure is to use Vieta's formulas to create a system of equations and then eliminate the root variables to solve for the parameter.

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12. The sum of the first 5 terms of a geometric progression is the same as the sum of the first 7 terms of the same progression. If the sum of the first 9 terms is 24, then the 4th term of the progression is

(1) 24
(2) -24
(3) -48
(4) 48

Correct Answer: (2) -24

Solution:

Step 1: Understanding the Concept:
This problem deals with the properties of a geometric progression (GP), specifically the sum of its terms.

Step 2: Key Formula or Approach:
Let the first term of the GP be 'a' and the common ratio be 'r'. The n-th term is $T_n = ar^{n-1}$. The sum of the first n terms is $S_n = \frac{a(r^n-1)}{r-1}$ (for $r \neq 1$). The given conditions are $S_5 = S_7$ and $S_9 = 24$. We need to find T_4.

Step 3: Detailed Explanation:
We are given the condition $S_5 = S_7$. This can be written as $S_7 - S_5 = 0$. The difference $S_7 - S_5$ is the sum of the terms from the 6th to the 7th, i.e., $T_6 + T_7$. So, $T_6 + T_7 = 0$. Using the formula for the n-th term:

$$ar^5 + ar^6 = 0$$

Factor out the common term ar^5:

$$ar^5(1 + r) = 0$$

For a non-trivial GP, we assume the first term $a \neq 0$. If $r = 0$, the condition $S_5 = S_7 = a$ holds, but then $S_9 = a = 24$ and $T_4 = ar^3 = 0$, which is not an option. So, we must have:

$$1 + r = 0 \implies r = -1$$

The common ratio of the GP is -1. This means the progression is an alternating series: $a, -a, a, -a, \ldots$
Now we use the second condition, $S_9 = 24$. Let's write out the sum of the first 9 terms with $r = -1$:

$$S_9 = a + a(-1) + a(-1)^2 + \cdots + a(-1)^8$$

$$S_9 = a - a + a - a + a - a + a - a + a$$

The pairs of terms cancel out, and since there is an odd number of terms (9), the sum is simply the first term.

$$S_9 = a$$

We are given $S_9 = 24$, so $a = 24$.
Finally, we need to find the 4th term of the progression, T_4.

$$T_4 = ar^{4-1} = ar^3$$

Substitute the values of a and r we found:

$$T_4 = (24)(-1)^3 = (24)(-1) = -24$$

Step 4: Final Answer:
The 4th term of the progression is -24.

Quick Tip

The condition $S_m = S_n$ (with $m < n$) for a GP implies that the sum of the terms from T_{m+1} to T_n is zero. This can often lead to a quick deduction about the common ratio, r.

---

13. A and B take part in a rifle shooting match. The probability of A hitting the target is 0.4, while the probability of B hitting the target is 0.6. If A has the first shot, post which both strike alternately, then the probability that A hits the target before B hits it is

(1) $\frac{9}{19}$
(2) $\frac{2}{3}$
(3) $\frac{1}{2}$
(4) $\frac{10}{19}$

Correct Answer: (4) $\frac{10}{19}$

Solution:

Step 1: Understanding the Concept:
This is a probability problem involving an infinite geometric series. A wins if he hits the target on his turn, provided no one has hit the target before. We need to sum the probabilities of all the disjoint events in which A can win.

Step 2: Key Formula or Approach:
Let P(A) be the probability of A hitting and P(B) be the probability of B hitting. P(A') is A missing, P(B') is B missing. A can win on his 1st shot, or 2nd shot (which is the 3rd shot of the game), or 3rd shot (5th shot of the game), and so on. The total probability is the sum of

these probabilities: $P(\text{A wins}) = P(A) + P(A' \cap B' \cap A) + P(A' \cap B' \cap A' \cap B' \cap A) + \ldots$ This forms an infinite geometric progression. The sum is given by $S = \frac{a}{1-r}$, where 'a' is the first term and 'r' is the common ratio.

Step 3: Detailed Explanation:
We are given: P(A hits) = P(A) = 0.4 P(A misses) = P(A') = 1 - 0.4 = 0.6 P(B hits) = P(B) = 0.6 P(B misses) = P(B') = 1 - 0.6 = 0.4
A shoots first. The sequence of events for A to win are:

- A hits on the 1st shot: The probability is P(A) = 0.4.
- A wins on the 3rd shot: This means A misses (1st), B misses (2nd), and A hits (3rd). The probability is $P(A') \times P(B') \times P(A) = 0.6 \times 0.4 \times 0.4 = 0.096$.
- A wins on the 5th shot: This means (A misses, B misses) happens twice, then A hits. The probability is $(P(A') \times P(B'))^2 \times P(A) = (0.6 \times 0.4)^2 \times 0.4 = (0.24)^2 \times 0.4$.

The total probability of A winning is the sum of this infinite series:

$$P(\text{A wins}) = 0.4 + (0.6 \times 0.4 \times 0.4) + ((0.6 \times 0.4)^2 \times 0.4) + \ldots$$

$$P(\text{A wins}) = 0.4 + (0.24 \times 0.4) + (0.24^2 \times 0.4) + \ldots$$

This is a geometric progression with: First term, $a = 0.4$ Common ratio, $r = 0.6 \times 0.4 = 0.24$
The sum of an infinite geometric progression is $S = \frac{a}{1-r}$.

$$P(\text{A wins}) = \frac{0.4}{1 - 0.24} = \frac{0.4}{0.76}$$

To simplify the fraction:

$$\frac{0.4}{0.76} = \frac{40}{76} = \frac{10}{19}$$

Step 4: Final Answer:
The probability that A hits the target before B hits it is $\frac{10}{19}$.

Quick Tip

In alternate-turn probability problems, identify the event that constitutes one full 'round' where the state resets (e.g., both players miss). The probability of this event is the common ratio 'r' of the geometric series. The first term 'a' is the probability of the player winning on their very first turn.

---

14. Two swimmers, Ankit and Bipul, start swimming from the opposite ends of a swimming pool at the same time. Ankit can cover the length of the pool once in 10 minutes. Bipul can cover the length of the pool once in 15 minutes. They swim back and forth for 80 minutes without stopping. The number of times they meet each other is _____

(1) 5
(2) 7
(3) 6
(4) 8

Correct Answer: (2) 7

Solution:

Step 1: Understanding the Concept:
This is a relative speed problem. When two objects move towards each other, their relative speed is the sum of their individual speeds. We can find the time taken for each meeting and count how many meetings occur within the given 80-minute timeframe.

Step 2: Key Formula or Approach:
1. Let the length of the pool be L. Calculate the speeds of Ankit (v_A) and Bipul (v_B). 2. For the first meeting, they collectively cover a distance of L. Time = Distance / Relative Speed. 3. For all subsequent meetings, they collectively cover a distance of 2L (since they each swim one length to return to their starting sides and then move towards each other again). 4. Calculate the times of each meeting and count how many fall within 80 minutes.

Step 3: Detailed Explanation:
Let the length of the pool be L units. Speed of Ankit, $v_A = \frac{L}{10}$ units/minute. Speed of Bipul, $v_B = \frac{L}{15}$ units/minute. They are swimming towards each other, so their relative speed is:

$$v_{rel} = v_A + v_B = \frac{L}{10} + \frac{L}{15} = \frac{3L + 2L}{30} = \frac{5L}{30} = \frac{L}{6} \text{ units/minute}$$

First Meeting: To meet for the first time, they need to cover a total distance of L.

$$\text{Time for 1st meeting} = \frac{\text{Distance}}{\text{Relative Speed}} = \frac{L}{L/6} = 6 \text{ minutes}$$

Subsequent Meetings: After the first meeting, for them to meet again, they must collectively swim a total distance of 2L. For example, Ankit swims to one end and turns back, while Bipul swims to the other end and turns back.

$$\text{Time between subsequent meetings} = \frac{2L}{L/6} = 12 \text{ minutes}$$

Now, let's find the times at which the meetings occur:

- 1st meeting: 6 minutes
- 2nd meeting: 6 + 12 = 18 minutes
- 3rd meeting: 18 + 12 = 30 minutes
- 4th meeting: 30 + 12 = 42 minutes
- 5th meeting: 42 + 12 = 54 minutes
- 6th meeting: 54 + 12 = 66 minutes

- 7th meeting: $66 + 12 = 78$ minutes
- 8th meeting: $78 + 12 = 90$ minutes (This is after the 80-minute duration)

Since the 7th meeting occurs at 78 minutes, which is within the 80-minute timeframe, they meet a total of 7 times.

Step 4: Final Answer:
The number of times they meet each other is 7.

Quick Tip

A quick formula for this type of problem is: Total time / Time between meetings (after the first). Let t_1 and t_2 be the times taken by each person. Time for 1st meeting = $\frac{t_1 t_2}{t_1+t_2}$. Time for subsequent meetings = $\frac{2t_1 t_2}{t_1+t_2}$. Here, time for 1st meeting = 6 min, time for subsequent = 12 min. Number of meetings = 1 (for the first) + floor((80-6)/12) = 1 + floor(74/12) = 1 + 6 = 7.

---

15. Let $S_1 = \{100, 105, 110, 115, \dots\}$ and $S_2 = \{100, 95, 90, 85, \dots\}$ be two series in arithmetic progression. If a_k and b_k are the k-th terms of S_1 and S_2, respectively, then $\sum_{k=1}^{20} a_k b_k$ equals _____

(1) 138250
(2) 137275
(3) 135375
(4) 137225

Correct Answer: (1) 138250

Solution:

Step 1: Understanding the Concept:
The problem asks for the sum of the product of corresponding terms of two arithmetic progressions. We need to find the general formula for the k-th terms, find the product, and then use summation formulas for powers of k.

Step 2: Key Formula or Approach:
1. The k-th term of an AP is $T_k = a + (k-1)d$, where 'a' is the first term and 'd' is the common difference. 2. Find the expressions for a_k and b_k. 3. Find the expression for the product $a_k b_k$. 4. Use the formulas for the sum of series: $\sum_{k=1}^{n} c = cn$ $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

Step 3: Detailed Explanation:
For series S_1: First term $a_{S1} = 100$, common difference $d_1 = 5$. The k-th term is $a_k = 100 + (k-1)5 = 100 + 5k - 5 = 95 + 5k$.

For series S_2: First term $b_{S2} = 100$, common difference $d_2 = -5$. The k-th term is $b_k = 100 + (k-1)(-5) = 100 - 5k + 5 = 105 - 5k$.
Now, find the product $a_k b_k$:

$$a_k b_k = (95 + 5k)(105 - 5k)$$

We can factor out 5 from each term:

$$a_k b_k = 5(19 + k) \cdot 5(21 - k) = 25(19 + k)(21 - k)$$

Expand the product in the parenthesis:

$$(19 + k)(21 - k) = 19 \times 21 - 19k + 21k - k^2 = 399 + 2k - k^2$$

So, $a_k b_k = 25(399 + 2k - k^2)$.
Now we need to compute the sum $\sum_{k=1}^{20} a_k b_k$:

$$\sum_{k=1}^{20} 25(399 + 2k - k^2) = 25 \left[\sum_{k=1}^{20} 399 + 2 \sum_{k=1}^{20} k - \sum_{k=1}^{20} k^2 \right]$$

Let's calculate each sum with n=20:

- $\sum_{k=1}^{20} 399 = 399 \times 20 = 7980$
- $\sum_{k=1}^{20} k = \frac{20(20+1)}{2} = \frac{20 \times 21}{2} = 210$
- $\sum_{k=1}^{20} k^2 = \frac{20(20+1)(2 \times 20+1)}{6} = \frac{20 \times 21 \times 41}{6} = 10 \times 7 \times 41 = 2870$

Substitute these values back into the main expression:

$$\text{Sum} = 25[7980 + 2(210) - 2870]$$

$$\text{Sum} = 25[7980 + 420 - 2870]$$

$$\text{Sum} = 25[8400 - 2870]$$

$$\text{Sum} = 25[5530]$$

$$\text{Sum} = 138250$$

Step 4: Final Answer:
The value of $\sum_{k=1}^{20} a_k b_k$ is 138250.

Quick Tip

When dealing with products of AP terms, look for patterns. Here, $a_k + b_k = (95 + 5k) + (105 - 5k) = 200$, which is a constant. While not directly used in this solution, such symmetries can sometimes offer shortcuts in similar problems.

---

16. Let A and B be two finite sets such that n(A - B), n(A ∩ B), n(B - A) are in an arithmetic progression. Here n(X) denotes the number of elements in a finite

set X. If n(A ∪ B) = 18, then n(A) + n(B) is _____

(1) 36
(2) 30
(3) 27
(4) 24

Correct Answer: (4) 24

Solution:

Step 1: Understanding the Concept:
This problem combines set theory with arithmetic progressions. We need to use the fundamental formulas of set theory and the property of an AP to solve for the required quantity.

Step 2: Key Formula or Approach:
1. If three numbers x, y, z are in AP, then $2y = x + z$. 2. The formula for the union of two sets is $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$. 3. The formula relating union and intersection is $n(A) + n(B) = n(A \cup B) + n(A \cap B)$.

Step 3: Detailed Explanation:
Let $x = n(A - B)$, $y = n(A \cap B)$, and $z = n(B - A)$. We are given that x, y, z are in an arithmetic progression. This means the middle term is the average of the other two:

$$2y = x + z$$

$$2 \cdot n(A \cap B) = n(A - B) + n(B - A) \quad \text{(Equation 1)}$$

We are also given that $n(A \cup B) = 18$. Using the formula for the union based on disjoint partitions:

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$$

$$18 = (n(A - B) + n(B - A)) + n(A \cap B) \quad \text{(Equation 2)}$$

Now, we can substitute the relationship from Equation 1 into Equation 2.

$$18 = (2 \cdot n(A \cap B)) + n(A \cap B)$$

$$18 = 3 \cdot n(A \cap B)$$

Solving for the size of the intersection:

$$n(A \cap B) = \frac{18}{3} = 6$$

The question asks for $n(A) + n(B)$. We can use the principle of inclusion-exclusion:

$$n(A) + n(B) = n(A \cup B) + n(A \cap B)$$

We are given $n(A \cup B) = 18$ and we just found $n(A \cap B) = 6$.

$$n(A) + n(B) = 18 + 6 = 24$$

Step 4: Final Answer:
The value of n(A) + n(B) is 24.

Quick Tip

Drawing a Venn diagram can be extremely helpful to visualize the relationship between $n(A-B)$, $n(B-A)$, and $n(A \cap B)$. These three quantities represent the three disjoint regions that make up the union $A \cup B$.

---

17. If $\log_{25}[5\log_3(1+\log_3(1+2\log_2 x))] = 1/2$ then x is

(1) 4
(2) 16
(3) 8
(4) 2

Correct Answer: (2) 16

Solution:

Step 1: Understanding the Concept:
This problem involves solving a nested logarithmic equation. We need to work from the outermost logarithm inwards, using the fundamental definition of a logarithm at each step.

Step 2: Key Formula or Approach:
The definition of a logarithm states that if $\log_b(y) = z$, then $y = b^z$. We will apply this rule repeatedly.

Step 3: Detailed Explanation:
The given equation is $\log_{25}[5\log_3(1+\log_3(1+2\log_2 x))] = 1/2$.
Step 1: Outermost Logarithm (base 25) Using the definition $\log_b(y) = z \implies y = b^z$:

$$5\log_3(1+\log_3(1+2\log_2 x)) = 25^{1/2}$$

Since $25^{1/2} = \sqrt{25} = 5$:

$$5\log_3(1+\log_3(1+2\log_2 x)) = 5$$

Divide both sides by 5:

$$\log_3(1+\log_3(1+2\log_2 x)) = 1$$

Step 2: Second Logarithm (base 3) Applying the definition again:

$$1+\log_3(1+2\log_2 x) = 3^1$$

$$1+\log_3(1+2\log_2 x) = 3$$

Subtract 1 from both sides:

$$\log_3(1+2\log_2 x) = 2$$

Step 3: Third Logarithm (base 3) Applying the definition again:

$$1 + 2\log_2 x = 3^2$$

$$1 + 2\log_2 x = 9$$

Subtract 1 from both sides:

$$2\log_2 x = 8$$

Divide by 2:

$$\log_2 x = 4$$

Step 4: Innermost Logarithm (base 2) Applying the definition one last time:

$$x = 2^4$$

$$x = 16$$

Step 4: Final Answer:
The value of x is 16.

Quick Tip

When solving nested logarithmic equations, think of it like peeling an onion. Start with the outermost function and apply the inverse operation (in this case, exponentiation) to remove one layer at a time until you isolate the variable.

---

18. Area of a regular octagon inscribed in a circle of radius 1 unit is

(1) $2\sqrt{2}$
(2) $\frac{9}{2\sqrt{2}}$
(3) $\sqrt{10}$
(4) $2 + \sqrt{2}$

Correct Answer: (1) $2\sqrt{2}$

Solution:

Step 1: Understanding the Concept:
A regular octagon inscribed in a circle can be divided into 8 congruent isosceles triangles, with their common vertex at the center of the circle. The area of the octagon is the sum of the areas of these triangles.

Step 2: Key Formula or Approach:
1. The angle at the center of the circle subtended by each side of the octagon is $\frac{360^\circ}{8} = 45^\circ$. 2. The area of a triangle can be calculated using the formula Area $= \frac{1}{2}ab\sin(C)$, where a and b are two sides and C is the included angle. 3. Total Area $= 8 \times$ Area of one triangle.

Step 3: Detailed Explanation:
Let the circle have its center at the origin O and radius r = 1. The regular octagon can be divided into 8 identical isosceles triangles, such as $\triangle OAB$, where A and B are adjacent vertices of the octagon. The sides OA and OB are radii of the circle, so $OA = OB = r = 1$. The angle between these two sides, $\angle AOB$, is one-eighth of the total angle around the center:

$$\angle AOB = \frac{360^\circ}{8} = 45^\circ$$

Now, we can find the area of one of these triangles using the formula Area $= \frac{1}{2}ab\sin(C)$:

$$\text{Area of } \triangle OAB = \frac{1}{2} \times OA \times OB \times \sin(\angle AOB)$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times 1 \times 1 \times \sin(45^\circ)$$

We know that $\sin(45^\circ) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

$$\text{Area of } \triangle OAB = \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$$

The total area of the octagon is 8 times the area of one triangle.

$$\text{Area of Octagon} = 8 \times (\text{Area of } \triangle OAB) = 8 \times \frac{\sqrt{2}}{4}$$

$$\text{Area of Octagon} = 2\sqrt{2}$$

Step 4: Final Answer:
The area of the regular octagon is $2\sqrt{2}$ square units.

Quick Tip

The general formula for the area of a regular n-gon inscribed in a circle of radius r is $\frac{1}{2}nr^2\sin(\frac{360^\circ}{n})$. You can use this as a direct formula for such problems. For an octagon, n=8 and r=1, giving $\frac{1}{2}(8)(1)^2\sin(45^\circ) = 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2}$.

---

19. The number of integers greater than 5000 and divisible by 5 that can be formed with the digits 1, 3, 5, 7, 8, 9 where no digit is repeated is

(1) 240
(2) 180
(3) 120
(4) 276

Correct Answer: (4) 276

Solution:

Step 1: Understanding the Concept:
This is a counting problem using principles of permutations. We need to count the number of possible integers that can be formed under several constraints: range (greater than 5000), divisibility, available digits, and no repetition.

Step 2: Key Formula or Approach:
We should break down the problem into cases based on the number of digits the integer can have. The constraints on the first and last digits must be handled carefully due to the overlapping use of digits.

Step 3: Detailed Explanation:
The available digits are {1, 3, 5, 7, 8, 9}. **Constraint 1: Divisible by 5** The last digit must be 5. So, the units place is fixed. Available digits for other places: {1, 3, 7, 8, 9}.
Constraint 2: Greater than 5000 This means the number can have 4, 5, or 6 digits.
Case 1: 4-digit numbers The number has the form _ _ _ 5.

- **Units place:** Fixed as 5 (1 choice).
- **Thousands place:** Must be greater than or equal to 5 to make the number ¿ 5000. The available digits are {1, 3, 7, 8, 9}. So, the choices are {7, 8, 9} (3 choices). (Note: 5 is already used).
- **Hundreds place:** We have used two digits (5 and one from {7,8,9}). From the original 6 digits, 4 are left. (4 choices).
- **Tens place:** Now 3 digits are left. (3 choices).

Number of 4-digit numbers = $3 \times 4 \times 3 \times 1 = 36$.
Case 2: 5-digit numbers The number has the form _ _ _ _ 5. Any 5-digit number formed from these digits will be greater than 5000.

- **Units place:** Fixed as 5 (1 choice).
- **Remaining 4 places:** We need to arrange the remaining 5 digits {1, 3, 7, 8, 9} in 4 places.

Number of ways = $P(5,4) = \frac{5!}{(5-4)!} = 5! = 120$.
Case 3: 6-digit numbers The number has the form _ _ _ _ _ 5. Any 6-digit number formed from these digits will be greater than 5000.

- **Units place:** Fixed as 5 (1 choice).
- **Remaining 5 places:** We need to arrange the remaining 5 digits {1, 3, 7, 8, 9} in 5 places.

Number of ways = $P(5,5) = 5! = 120$.
Total Number of Integers Total = (Number of 4-digit numbers) + (Number of 5-digit numbers) + (Number of 6-digit numbers) Total = 36 + 120 + 120 = 276.

Step 4: Final Answer:
The total number of such integers is 276.

Quick Tip

In permutation problems with multiple constraints, always handle the most restrictive positions first. Here, the units digit (divisibility) and the first digit (range) are the most constrained, so they should be filled first.

---

20. Let $f(x) = a^2x^2 + 2bx + c$ where, a $\neq$ 0, b, c are real numbers and x is a real variable then

(1) f(x) has no minimum and no maximum
(2) f(x) has a maximum and a minimum
(3) f(x) has a minimum and no maximum
(4) f(x) has a maximum and no minimum

Correct Answer: (3) f(x) has a minimum and no maximum

Solution:

Step 1: Understanding the Concept:
The function $f(x) = a^2x^2 + 2bx + c$ is a quadratic function. The graph of a quadratic function is a parabola. The existence of a maximum or minimum value depends on the orientation of this parabola, which is determined by the sign of the leading coefficient (the coefficient of the x^2 term).

Step 2: Key Formula or Approach:
For a quadratic function $F(x) = Ax^2 + Bx + C$:

- If the leading coefficient $A > 0$, the parabola opens upwards. It has a global minimum at its vertex and no maximum.
- If the leading coefficient $A < 0$, the parabola opens downwards. It has a global maximum at its vertex and no minimum.

Step 3: Detailed Explanation:
The given function is $f(x) = a^2x^2 + 2bx + c$. This is a quadratic function of the form $Ax^2 + Bx + C$, where:

- The leading coefficient is $A = a^2$.
- The linear coefficient is $B = 2b$.
- The constant term is $C = c$.

We are given that a, b, and c are real numbers and $a \neq 0$. Let's analyze the leading coefficient, $A = a^2$. Since a is a non-zero real number, its square, a^2, must be strictly positive.

$$a^2 > 0$$

Because the leading coefficient is positive, the graph of the function $f(x)$ is a parabola that opens upwards. A parabola that opens upwards extends infinitely in the positive y-direction, so it has no maximum value. It has a single lowest point at its vertex, which represents the global minimum value of the function.
Therefore, the function f(x) has a minimum and no maximum.

Step 4: Final Answer:
f(x) has a minimum and no maximum.

Quick Tip

The key to determining the extrema of a quadratic function is the sign of the coefficient of the x^2 term. If it's positive, think of a "smiley face" parabola ($\cup$), which has a minimum. If it's negative, think of a "frowny face" parabola ($\cap$), which has a maximum.

---

21. Anindita invests a total of 1 lakh rupees distributed across three schemes A, B and C for a period of two years. These schemes offer an interest rate of 10%, 8% and 12% per annum, respectively, each compounded annually. If the initial investment amount in scheme A is 30000 rupees and the total interest earned from all the three schemes during the first year is 10600 rupees, then the total interest earned, in rupees, from all the three schemes for the second year is _____

(1) 10308
(2) 19708
(3) 11748
(4) 22348

Correct Answer: (3) 11748

Solution:

Step 1: Understanding the Concept:
This problem involves compound interest and solving a system of linear equations. First, we need to determine the initial investment in each scheme using the given information. Then, we calculate the interest for the second year based on the compounded principal.

Step 2: Key Formula or Approach:
1. Set up equations for the total investment and total first-year interest. 2. Solve the system of equations to find the investment in schemes B and C. 3. For compound interest, the interest

for the second year is calculated on the principal of the first year plus the interest earned in the first year. Interest for 2nd year = (Principal + Interest for 1st year) × rate.

Step 3: Detailed Explanation:
Let the investments in schemes A, B, and C be P_A, P_B, P_C.
Total Investment: $P_A + P_B + P_C = 100000$.
Given $P_A = 30000$, so $30000 + P_B + P_C = 100000 \implies P_B + P_C = 70000$ (Eq. 1).

Total interest in the first year is 10600.
Interest = $P_A \times 0.10 + P_B \times 0.08 + P_C \times 0.12$.
$10600 = 30000 \times 0.10 + 0.08P_B + 0.12P_C$
$10600 = 3000 + 0.08P_B + 0.12P_C$
$7600 = 0.08P_B + 0.12P_C$ (Eq. 2).

Now, solve the system of equations:
From Eq. 1, $P_C = 70000 - P_B$. Substitute this into Eq. 2:
$7600 = 0.08P_B + 0.12(70000 - P_B)$
$7600 = 0.08P_B + 8400 - 0.12P_B$
$7600 - 8400 = -0.04P_B$
$-800 = -0.04P_B$
$P_B = \frac{800}{0.04} = \frac{80000}{4} = 20000$.
So, $P_C = 70000 - 20000 = 50000$.
The initial investments are: $P_A = 30000, P_B = 20000, P_C = 50000$.

Now, calculate the interest for the second year. The principal for the second year is the initial principal plus the first year's interest.

- **Scheme A:** Principal for 2nd year = $30000 + (30000 \times 0.10) = 30000 + 3000 = 33000$. Interest for 2nd year = $33000 \times 0.10 = 3300$.
- **Scheme B:** Principal for 2nd year = $20000 + (20000 \times 0.08) = 20000 + 1600 = 21600$. Interest for 2nd year = $21600 \times 0.08 = 1728$.
- **Scheme C:** Principal for 2nd year = $50000 + (50000 \times 0.12) = 50000 + 6000 = 56000$. Interest for 2nd year = $56000 \times 0.12 = 6720$.

Total interest earned in the second year = $3300 + 1728 + 6720 = 11748$.

Step 4: Final Answer:
The total interest earned for the second year is 11748 rupees.

Quick Tip

An alternative way to calculate the second year's interest is to realize it's simply the first year's interest, plus interest on the first year's interest. Total 2nd year interest = (Total 1st year interest) * (1 + blended rate). A simpler method is as shown above: calculate the new principal for each scheme and then find the interest for the next year.

22. If $a_1, a_2, \ldots, a_8$ are the roots of the equation $x^8 + x^7 + \cdots + x + 1 = 0$, then the value of $a_1^{2025} + a_2^{2025} + \cdots + a_8^{2025}$ is

(1) 0
(2) 8
(3) 4
(4) 2

Correct Answer: (2) 8

Solution:

Step 1: Understanding the Concept:
The given polynomial is a geometric series. We can use the formula for the sum of a geometric series to find a simpler equation that the roots must satisfy. This will reveal the nature of the roots (roots of unity).

Step 2: Key Formula or Approach:
1. The sum of a geometric series $1 + x + x^2 + \cdots + x^{n-1}$ is $\frac{x^n - 1}{x - 1}$. 2. If ω is an n-th root of unity, then $\omega^n = 1$. 3. We need to evaluate $\sum_{k=1}^{8} a_k^{2025}$. We can simplify a_k^{2025} using the properties of the roots.

Step 3: Detailed Explanation:
The given equation is $x^8 + x^7 + \cdots + x + 1 = 0$. This is the sum of a geometric progression. For $x \neq 1$, we can write the sum as:

$$\frac{x^9 - 1}{x - 1} = 0$$

This implies that $x^9 - 1 = 0$, with the condition that $x \neq 1$. So, the roots $a_1, a_2, \ldots, a_8$ are the 9th roots of unity, excluding the root 1. This means that for each root a_k, we have the property:

$$a_k^9 = 1$$

We need to find the value of the sum $S = a_1^{2025} + a_2^{2025} + \cdots + a_8^{2025}$. Let's analyze the exponent, 2025. We can check its relationship with 9. A number is divisible by 9 if the sum of its digits is divisible by 9. Sum of digits of $2025 = 2 + 0 + 2 + 5 = 9$. Since the sum of digits is 9, 2025 is divisible by 9. Let's find the quotient: $2025 \div 9 = 225$. So, $2025 = 9 \times 225$. Now we can simplify each term in the sum:

$$a_k^{2025} = a_k^{9 \times 225} = (a_k^9)^{225}$$

Since $a_k^9 = 1$, we have:

$$a_k^{2025} = (1)^{225} = 1$$

This is true for every root a_k from $k = 1$ to 8. Therefore, the sum becomes:

$$S = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$

There are 8 terms in the sum.

$$S = 8$$

Step 4: Final Answer:
The value of the sum is 8.

Quick Tip

Recognizing that the polynomial $1 + x + \cdots + x^{n-1}$ is related to the roots of unity is a crucial shortcut. The roots of this polynomial are the $(n+1)$-th roots of unity, except for 1.

---

23. The area of the triangle, formed by the straight lines $y = 0$, $12x - 5y = 0$, and $3x + 4y = 7$, is _____

(1) $\frac{28}{9}$
(2) $\frac{14}{9}$
(3) $\frac{35}{27}$
(4) $\frac{35}{54}$

Correct Answer: (2) $\frac{14}{9}$

Solution:

Step 1: Understanding the Concept:
To find the area of a triangle formed by three lines, we first need to find the coordinates of its vertices. The vertices are the points of intersection of the lines, taken two at a time.

Step 2: Key Formula or Approach:
1. Find the three vertices by solving the systems of linear equations for each pair of lines. 2. Use the determinant formula for the area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Step 3: Detailed Explanation:
Let the three lines be: L1: $y = 0$ L2: $12x - 5y = 0$ L3: $3x + 4y = 7$
Finding the Vertices:

- **Vertex A (Intersection of L1 and L2):** Substitute $y = 0$ into L2: $12x - 5(0) = 0 \implies 12x = 0 \implies x = 0$. So, Vertex A is (0, 0).
- **Vertex B (Intersection of L1 and L3):** Substitute $y = 0$ into L3: $3x + 4(0) = 7 \implies 3x = 7 \implies x = \frac{7}{3}$. So, Vertex B is $(\frac{7}{3}, 0)$.

- **Vertex C (Intersection of L2 and L3):** From L2, we have $12x = 5y \implies y = \frac{12}{5}x$. Substitute this into L3: $3x + 4\left(\frac{12}{5}x\right) = 7$.

$$3x + \frac{48}{5}x = 7$$

$$\frac{15x + 48x}{5} = 7$$

$$\frac{63x}{5} = 7 \implies 9x = 5 \implies x = \frac{5}{9}$$

Now find y: $y = \frac{12}{5}x = \frac{12}{5}\left(\frac{5}{9}\right) = \frac{12}{9} = \frac{4}{3}$. So, Vertex C is $(\frac{5}{9}, \frac{4}{3})$.

Calculating the Area: The vertices are A(0,0), B($\frac{7}{3}$,0), and C($\frac{5}{9}, \frac{4}{3}$). Since one vertex is at the origin, the area formula simplifies to $\frac{1}{2}|x_B y_C - x_C y_B|$.

$$\text{Area} = \frac{1}{2}\left|\left(\frac{7}{3}\right)\left(\frac{4}{3}\right) - \left(\frac{5}{9}\right)(0)\right|$$

$$\text{Area} = \frac{1}{2}\left|\frac{28}{9} - 0\right| = \frac{1}{2} \times \frac{28}{9} = \frac{14}{9}$$

Alternatively, we can consider the base of the triangle to be the segment AB, which lies on the x-axis ($y = 0$). Base length $= |\frac{7}{3} - 0| = \frac{7}{3}$. The height of the triangle is the perpendicular distance from vertex C to the base (the x-axis), which is simply the y-coordinate of C. Height $= \frac{4}{3}$.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \frac{7}{3} \times \frac{4}{3} = \frac{28}{18} = \frac{14}{9}$$

Step 4: Final Answer:
The area of the triangle is $\frac{14}{9}$ square units.

Quick Tip

When one of the lines is an axis (like y=0 or x=0), calculating the area is often easier. You can use that line as the base of the triangle, and the height is simply the corresponding coordinate of the third vertex.

---

24. Given that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$, the value of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$ is

(1) $\frac{\pi^2}{6} - 1$
(2) $\frac{\pi^2}{12}$
(3) $\frac{\pi^2}{8}$
(4) $\frac{\pi}{6}$

Correct Answer: (3) $\frac{\pi^2}{8}$

Solution:

Step 1: Understanding the Concept:
This problem involves manipulating the famous Basel problem series sum. The given series includes all natural numbers. The series we need to find includes only the odd natural numbers. The key is to separate the given series into its odd and even components.

Step 2: Key Formula or Approach:
Let $S = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. We can split S into the sum of odd terms and even terms:

$$S = \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots\right)$$

Let S_{odd} be the sum of the odd terms (which we want to find) and S_{even} be the sum of the even terms.

$$S = S_{odd} + S_{even}$$

We can express S_{even} in terms of S.

Step 3: Detailed Explanation:
Let's analyze the sum of the even terms, S_{even}:

$$S_{even} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots = \sum_{k=1}^{\infty} \frac{1}{(2k)^2}$$

$$S_{even} = \sum_{k=1}^{\infty} \frac{1}{4k^2}$$

We can factor out the constant $\frac{1}{4}$ from the summation:

$$S_{even} = \frac{1}{4} \sum_{k=1}^{\infty} \frac{1}{k^2}$$

The summation $\sum_{k=1}^{\infty} \frac{1}{k^2}$ is just the original series S. So, $S_{even} = \frac{1}{4}S$.
Now substitute this back into the split equation:

$$S = S_{odd} + S_{even}$$

$$S = S_{odd} + \frac{1}{4}S$$

We can now solve for S_{odd}:

$$S_{odd} = S - \frac{1}{4}S = \frac{3}{4}S$$

We are given that $S = \frac{\pi^2}{6}$.

$$S_{odd} = \frac{3}{4}\left(\frac{\pi^2}{6}\right) = \frac{3\pi^2}{24} = \frac{\pi^2}{8}$$

Step 4: Final Answer:
The value of the series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$ is $\frac{\pi^2}{8}$.

Quick Tip

This is a standard result worth remembering. The sum of the reciprocals of the squares of all natural numbers is $\pi^2/6$. The sum for just the odd numbers is $\pi^2/8$, and the sum for just the even numbers is $\pi^2/24$.

---

25. Which of the following numbers is divisible by $3^{10} + 2$?

(1) $3^{30} + 2$
(2) $3^{20} + 4$
(3) $3^{30} + 8$
(4) $3^{20} + 8$

Correct Answer: (3) $3^{30} + 8$

Solution:

Step 1: Understanding the Concept:
This problem tests the knowledge of algebraic factorization, specifically the sum of cubes identity. We need to check which of the given options can be expressed as a product that includes $(3^{10} + 2)$ as a factor.

Step 2: Key Formula or Approach:
The key algebraic identity is the sum of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

We will try to apply this identity to the options by letting $a = 3^{10}$.

Step 3: Detailed Explanation:
Let $x = 3^{10}$. The question asks which of the options is divisible by $x + 2$.
Let's examine each option by substituting $x = 3^{10}$:

1. $3^{30} + 2 = (3^{10})^3 + 2 = x^3 + 2$. This is not directly divisible by $x + 2$. (According to the Factor Theorem, for $x^3 + 2$ to be divisible by $x + 2$, $x = -2$ should be a root, i.e., $(-2)^3 + 2 = -8 + 2 = -6 \neq 0$).
2. $3^{20} + 4 = (3^{10})^2 + 4 = x^2 + 4$. This is a sum of squares, which does not generally factor into $(x + 2)$.
3. $3^{30} + 8 = (3^{10})^3 + 8 = x^3 + 8$. We can write 8 as 2^3. So the expression is $x^3 + 2^3$. This is a sum of cubes. We can use the identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with $a = x$ and $b = 2$.
$$x^3 + 2^3 = (x + 2)(x^2 - 2x + 2^2) = (x + 2)(x^2 - 2x + 4)$$
Substituting back $x = 3^{10}$:
$$3^{30} + 8 = (3^{10} + 2)((3^{10})^2 - 2(3^{10}) + 4) = (3^{10} + 2)(3^{20} - 2 \cdot 3^{10} + 4)$$

Since $3^{30} + 8$ can be written as a product of $(3^{10} + 2)$ and another integer, it is divisible by $3^{10} + 2$.

4. $3^{20} + 8 = (3^{10})^2 + 8 = x^2 + 8$. This does not factor into $(x + 2)$.

Step 4: Final Answer:
The number $3^{30} + 8$ is divisible by $3^{10} + 2$.

Quick Tip

Remember the factorization formulas for sums and differences of powers, especially for cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. These are very useful in divisibility problems.

Comprehension for Q.26 to Q.30
The table given below provides the details of monthly sales (in lakhs of rupees) and the value of products returned by the customers (as a percentage of sales) of an e-commerce company for three product categories for the year 2024. Net sales (in lakhs of rupees) is defined as the difference between sales (in lakhs of rupees) and the value of products returned (in lakhs of rupees).

	Sales (in lakhs of rupees)			Value of products returned (as a percentage of sales)		
Month	**Apparel**	**Footwear**	**Electronics**	**Apparel**	**Footwear**	**Electronics**
January	262	104	289	13%	7%	2%
February	279	113	387	16%	9%	3%
March	236	121	283	20%	7%	2%
April	258	58	325	16%	8%	1%
May	249	69	359	12%	6%	4%
June	230	111	321	19%	5%	3%
July	244	119	341	17%	9%	4%
August	252	60	336	16%	6%	2%
September	288	118	355	10%	9%	5%
October	222	108	383	15%	8%	2%
November	228	93	282	14%	9%	4%
December	221	86	268	18%	10%	1%

---

26. Among the following four months, for which month the value of the Footwear returned (in lakhs of rupees) was the highest?

(1) March
(2) July
(3) June
(4) September

Correct Answer: (2) July

Solution:

Step 1: Understanding the Concept:
The question asks to find the month with the highest value of returned Footwear. The value of returned products is calculated by multiplying the sales of that month by the percentage of products returned.

Step 2: Key Formula or Approach:
Value of Returned Footwear = (Sales of Footwear) $\times$ (Percentage of Footwear Returned)

Step 3: Detailed Explanation:
We need to calculate the value of returned Footwear for each of the four months listed in the options.

- **March:** Sales = 121 lakhs, Return % = 7%. Value returned = $121 \times 0.07 = 8.47$ lakhs.
- **July:** Sales = 119 lakhs, Return % = 9%. Value returned = $119 \times 0.09 = 10.71$ lakhs.
- **June:** Sales = 111 lakhs, Return % = 5%. Value returned = $111 \times 0.05 = 5.55$ lakhs.
- **September:** Sales = 118 lakhs, Return % = 9%. Value returned = $118 \times 0.09 = 10.62$ lakhs.

Comparing the calculated values: March: 8.47 lakhs July: 10.71 lakhs June: 5.55 lakhs September: 10.62 lakhs The highest value is 10.71 lakhs, which occurred in July.

Step 4: Final Answer:
The value of the Footwear returned was the highest in July.

Quick Tip

When comparing values based on percentages, don't just look at the highest percentage or the highest sales figure. You must always calculate the actual value, as a lower percentage of a higher base can be greater than a higher percentage of a lower base.

---

27. By what percentage the net sales for June increased as compared to May in the Footwear category?

(1) 7.21 percent
(2) 18.97 percent
(3) 62.58 percent
(4) 60.87 percent

Correct Answer: (3) 62.58 percent

Solution:

Step 1: Understanding the Concept:
The question asks for the percentage increase in net sales from May to June for the Footwear category. First, we need to calculate the net sales for both months. Net sales is defined as Sales minus the Value of products returned.

Step 2: Key Formula or Approach:
1. Calculate Net Sales = Sales × (1 - Return Percentage). 2. Calculate Percentage Increase = $\frac{\text{Net Sales in June} - \text{Net Sales in May}}{\text{Net Sales in May}} \times 100\%$.

Step 3: Detailed Explanation:
Calculate Net Sales for May (Footwear):

- Sales in May = 69 lakhs
- Return Percentage in May = 6%
- Value Returned in May = $69 \times 0.06 = 4.14$ lakhs
- Net Sales in May = $69 - 4.14 = 64.86$ lakhs

Calculate Net Sales for June (Footwear):

- Sales in June = 111 lakhs
- Return Percentage in June = 5%
- Value Returned in June = $111 \times 0.05 = 5.55$ lakhs
- Net Sales in June = $111 - 5.55 = 105.45$ lakhs

Calculate Percentage Increase:

- Increase in Net Sales = $105.45 - 64.86 = 40.59$ lakhs
- Percentage Increase = $\frac{40.59}{64.86} \times 100\%$
- Percentage Increase $\approx 0.62583 \times 100\% \approx 62.58\%$

Step 4: Final Answer:
The net sales for June increased by approximately 62.58 percent as compared to May.

Quick Tip

To calculate net sales more quickly, use the formula: Net Sales = Gross Sales × (1 - Return Rate). For May: $69 \times 0.94 = 64.86$. For June: $111 \times 0.95 = 105.45$. This saves one calculation step.

---

28. Which month had highest percentage decline in monthly sales as compared to previous month for the Apparel category?

(1) October
(2) June
(3) March
(4) December

Correct Answer: (1) October

Solution:

Step 1: Understanding the Concept:
We need to calculate the percentage decline in sales for the Apparel category for each of the given months compared to its preceding month and identify the month with the highest percentage decline.

Step 2: Key Formula or Approach:
Percentage Decline $= \frac{\text{Sales in Previous Month} - \text{Sales in Current Month}}{\text{Sales in Previous Month}} \times 100\%$. We only need to perform this calculation for months that showed a decline.

Step 3: Detailed Explanation:
Let's calculate the percentage decline for each of the months listed in the options.

- **October:** Sales in October = 222 lakhs. Sales in September (previous month) = 288 lakhs. Decline = $288 - 222 = 66$ lakhs. Percentage Decline = $\frac{66}{288} \times 100\% \approx 22.92\%$.
- **June:** Sales in June = 230 lakhs. Sales in May (previous month) = 249 lakhs. Decline = $249 - 230 = 19$ lakhs. Percentage Decline = $\frac{19}{249} \times 100\% \approx 7.63\%$.
- **March:** Sales in March = 236 lakhs. Sales in February (previous month) = 279 lakhs. Decline = $279 - 236 = 43$ lakhs. Percentage Decline = $\frac{43}{279} \times 100\% \approx 15.41\%$.
- **December:** Sales in December = 221 lakhs. Sales in November (previous month) = 228 lakhs. Decline = $228 - 221 = 7$ lakhs. Percentage Decline = $\frac{7}{228} \times 100\% \approx 3.07\%$.

Comparing the percentage declines: October ($\approx 22.92\%$), June ($\approx 7.63\%$), March ($\approx 15.41\%$), and December ($\approx 3.07\%$). The highest percentage decline occurred in October.

Step 4: Final Answer:
October had the highest percentage decline in monthly sales as compared to the previous month for the Apparel category.

Quick Tip

When comparing percentage changes, be careful to use the correct base for each calculation. The base is always the value of the earlier period (the 'previous month' in this case).

29. For which categories the value of the products returned (as a percentage of sales) increased for three consecutive months?

(1) Only Footwear
(2) Both Apparel and Footwear
(3) Only Apparel
(4) Only Electronics

Correct Answer: (1) Only Footwear

Solution:

Step 1: Understanding the Concept:
We need to examine the "Value of products returned (as a percentage of sales)" columns for each of the three categories and find if there is any instance where the percentage increases for three months in a row.

Step 2: Key Formula or Approach:
Scan each category's return percentage data month by month, looking for a sequence of the form $P_{month\ N} < P_{month\ N+1} < P_{month\ N+2}$.

Step 3: Detailed Explanation:
Let's check each category:

- **Apparel Return %**: 13, 16, 20, 16, 12, 19, 17, 16, 10, 15, 14, 18. The sequence increases from Jan to March (13% to 16% to 20%), which is a two-month increase, not three. No other sequence shows three consecutive increases.
- **Footwear Return %**: 7, 9, 7, 8, 6, 5, 9, 6, 9, 8, 9, 10. Let's check the sequence from October to December: October: 8% November: 9% (Increase from October) December: 10% (Increase from November) This is an increase for three consecutive months (from the value in September to October is a decrease, but the question asks for the value *increasing* for three months. So, we look at the values themselves: 8 ¡ 9 ¡ 10. This trend occurs over the period Oct-Nov-Dec).
- **Electronics Return %**: 2, 3, 2, 1, 4, 3, 4, 2, 5, 2, 4, 1. There is no sequence of three consecutive increases in this category. For example, 1% -¿ 4% (Apr-May) is followed by 4% -¿ 3% (May-Jun).

Only the Footwear category shows an increase in the return percentage for three consecutive months (October, November, December).

Step 4: Final Answer:
The value of products returned (as a percentage of sales) increased for three consecutive months for Only Footwear.

Quick Tip

Read the question carefully. "Increased for three consecutive months" means you are looking for four data points P_1, P_2, P_3, P_4 such that $P_1 < P_2 < P_3 < P_4$. The increase from P_1 to P_2 is the first, P_2 to P_3 is the second, and P_3 to P_4 is the third. Here the question seems to be interpreted as finding a sequence of three months where the value itself is strictly increasing.

---

30. Among the following four months, for which month the contribution of the Apparel category in the total monthly sales was the highest?

(1) January
(2) April
(3) August
(4) December

Correct Answer: (2) April

Solution:

Step 1: Understanding the Concept:
The question asks for the month where Apparel sales constituted the largest percentage of the total sales for that month. We need to calculate this percentage for each of the four given months.

Step 2: Key Formula or Approach:
Contribution % $= \frac{\text{Apparel Sales}}{\text{Total Sales (Apparel + Footwear + Electronics)}} \times 100\%$.

Step 3: Detailed Explanation:
We need to calculate the total sales for each specified month and then find the contribution of Apparel sales.

- **January:** Apparel Sales = 262. Total Sales = 262 (Apparel) + 104 (Footwear) + 289 (Electronics) = 655 lakhs. Contribution = $\frac{262}{655} \times 100\% \approx 40.00\%$.
- **April:** Apparel Sales = 258. Total Sales = 258 (Apparel) + 58 (Footwear) + 325 (Electronics) = 641 lakhs. Contribution = $\frac{258}{641} \times 100\% \approx 40.25\%$.
- **August:** Apparel Sales = 252. Total Sales = 252 (Apparel) + 60 (Footwear) + 336 (Electronics) = 648 lakhs. Contribution = $\frac{252}{648} \times 100\% \approx 38.89\%$.
- **December:** Apparel Sales = 221. Total Sales = 221 (Apparel) + 86 (Footwear) + 268 (Electronics) = 575 lakhs. Contribution = $\frac{221}{575} \times 100\% \approx 38.43\%$.

Comparing the contribution percentages: January: $\approx 40.00\%$ April: $\approx 40.25\%$ August: $\approx 38.89\%$ December: $\approx 38.43\%$ The highest contribution was in April.

Step 4: Final Answer:
The contribution of the Apparel category to the total monthly sales was the highest in April.

Quick Tip

When comparing fractions like a/b, c/d, etc., you don't always need to calculate the exact percentage. You can use approximation or cross-multiplication. For instance, to compare Jan (262/655) and April (258/641), you can estimate $262/655 \approx 260/650 \approx 40\%$ and $258/641 \approx 256/640 = 40\%$. Since the approximations are close, a precise calculation is necessary here.

Verbal Ability

Comprehension for Q.1 to Q.6
Read the following passage and choose the answer that is closest to each of the questions that are based on the passage.
Meta is recalibrating content on its social media platforms as the political tide has turned in Washington, with Mark Zuckerberg announcing last week that his company plans to fire its US fact-checkers. Fact-checking evolved in response to allegations of misinformation and is being watered down in response to accusations of censorship. Social media does not have solutions to either. Community review — introduced by Elon Musk at X and planned by Zuckerberg for Facebook and Instagram — is not a significant improvement over fact-checking. Having Washington lean on foreign governments over content moderation does not benefit free speech. Yet, that is the nature of the social media beast, designed to amplify bias.
Information and misinformation continue to jostle on social media at the mercy of user discretion. Social media now has enough control over all other forms of media to broaden its reach. It is the connective tissue for mass consumption of entertainment, and alternative platforms are reworking their engagement with social media. Technologies are shaping up to drive this advantage further through synthetic content targeted precisely at its intended audience. Meta's algorithm will now play up politics because it is the flavour of the season.
The Achilles' Heel of social media is informed choice which could turn against misinformation. Its move away from content moderation is driven by the need to be more inclusive, yet unfiltered content can push users away from social media towards legacy forms that have better moderation systems in place. Lawmakers across the world are unlikely to give social media a free run, even if Donald Trump is working on their case. Protections have already been put in place across jurisdictions over misinformation. These may be difficult to dismantle, even if the Republicans pull US-owned social media companies further to the right.
Media consumption is, in essence, evidence-based judgement that mediums must adapt to. Content moderation, not free speech, is the adaptation mechanism. Musk and Zuckerberg are not exempt.

---

1. The writer's conclusion is that information available on social media is linked to

(1) the policies of the governments in power.
(2) the global legal systems' support of free speech.
(3) the need for deregulation.
(4) the individual's right to free speech.

Correct Answer: (1) the policies of the governments in power.

Solution:

Step 1: Understanding the Concept:
This question asks to identify the primary factor that, according to the writer, influences the information presented on social media.

Step 2: Detailed Explanation:
The passage begins by stating, "Meta is recalibrating content on its social media platforms as the political tide has turned in Washington..." This directly links the changes in content policy to the political climate.
It further mentions that Meta's algorithm will now "play up politics because it is the flavour of the season," and discusses the possibility of "Republicans pull[ing] US-owned social media companies further to the right."
These statements collectively support the conclusion that the information and content moderation policies on social media are heavily influenced by the political parties or governments currently in power. The other options are either contradicted or not supported by the text. The passage argues that content moderation, not free speech, is the key adaptation mechanism.

Step 3: Final Answer:
The writer's conclusion is that information available on social media is linked to the policies of the governments in power.

Quick Tip

In reading comprehension, the opening and closing sentences of a passage often contain the main idea or conclusion. The first sentence here immediately connects social media content to the "political tide."

---

2. The writer argues that social media

(1) is in a difficult position because it cannot adapt to new policies.
(2) remains unaffected by global debates amongst lawmakers on misinformation.
(3) has become the preferred way to access entertainment.
(4) flourishes because it can publish any material.

Correct Answer: (3) has become the preferred way to access entertainment.

Solution:

Step 1: Understanding the Concept:
This question asks to identify a specific argument the writer makes about the role and status of social media.

Step 2: Detailed Explanation:
The passage states, "Social media now has enough control over all other forms of media to broaden its reach. It is the connective tissue for mass consumption of entertainment...".
This phrasing, particularly "connective tissue for mass consumption of entertainment," strongly suggests that social media has become a central and preferred channel for accessing entertainment.
Option (1) is incorrect as the passage says Meta is "recalibrating," which is a form of adaptation.
Option (2) is incorrect as the passage discusses lawmakers and protections against misinformation.
Option (4) is incorrect as the central theme is the struggle with content moderation, not the freedom to publish anything.

Step 3: Final Answer:
The writer argues that social media has become the preferred way to access entertainment.

Quick Tip

Look for strong, descriptive phrases in the passage. "Connective tissue for mass consumption" is a powerful metaphor that directly supports the idea of social media being a primary gateway to entertainment.

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3. Technologies are enabling social media to

(1) accept the current trends as emphasised by algorithms.
(2) readjust its interaction with competitors.
(3) enlarge its sphere of influence and persuasion.
(4) understand that algorithms cannot control its content.

Correct Answer: (3) enlarge its sphere of influence and persuasion.

Solution:

Step 1: Understanding the Concept:
The question asks about the role of technology in shaping the capabilities of social media, according to the passage.

Step 2: Detailed Explanation:
The passage states, "Technologies are shaping up to drive this advantage further through synthetic content targeted precisely at its intended audience."
The "advantage" mentioned refers to social media's control and reach. By using technology to create and target synthetic content, social media platforms can more effectively persuade and influence their users. This directly corresponds to enlarging their "sphere of influence and persuasion."
The other options do not accurately reflect this statement. Technology is not just about accepting trends (1) or understanding limitations (4), but actively using tools to expand its power.

Step 3: Final Answer:
Technologies are enabling social media to enlarge its sphere of influence and persuasion.

Quick Tip

Pay attention to cause-and-effect language. The phrase "drive this advantage further" clearly indicates that technology is a tool for expansion and strengthening of existing power.

---

4. The writer implies that

(1) older forms of media will regain users because of their controls.
(2) social media can never be discarded by its users.
(3) social media's innate strength is the user's inability to fact check.
(4) uncensored content will always have more appeal than controlled content.

Correct Answer: (1) older forms of media will regain users because of their controls.

Solution:

Step 1: Understanding the Concept:
This question asks for an inference based on the writer's statements about user behavior in response to changes in social media.

Step 2: Detailed Explanation:
The passage makes a specific point in the third paragraph: "...unfiltered content can push users away from social media towards legacy forms that have better moderation systems in place."
This statement directly implies that users might leave social media in favor of "legacy forms" (older forms of media) precisely because those older forms have better "controls" (moderation systems). This perfectly matches option (1).
The other options are contradicted by the text. Option (2) is contradicted by the statement above. The passage does not support options (3) or (4).

Step 3: Final Answer:
The writer implies that older forms of media will regain users because of their controls.

Quick Tip

For "implication" questions, look for sentences that present a potential consequence or reaction. The "push users away from... towards..." structure is a strong indicator of the writer's implied prediction.

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5. The inherent downside associated with social media is that it

(1) reinforces existing objectivity among the users.
(2) creates and spreads much innate and acquired prejudice.
(3) does not address the problem of the digital divide.
(4) results in unremitting expansion of freedom of expression.

Correct Answer: (2) creates and spreads much innate and acquired prejudice.

Solution:

Step 1: Understanding the Concept:
The question asks to identify a fundamental, built-in negative aspect of social media as described by the author.

Step 2: Detailed Explanation:
At the end of the first paragraph, the author describes the "nature of the social media beast," stating it is "designed to amplify bias."
Bias is a synonym for prejudice. Therefore, the statement that social media is designed to amplify bias is equivalent to saying its inherent nature is to create and spread prejudice. This makes option (2) the correct answer.
Option (1) is the opposite of what the passage states. Option (3) is not discussed in the passage. Option (4) is also argued against, as the author emphasizes the need for content moderation over absolute free speech.

Step 3: Final Answer:
The inherent downside associated with social media is that it creates and spreads much innate and acquired prejudice.

Quick Tip

Look for definitive statements about the "nature" or "design" of the subject. When the author says social media is "designed to amplify bias," it's a strong clue about what they see as an inherent, fundamental characteristic.

6. Social media has succeeded in

(1) finding alternative means for fact-checking.
(2) ignoring technology and artificial content.
(3) becoming independent of other media.
(4) controlling other media that depend on it.

Correct Answer: (4) controlling other media that depend on it.

Solution:

Step 1: Understanding the Concept:
This question asks to identify an area where, according to the passage, social media has been successful.

Step 2: Detailed Explanation:
The second paragraph states, "Social media now has enough control over all other forms of media to broaden its reach. It is the connective tissue for mass consumption of entertainment, and alternative platforms are reworking their engagement with social media."
This indicates that social media has achieved a dominant position, effectively controlling or influencing other media platforms that now must rework their strategies based on social media's engagement models. This directly supports option (4).
Option (1) is incorrect, as the passage states Meta is firing fact-checkers. Option (2) is incorrect as it is embracing technology. Option (3) is incorrect because being the "connective tissue" implies interdependence, not independence.

Step 3: Final Answer:
Social media has succeeded in controlling other media that depend on it.

Quick Tip

Questions about "success" or "achievements" can be answered by looking for words that denote power, control, or a central role. Phrases like "has enough control over all other forms" and "connective tissue" are clear indicators of success in this context.

Comprehension for Q.7 to Q.12
Read the following passage and choose the answer that is closest to each of the questions that are based on the passage.
According to the French philosopher Jean Baudrillard, commodities available for consumption are not inherently negative things. Baudrillard tried to interpret consumption in modern societies by engaging with the 'cargo myth' prevalent among the indigenous Melanesian people living in the South Pacific. The Melanesians did not know what aeroplanes were. However,

they saw that these winged entities descended from the air for white people and appeared to make them happy. They also noted that aeroplanes never descended for the Melanesian people. The Melanesian natives noted that the white people had placed objects similar to the aeroplane on the ground. They concluded that these objects were attracting the aeroplanes in the air and bringing them to the ground. Through a magical process, the aeroplanes were bringing plenty to the white people and making them happy. The Melanesian people concluded that they would need to place objects that simulated the aeroplane on the ground and attract them from the air. Baudrillard believes that the cargo myth holds an important analogy for the ways in which consumers engage with objects of consumption.
According to Baudrillard, the modern consumer "sets in place a whole array of sham objects, of characteristic signs of happiness, and then waits for happiness to alight". For instance, modern consumers believe that they will get happiness if they buy the latest available version of a mobile phone or automobile. However, consumption does not usually lead to happiness. While consumers should ideally be blaming their heightened expectations for their lack of happiness, they blame the commodity instead.
They feel that they should have waited for the next version of a mobile phone or automobile before buying the one they did. The version they bought is somehow inferior and therefore cannot make them happy. Baudrillard argues that consumers have replaced 'real' happiness with 'signs' of happiness. This results in the endless deferment of the arrival of total happiness. In Baudrillard's words, "in everyday practice, the blessings of consumption are not experienced as resulting from work or from a production process; they are experienced as a miracle". Modern consumers view consumption in the same magical way as the Melanesian people viewed the aeroplanes in the cargo myth. Television commercials also present objects of consumption as miracles. As a result, commodities appear to be distanced from the social processes which lead to their production. In effect, objects of consumption are divorced from the reality which produces them.

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7. Why are consumers unhappy with commodities that they have just bought?

(1) Because television commercials do not create enough hype about commodities.
(2) Because they have exaggerated expectations of commodities.
(3) Because the Law of Diminishing Marginal Commodities comes into play.
(4) Because they focus on improved functionality of commodities.

Correct Answer: (2) Because they have exaggerated expectations of commodities.

Solution:

Step 1: Understanding the Concept:
This question asks for the reason behind consumer dissatisfaction with their purchases, according to Baudrillard's argument in the passage.

Step 2: Detailed Explanation:
The third paragraph explicitly addresses this: "However, consumption does not usually lead to happiness. While consumers should ideally be blaming their heightened expectations for their lack of happiness, they blame the commodity instead." This directly states that the cause of

unhappiness is their "heightened expectations," which is synonymous with "exaggerated expectations."

Step 3: Final Answer:
Consumers are unhappy with commodities they have just bought because they have exaggerated expectations of commodities.

Quick Tip

When a question asks "why," scan the text for causal words and phrases like "because," "as a result," "due to," or sentences that explain a cause-and-effect relationship, as seen in the passage.

---

8. Which of the following is an argument made by Baudrillard?

(1) Production and consumption are magical processes.
(2) Consumers value signs more than the real.
(3) Melanesian people coped with the inequality of colonialism by creating myths.
(4) Television commercials are at the heart of unhappiness experienced by consumers.

Correct Answer: (2) Consumers value signs more than the real.

Solution:

Step 1: Understanding the Concept:
This question requires identifying a central argument of Baudrillard's theory as presented in the text.

Step 2: Detailed Explanation:
The fourth paragraph contains a key argument: "Baudrillard argues that consumers have replaced 'real' happiness with 'signs' of happiness." This directly supports option (2), which states that consumers value signs more than the real. The "signs" are the commodities they buy, which they believe will bring happiness, but the "real" happiness never arrives.
Option (1) misinterprets the passage; Baudrillard argues consumers *experience* consumption as magical, not that it *is* magical. Option (3) is an interpretation of the cargo myth's origin but not Baudrillard's central argument about modern consumption. Option (4) is a contributing factor, but the core argument is the replacement of the real with signs.

Step 3: Final Answer:
An argument made by Baudrillard is that consumers value signs more than the real.

Quick Tip

Look for sentences where the author explicitly attributes an argument to the subject, such as "Baudrillard argues that..." These phrases directly point to the core ideas being presented.

---

9. How can consumption be made more satisfying?

(1) By banning television commercials that promise real happiness.
(2) By understanding the connection between production and consumption.
(3) By recognising that commodities produce miraculous change.
(4) By rejecting colonialism and all other forms of economic oppression.

Correct Answer: (2) By understanding the connection between production and consumption.

Solution:

Step 1: Understanding the Concept:
This question asks for the implied solution to the problem of unsatisfying consumption, based on Baudrillard's critique.

Step 2: Detailed Explanation:
The final paragraph highlights the core problem: "commodities appear to be distanced from the social processes which lead to their production. In effect, objects of consumption are divorced from the reality which produces them." The dissatisfaction arises because consumption is seen as a "miracle," disconnected from the reality of work and production. The logical solution, therefore, is to bridge this gap. If the problem is the divorce between consumption and production, the solution is to understand their connection. This makes option (2) the most logical answer derived from the text.

Step 3: Final Answer:
Consumption can be made more satisfying by understanding the connection between production and consumption.

Quick Tip

For "how-to" or "solution" questions, first identify the problem as described in the text. The solution is often the direct opposite or remedy of the stated problem. Problem: a "divorce" between production and consumption. Solution: reuniting them through understanding.

---

10. How does Baudrillard engage with the cargo myth?

(1) He uses it to describe the suffering of Indigenous people.
(2) He uses it to show that consumers should consume more serious objects.
(3) He uses it as a metaphor to critique modern consumption.
(4) He uses it to show that consumption is a blessing.

Correct Answer: (3) He uses it as a metaphor to critique modern consumption.

Solution:

Step 1: Understanding the Concept:
The question asks about the purpose of introducing the "cargo myth" in Baudrillard's analysis.

Step 2: Detailed Explanation:
The second paragraph ends with a crucial sentence: "Baudrillard believes that the cargo myth holds an important analogy for the ways in which consumers engage with objects of consumption." The word "analogy" is a key indicator. An analogy is a comparison or metaphor used to explain or illustrate a point. The passage then explains how modern consumers, like the Melanesians, set up "sham objects" (signs) and wait for a "miracle" (happiness). Therefore, Baudrillard uses the myth as a metaphorical framework to understand and critique modern consumer behavior.

Step 3: Final Answer:
Baudrillard engages with the cargo myth by using it as a metaphor to critique modern consumption.

Quick Tip

Pay close attention to words that signal relationships between ideas, such as "analogy," "metaphor," "for instance," and "in contrast." These words often reveal the author's or subject's argumentative strategy.

---

11. What is Baudrillard's position on total happiness?

(1) It comes with patience and waiting.
(2) It results from ethical consumption.
(3) It prioritises production over consumption.
(4) It is perpetually delayed.

Correct Answer: (4) It is perpetually delayed.

Solution:

Step 1: Understanding the Concept:
The question asks to identify Baudrillard's view on the attainment of "total happiness" through consumption.

Step 2: Detailed Explanation:
The fourth paragraph provides a direct answer. It describes how consumers blame the commodity for their unhappiness and feel they should have waited for the next version. The paragraph concludes, "This results in the endless deferment of the arrival of total happiness." "Endless deferment" is synonymous with being "perpetually delayed."
Option (1) is incorrect because while consumers wait, happiness does not actually arrive. The other options are not supported by the text.

Step 3: Final Answer:
Baudrillard's position on total happiness is that it is perpetually delayed.

Quick Tip

Look for summary statements at the end of paragraphs. These often encapsulate the main point of the preceding sentences. The final sentence of the fourth paragraph directly summarizes the outcome of the consumer cycle.

---

12. What is Baudrillard's position on consumption?

(1) It is a positive process.
(2) It is an egalitarian process.
(3) It is an irrational process.
(4) It is a utilitarian process.

Correct Answer: (3) It is an irrational process.

Solution:

Step 1: Understanding the Concept:
This question asks for Baudrillard's overall characterization of modern consumption based on the passage.

Step 2: Detailed Explanation:
The entire passage describes consumption as a process based on a "magical" belief system, similar to the "cargo myth." Consumers engage with "sham objects" and wait for happiness "to alight." They view commodities as "miracles," divorced from the reality of their production. A process based on magic, miracles, and a fundamental misunderstanding of reality is, by definition, an irrational process. It is not based on logic or reason. The passage clearly shows consumption is not positive (it leads to unhappiness), not egalitarian (the myth starts from

inequality), and not utilitarian (it's about signs, not utility).

Step 3: Final Answer:
Baudrillard's position on consumption is that it is an irrational process.

Quick Tip

Synthesize the overall tone and argument of the passage. The repeated use of words like "myth," "magical," and "miracle" to describe modern consumer behavior points towards a critique of its rationality.

---

13. There are so many instances of one or more deer crossing the road, or just standing in the middle of the road, or else _____; it is like the deer cannot hear the noise of the engines or see the headlights.

(1) jumping under the road
(2) staggering with the road
(3) foraging beneath the road
(4) bounding across the road

Correct Answer: (4) bounding across the road

Solution:

Step 1: Understanding the Concept:
The question asks to complete the sentence with the most appropriate phrase describing a deer's movement. The sentence lists different behaviors of deer near a road.

Step 2: Detailed Explanation:
The sentence lists actions like "crossing the road" and "standing in the middle of the road." We need a phrase that describes another common way deer move across a road.

- "Bounding" means to move quickly with large jumps or leaps, which is a very characteristic way deer move. "Bounding across the road" fits the context perfectly.
- "Jumping under the road" and "foraging beneath the road" are physically impossible or nonsensical.
- "Staggering with the road" is grammatically awkward and doesn't describe a deer's movement.

Therefore, "bounding across the road" is the most logical and descriptive choice.

Step 3: Final Answer:
The correct phrase is "bounding across the road".

Quick Tip

In sentence completion questions, consider the typical collocations (words that often go together) and the specific context. "Bounding" is a verb strongly associated with the movement of animals like deer.

---

14. We hope that the government's new policies will _____ a period of economic growth.

(1) turn up
(2) usher in
(3) set in
(4) set forth

Correct Answer: (2) usher in

Solution:

Step 1: Understanding the Concept:
This question tests the knowledge of phrasal verbs and their appropriate usage in a formal context. We need to choose the phrasal verb that means "to introduce" or "to cause to begin."

Step 2: Detailed Explanation:

- **turn up:** means to arrive or appear unexpectedly. (e.g., "He turned up late for the meeting.")
- **usher in:** means to mark the beginning of something new, or to cause it to begin. (e.g., "The new technology ushered in an era of change.") This fits the context perfectly.
- **set in:** means to begin and seem likely to continue, often used for something unpleasant. (e.g., "The rain set in for the day.")
- **set forth:** means to start a journey or to state or explain ideas. (e.g., "He set forth on his adventure." or "She set forth her reasons.")

The most appropriate phrasal verb to describe new policies starting a period of economic growth is "usher in."

Step 3: Final Answer:
The correct phrasal verb is "usher in".

Quick Tip

Pay attention to the connotation of phrasal verbs. "Usher in" has a positive and formal tone, suitable for topics like economic growth and new policies. "Set in," in contrast, often has a negative or neutral but persistent connotation.

---

15. When she inherited some jewellery from a distant relative, she had no idea of its worth and decided _____.

(1) to have it appraised
(2) to get an approval
(3) to get it appreciated
(4) to have it apprised

Correct Answer: (1) to have it appraised

Solution:

Step 1: Understanding the Concept:
This question tests vocabulary, specifically the difference between similar-sounding words (confusables). The context is determining the monetary worth of jewellery.

Step 2: Detailed Explanation:

- **to appraise:** means to assess the value or quality of something. This is the correct term for having an expert determine the monetary worth of an item like jewellery.
- **to get an approval:** means to get permission or agreement for something. This does not fit the context of determining value.
- **to get it appreciated:** means to be grateful for something or for something to increase in value. While the jewellery might be appreciated, the action one takes to find out its value is not "getting it appreciated."
- **to apprise:** means to inform or tell someone about something. (e.g., "Please apprise me of the situation.") It is often confused with "appraise."

Given the context of not knowing the "worth" of the jewellery, the logical action is to have its value assessed, which is "to have it appraised."

Step 3: Final Answer:
The correct phrase is "to have it appraised".

Quick Tip

"Appraise" (with an 'a') is for assessing value. "Apprise" (with an 'i') is for informing. Remembering this simple distinction can help you avoid common errors with these two words.

---

16. The labourers who were fired broke into the office building and destroyed some of the machinery. Rather than finding a solution to their problems, they _____.

(1) extended their troubles
(2) exaggerated their hardships
(3) exacerbated the situation
(4) extenuated the circumstance

Correct Answer: (3) exacerbated the situation

Solution:

Step 1: Understanding the Concept:
This question tests vocabulary, specifically the ability to choose the most precise verb to describe the outcome of the labourers' actions. The context implies that their actions made a bad situation even worse.

Step 2: Detailed Explanation:
Let's analyze the options:

- **extended their troubles:** This is grammatically correct but lacks the specific meaning of making something actively worse. It's a bit too general.
- **exaggerated their hardships:** This means to represent their difficulties as being greater than they actually were. Their actions were destructive, not just descriptive.
- **exacerbated the situation:** The verb "exacerbate" means to make a problem, bad situation, or negative feeling worse. By destroying machinery, the labourers worsened their conflict with the company, which is a perfect fit for this word.
- **extenuated the circumstance:** The verb "extenuate" means to make a fault or offense seem less serious. This is the opposite of what their actions did.

The most precise and fitting word is "exacerbated."

Step 3: Final Answer:
Rather than finding a solution to their problems, they exacerbated the situation.

Quick Tip

Pay attention to nuances in meaning. While "extended their troubles" is not entirely wrong, "exacerbated the situation" is a more sophisticated and precise choice that captures the essence of making a bad situation worse, which is common in formal English.

---

17. Without a doubt, the widespread use of renewable energy is a key solution to climate change. However, it is not a _____ as efforts in conservation are equally crucial.

(1) silver lining
(2) silver bullet
(3) red herring
(4) dead ringer

Correct Answer: (2) silver bullet

Solution:

Step 1: Understanding the Concept:
This question tests the knowledge of common English idioms. The context requires an idiom that means a simple, complete, or magical solution to a complex problem.

Step 2: Detailed Explanation:
The sentence structure "it is not a _____ as [other things] are equally crucial" implies that renewable energy is not the *only* or *perfect* solution.

- **silver lining:** A positive aspect that can be found in a negative situation. This does not fit the context.
- **silver bullet:** A simple and seemingly magical solution to a complex problem. The sentence correctly states that renewable energy is not a "silver bullet" because other efforts are also needed. This is the perfect fit.
- **red herring:** Something that is intended to be misleading or distracting. This does not fit the context.
- **dead ringer:** A person or thing that is a perfect duplicate of another. This is completely irrelevant to the context.

Step 3: Final Answer:
However, it is not a silver bullet as efforts in conservation are equally crucial.

Quick Tip

Associate "silver bullet" with a single, easy solution for a difficult problem. This idiom is often used in the negative (e.g., "there is no silver bullet") to emphasize the complexity of an issue.

---

18. Everyone wondered how the travel vlogger could go around the world all through the year and _____.

(1) manage his iterative life style
(2) manage his itinerary life style
(3) manage his itinerary in his life style
(4) manage his itinerant life style

Correct Answer: (4) manage his itinerant life style

Solution:

Step 1: Understanding the Concept:
This question tests vocabulary, specifically the difference between several similar-sounding words. The correct word must describe a lifestyle of continuous travel.

Step 2: Detailed Explanation:
Let's define the key words:

- **iterative:** Involving repetition of a process and procedures. It does not relate to travel.
- **itinerary:** A planned route or journey. An "itinerary life style" is not a standard phrase; an itinerary is a plan for travel, not the lifestyle itself.
- **itinerant:** Traveling from place to place. This word is an adjective that perfectly describes a lifestyle characterized by constant travel. An "itinerant life style" is the correct and standard expression.

The vlogger's lifestyle involves traveling around the world, so "itinerant" is the most appropriate adjective.

Step 3: Final Answer:
Everyone wondered how the travel vlogger could go around the world all through the year and manage his itinerant life style.

Quick Tip

Remember the difference: "Itinerary" is a noun meaning a travel plan. "Itinerant" is an adjective meaning traveling. If you are describing a lifestyle of travel, you need the adjective "itinerant."

19. Deepak is an unpleasant person, but we all _____ because his sister is a close friend of ours.

(1) put along with him
(2) put him aside
(3) put up with him
(4) put him down

Correct Answer: (3) put up with him

Solution:

Step 1: Understanding the Concept:
This question tests knowledge of phrasal verbs. The sentence requires a phrasal verb that means to tolerate or endure someone who is unpleasant.

Step 2: Detailed Explanation:
Let's analyze the phrasal verbs:

- **put along with him:** This is not a standard English phrasal verb. The correct expression is "get along with someone," which means to have a good relationship.
- **put him aside:** This means to ignore or disregard someone or something, but it doesn't convey the sense of active tolerance in a social situation.
- **put up with him:** This phrasal verb means to tolerate or endure something or someone patiently. This fits the context perfectly, as the speaker is enduring Deepak's unpleasantness because of his sister.
- **put him down:** This means to insult or criticize someone. This is the opposite of what the speaker is doing.

Step 3: Final Answer:
Deepak is an unpleasant person, but we all put up with him because his sister is a close friend of ours.

Quick Tip

"Put up with" is a very common phrasal verb for "tolerate." Associate the "up" with the effort required to endure something you don't like.

---

20. Although the new policy aims to increase efficiency, reducing costs, and enhancing emplo some employees feel that the changes are too abrupt and poorly communicated.

(1) increasing efficiency, reducing of costs, and enhancing of employee satisfaction
(2) increase efficiency, reduce costs, and enhance employee satisfaction
(3) increase the efficiency, reduce the costs and enhancing employee satisfaction
(4) increase efficiency, reducing the costs and enhanced employee satisfaction

Correct Answer: (2) increase efficiency, reduce costs, and enhance employee satisfaction

Solution:

Step 1: Understanding the Concept:
This question tests the principle of parallel structure (or parallelism). When listing items in a series, each item should be in the same grammatical form.

Step 2: Detailed Explanation:
The original underlined phrase is "increase efficiency, reducing costs, and enhancing employee satisfaction". The introductory verb is "aims to," which should be followed by a verb in its base form (infinitive without 'to'). The list starts with "increase" (base form), but the next two items are "reducing" and "enhancing" (present participles/gerunds). This violates the rule of parallelism. To correct this, all three items in the list should be in the base form to match the first item.

- Option (1) uses participles and unnecessarily adds "of".
- Option (2) correctly lists all three items in their base verb form: "increase..., reduce..., and enhance...". This maintains a parallel structure.
- Option (3) mixes base form ("reduce") and a participle ("enhancing").
- Option (4) mixes a participle ("reducing") and a past participle ("enhanced").

The only option that maintains a consistent parallel structure is (2).

Step 3: Final Answer:
The correct phrase is "increase efficiency, reduce costs, and enhance employee satisfaction".

Quick Tip

When you see a list of actions or items joined by "and" or "or," check that they all have the same grammatical form (e.g., all nouns, all infinitives, all "-ing" forms). This is the essence of parallel structure.

---

21. Thank goodness, the damage to the car was neglectful.

(1) was negligible
(2) was neglectable
(3) was negligent

(4) was neglecting

Correct Answer: (1) was negligible

Solution:

Step 1: Understanding the Concept:
This question tests the difference between commonly confused words (confusables) derived from the same root ("neglect"). The context requires a word that describes the extent of the damage.

Step 2: Detailed Explanation:

- **neglectful:** This is an adjective used to describe a person or their behavior, meaning they fail to give proper care or attention to someone or something. (e.g., "The owner was neglectful of the pet.") It cannot describe an inanimate object like damage.
- **negligible:** This adjective means so small, unimportant, or insignificant that it is not worth considering. This word correctly describes damage that is very minor.
- **neglectable:** This is not a standard English word. The correct form is "negligible".
- **negligent:** This adjective describes a person or action that shows a failure to take proper care, often leading to harm. (e.g., "The negligent driver caused the accident.") It describes the cause, not the effect.
- **neglecting:** This is a verb form and does not fit grammatically.

The appropriate word to describe the amount of damage is "negligible."

Step 3: Final Answer:
Thank goodness, the damage to the car was negligible.

Quick Tip

Remember the difference: "Negligible" describes a very small amount or degree (think "insignificant"). "Negligent" and "neglectful" describe the careless behavior of a person.

---

22. Among scientists, the discovery of the double helix structure of DNA and the genetic code it incorporates is widely regarded to be one of the most significant scientific discovery of the twentieth century.

(1) regarded being one of the most significant scientific discoveries
(2) regarded like one of the most significant scientific discovery
(3) regarded for being one of the most significant scientific discoveries
(4) regarded as one of the most significant scientific discoveries

Correct Answer: (4) regarded as one of the most significant scientific discoveries

Solution:

Step 1: Understanding the Concept:
This question tests two grammatical points: the correct idiom for the verb "regard" and the correct plural form for nouns in a "one of the" construction.

Step 2: Detailed Explanation:

1. **Idiomatic Use of 'Regard':** The correct idiom is "regarded as," not "regarded to be," "regarded like," or "regarded for being." The verb "regard" in this context means "to consider or think of in a specified way," and it is followed by the preposition "as."
2. **"One of the" Construction:** The phrase "one of the most significant..." must be followed by a plural noun. The original sentence uses the singular "discovery," which is incorrect. It should be "discoveries."

Let's analyze the options:

- Option (1) uses the incorrect construction "regarded being" and the correct plural "discoveries."
- Option (2) uses the incorrect "regarded like" and the incorrect singular "discovery."
- Option (3) uses the incorrect "regarded for being" and the correct plural "discoveries."
- Option (4) uses the correct idiom "regarded as" and the correct plural noun "discoveries."

Therefore, option (4) is the only one that corrects both errors.

Step 3: Final Answer:
The correct phrase is "regarded as one of the most significant scientific discoveries".

Quick Tip

Remember these two rules: 1. The correct idiom is always "regard as." 2. The phrase "one of the" is always followed by a plural noun (e.g., "one of the reasons," not "one of the reason").

---

23. When I had to leave town due to office work, I had my brother to give food to my dog twice a day.

(1) had my brother feed my dog
(2) had my brother to feed my dog
(3) had my brother who fed my dog
(4) had my brother giving food to my dog

Correct Answer: (1) had my brother feed my dog

Solution:

Step 1: Understanding the Concept:
This question tests the use of causative verbs. The verb "have" is used here to mean "cause someone to do something." The grammatical structure for this usage is specific.

Step 2: Key Formula or Approach:
The structure for the active causative verb "have" is: **Subject + have/has/had + agent (person) + base form of verb (infinitive without 'to') + object.**

Step 3: Detailed Explanation:
In the given sentence, the structure is: I (Subject) + had + my brother (agent) + ... According to the rule, the verb that follows the agent ("my brother") should be in its base form.

- The original sentence uses "to give," which is the full infinitive. This is incorrect.
- Option (1) uses "feed," which is the base form of the verb. This correctly follows the causative structure: "I had my brother feed my dog."
- Option (2) uses "to feed," the full infinitive, which is incorrect.
- Option (3) uses a relative clause "who fed my dog," which changes the structure and meaning. It's not a causative construction.
- Option (4) uses "giving," a present participle, which is also incorrect for the causative "have." (The structure "have someone doing something" exists, but it has a different meaning, usually implying an ongoing experience.)

The only option that correctly applies the causative "have" structure is (1).

Step 3: Final Answer:
The correct phrase is "had my brother feed my dog".

Quick Tip

Memorize the patterns for the main causative verbs:

- **have someone do** something (base form)
- **make someone do** something (base form)
- **let someone do** something (base form)
- **get someone to do** something (full infinitive)

---

24. If the President knew that his allies would let him down so suddenly, he would have handled them with the greatest care.

(1) If the President could know beforehand that his allies would let him down
(2) Had the President known that his allies would let him down
(3) Had the President knowledge that his allies would let him down
(4) If the President knew that his allies can let him down

Correct Answer: (2) Had the President known that his allies would let him down

Solution:

Step 1: Understanding the Concept:
This question tests the ability to rephrase a sentence using a different grammatical structure, specifically converting a third conditional sentence into its inverted form. The original sentence is a hypothetical statement about a past event that did not happen.

Step 2: Key Formula or Approach:
A third conditional sentence has the structure: "If + subject + had + past participle, ... subject + would have + past participle." This can be inverted for emphasis or formality by removing "If" and inverting the subject and the auxiliary verb "had": "Had + subject + past participle, ... subject + would have + past participle."

Step 3: Detailed Explanation:
The original sentence is: "**If the President knew... he would have handled...**". There's a tense mismatch here. The "would have handled" part belongs to a third conditional (past unreal), which requires a past perfect "if" clause ("if he had known"). The original sentence is grammatically flawed. We must choose the option that creates a grammatically correct and logically equivalent sentence.
Let's assume the intended meaning is a third conditional: "If the President **had known** that his allies would let him down..., he would have handled them...". We need to find the correct inverted form of this.

- Option (1) changes "knew" to "could know," which alters the meaning and doesn't match the third conditional structure.
- Option (2) correctly inverts the third conditional structure: "Had the President known..." This is the standard, formal way to express the hypothetical past condition without using "if".
- Option (3) is grammatically awkward. "Had... knowledge" is not the correct structure for this inversion.
- Option (4) uses "can let him down," which is the wrong tense and mood for a past hypothetical situation.

Option (2) is the best and most grammatically sound alternative that captures the intended hypothetical meaning.

Step 4: Final Answer:
The correct alternative is "Had the President known that his allies would let him down".

Quick Tip

To invert a third conditional sentence, simply drop the "if" and swap the subject and "had". For example, "If I had seen you..." becomes "Had I seen you...".

---

25. On the first day of January 2025, the Indian Meteorological Department [IMD] announced that 2024 was the hottest year on record. A study by the Council on Energy, Environment and Water shows that nearly eight out of ten Indians live in districts that are at risk of either a flood, a cyclone, or a drought. Nearly twenty three States in India are heatwave-prone. In the summer of 2024, India recorded more than 44,000 cases of heatstroke and over 300 heat-related mortalities, as per the bulletin of the Ministry of Health and Family Welfare. Water reservoirs and the energy demand that keeps India powered are impacted too. During a ten-day long heatwave in Delhi, peak power demand was 16% higher than the previous year.

(1) According to the Council, more than 20% of the population is not affected by climate change
(2) The increasing heat stress remains a major challenge, affecting public health and economic productivity
(3) However, the people of these districts are given sufficient compensation for loss of life and property
(4) The record-breaking heat of the summer of 2024 resulted in an unpredictable and delayed monsoon

Correct Answer: (2) The increasing heat stress remains a major challenge, affecting public health and economic productivity

Solution:

Step 1: Understanding the Concept:
This question asks for the most accurate inference or summary that can be drawn from the provided passage. We need to evaluate each option against the information given in the text.

Step 2: Detailed Explanation:
The passage details the widespread impact of climate change in India in 2024. It mentions that 2024 was the hottest year, most of the population is at risk, states are heatwave-prone, and there were significant health impacts (heatstroke, mortalities). It also mentions economic impacts (water reservoirs, energy demand).

- Option (1) is a possible inference but is not the main point. The passage says "nearly eight out of ten" are at risk, which means "more than two out of ten" (or 20%) might not live in districts at risk of those specific events, but the passage doesn't say they are "not affected by climate change" in general. This is a weak inference.
- Option (2) accurately summarizes the core message of the passage. The "increasing heat stress" (hottest year, heatwaves) is presented as a "major challenge" with clear effects on

"public health" (mortalities, heatstroke) and "economic productivity" (impact on water and energy).

- Option (3) is not supported by the passage. There is no mention of compensation being given.
- Option (4) is not supported by the passage. There is no mention of the monsoon being delayed or unpredictable.

Option (2) provides the best and most comprehensive summary of the information presented in the paragraph.

Step 3: Final Answer:
The most accurate inference is that the increasing heat stress remains a major challenge, affecting public health and economic productivity.

Quick Tip

For inference questions, choose the statement that synthesizes multiple points from the text. A good inference often acts as a summary of the main idea, rather than just re-stating a single detail or making an unsupported leap.

---

26. As globalization held sway over the world, communities, which used to live in relative isolation, sought access to the wider world, and in the process, they parted with their own language and adopted a new lingua franca. The loss of language, however, does not merely mean the loss of a mode of communication or the loss of a few thousand words. _____. So, when a language dies, a way of thinking dies with it.

(1) A certain school of thought regrets the demise of local languages but in recent times revival movements have emerged across the world, especially in India
(2) Since evolution and change in languages is a part of history, most of the languages spoken today would be scarcely recognizable from what they were a few thousand or maybe even a few hundred years ago
(3) A potentially endangered language can sometimes appear to be thriving, or on the other hand, it can show signs of declining
(4) Languages exist not only for the purposes of practical communication; they convey a linguistic community's entire mindset and its culture

Correct Answer: (4) Languages exist not only for the purposes of practical communication; they convey a linguistic community's entire mindset and its culture

Solution:

Step 1: Understanding the Concept:
This question requires finding a sentence that logically connects the preceding sentence with the concluding one. The paragraph argues that losing a language is more than just losing words.

Step 2: Detailed Explanation:

- **Preceding Sentence:** "The loss of language, however, does not merely mean the loss of a mode of communication or the loss of a few thousand words." This sentence sets up the idea that there is a deeper loss involved.
- **Concluding Sentence:** "So, when a language dies, a way of thinking dies with it." This sentence explains what that deeper loss is.
- **The Missing Link:** We need a sentence that explains *why* the loss is more than just words and how it connects to a "way of thinking."

Option (4) provides this link perfectly: "Languages exist not only for the purposes of practical communication; they convey a linguistic community's entire mindset and its culture." This explains that language embodies a mindset and culture, which directly leads to the conclusion that losing the language means losing that "way of thinking." The other options introduce irrelevant ideas about revival movements, historical evolution, or the state of endangered languages.

Step 3: Final Answer:
The most logical sentence to complete the paragraph is: "Languages exist not only for the purposes of practical communication; they convey a linguistic community's entire mindset and its culture".

Quick Tip

In paragraph completion, look for the "bridge" sentence. The missing sentence must logically connect the idea before it to the idea after it, often by explaining or elaborating on the first idea to set up the conclusion.

---

27. An island in Japan boasts of numerous dairy farms that own nearly one million cows, and supplies 70% of the milk sold in the country. These dairy farms have now begun to use cow manure to produce hydrogen. The methane from cow manure mingles with steam in a high temperature environment to produce hydrogen, which is used to electrify the local zoo. _____.

(1) This shows how Japan has always used technology to help animals
(2) The Indian government too, should replicate this, and use such technology to produce hydrogen
(3) It is a case study of a certain animal that is useful in providing energy for several other animals

(4) This is an exemplary way of creating a sustainable source of energy using innovative technology

Correct Answer: (4) This is an exemplary way of creating a sustainable source of energy using innovative technology

Solution:

Step 1: Understanding the Concept:
The question asks for a concluding sentence that best summarizes the significance of the process described in the paragraph.

Step 2: Detailed Explanation:
The paragraph describes a process where a waste product (cow manure) is converted into a useful energy source (hydrogen) to power a local facility (the zoo). This is a clear example of innovation aimed at sustainability.

- Option (1) makes a broad, unsupported claim about what Japan has "always" done.
- Option (2) introduces a recommendation for another country (India), which is outside the scope of the descriptive paragraph.
- Option (3) is too narrow and slightly inaccurate. The energy is for the zoo, not "several other animals."
- Option (4) provides an excellent summary. It correctly identifies the process as "exemplary" (a good example), "sustainable" (using waste to create energy), and involving "innovative technology." This captures the essence of the entire paragraph.

Step 3: Final Answer:
The best concluding sentence is: "This is an exemplary way of creating a sustainable source of energy using innovative technology".

Quick Tip

A good concluding sentence for a descriptive paragraph often summarizes the main point or highlights its broader significance. Look for the option that captures the "so what?" of the information presented.

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28. A report published in Lancet Diabetes and Endocrinology has called for an overhaul of our understanding of obesity. An over-reliance on using Body Mass Index [BMI] as a metric has the peculiar effect of leading to both underdiagnosis and overdiagnosis of the condition. _____. BMI does not give accurate information about how fat is distributed in an individual's body. It frequently fails to capture the true state of health of an individual. A person's BMI may indicate they are "obese", but their organs and bodily functions may be absolutely normal.

Every individual is a unique constellation - not only of genes and other biological variables, but also socio-economic conditions and habits.

(1) BMI reading can help the doctor to accurately prescribe the appropriate dosage to reduce fat
(2) This is because BMI does not provide a reliable picture of health, nor any direct measure of fat
(3) Obesity is the end result of multiple factors and BMI can pinpoint the cause of the problem
(4) Further, much of the information on diabetes, obesity or BMI available on social media is misleading

Correct Answer: (2) This is because BMI does not provide a reliable picture of health, nor any direct measure of fat

Solution:

Step 1: Understanding the Concept:
We need to find a sentence that fits logically between the statement that BMI leads to misdiagnosis and the explanation of BMI's specific failures.

Step 2: Detailed Explanation:

- **Preceding Sentence:** "An over-reliance on using Body Mass Index [BMI] as a metric has the peculiar effect of leading to both underdiagnosis and overdiagnosis of the condition." This sentence states the problem: BMI is unreliable for diagnosis.
- **Following Sentence:** "BMI does not give accurate information about how fat is distributed..." This sentence begins the specific explanation of *why* BMI is unreliable.
- **The Missing Link:** The missing sentence should act as a bridge, stating the general reason for BMI's unreliability before the text dives into specifics.

Option (2) serves this purpose perfectly: "This is because BMI does not provide a reliable picture of health, nor any direct measure of fat." It provides a general reason ("not a reliable picture of health") that neatly summarizes the problem of misdiagnosis and logically introduces the subsequent, more detailed explanations about fat distribution and organ function. The other options contradict the paragraph's main argument or introduce irrelevant topics.

Step 3: Final Answer:
The most logical sentence is: "This is because BMI does not provide a reliable picture of health, nor any direct measure of fat".

Quick Tip

Look at the flow of logic from general to specific. The paragraph starts with a general problem (misdiagnosis), and the missing sentence should provide a general reason, which is then supported by the specific details that follow.

29. Since chronic stress can _____ the immune system, making individuals more susceptible to illness and _____ their overall well-being, healthcare practitioners often recommend mindfulness practices and proper sleep to _____ these negative effects.

(1) compromise; impair; counter
(2) endanger; preserve; decrease
(3) paralyse; improve; diminish
(4) undermine; elevate; impede

Correct Answer: (1) compromise; impair; counter

Solution:

Step 1: Understanding the Concept:
This question requires filling in three blanks with words that fit the logical and contextual meaning of the sentence. The sentence describes the negative effects of stress and the positive effects of certain practices.

Step 2: Detailed Explanation:
Let's analyze the blanks one by one:

- **Blank 1:** Stress has a negative effect on the immune system. Words like "compromise," "endanger," "paralyse," or "undermine" would fit.
- **Blank 2:** Stress also has a negative effect on well-being. A word like "impair" (weaken or damage) fits well. "Preserve" and "improve" are positive and incorrect. "Elevate" doesn't make sense in this context.
- **Blank 3:** Mindfulness and sleep are recommended to deal with the negative effects. A word like "counter" (act against) or "mitigate" would be appropriate. "Decrease," "diminish," or "impede" (hinder) do not fit the meaning of actively fighting the effects.

Now let's evaluate the options as a whole:

- (1) **compromise; impair; counter:** "Compromise the immune system" (weaken it), "impair their overall well-being" (damage it), and "counter these negative effects" (act against them). All three words fit perfectly.
- (2) endanger; preserve; decrease: "Preserve" is incorrect.
- (3) paralyse; improve; diminish: "Improve" is incorrect.
- (4) undermine; elevate; impede: "Elevate" and "impede" are incorrect.

The only option where all three words are logically and contextually correct is (1).

Step 3: Final Answer:
The correct words are compromise; impair; counter.

Quick Tip

In multiple-blank questions, use the process of elimination. Even if you're unsure about the first blank, you can often eliminate options based on a word that clearly doesn't fit in the second or third blank.

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30. Astronauts who stayed for an _____ period of time at the International Space Station displayed a remarkable level of _____ endurance and mental _____.

(1) expanded; stern; acuity
(2) extended; physical; resilience
(3) explicit; stoic; integrity
(4) extensive; dysfunctional; agility

Correct Answer: (2) extended; physical; resilience

Solution:

Step 1: Understanding the Concept:
This is a sentence completion question that requires choosing a set of three words that are contextually appropriate for describing a long stay in space.

Step 2: Detailed Explanation:
Let's break down the sentence:

- **Blank 1:** Describes the period of time. "Extended" (meaning lasting longer than usual) is the most suitable word. "Expanded," "explicit," and "extensive" are less appropriate for describing a duration of time.
- **Blank 2:** Describes a type of endurance. In the context of space travel, "physical" endurance is a key requirement. "Stern" (severe), "stoic" (enduring pain without complaint), and "dysfunctional" are not the right adjectives for "endurance" here.
- **Blank 3:** Describes a mental quality. "Resilience" (the capacity to recover quickly from difficulties) is a crucial mental attribute for astronauts facing the challenges of long-duration space missions. "Acuity" (sharpness of thought), "integrity" (moral principle), and "agility" (ability to move or think quickly) are also positive qualities, but "resilience" best captures the idea of coping with a long, demanding period.

Let's check the options:

- (1) "expanded" is awkward for time; "stern endurance" is an odd pairing.
- (2) **extended; physical; resilience:** All three words fit perfectly. An "extended period" requires "physical endurance" and "mental resilience."
- (3) "explicit period" doesn't make sense; "stoic integrity" is a possible phrase but less relevant than resilience.

- (4) "dysfunctional endurance" is contradictory.

Option (2) provides the most coherent and logical set of words.

Step 3: Final Answer:
The correct words are extended; physical; resilience.

Quick Tip

Consider the collocations (words that frequently appear together). "Extended period," "physical endurance," and "mental resilience" are all strong, common collocations, which is a good indicator that they form the correct answer.

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31. Psychologists urge users to remember that social media rarely reflects the full complexity of real life. Influencers often _____ a carefully curated online persona, which can _____ unrealistic standards and occasionally _____ negative self-comparisons amongst their followers.

(1) profess; inspire; release
(2) advocate; perpetuate; stimulate
(3) endorse; foster; provoke
(4) maintain; generate; trigger

Correct Answer: (4) maintain; generate; trigger

Solution:

Step 1: Understanding the Concept:
This sentence completion requires selecting three verbs that accurately describe the actions and consequences of influencers' online personas.

Step 2: Detailed Explanation:
Let's analyze the blanks in context:

- **Blank 1:** Describes what influencers do with their online persona. "Maintain" fits perfectly, as a persona is something that is kept up over time. "Profess," "advocate," and "endorse" are plausible but "maintain" is the most direct fit for "persona."
- **Blank 2:** Describes what the persona does with unrealistic standards. "Generate" (create) or "perpetuate" (cause to continue) would fit. The persona creates or gives rise to these standards.
- **Blank 3:** Describes what the persona does with negative self-comparisons. "Trigger" (cause an event or situation to happen) is an excellent choice, as seeing a curated life can trigger feelings of inadequacy in others. "Provoke" is also a good fit.

Now let's look at the options as complete sets:

- (1) profess; inspire; release: "Release comparisons" is not idiomatic.
- (2) advocate; perpetuate; stimulate: "Advocate a persona" is slightly odd. "Stimulate comparisons" is possible but "trigger" is stronger.
- (3) endorse; foster; provoke: "Foster unrealistic standards" and "provoke comparisons" works well. "Endorse a persona" is also good. This is a strong contender.
- (4) **maintain; generate; trigger:** "Maintain a persona," "generate unrealistic standards," and "trigger negative self-comparisons." All three words are precise, idiomatic, and fit the psychological context of the sentence perfectly.

Comparing (3) and (4), the set in (4) offers slightly more precise vocabulary for the entire sequence of actions and consequences. "Maintain" is better than "endorse" for a persona, and "generate" and "trigger" are very specific and common terms in this context.

Step 3: Final Answer:
The correct words are maintain; generate; trigger.

Quick Tip

In sentence completion, look for the option that is not just plausible, but the most precise and idiomatic. Words commonly used in a specific field (like "trigger" in psychology) are often a clue to the correct answer.

---

32. Art can be _____ because it encourages individuals to express their emotions through a creative outlet, allowing them to process complex feelings, reduce stress, and _____ self-awareness.

(1) acceptable; disturb
(2) therapeutic; enhance
(3) pleasing; decrease
(4) avoidable; mitigate

Correct Answer: (2) therapeutic; enhance

Solution:

Step 1: Understanding the Concept:
This question asks to fill two blanks to complete a sentence about the benefits of art. The words chosen must reflect the positive outcomes described in the rest of the sentence.

Step 2: Detailed Explanation:

- **Blank 1:** The word must describe the quality of art that leads to the benefits listed (expressing emotions, processing feelings, reducing stress). The word "therapeutic" (having a healing effect) perfectly summarizes these benefits. "Acceptable," "pleasing," and "avoidable" are much weaker and less specific.
- **Blank 2:** The word must describe what art does to "self-awareness." Since expressing emotions and processing feelings lead to better self-understanding, the verb should be positive. "Enhance" (increase or improve) is the ideal choice. "Disturb" and "decrease" are negative and illogical. "Mitigate" (make less severe) does not make sense with "self-awareness."

The pair "therapeutic; enhance" is the only one where both words fit the context logically and positively.

Step 3: Final Answer:
The correct words are therapeutic; enhance.

Quick Tip

The word "because" is a key signal. The first blank should be a word that is explained or defined by the clause that follows "because." The list of benefits clearly points to a "therapeutic" quality.

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33. The notion of personhood is _____ on something more than a particular type of genetic material within human beings: it arises only with the larger-scale structural _____ of that material, which permits capacities like _____ thought and moral agency.

(1) interdependent; division; differentiation
(2) dependent; disorganisation; deconstruction
(3) premised; organisation; consciousness
(4) built; distribution; calibration

Correct Answer: (3) premised; organisation; consciousness

Solution:

Step 1: Understanding the Concept:
This is a complex sentence completion question that requires understanding philosophical and biological concepts. We need to choose three words that create a coherent argument about the basis of personhood.

Step 2: Detailed Explanation:

- **Blank 1:** The sentence states that personhood is based ("___ on") something more than just genes. "Premised on" (based on the belief or idea that) is a very fitting and formal choice. "Dependent on," "interdependent on," and "built on" are also plausible.
- **Blank 2:** This blank describes the "larger-scale structural ___" of genetic material that allows for personhood. The word should suggest a complex and ordered structure. "Organisation" is the perfect word, contrasting with just the "particular type" of material. "Disorganisation" is the opposite of what's needed. "Division" and "distribution" are too simplistic.
- **Blank 3:** This blank needs a capacity similar to "moral agency" that arises from the complex organization. "Consciousness" is a high-level capacity that, along with thought and moral agency, is often seen as a cornerstone of personhood. "Differentiation," "deconstruction," and "calibration" do not fit as capacities of a person in this context.

Evaluating the options as sets:

- (1) "division" and "differentiation" are not quite right for blanks 2 and 3.
- (2) "disorganisation" and "deconstruction" are logically incorrect.
- (3) **premised; organisation; consciousness:** All three words fit perfectly. Personhood is "premised on" the "organisation" of genetic material, which permits "consciousness."
- (4) "distribution" and "calibration" are weak fits for blanks 2 and 3.

Option (3) creates the most philosophically and logically sound sentence.

Step 3: Final Answer:
The correct words are premised; organisation; consciousness.

Quick Tip

For abstract or academic sentences, focus on the logical relationships. The sentence contrasts "a particular type of material" with a "larger-scale structural ___," implying order and complexity, which strongly points to "organisation."

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34. While Curcumin, which is an _____ found in turmeric helps to reduce _____, extremely high doses of it can _____ headache and nausea.

(1) alchemy; injury; cause
(2) ingredient; inflammation; induce
(3) enzyme; abrasion; infuse
(4) alkali; infection; promote

Correct Answer: (2) ingredient; inflammation; induce

Solution:

Step 1: Understanding the Concept:
This sentence completion requires selecting three words related to health and medicine that correctly describe the properties and effects of Curcumin.

Step 2: Detailed Explanation:

- **Blank 1:** Describes what Curcumin is in relation to turmeric. "Ingredient" is the most accurate and common term. "Alchemy," "enzyme," and "alkali" are scientifically incorrect in this context.
- **Blank 2:** Describes a condition that Curcumin helps to reduce. Curcumin is famously known for its anti-inflammatory properties. Thus, it helps to reduce "inflammation." "Injury," "abrasion," and "infection" are less specific or incorrect.
- **Blank 3:** Describes what high doses of something can do to produce negative symptoms like headache. "Induce" (to cause or bring about) is the correct medical term. "Cause" is also possible but "induce" is often used for side effects. "Infuse" and "promote" do not fit.

Let's review the options:

- (1) "alchemy" is incorrect.
- (2) **ingredient; inflammation; induce:** All three words are perfectly suited to the context. Curcumin is an "ingredient" that reduces "inflammation" and in high doses can "induce" side effects.
- (3) "enzyme" and "abrasion" are incorrect.
- (4) "alkali" and "infection" are incorrect.

Option (2) is the only one where all three words are correct.

Step 3: Final Answer:
The correct words are ingredient; inflammation; induce.

Quick Tip

Use your general knowledge to help with sentence completion. Knowing that turmeric is an ingredient used in cooking and that it is known for fighting inflammation can quickly lead you to the correct answer.

Comprehension for Q.35 to Q.40
CONVERSATION ANALYSIS: Read the following transcript and choose the answer that is closest to each of the questions that are based on the transcript.

Lucia Rahilly (Global Editorial Director, The McKinsey Podcast): Today we're talking about the next big arenas of competition, about the industries that will matter most in the global business landscape, which you describe as arenas of competition. What do we mean when we use this term?
Chris Bradley (Director, McKinsey Global Institute): If I go back and look at the top ten companies in 2005, they were in traditional industries such as oil and gas, retail, industrials, and pharmaceuticals. The average company was worth about $250 billion. If I advance the clock forward to 2020, nine in ten of those companies have been replaced, and by companies that are eight times bigger than the old guards.
And this new batch of companies comes from these new arenas or competitive sectors. In fact, they're so different that we have a nickname for them. If you're a fan of Harry Potter, it's wizards versus muggles.
Arena industries are wizard-ish; we found that there's a set of industries that play by very different set of economic rules and get very different results, while the rest, the muggles (even though they run the world, finance the world, and energize the world), play by a more traditional set of economic rules.
Lucia Rahilly: Could we put a finer point on what is novel or different about the lens that you applied to determine what's a wizard and what's a muggle?
Chris Bradley: Wizards are defined by growth and dynamism. We looked at where value is flowing and the places where value is moving. And where is the value flowing? What we see is that this set of wizards, which represent about ten percent of industries, hog 45 percent of the growth in market cap. But there's another dimension or axis too, which is dynamism. That is measured by a new metric we've come up with called the "shuffle rate." How much does the bottom move to the top? It turns out that in this set of wizard-ish industries, or arenas, the shuffle rate is much higher than it is in the traditional industry.
Lucia Rahilly: So, where are we seeing the most profit?
Chris Bradley: The economic profit, which is the profit you make minus the cost for the capital you employ is in the wizard industries. It's where R&D happens; they're two times more R&D intensive. They're big stars, the nebulae, where new business is born.

---

35. In the context of the conversation, "dynamism" most closely refers to

(1) the never-changing reliance on established and unchanging business practices.
(2) the stability and predictability of traditional industries.
(3) the rapid and frequent changes in leadership and market position within an industry.
(4) the slow, gradual growth and morphing of established companies.

Correct Answer: (3) the rapid and frequent changes in leadership and market position within an industry.

Solution:

Step 1: Understanding the Concept:
This question asks for the definition of "dynamism" as it is used in the conversation.

Step 2: Detailed Explanation:
Chris Bradley explicitly defines "dynamism" in the conversation. He says, "...another dimension or axis too, which is dynamism. That is measured by a new metric we've come up with called the 'shuffle rate.' How much does the bottom move to the top? It turns out that in this set of wizard-ish industries, or arenas, the shuffle rate is much higher than it is in the traditional industry." The "shuffle rate," described as measuring how much "the bottom move to the top," is a measure of churn and change in market leadership. This directly corresponds to "rapid and frequent changes in leadership and market position within an industry." Options 1, 2, and 4 describe the opposite of dynamism.

Step 3: Final Answer:
In the context of the conversation, "dynamism" most closely refers to the rapid and frequent changes in leadership and market position within an industry.

Quick Tip

When a term is introduced in a text, look for an immediate definition or an example that clarifies its meaning. Here, "dynamism" is directly explained by the concept of the "shuffle rate."

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36. In the context of the conversation, the term "arenas of competition" refers to

(1) government regulations that control business competition.
(2) broad categories of industries where companies engage in competitive activities.
(3) physical locations where businesses compete.
(4) specific companies that are considered to be powerful competitors.

Correct Answer: (2) broad categories of industries where companies engage in competitive activities.

Solution:

Step 1: Understanding the Concept:
The question asks for the meaning of the phrase "arenas of competition" based on the transcript.

Step 2: Detailed Explanation:
In the third paragraph, Chris Bradley says, "And this new batch of companies comes from these new arenas or competitive sectors." By using "or," he equates "arenas" with "competitive sectors." A sector is a broad category of industry. This aligns perfectly with option (2). The entire conversation is about different types of industries (oil and gas, retail, etc.), not specific companies (4), physical locations (3), or regulations (1).

Step 3: Final Answer:
The term "arenas of competition" refers to broad categories of industries where companies

engage in competitive activities.

Quick Tip

Pay attention to synonyms and rephrasing within a text. When a speaker says "X or Y," they are often using Y to clarify the meaning of X. Here, "arenas" is clarified by "competitive sectors."

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37. "Muggles" refers to industries that

(1) operate under traditional economic principles.
(2) are primarily focused on technological innovation.
(3) are characterized by rapid and frequent changes.
(4) exhibit high levels of market capitalization growth.

Correct Answer: (1) operate under traditional economic principles.

Solution:

Step 1: Understanding the Concept:
This question asks for the definition of "muggles" as used in the conversation to describe certain industries.

Step 2: Detailed Explanation:
Chris Bradley defines "muggles" in the fourth paragraph: "...while the rest, the muggles (even though they run the world, finance the world, and energize the world), play by a more traditional set of economic rules." This is a direct definition that matches option (1). Options (2), (3), and (4) are descriptions of the "wizard" industries, which are contrasted with the muggles.

Step 3: Final Answer:
"Muggles" refers to industries that operate under traditional economic principles.

Quick Tip

In texts that use metaphors or nicknames (like wizards vs. muggles), the definitions are almost always provided immediately after the terms are introduced. Locate the term in the text to find its meaning.

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38. Which one of the following does "shuffle rate" not measure?

(1) Volatility of market leadership.
(2) Overall profitability of traditional industries.
(3) Relative change within an industry.
(4) Churn within the arena of competition.

Correct Answer: (2) Overall profitability of traditional industries.

Solution:

Step 1: Understanding the Concept:
The question asks to identify what the "shuffle rate" metric does NOT measure, based on its description in the transcript.

Step 2: Detailed Explanation:
Chris Bradley describes the "shuffle rate" as a measure of dynamism, answering the question, "How much does the bottom move to the top?". This definition directly implies that the shuffle rate measures change, movement, and churn.

- (1) Volatility of market leadership: This is what "bottom move to the top" describes. So, shuffle rate *does* measure this.
- (3) Relative change within an industry: This is another way of describing the same concept. Shuffle rate *does* measure this.
- (4) Churn within the arena of competition: "Churn" is a synonym for this kind of movement. Shuffle rate *does* measure this.
- (2) Overall profitability of traditional industries: Profitability is discussed as a separate characteristic. Chris Bradley says, "So, where are we seeing the most profit? ... The economic profit... is in the wizard industries." He presents dynamism (measured by shuffle rate) and profitability as two different dimensions. Therefore, shuffle rate is not a direct measure of profitability.

Step 3: Final Answer:
"Shuffle rate" does not measure the overall profitability of traditional industries.

Quick Tip

For "what is NOT" questions, use the process of elimination. Verify which options are supported by the text. The one that is not mentioned or is presented as a separate concept is the correct answer.

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39. Which of the following best and correctly summarizes the main idea of the conversation?

(1) Newer, dynamic industries, termed "wizards," are experiencing significantly greater growth and profit compared to traditional industries.
(2) The global economy is shifting back towards traditional industries, as they offer more stable returns.
(3) Traditional industries are consistently more profitable than newer, "wizard-ish" industries.
(4) The terms "wizard" and "muggle" are used to describe the magical elements of business success.

Correct Answer: (1) Newer, dynamic industries, termed "wizards," are experiencing significantly greater growth and profit compared to traditional industries.

Solution:

Step 1: Understanding the Concept:
This question requires summarizing the central argument of the entire conversation.

Step 2: Detailed Explanation:
The conversation introduces a distinction between two types of industries: traditional "muggles" and new "wizards." It then details how the "wizards" are outperforming the "muggles" in key areas. They are characterized by "growth" (hogging 45% of market cap growth), "dynamism" (high shuffle rate), and higher "economic profit." Option (1) perfectly encapsulates this main idea. Options (2) and (3) are directly contradicted by the text, which states that wizards are growing faster and are more profitable. Option (4) is a misinterpretation; the terms are a metaphor for economic principles, not literal magic.

Step 3: Final Answer:
The best summary is: Newer, dynamic industries, termed "wizards," are experiencing significantly greater growth and profit compared to traditional industries.

Quick Tip

A good summary should capture the core contrast or argument of the text. Here, the central theme is the comparison between "wizard" and "muggle" industries and the superior performance of the former.

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40. "Wizard" industries are characterized by

(1) a reliance on traditional economic rules and practices.
(2) a higher concentration of economic profit and research and development.
(3) lower research and development spending.
(4) a slower rate of market capitalization growth.

Correct Answer: (2) a higher concentration of economic profit and research and development.

Solution:

Step 1: Understanding the Concept:
The question asks to identify the characteristics of "wizard" industries based on the transcript.

Step 2: Detailed Explanation:
In the last paragraph, Chris Bradley provides a clear summary of the characteristics of wizard industries: "The economic profit... is in the wizard industries. It's where R&D happens; they're two times more R&D intensive. They're big stars, the nebulae, where new business is born." This statement directly supports option (2), as it highlights both "economic profit" and high "research and development" (R&D) spending. Option (1) describes "muggles." Option (3) is the opposite of what is stated ("two times more R&D intensive"). Option (4) is also the opposite of what is stated (wizards "hog 45 percent of the growth in market cap").

Step 3: Final Answer:
"Wizard" industries are characterized by a higher concentration of economic profit and research and development.

Quick Tip

To answer questions about the characteristics of a specific concept, collect all the descriptive phrases and facts associated with that concept throughout the text. Then, match this collection of details with the given options.

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41. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.

1. **In drought conditions, water often depletes in the topsoil and remains accessible only in the deeper subsoil layers.**
2. **A new study gives new insights into how the acid changes root growth angles to enable plants to reach out deeper subsoils in search of water.**
3. **Plants rely on their root systems, the primary organs for interacting with soil, to actively seek water.**
4. **Abscisic acid plays an important role in helping plants adapt to these challenging conditions.**

Correct Answer: 3142

Solution:

Step 1: Understanding the Concept:
This is a paragraph jumble (parajumble) question. The goal is to arrange the sentences in a

logical sequence to form a coherent paragraph. We should look for an introductory sentence, logical links (cause-effect, problem-solution), and a concluding statement.

Step 2: Detailed Explanation:

- **Sentence 3** is the most general and introductory statement. It introduces the main topic: how plants use their root systems to find water. This is the best starting point.
- **Sentence 1** logically follows Sentence 3. It specifies a problem related to water-seeking: drought conditions. It sets up a specific challenge that the plants need to overcome.
- **Sentence 4** introduces a key element in the plant's response to this challenge: Abscisic acid. It states the function of this acid in adapting to "these challenging conditions" (referring to the drought mentioned in Sentence 1).
- **Sentence 2** provides the specific detail of how the acid works, linking back to Sentence 4. It explains that a "new study" shows *how* the acid helps plants reach deeper water, which directly elaborates on the role of Abscisic acid mentioned in Sentence 4.

The logical flow is: General statement about roots (3) → Specific problem of drought (1) → Introduction of the solution agent, an acid (4) → Specifics of how the acid works (2).
Therefore, the correct sequence is 3142.

Step 3: Final Answer:
The correct sequence is 3142.

Quick Tip

In parajumbles, look for a "general to specific" structure. Start with the broadest statement, then narrow down to a specific problem, then introduce a solution, and finally, provide details about that solution.

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42. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.

1. **Among its major urban centres, Harappa and Mohenjo-Daro stand out as prime examples of this architectural prowess, revealing large public structures, residential areas, and sophisticated water management systems indicative of a complex societal structure.**
2. **Showing remarkable sophistication for its time, this ancient culture developed meticulously planned cities, complete with advanced sanitation systems and intricate grid layouts that underscore their profound understanding of urban design and engineering.**

3. **Economically, the civilization thrived on a foundation of extensive trade networks, connecting them with distant lands, alongside a robust agricultural system that sustained their large populations and facilitated surplus production.**
4. **Despite its impressive achievements and longevity, the reasons behind the eventual decline of this remarkable civilization remain largely enigmatic, prompting ongoing research and speculation among historians and archaeologists.**
5. **The Indus Valley Civilization, flourishing in the Bronze Age, represents one of humanity's earliest urban societies, evidenced by archaeological discoveries dating back thousands of years.**

Correct Answer: 52134

Solution:

Step 1: Understanding the Concept:
This is a parajumble question. We need to arrange the five sentences about the Indus Valley Civilization into a logical and coherent paragraph.

Step 2: Detailed Explanation:

- **Sentence 5** is the ideal introductory sentence. It introduces the subject—the Indus Valley Civilization—and provides broad context (Bronze Age, earliest urban societies).
- **Sentence 2** naturally follows Sentence 5. It begins to describe the achievements of "this ancient culture" (referring back to the IVC) in general terms, focusing on its planned cities and engineering.
- **Sentence 1** provides specific examples for the general claim made in Sentence 2. Sentence 2 mentions "meticulously planned cities," and Sentence 1 names two of "its major urban centres," Harappa and Mohenjo-Daro, as examples of this "architectural prowess."
- **Sentence 3** shifts the focus from urban planning to another aspect of the civilization: its economy. This is a logical progression, broadening the description of the civilization's achievements.
- **Sentence 4** serves as the concluding sentence. It moves from the achievements to the decline of the civilization, a common way to conclude a historical summary. The phrase "Despite its impressive achievements" links back to all the previous points (2, 1, 3).

The logical flow is: Introduction (5) $\rightarrow$ General achievement in urban planning (2) $\rightarrow$ Specific examples of urban planning (1) $\rightarrow$ Another achievement in economics (3) $\rightarrow$ Conclusion about its decline (4).
Therefore, the correct sequence is 52134.

Step 3: Final Answer:
The correct sequence is 52134.

Quick Tip

Look for pronoun references and demonstrative adjectives. "This ancient culture" in sentence 2 must refer to something mentioned just before, making sentence 5 the logical predecessor. Similarly, "this architectural prowess" in sentence 1 refers back to the engineering mentioned in sentence 2.

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43. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.

1. **Seven of the ten worst affected countries (including India) are low- and middle-income countries.**
2. **Between 1993 and 2022, India was the sixth worst-affected country in terms of fatalities and damage sustained from extreme weather events wrought by the climate crisis.**
3. **High income nations, whose economies are founded in industrial era use of fossil fuels, meanwhile, insist that growing economies, especially India and China, shoulder greater responsibility.**
4. **This reinforces the developing world's contention that it has had to bear a disproportionate burden of climate afflictions despite having contributed little to the crisis.**

Correct Answer: 2143

Solution:

Step 1: Understanding the Concept:
This is a parajumble question about the disproportionate impact of the climate crisis on developing nations. We need to find the most logical sequence for the four sentences.

Step 2: Detailed Explanation:

- **Sentence 2** is a good starting point. It provides a specific, factual statement about India's vulnerability to the climate crisis over a defined period.
- **Sentence 1** logically follows Sentence 2. Sentence 2 focuses on India specifically. Sentence 1 broadens this focus by placing India in the context of other "low- and middle-income countries," showing that this is a widespread issue among developing nations.
- **Sentence 4** builds directly on the idea presented in sentences 2 and 1. The fact that India (2) and other developing countries (1) are the worst affected "reinforces the developing world's contention that it has had to bear a disproportionate burden." The word "This" in sentence 4 clearly refers to the facts presented in 2 and 1.

- **Sentence 3** introduces a contrast and the position of "High income nations." It presents the opposing view, which is a common way to structure an argument after establishing one's own position. The word "meanwhile" signals this contrast. Placing it last creates a powerful concluding point highlighting the conflict in global climate politics.

The logical flow is: Specific fact about India (2) → Generalization to developing countries (1) → Conclusion about the disproportionate burden (4) → Contrasting view of developed nations (3).
Therefore, the correct sequence is 2143.

Step 3: Final Answer:
The correct sequence is 2143.

Quick Tip

In argumentative paragraphs, a common structure is: present evidence (2, 1), draw a conclusion from that evidence (4), and then present the counter-argument (3). Look for connecting words like "This" and "meanwhile" to find these links.

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44. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.

1. **Using the wonders of Artificial Intelligence (AI), they quickly improved upon those skills to become far more dexterous.**
2. **Inside a robotics laboratory of the Toyota Research Institute, a group of robots is busy cooking. There is nothing special about that; robotic chefs have been around for a while.**
3. **Despite their extraordinary culinary capabilities, these robots are not destined for a career in catering.**
4. **But these robots are more proficient than most: flipping pancakes, slicing vegetables, and making pizzas with ease.**
5. **The difference is that instead of being laboriously programmed to carry out their tasks, the Toyota robots have been taught only a basic set of skills.**

Correct Answer: 24513

Solution:

Step 1: Understanding the Concept:
This parajumble describes a group of cooking robots. We need to arrange the sentences to create a logical narrative that introduces the robots, explains what makes them special, and

concludes with their purpose.

Step 2: Detailed Explanation:

- **Sentence 2** is the clear introduction. It sets the scene ("Inside a robotics laboratory...") and introduces the subject ("a group of robots is busy cooking"). It also notes that this fact alone isn't special.
- **Sentence 4** logically follows. The word "But" signals a contrast to the end of sentence 2 ("There is nothing special about that"). Sentence 4 explains what *is* special: "But these robots are more proficient than most..." and gives examples.
- **Sentence 5** explains the reason for their proficiency, which was introduced in sentence 4. "The difference is..." directly addresses why these robots are more proficient than others. It explains their unique learning method (taught basic skills, not laboriously programmed).
- **Sentence 1** builds on sentence 5. Sentence 5 says they were taught a "basic set of skills." Sentence 1 explains how they improved from there: "Using... AI, they quickly improved upon those skills...". "Those skills" clearly refers to the basic skills from sentence 5.
- **Sentence 3** is the concluding sentence. After describing their amazing abilities, it provides a twist, stating that despite these skills, they have a different purpose ("not destined for a career in catering"). This is a natural way to end the description.

The logical sequence is 2-4-5-1-3.

Step 3: Final Answer:
The correct sequence is 24513.

Quick Tip

Look for contrast words like "But" and "Despite," and linking phrases like "The difference is..." and "those skills." These create a strong logical chain that helps to piece the paragraph together.

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45. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.

1. **When we take time to notice these moments, we discover hidden beauty that sparks our creative thoughts because creativity isn't just about rare, amazing events—it's also about finding the special in the ordinary.**
2. **Creativity is often seen as the ability to look at the world in a new way—to turn everyday sights, sounds, and experiences into art or ideas.**
3. **In fact, inspiration can come from small details of daily life: the gentle warmth of morning sunlight on a kitchen counter, the steady sound of traffic outside, or the brief smile of a stranger on a busy street.**

4. **Many people wrongly think that true creativity only comes from big ideas or exciting adventures.**

Correct Answer: 2431

Solution:

Step 1: Understanding the Concept:
This is a parajumble question about the nature of creativity. The sentences present a common misconception and then contrast it with a different perspective.

Step 2: Detailed Explanation:

- **Sentence 2** is a good, general definition of creativity that can serve as the topic sentence for the paragraph.
- **Sentence 4** follows sentence 2 by introducing a common misconception that contrasts with the definition. Sentence 2 defines creativity as turning "everyday" experiences into art, while sentence 4 presents the wrong idea that it only comes from "big ideas or exciting adventures." This contrast is a strong logical link.
- **Sentence 3** directly refutes the misconception in sentence 4. The phrase "In fact" signals this refutation. Sentence 4 says creativity doesn't come from small things, and sentence 3 says, "In fact, inspiration can come from small details of daily life," providing specific examples.
- **Sentence 1** serves as the conclusion. It synthesizes the ideas from the previous sentences. "These moments" refers to the small details mentioned in sentence 3. It then explains the broader principle: "creativity... is also about finding the special in the ordinary," which echoes the initial definition in sentence 2 and summarizes the argument against the misconception in sentence 4.

The logical flow is: Definition of creativity (2) $\rightarrow$ Misconception about creativity (4) $\rightarrow$ Refutation with examples (3) $\rightarrow$ Concluding summary (1).
Therefore, the correct sequence is 2431.

Step 3: Final Answer:
The correct sequence is 2431.

Quick Tip

Paragraphs are often structured as "Argument - Counterargument - Refutation - Conclusion." Identifying this pattern can be a powerful strategy for solving parajumbles. Here, (2) is the argument, (4) is the counterargument/misconception, (3) is the refutation, and (1) is the conclusion.