IPMAT 2025 Question Paper with Solutions

Time Allowed :2 Hours	**Maximum Marks :**360	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper is divided into three sections:
   - **Quantitative Ability (SA):** 15 questions (15 questions × 4 mark) for a total of 60 marks.
   - **Quantitative Ability (MCQ):** This section comprises of total 30 questions for 120 marks.
   - **Verbal Ability:** This section comprises of total 45 questions for 180 marks.
2. The total number of questions is **90**, carrying a maximum of **360 marks**.
3. The duration of the exam is **2 hours**.
4. Marking scheme:
   - For Quantitative Aptitude (MCQ) and Verbal Ability, 1 mark will be deducted for every incorrect response.
   - No negative marking for Quantitative Aptitude (SA).
   - No marks will be awarded for unanswered questions.
5. Follow the instructions provided during the exam for submitting your answers.

Quantitative Ability SA

1. A circle of radius 13 cm touches the adjacent sides AB and BC of a square ABCD at M and N, respectively. If AB = 18 cm and the circle intersects the other two sides CD and DA at P and Q, respectively, then the area, in sq. cm, of triangle PMD is

Correct Answer: 153

Solution:

Step 1: Understanding the Question and Setting up Coordinates:
We are given a square ABCD with side length 18 cm. A circle with radius 13 cm is placed such that it is tangent to two adjacent sides, AB and BC. It also intersects the other two sides, CD and DA. We need to find the area of the triangle formed by points P, M, and D.
Let's set up a coordinate system with the vertex B at the origin (0,0).

The vertices of the square will be:
B = (0, 0)
A = (0, 18)
C = (18, 0)
D = (18, 18)
The sides of the square are on the lines: AB is on x=0, BC is on y=0, CD is on x=18, and DA is on y=18.

Step 2: Finding the Equation of the Circle:
The circle touches side AB (line x=0) and side BC (line y=0). The center of the circle (h, k) must be at a distance of the radius (r=13) from both axes. Since the side of the square is 18 cm, the center (13, 13) is inside the square.
So, the center of the circle is O = (13, 13).
The equation of the circle is $(x-h)^2 + (y-k)^2 = r^2$, which is:

$$(x-13)^2 + (y-13)^2 = 13^2 = 169$$

Step 3: Finding the Coordinates of M, P, and D:
Point M: M is the point of tangency on side AB (x=0).
Substitute x=0 into the circle's equation:

$$(0-13)^2 + (y-13)^2 = 169$$
$$169 + (y-13)^2 = 169$$
$$(y-13)^2 = 0 \implies y = 13$$

So, the coordinates of M are (0, 13).

Point P: P is the intersection point on side CD (x=18).
Substitute x=18 into the circle's equation:

$$(18-13)^2 + (y-13)^2 = 169$$
$$5^2 + (y-13)^2 = 169$$
$$25 + (y-13)^2 = 169$$
$$(y-13)^2 = 144 \implies y - 13 = \pm 12$$

So, y = 13 + 12 = 25 or y = 13 - 12 = 1.
Since P lies on the segment CD, its y-coordinate must be between 0 and 18. So, we choose y=1.
The coordinates of P are (18, 1).

Point D: D is a vertex of the square.
The coordinates of D are (18, 18).

Step 4: Calculating the Area of Triangle PMD:
The vertices of the triangle are P(18, 1), M(0, 13), and D(18, 18).
We can take the segment PD as the base of the triangle. Since both P and D have the same x-coordinate (x=18), the base is a vertical line.

Length of base PD = Difference in y-coordinates = $|18 - 1| = 17$ cm.
The height of the triangle corresponding to the base PD is the perpendicular distance from point M(0, 13) to the line x=18.
Height = Difference in x-coordinates = $|18 - 0| = 18$ cm.
The area of a triangle is given by the formula:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area of } \triangle PMD = \frac{1}{2} \times 17 \times 18 = 17 \times 9 = 153$$

The area of triangle PMD is 153 sq. cm.

Quick Tip

For geometry problems involving shapes and circles, setting up a coordinate system is often the most effective strategy. Place one vertex at the origin (0,0) to simplify the equations of the lines representing the sides of the shape. Be careful to correctly identify the equations for each side (e.g., x=constant or y=constant).

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2. Monica, who is 18 years old, is one-third the age of her father. The age at which she will be half the age of her father is _____

Correct Answer: 36

Solution:

Step 1: Understanding the Question and Finding Present Ages:
We are given Monica's current age and its relation to her father's current age. We need to find Monica's age in the future when a new relationship between their ages is established.
Let Monica's current age be M and her father's current age be F.
Given: $M = 18$ years.
Also given: $M = \frac{1}{3}F$.
Substituting Monica's age, we find her father's current age:

$$18 = \frac{1}{3}F \implies F = 18 \times 3 = 54$$

So, Monica is currently 18 and her father is 54.

Step 2: Setting up the Future Ages Scenario:
Let's assume it takes 'x' years for Monica to be half her father's age.
In 'x' years:
Monica's future age will be $18 + x$.
Her father's future age will be $54 + x$.

According to the condition in the question, at that time:

$$\text{Monica's future age} = \frac{1}{2} \times \text{Father's future age}$$

$$18 + x = \frac{1}{2}(54 + x)$$

Step 3: Solving for x:
Now, we solve the equation for x.

$$2 \times (18 + x) = 54 + x$$

$$36 + 2x = 54 + x$$

$$2x - x = 54 - 36$$

$$x = 18$$

So, this will happen in 18 years.

Step 4: Final Answer:
The question asks for "The age at which she will be half the age of her father", which is Monica's age at that future time.
Monica's age $= 18 + x = 18 + 18 = 36$ years.
Let's verify: In 18 years, Monica will be 36, and her father will be $54 + 18 = 72$. 36 is indeed half of 72.

Quick Tip

In age-related problems, always clearly define variables for the current ages first. Then, for future or past scenarios, add or subtract the time difference ('x' years) to the current ages. The difference in ages between two people always remains constant. Here, the difference is $54 - 18 = 36$ years. When Monica is half her father's age, if her age is A, his is 2A. The difference is $2A - A = A = 36$. So, Monica's age will be 36. This is a quick way to solve.

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3. Five teams—A, B, C, D, and E—each consisting of 15 members, are going on expeditions to five different locations. Each team includes members from three different skill sets: biologists, geologists, and explorers. However, the number of members from each skill set varies by team and each member has only one specialty. The total number of biologists, geologists, and explorers are equal.
The following additional information is available
• Every team has at least 2 members from each of the three skill sets.
• Teams C and D have 6 biologists each, and Team A has 6 geologists.
• Every team except A has more biologists than explorers.
• The number of explorers in each team is distinct and decreases in the order A, B, C, D, and E.

The number of biologists in team E is _____

Correct Answer: 4

Solution:

Step 1: Understanding the Question and Initial Setup:
We have 5 teams (A, B, C, D, E), each with 15 members. Total members = 5 x 15 = 75.
The members are from three skill sets: Biologists (B), Geologists (G), and Explorers (E).
The total number of members in each skill set is equal: Total B = Total G = Total E = 75 / 3 = 25.
Let's create a table to organize the information, where B_i, G_i, E_i are the number of members in team i.
For each team i, $B_i + G_i + E_i = 15$ and $B_i, G_i, E_i \geq 2$.

Step 2: Decoding the Constraints:
1. $B_C = 6$, $B_D = 6$, $G_A = 6$.
2. For teams B, C, D, E: $B_i > E_i$.
3. The number of explorers $(E_A, E_B, E_C, E_D, E_E)$ are distinct integers, and $E_A > E_B > E_C > E_D > E_E \geq 2$.

Step 3: Finding the Number of Explorers in Each Team:
From constraint 2, for Team C: $B_C > E_C \implies 6 > E_C$. So, $E_C \in \{2, 3, 4, 5\}$.
For Team D: $B_D > E_D \implies 6 > E_D$. So, $E_D \in \{2, 3, 4, 5\}$.
From constraint 3, we know $E_C > E_D > E_E \geq 2$. This puts tight restrictions on these values.
From Team A: $B_A + G_A + E_A = 15 \implies B_A + 6 + E_A = 15 \implies B_A + E_A = 9$. Since $B_A \geq 2$, we have $E_A \leq 7$.
We have a sequence of 5 distinct integers $E_A > E_B > E_C > E_D > E_E \geq 2$. Their sum must be 25.
Also, $E_A \leq 7$ and $E_C \leq 5$.
Let's try to construct this sequence. Let the sequence be e_1, e_2, e_3, e_4, e_5. We know $e_1 \leq 7$ and $e_3 \leq 5$. If $e_3 = 5$, then e_4 could be 4, and e_5 could be 3 or 2.
Let's test the combination: $E_C = 5, E_D = 4, E_E = 3$. This fits $E_C > E_D > E_E \geq 2$ and $6 > E_C, 6 > E_D$. Now we need $E_A > E_B > 5$ with $E_A \leq 7$. The only possibility is $E_B = 6$ and $E_A = 7$. Let's check the sum of this unique sequence of explorer numbers: E = (7, 6, 5, 4, 3). Sum = 7 + 6 + 5 + 4 + 3 = 25. This matches the total number of explorers. So, this is the correct distribution.
$E_A = 7, E_B = 6, E_C = 5, E_D = 4, E_E = 3$.

Step 4: Completing the Table to Find B_E:
We can now fill in more details in our table.

Team	Biologists (B)	Geologists (G)	Explorers (E)
A	B_A	6	7
B	B_B	G_B	6
C	6	G_C	5
D	6	G_D	4
E	B_E	G_E	3
Total	**25**	**25**	**25**

From Team B's row: $B_B + G_B + 6 = 15 \implies B_B + G_B = 9$. We also know $B_B > E_B$, so $B_B > 6$. Since $G_B \geq 2$, B_B can be at most 7. Thus, $B_B = 7$ and $G_B = 2$.

Now use the total for Biologists: $B_A + B_B + B_C + B_D + B_E = 25$.
First, find B_A: From $B_A + E_A = 9$, we have $B_A + 7 = 9 \implies B_A = 2$.
Now, substitute the known biologist numbers into the sum:

$$2 + 7 + 6 + 6 + B_E = 25$$

$$21 + B_E = 25$$

$$B_E = 4$$

The number of biologists in team E is 4.
We can also find the remaining values for completeness: $G_C = 15-6-5 = 4$. $G_D = 15-6-4 = 5$. $G_E = 15 - 4 - 3 = 8$. Total Geologists: $6 + 2 + 4 + 5 + 8 = 25$. The table is consistent.

Quick Tip

For complex logical reasoning problems with multiple constraints, creating a table is the best approach. Start by filling in the direct information, then use the constraints to deduce other values. The most restrictive constraints (like the ordered, distinct numbers for explorers) are often the key to unlocking the entire puzzle.

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4. If $log_3(x^2 - 1)$, $log_3(2x^2 + 1)$ and $log_3(6x^2 + 3)$ are the first three terms of an arithmetic progression, then the sum of the next three terms of the progression is

Correct Answer: 15

Solution:

Step 1: Understanding the Condition for an Arithmetic Progression (AP):
If three terms a, b, and c are in an AP, then the middle term is the arithmetic mean of the other two, i.e., $2b = a + c$.
Let $a = \log_3(x^2 - 1)$, $b = \log_3(2x^2 + 1)$, and $c = \log_3(6x^2 + 3)$.
Applying the AP condition:

$$2\log_3(2x^2 + 1) = \log_3(x^2 - 1) + \log_3(6x^2 + 3)$$

Step 2: Using Logarithm Properties to Solve for x:
Using the properties $n\log(a) = \log(a^n)$ and $\log(a) + \log(b) = \log(ab)$:

$$\log_3((2x^2+1)^2) = \log_3((x^2-1)(6x^2+3))$$

Since the logarithms are equal, their arguments must be equal:

$$(2x^2+1)^2 = (x^2-1)(6x^2+3)$$
$$4x^4 + 4x^2 + 1 = 6x^4 + 3x^2 - 6x^2 - 3$$
$$4x^4 + 4x^2 + 1 = 6x^4 - 3x^2 - 3$$

Rearranging the terms to form a quadratic-like equation:

$$2x^4 - 7x^2 - 4 = 0$$

Let $y = x^2$. The equation becomes $2y^2 - 7y - 4 = 0$.
Factoring the quadratic: $2y^2-8y+y-4 = 0 \implies 2y(y-4)+1(y-4) = 0 \implies (2y+1)(y-4) = 0$.
The solutions for y are $y = 4$ or $y = -1/2$.
Since $y = x^2$, y must be non-negative. So, $x^2 = 4$.
For the logarithms to be defined, $x^2 - 1 > 0$, which means $x^2 > 1$. Our solution $x^2 = 4$ satisfies this condition.

Step 3: Finding the Terms of the AP and the Sum:
Now, substitute $x^2 = 4$ back into the expressions for the terms:
Term 1: $\log_3(4-1) = \log_3(3) = 1$.
Term 2: $\log_3(2(4)+1) = \log_3(9) = 2$.
Term 3: $\log_3(6(4)+3) = \log_3(27) = 3$.
The AP is 1, 2, 3, ... with a common difference $d = 1$.
The next three terms are the 4th, 5th, and 6th terms.
Term $4 = 3 + 1 = 4$.
Term $5 = 4 + 1 = 5$.
Term $6 = 5 + 1 = 6$.

Step 4: Final Answer:
The sum of the next three terms is:

$$4 + 5 + 6 = 15$$

Quick Tip

When you see terms with logarithms in an AP, immediately use the property $2b = a + c$. This will allow you to combine the log terms and solve for the unknown variable by equating the arguments. Also, remember to check the domain of the logarithm functions.

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5. English exam and Math exam were conducted separately for a class of 120 students. The number of students who did not appear for the English exam is twice

the number of students who did not appear for the Math exam. The number of students who passed the Math exam is twice the number of students who appeared but failed the English exam. If the number of students who passed the English exam is twice the number of students who appeared but failed the Math exam, then the number of students who appeared but failed the English exam is _____

Correct Answer: 40

Solution:

Step 1: Define Variables:
Let the total number of students be $T = 120$.
Let N_E and N_M be the number of students who did not appear for the English and Math exams, respectively.
Let A_E and A_M be the number of students who appeared for the English and Math exams.
Let P_E and F_E be the number of students who passed and failed the English exam (among those who appeared).
Let P_M and F_M be the number of students who passed and failed the Math exam (among those who appeared).
We have the following relationships:
$A_E = T - N_E = 120 - N_E$
$A_M = T - N_M = 120 - N_M$
$A_E = P_E + F_E$
$A_M = P_M + F_M$

Step 2: Formulate Equations from the Given Information:
(i) $N_E = 2N_M$
(ii) $P_M = 2F_E$
(iii) $P_E = 2F_M$
We need to find the value of F_E.

Step 3: Establish a Relationship Between A_E and A_M:
From (i), we have $N_E = 2N_M$.
We can write $N_M = 120 - A_M$.
Substitute this into the expression for A_E:
$A_E = 120 - N_E = 120 - (2N_M) = 120 - 2(120 - A_M)$
$A_E = 120 - 240 + 2A_M$
$A_E = 2A_M - 120$

Step 4: Solve for F_E:
Now, let's express A_E and A_M in terms of F_E and F_M using the given relations.
$A_E = P_E + F_E$. Using (iii), $A_E = 2F_M + F_E$.
$A_M = P_M + F_M$. Using (ii), $A_M = 2F_E + F_M$.
Now substitute these into the relationship $A_E = 2A_M - 120$:

$$(2F_M + F_E) = 2(2F_E + F_M) - 120$$

$$2F_M + F_E = 4F_E + 2F_M - 120$$

The $2F_M$ terms on both sides cancel out:

$$F_E = 4F_E - 120$$
$$3F_E = 120$$
$$F_E = 40$$

Step 5: Final Answer:
The number of students who appeared but failed the English exam is F_E, which is 40.

Quick Tip

This problem involves many variables and relationships. The key is to systematically define each variable and write down all the given conditions as equations. Then, look for a way to connect the different sets of variables (e.g., connecting the 'appeared' students with the 'not appeared' students) to reduce the number of unknowns and solve for the required value.

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6. Five teams—A, B, C, D, and E—each consisting of 15 members, are going on expeditions to five different locations. Each team includes members from three different skill sets: biologists, geologists, and explorers. However, the number of members from each skill set varies by team and each member has only one speciality. The total number of biologists, geologists, and explorers are equal.
The following additional information is available
• Every team has at least 2 members from each of the three skill sets.
• Teams C and D have 6 biologists each, and Team A has 6 geologists.
• Every team except A has more biologists than explorers.
• The number of explorers in each team is distinct and decreases in the order A, B, C, D, and E.
The number of teams having more geologists than biologists is _____

Correct Answer: 2

Solution:

Step 1: Recap of Deduced Information from Q.3:
This question is based on the same data as Q.3. From the solution to Q.3, we have already determined the complete distribution of members in all teams. Let's reconstruct the final table.
Total members = 75.
Total Biologists = Total Geologists = Total Explorers = 25.
For each team i, $B_i + G_i + E_i = 15$ and $B_i, G_i, E_i \geq 2$.
Distribution of explorers (distinct, decreasing, sum=25): $E_A = 7, E_B = 6, E_C = 5, E_D = 4, E_E = 3$.

Given: $B_C = 6, B_D = 6, G_A = 6$.
Deduced: $B_A = 2, B_B = 7, B_E = 4$.
Deduced: $G_A = 6, G_B = 2, G_C = 4, G_D = 5, G_E = 8$.

The final table is:

Team	Biologists (B)	Geologists (G)	Explorers (E)
A	2	6	7
B	7	2	6
C	6	4	5
D	6	5	4
E	4	8	3
Total	**25**	**25**	**25**

Step 2: Identifying Teams with More Geologists than Biologists:
The question asks for the number of teams where the number of geologists is greater than the number of biologists ($G_i > B_i$). We will check this condition for each team.
Team A: Geologists (6) > Biologists (2). Yes.
Team B: Geologists (2) > Biologists (7). No.
Team C: Geologists (4) > Biologists (6). No.
Team D: Geologists (5) > Biologists (6). No.
Team E: Geologists (8) > Biologists (4). Yes.

Step 3: Final Answer:
Two teams, A and E, have more geologists than biologists.
Therefore, the answer is 2.

Quick Tip

When a set of questions is based on a single data block, solve the entire logic puzzle first and create a master table. This initial investment of time will make answering all subsequent questions much faster and more accurate. Double-check your table for consistency with all given conditions before proceeding.

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7. If $A = \begin{bmatrix} 2 & n \\ 4 & 1 \end{bmatrix}$ **such that** $A^3 = 27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix}$ **then** $p + q + r$ **equals** _____

Correct Answer: 12

Solution:

Step 1: Find the Characteristic Equation of Matrix A:

The characteristic equation of a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by $|A - \lambda I| = 0$, which simplifies to $\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0$.

Here, $A = \begin{bmatrix} 2 & n \\ 4 & 1 \end{bmatrix}$.

Trace of A, $\text{tr}(A) = 2 + 1 = 3$.

Determinant of A, $\det(A) = (2)(1) - (4)(n) = 2 - 4n$.

The characteristic equation is $\lambda^2 - 3\lambda + (2 - 4n) = 0$.

By the Cayley-Hamilton theorem, the matrix A satisfies its own characteristic equation:

$$A^2 - 3A + (2 - 4n)I = 0$$
$$A^2 = 3A - (2 - 4n)I$$

Step 2: Calculate A^3:

Multiply the equation by A:

$$A^3 = A(A^2) = A(3A - (2 - 4n)I) = 3A^2 - (2 - 4n)A$$

Now substitute the expression for A^2 again:

$$A^3 = 3(3A - (2 - 4n)I) - (2 - 4n)A$$
$$A^3 = 9A - 3(2 - 4n)I - (2 - 4n)A$$
$$A^3 = (9 - (2 - 4n))A - (6 - 12n)I$$
$$A^3 = (7 + 4n)A - (6 - 12n)I$$

The question states $A^3 = 27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix}$. For this to be a multiple of A, the term with the identity matrix I should be zero.

This means $6 - 12n = 0 \implies 12n = 6 \implies n = 1/2$.

Step 3: Finding p, q, r assuming n=1/2:

If $n = 1/2$, the equation for A^3 becomes:

$$A^3 = (7 + 4(1/2))A = (7 + 2)A = 9A$$

So, $A^3 = 9 \begin{bmatrix} 2 & 1/2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 18 & 9/2 \\ 36 & 9 \end{bmatrix}$.

The problem states $A^3 = 27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix}$. This gives $9A = 27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix}$, which means $A = 3 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix}$.

$$\begin{bmatrix} 2 & 1/2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 3q \\ 3p & 3r \end{bmatrix}$$

This leads to contradictions (e.g., 2=12). The initial assumption about the typo must be incorrect.

Let's re-evaluate the problem. Maybe there is no typo. Let's compute A^2 and A^3 directly.

$$A^2 = \begin{bmatrix} 2 & n \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & n \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4+4n & 2n+n \\ 8+4 & 4n+1 \end{bmatrix} = \begin{bmatrix} 4+4n & 3n \\ 12 & 4n+1 \end{bmatrix}.$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4+4n & 3n \\ 12 & 4n+1 \end{bmatrix} \begin{bmatrix} 2 & n \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2(4+4n)+4(3n) & n(4+4n)+1(3n) \\ 2(12)+4(4n+1) & n(12)+1(4n+1) \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 8+8n+12n & 4n+4n^2+3n \\ 24+16n+4 & 12n+4n+1 \end{bmatrix} = \begin{bmatrix} 8+20n & 4n^2+7n \\ 28+16n & 16n+1 \end{bmatrix}.$$

We are given $A^3 = 27 \begin{bmatrix} 4 & q \\ p & r \end{bmatrix} = \begin{bmatrix} 108 & 27q \\ 27p & 27r \end{bmatrix}$.

Equating the elements:

1) $8 + 20n = 108 \implies 20n = 100 \implies n = 5$.

2) $28 + 16n = 27p \implies 28 + 16(5) = 27p \implies 28 + 80 = 27p \implies 108 = 27p \implies p = 4$.

3) $4n^2 + 7n = 27q \implies 4(5^2) + 7(5) = 27q \implies 4(25) + 35 = 27q \implies 100 + 35 = 27q \implies 135 = 27q \implies q = 5$.

4) $16n + 1 = 27r \implies 16(5) + 1 = 27r \implies 80 + 1 = 27r \implies 81 = 27r \implies r = 3$.

We found consistent values: n=5, p=4, q=5, r=3.

Step 4: Final Answer:

We need to find the value of $p + q + r$.

$$p + q + r = 4 + 5 + 3 = 12$$

Quick Tip

For matrix problems, if a standard theorem (like Cayley-Hamilton) doesn't seem to fit or leads to contradictions, don't be afraid to use direct computation. Calculate the powers of the matrix step-by-step and then equate the corresponding elements. This method is straightforward and avoids potential misinterpretations of the question's intent.

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8. Five teams—A, B, C, D, and E—each consisting of 15 members, are going on expeditions to five different locations. Each team includes members from three different skill sets: biologists, geologists, and explorers. However, the number of members from each skill set varies by team and each member has only one specialty. The total number of biologists, geologists, and explorers are equal.

The following additional information is available

- **Every team has at least 2 members from each of the three skill sets.**
- **Teams C and D have 6 biologists each, and Team A has 6 geologists.**
- **Every team except A has more biologists than explorers.**
- **The number of explorers in each team is distinct and decreases in the order A, B, C, D, and E.**

The median number of biologists across five teams is _____

Correct Answer: 6

Solution:

Step 1: Recap of Deduced Information from Q.3:
This question is also based on the same data as Q.3 and Q.6. We use the completed table from the previous solutions.

Team	Biologists (B)	Geologists (G)	Explorers (E)
A	2	6	7
B	7	2	6
C	6	4	5
D	6	5	4
E	4	8	3

Step 2: Finding the Median Number of Biologists:
The question asks for the median number of biologists across the five teams.
The number of biologists in the five teams are:
Team A: 2
Team B: 7
Team C: 6
Team D: 6
Team E: 4
First, arrange these numbers in ascending order:

$$2, 4, 6, 6, 7$$

The median is the middle value in an ordered set of numbers. For a set of 5 numbers, the median is the $\frac{5+1}{2} = 3$-rd term.

Step 3: Final Answer:
The 3rd term in the ordered list is 6.
Therefore, the median number of biologists across the five teams is 6.

Quick Tip

To find the median of a set of numbers, always remember the first step is to arrange the data in ascending or descending order. For an odd number of observations (n), the median is the $(\frac{n+1}{2})^{th}$ term. For an even number of observations, it is the average of the two middle terms.

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9. If the polynomial $ax^2 + bx + 5$ leaves a remainder 3 when divided by $x - 1$, and a remainder 2 when divided by $x + 1$, then $2b - 4a$ equals _____

Correct Answer: 11

Solution:

Step 1: Understanding the Remainder Theorem:
The Remainder Theorem states that if a polynomial $P(x)$ is divided by a linear factor $(x - c)$, the remainder is $P(c)$.
Let the given polynomial be $P(x) = ax^2 + bx + 5$.

Step 2: Applying the Remainder Theorem to the Given Conditions:
Condition 1: When $P(x)$ is divided by $(x - 1)$, the remainder is 3.
According to the Remainder Theorem, this means $P(1) = 3$.

$$P(1) = a(1)^2 + b(1) + 5 = 3$$
$$a + b + 5 = 3$$
$$a + b = -2 \quad \text{(Equation 1)}$$

Condition 2: When $P(x)$ is divided by $(x + 1)$, which is $(x - (-1))$, the remainder is 2.
According to the Remainder Theorem, this means $P(-1) = 2$.

$$P(-1) = a(-1)^2 + b(-1) + 5 = 2$$
$$a - b + 5 = 2$$
$$a - b = -3 \quad \text{(Equation 2)}$$

Step 3: Solving the System of Linear Equations for a and b:
We have two linear equations with two variables:
1) $a + b = -2$
2) $a - b = -3$
Adding Equation 1 and Equation 2:

$$(a + b) + (a - b) = -2 + (-3)$$
$$2a = -5 \implies a = -5/2$$

Subtracting Equation 2 from Equation 1:

$$(a + b) - (a - b) = -2 - (-3)$$
$$2b = 1 \implies b = 1/2$$

Step 4: Final Answer:
We need to find the value of the expression $2b - 4a$.
Substitute the values of a and b we found:

$$2b - 4a = 2(1/2) - 4(-5/2)$$
$$= 1 - (-10)$$
$$= 1 + 10 = 11$$

Quick Tip

The Remainder Theorem is a powerful shortcut for problems involving polynomial division. Instead of performing long division, simply evaluate the polynomial at the root of the divisor ($x = c$ for divisor $x - c$). This quickly gives you the remainder and often leads to a system of linear equations that is easy to solve.

---

10. If m and n are two positive integers such that $7m+11n = 200$, then the minimum possible value of $m + n$ is _____

Correct Answer: 20

Solution:

Step 1: Understanding the Diophantine Equation:
We are given a linear Diophantine equation $7m + 11n = 200$, where m and n are positive integers. We need to find the pair (m, n) that minimizes the sum $m + n$.

Step 2: Finding a Particular Solution:
We can rearrange the equation to find possible values for m and n.

$$7m = 200 - 11n$$

Since m is a positive integer, $7m > 0$, which implies $200 - 11n > 0 \implies 11n < 200 \implies n < 200/11 \approx 18.18$.
So, n can take integer values from 1 to 18.
Also, $200 - 11n$ must be a multiple of 7. Let's test values of n.
We can write $200 = 7 \times 28 + 4$ and $11 = 7 \times 1 + 4$.
The equation becomes $7m = (7 \times 28 + 4) - (7 \times 1 + 4)n$.

$$7m = 7(28 - n) + 4 - 4n$$

For the left side to be a multiple of 7, the right side must also be a multiple of 7. This means $4 - 4n$ must be a multiple of 7.

$$4(1 - n) \equiv 0 \pmod 7$$

Since 4 and 7 are coprime, $(1 - n)$ must be a multiple of 7.

$$1 - n = 7k$$

for some integer k.

$$n = 1 - 7k$$

Since n is a positive integer between 1 and 18, we can find possible values for k.
If k=0, $n = 1$. Check: $4(1-1) = 0$, which is a multiple of 7. If k=-1, $n = 1-7(-1) = 8$. Check: $4(1-8) = -28$, which is a multiple of 7. If k=-2, $n = 1-7(-2) = 15$. Check: $4(1-15) = -56$,

which is a multiple of 7. If k=-3, $n = 1 - 7(-3) = 22$, which is too large.
So, the possible values for n are 1, 8, and 15.

Step 3: Finding Corresponding m values and the sum m+n:
Case 1: n = 1
$7m = 200 - 11(1) = 189 \implies m = 189/7 = 27.$
Solution (m, n) = (27, 1). Both are positive integers.
Sum $m + n = 27 + 1 = 28.$

Case 2: n = 8
$7m = 200 - 11(8) = 200 - 88 = 112 \implies m = 112/7 = 16.$
Solution (m, n) = (16, 8). Both are positive integers.
Sum $m + n = 16 + 8 = 24.$

Case 3: n = 15
$7m = 200 - 11(15) = 200 - 165 = 35 \implies m = 35/7 = 5.$
Solution (m, n) = (5, 15). Both are positive integers.
Sum $m + n = 5 + 15 = 20.$

Step 4: Final Answer:
Comparing the sums from the three possible cases (28, 24, 20), the minimum possible value of $m + n$ is 20.

Quick Tip

To minimize a sum like $m + n$ for an equation $ax + by = c$ (where a, b, c ¿ 0), you generally want to maximize the variable with the larger coefficient. In $7m + 11n = 200$, you should test the largest possible values of 'n' first, as this will lead to smaller values of 'm' and likely a smaller sum. This intuition helps you find the minimal sum faster.

---

11. If the sum of the first 21 terms of the sequence $\ln \frac{a}{b}, \ln \frac{a}{b\sqrt{b}}, \ln \frac{a}{b^2}, \ln \frac{a}{b^2\sqrt{b}}, \ldots$ is $\ln \frac{a^m}{b^n}$, then the value of $m + n$ is _____

Correct Answer: 147

Solution:

Step 1: Analyzing the Sequence
The given sequence is $\ln \frac{a}{b}, \ln \frac{a}{b\sqrt{b}}, \ln \frac{a}{b^2}, \ldots$.
Let's write out the terms using exponent notation and logarithm properties to identify the pattern.
Term 1: $T_1 = \ln\left(\frac{a}{b^1}\right) = \ln a - 1 \cdot \ln b$
Term 2: $T_2 = \ln\left(\frac{a}{b\sqrt{b}}\right) = \ln\left(\frac{a}{b^{3/2}}\right) = \ln a - \frac{3}{2}\ln b$

Term 3: $T_3 = \ln\left(\frac{a}{b^2}\right) = \ln a - 2\ln b$
The numerator 'a' is constant in all terms. The powers of the denominator 'b' are $1, \frac{3}{2}, 2, \ldots$. This is an arithmetic progression with the first term being 1 and the common difference being $\frac{1}{2}$.
The exponent of 'b' in the k-th term, let's call it p_k, is given by the formula for an AP: $p_k = 1 + (k-1)\frac{1}{2} = \frac{2+k-1}{2} = \frac{k+1}{2}$.
Therefore, the k-th term of the sequence is $T_k = \ln\left(\frac{a}{b^{(k+1)/2}}\right) = \ln a - \frac{k+1}{2}\ln b$.

Step 2: Calculating the Sum of the First 21 Terms
We need to find the sum $S_{21} = \sum_{k=1}^{21} T_k$.

$$S_{21} = \sum_{k=1}^{21} \left(\ln a - \frac{k+1}{2}\ln b\right)$$

We can split the summation into two parts:

$$S_{21} = \left(\sum_{k=1}^{21} \ln a\right) - \left(\sum_{k=1}^{21} \frac{k+1}{2}\ln b\right)$$

For the first part (summing a constant 21 times):

$$\sum_{k=1}^{21} \ln a = 21\ln a$$

For the second part:

$$\sum_{k=1}^{21} \frac{k+1}{2}\ln b = \frac{\ln b}{2}\sum_{k=1}^{21}(k+1)$$

The sum $\sum_{k=1}^{21}(k+1)$ is the sum of the arithmetic progression $2, 3, \ldots, 22$.
Using the sum formula $\frac{\text{n}}{2}$(first term + last term):
Sum $= \frac{21}{2}(2+22) = \frac{21\times 24}{2} = 252$.
So, the second part of the sum evaluates to $\frac{\ln b}{2} \times 252 = 126\ln b$.

Step 3: Combining the Parts and Finding m and n
The total sum is the difference between the two parts:

$$S_{21} = 21\ln a - 126\ln b$$

Using logarithm properties ($c\ln x = \ln x^c$ and $\ln x - \ln y = \ln(x/y)$) to write this as a single logarithm:

$$S_{21} = \ln(a^{21}) - \ln(b^{126}) = \ln\left(\frac{a^{21}}{b^{126}}\right)$$

We are given that this sum is equal to $\ln\left(\frac{a^m}{b^n}\right)$.
By comparing our result with the given form, we can determine m and n:
$m = 21$ and $n = 126$.

Step 4: Final Answer
The question asks for the value of $m + n$.

$$m + n = 21 + 126 = 147$$

Quick Tip

When dealing with a sum of logarithms, use the property $\ln x + \ln y = \ln(xy)$. However, if the terms form an arithmetic progression like in this case, it is often easier to sum the terms algebraically first. Breaking down each log term into simpler parts (e.g., $\ln(u/v) = \ln u - \ln v$) can reveal the underlying AP structure, which you can then sum using standard formulas.

---

12. Arpita and Nikita, working together, can complete an assigned job in 12 days. If Arpita works initially to complete 40% of the job, and the remaining job is completed by Nikita alone, then it takes 24 days to complete the job. The possible number of days that Nikita requires to complete the entire job, working alone, is _____

Correct Answer: 20

Solution:

Step 1: Define variables for work rates.
Let A be the number of days Arpita takes to complete the job alone. Her daily work rate is $1/A$.
Let N be the number of days Nikita takes to complete the job alone. Her daily work rate is $1/N$.
The total work is considered as 1 unit.

Step 2: Formulate equations from the given information.
Condition 1: Working together, they complete the job in 12 days.
Their combined daily work rate is $1/A + 1/N$.
So, $(1/A + 1/N) \times 12 = 1$, which gives:

$$\frac{1}{A} + \frac{1}{N} = \frac{1}{12} \quad \text{(Equation 1)}$$

Condition 2: Arpita completes 40% (0.4) of the job, and Nikita completes the remaining 60% (0.6). The total time taken is 24 days.
Time taken by Arpita to do 0.4 of the job $= 0.4 \times A$ days.
Time taken by Nikita to do 0.6 of the job $= 0.6 \times N$ days.
The total time is the sum of these two durations:

$$0.4A + 0.6N = 24 \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations.
From Equation 1, we can express A in terms of N:

$$\frac{1}{A} = \frac{1}{12} - \frac{1}{N} = \frac{N-12}{12N} \implies A = \frac{12N}{N-12}$$

Now, substitute this expression for A into Equation 2:

$$0.4\left(\frac{12N}{N-12}\right) + 0.6N = 24$$

Divide the entire equation by 0.2 to simplify:

$$2\left(\frac{12N}{N-12}\right) + 3N = 120$$

$$\frac{24N}{N-12} + 3N = 120$$

Multiply by $(N-12)$ to clear the denominator:

$$24N + 3N(N-12) = 120(N-12)$$

$$24N + 3N^2 - 36N = 120N - 1440$$

$$3N^2 - 12N = 120N - 1440$$

Rearrange into a standard quadratic equation form:

$$3N^2 - 132N + 1440 = 0$$

Divide by 3:

$$N^2 - 44N + 480 = 0$$

Factor the quadratic equation. We need two numbers that multiply to 480 and add up to 44. Let's consider factors of 480: (10, 48), (12, 40), (20, 24). The pair (20, 24) sums to 44.

$$(N-20)(N-24) = 0$$

So, the possible values for N are $N = 20$ or $N = 24$.

Step 4: Check the validity of the solutions.
We need to find the corresponding values of A for each possible N and check if they are valid (A must be positive). **Case 1: N = 20**

$$A = \frac{12(20)}{20-12} = \frac{240}{8} = 30$$

This is a valid solution. A=30, N=20. **Case 2: N = 24**

$$A = \frac{12(24)}{24-12} = \frac{288}{12} = 24$$

This is also a valid solution. A=24, N=24.
Final Answer:
The possible values for the number of days Nikita requires are 20 and 24. Since the provided answer key indicates 20, we choose 20. There might be an unstated assumption, or the question

is asking for one of the possible values.
The number of days Nikita requires to complete the entire job, working alone, is 20.

Quick Tip

In 'Work and Time' problems, always work with rates (work done per unit of time). This approach converts the problem into a system of algebraic equations. When solving, if you get a quadratic equation with two positive roots, both are mathematically valid solutions unless a specific condition in the problem rules one out.

---

13. Eight teams take part in a tournament where each team plays against every other team exactly once. In a particular year, one team got suspended after playing 3 matches, due to a disciplinary issue. The organizers decide to proceed, nonetheless, with the remaining matches. The total number of matches that were played in the tournament that year is _____

Correct Answer: 24

Solution:

Step 1: Calculate the total number of matches originally scheduled.
This is a round-robin tournament. The number of matches when n teams play each other once is given by the combination formula $\binom{n}{2} = \frac{n(n-1)}{2}$.
With 8 teams, the total number of scheduled matches was:

$$\binom{8}{2} = \frac{8 \times (8-1)}{2} = \frac{8 \times 7}{2} = 28$$

So, 28 matches were supposed to be played in total.

Step 2: Calculate the number of matches cancelled due to suspension.
Let's call the suspended team 'S'. Team S was supposed to play against 7 other teams.
Team S played 3 matches before being suspended.
This means the remaining matches that Team S was supposed to play were cancelled.
Number of cancelled matches = (Total matches for Team S) - (Matches played by Team S)

$$\text{Cancelled matches} = 7 - 3 = 4$$

Step 3: Calculate the actual number of matches played.
The total number of matches played is the originally scheduled total minus the cancelled matches.

$$\text{Total matches played} = (\text{Total scheduled matches}) - (\text{Cancelled matches})$$

$$\text{Total matches played} = 28 - 4 = 24$$

Alternative Method:
We can also calculate the matches played in two parts:

1. Matches played by the suspended team: Given as 3.
2. Matches played among the remaining 7 teams. These 7 teams play a full round-robin tournament among themselves.
Number of matches among the 7 teams = $\binom{7}{2} = \frac{7\times(7-1)}{2} = \frac{7\times6}{2} = 21$.
Total matches played = (Matches involving the suspended team) + (Matches among the other teams)

$$\text{Total matches played} = 3 + 21 = 24$$

Step 4: Final Answer:
The total number of matches played in the tournament was 24.

Quick Tip

For problems involving tournaments or pairings, the combination formula $\binom{n}{k}$ is very useful. For a round-robin tournament (everyone plays everyone else once), the number of games is $\binom{n}{2}$. When there are exceptions, it's often easiest to calculate the total and subtract the exceptions, or to calculate the parts separately and add them up.

---

14. If a, b, c are three distinct natural numbers, all less than 100, such that $|a-b|+|b-c|=|c-a|$, then the maximum possible value of b is _____

Correct Answer: 98

Solution:

Step 1: Understanding the property of the absolute value equation.
The given equation is $|a-b|+|b-c|=|c-a|$.
Let's analyze this property. Consider three points x, y, z on a number line. The distance between x and z is $|x-z|$. The distance between x and y is $|x-y|$, and the distance between y and z is $|y-z|$.
The triangle inequality states that $|x-y|+|y-z| \geq |x-z|$.
The equality $|x-y|+|y-z|=|x-z|$ holds if and only if the point y lies between x and z (inclusive).
In our case, the equation $|a-b|+|b-c|=|c-a|$ means that the number 'b' lies between 'a' and 'c' on the number line.
This gives us two possible orderings:
1. $a < b < c$
2. $c < b < a$

Step 2: Setting up the constraints and the objective.
We are given that a, b, and c are:
1. Distinct natural numbers.
2. All less than 100. So, $a, b, c \in \{1, 2, ..., 99\}$.

Our objective is to find the maximum possible value of b.

Step 3: Analyzing the orderings to maximize b.
Let's consider the two possible orderings for a, b, c.
Case 1: $a < b < c$
To maximize b, we need to choose values for a, b, c that satisfy this inequality and the given constraints.
Since b must be less than c, and c is at most 99, the maximum value b can take is 98.
If we let $b = 98$, we need to find an 'a' and a 'c' such that $a < 98 < c$.
We can choose $c = 99$ (the largest possible value for c).
We can choose any natural number 'a' that is less than 98, for example, $a = 1$.
The triplet (a, b, c) = (1, 98, 99) satisfies all conditions: - They are distinct natural numbers (1, 98, 99). - They are all less than 100 (1, 98, 99 are all ≤ 99). - The ordering $a < b < c$ is satisfied (1 ¡ 98 ¡ 99). In this case, the maximum value of b is 98.
Case 2: $c < b < a$
To maximize b, we need to choose values for a, b, c that satisfy this inequality. Since b must be less than a, and a is at most 99, the maximum value b can take is 98. If we let $b = 98$, we need to find 'c' and 'a' such that $c < 98 < a$. We can choose $a = 99$ (the largest possible value for a). We can choose any natural number 'c' that is less than 98, for example, $c = 1$. The triplet (a, b, c) = (99, 98, 1) satisfies all conditions. In this case, the maximum value of b is also 98.
Step 4: Final Answer:
In both possible scenarios, the maximum possible value for b is 98.

Quick Tip

Recognize that the equation $|x-y|+|y-z| = |x-z|$ is a standard property that implies collinearity on the number line, with 'y' lying between 'x' and 'z'. Once you understand this geometric interpretation, the problem simplifies to finding the maximum value for a number 'b' that is strictly between two other distinct numbers within a given range.

---

15. The number of factors of $3^5 \times 5^8 \times 7^2$ that are perfect squares is _____

Correct Answer: 30

Solution:

Step 1: Understanding the condition for a factor to be a perfect square.
Let the given number be $N = 3^5 \times 5^8 \times 7^2$.
A factor of N will be of the form $3^a \times 5^b \times 7^c$, where:

$$0 \leq a \leq 5$$

$$0 \leq b \leq 8$$

$$0 \leq c \leq 2$$

For this factor to be a perfect square, the exponents (a, b, and c) must all be even integers.

Step 2: Finding the possible even exponents for each prime factor.
For the prime factor 3: The exponent 'a' must be an even number between 0 and 5, inclusive. Possible values for a are: 0, 2, 4. There are 3 possible choices for a.

For the prime factor 5: The exponent 'b' must be an even number between 0 and 8, inclusive. Possible values for b are: 0, 2, 4, 6, 8. There are 5 possible choices for b.

For the prime factor 7: The exponent 'c' must be an even number between 0 and 2, inclusive. Possible values for c are: 0, 2. There are 2 possible choices for c.

Step 3: Calculating the total number of perfect square factors.
The total number of factors that are perfect squares is the product of the number of choices for each exponent. This is due to the fundamental principle of counting.
Total number of perfect square factors = (Number of choices for a) $\times$ (Number of choices for b) $\times$ (Number of choices for c)

$$\text{Total} = 3 \times 5 \times 2 = 30$$

Step 4: Final Answer:
There are 30 factors of the given number that are perfect squares.

Quick Tip

For any number $N = p_1^{e_1} \times p_2^{e_2} \times \cdots \times p_k^{e_k}$, the number of factors that are perfect squares can be found quickly. For each prime p_i, the number of choices for its exponent in a square factor is $\lfloor e_i/2 \rfloor + 1$. The total number of square factors is the product of these values: $(\lfloor e_1/2 \rfloor + 1) \times (\lfloor e_2/2 \rfloor + 1) \times \ldots$. For this problem: $(\lfloor 5/2 \rfloor + 1) \times (\lfloor 8/2 \rfloor + 1) \times (\lfloor 2/2 \rfloor + 1) = (2+1)(4+1)(1+1) = 3 \times 5 \times 2 = 30$.

Quantitative Ability MCQ

1. A circle touches the y-axis at (0, 4) and passes through the point (–2, 0). Then the radius of the circle is

(A) 5
(B) 4
(C) 7
(D) 6

Correct Answer: (A) 5

Solution:

Step 1: Understanding the Question and Properties of the Circle
We are given a circle that is tangent to the y-axis (the line $x = 0$) at the point (0, 4).
Let the center of the circle be (h, k) and its radius be r.
Since the circle is tangent to the vertical line $x = 0$ at (0, 4), the center of the circle must lie on the horizontal line passing through (0, 4). This means the y-coordinate of the center is 4. So, $k = 4$.
The radius of the circle is the perpendicular distance from the center $(h, 4)$ to the tangent line $x = 0$. This distance is $|h|$. So, $r = |h|$.

Step 2: Forming the Equation of the Circle
The general equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$.
Substituting $k = 4$ and $r^2 = h^2$, we get:

$$(x - h)^2 + (y - 4)^2 = h^2$$

Step 3: Using the Second Point to Find the Center and Radius
We are given that the circle passes through the point (–2, 0). We can substitute these coordinates into the circle's equation to solve for h.

$$(-2 - h)^2 + (0 - 4)^2 = h^2$$
$$(-(2 + h))^2 + (-4)^2 = h^2$$
$$(h + 2)^2 + 16 = h^2$$

Expanding the equation:

$$(h^2 + 4h + 4) + 16 = h^2$$
$$h^2 + 4h + 20 = h^2$$

Subtracting h^2 from both sides:

$$4h + 20 = 0$$
$$4h = -20$$
$$h = -5$$

Step 4: Final Answer
The radius of the circle is $r = |h|$.

$$r = |-5| = 5$$

Quick Tip

For problems involving circles tangent to axes, remember that the distance from the center (h, k) to the y-axis is —h— and to the x-axis is —k—. If a circle is tangent to an axis, one of these distances will be equal to the radius. Drawing a quick sketch can help visualize the geometry and prevent errors.

2. In triangle ABC, AB = AC = x, $\angle ABC = \theta$ and the circumradius is equal to y. Then $\frac{x}{y}$ equals

(A) $2\cos\theta$
(B) $2\sin\theta$
(C) $\sin\theta$
(D) $\cos\theta$

Correct Answer: (B) $2\sin\theta$

Solution:

Step 1: Understanding the Question and the Sine Rule
We are given an isosceles triangle ABC with AB = AC = x. The base angle $\angle ABC$ is θ, and the circumradius R is y. We need to find the ratio $\frac{x}{y}$.
The Sine Rule in a triangle states that the ratio of the length of a side to the sine of its opposite angle is constant and equal to twice the circumradius (2R).

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Step 2: Applying the Sine Rule to Triangle ABC
In $\triangle ABC$:
Side AC has length x. The angle opposite to side AC is $\angle ABC$, which is θ.
The circumradius is given as $R = y$.
Let's apply the Sine Rule using side AC and its opposite angle $\angle ABC$:

$$\frac{\text{length of AC}}{\sin(\angle ABC)} = 2R$$

Substituting the given values:

$$\frac{x}{\sin\theta} = 2y$$

Step 3: Finding the required ratio $\frac{x}{y}$
We need to rearrange the equation from Step 2 to solve for $\frac{x}{y}$.

$$\frac{x}{\sin\theta} = 2y$$

Divide both sides by y and multiply by $\sin\theta$:

$$\frac{x}{y} = 2\sin\theta$$

Step 4: Final Answer
The value of $\frac{x}{y}$ is $2\sin\theta$, which corresponds to option (B).

Quick Tip

The Sine Rule is a fundamental tool for solving problems involving triangles, especially when the circumradius is mentioned. Always remember the formula $\frac{a}{\sin A} = 2R$. Identify a side and its opposite angle to quickly establish a relationship with the circumradius.

---

3. The remainder when $11^{1011} + 1011^{11}$ is divided by 9 is

(A) 0
(B) 7
(C) 9
(D) 8

Correct Answer: (D) 8

Solution:

Step 1: Understanding Modular Arithmetic
We need to find the value of $(11^{1011} + 1011^{11}) \pmod 9$. This involves finding the remainders of each part separately and then adding them.

Step 2: Finding the Remainder for the First Term (11^{1011})
First, find the remainder of the base, 11, when divided by 9.

$$11 = 1 \times 9 + 2 \implies 11 \equiv 2 \pmod 9$$

So, we need to compute $2^{1011} \pmod 9$. Let's find the cyclicity of powers of 2 modulo 9.
$2^1 \equiv 2 \pmod 9$
$2^2 \equiv 4 \pmod 9$
$2^3 \equiv 8 \pmod 9$
$2^4 \equiv 16 \equiv 7 \pmod 9$
$2^5 \equiv 14 \equiv 5 \pmod 9$
$2^6 \equiv 10 \equiv 1 \pmod 9$
The cycle of remainders has a length of 6. To find the remainder of 2^{1011}, we need to find the remainder of the exponent 1011 when divided by the cycle length, 6.

$$1011 \div 6 \implies 1011 = 6 \times 168 + 3 \implies 1011 \equiv 3 \pmod 6$$

Therefore, the remainder of 2^{1011} is the same as the remainder of 2^3.

$$11^{1011} \equiv 2^{1011} \equiv 2^3 \equiv 8 \pmod 9$$

Step 3: Finding the Remainder for the Second Term (1011^{11})
First, find the remainder of the base, 1011, when divided by 9. A number's remainder when divided by 9 is the same as the remainder of the sum of its digits.
Sum of digits of $1011 = 1 + 0 + 1 + 1 = 3$.
So, $1011 \equiv 3 \pmod 9$.

Now we need to compute $3^{11} \pmod 9$.
Let's look at the powers of 3 modulo 9.
$3^1 \equiv 3 \pmod 9$
$3^2 = 9 \equiv 0 \pmod 9$
For any power $k \geq 2$, $3^k = 3^{k-2} \times 3^2 = 3^{k-2} \times 9 \equiv 0 \pmod 9$.
Since the exponent is 11, which is greater than 2, we have:

$$1011^{11} \equiv 3^{11} \equiv 0 \pmod 9$$

Step 4: Final Answer
Add the remainders of the two terms.

$$(11^{1011} + 1011^{11}) \pmod 9 \equiv (8 + 0) \pmod 9 \equiv 8 \pmod 9$$

The final remainder is 8.

Quick Tip

To find remainders with large powers ($a^b \pmod n$), first find the remainder of the base ($a \pmod n$). Then, find the repeating cycle of the powers of this new base. The exponent 'b' can be reduced by taking it modulo the cycle length. This method, known as finding cyclicity, greatly simplifies calculations.

---

4. A natural number n lies between 100 and 400, and the sum of its digits is 10. The probability that n is divisible by 4, is

(A) $\frac{7}{27}$
(B) $\frac{1}{4}$
(C) $\frac{2}{9}$
(D) $\frac{1}{3}$

Correct Answer: (A) $\frac{7}{27}$

Solution:

Step 1: Finding the Total Number of Possible Outcomes (Sample Space)
Let the 3-digit number be $n = 100h + 10t + u$.
Constraints:
1. $100 < n < 400$, so the hundreds digit h can be 1, 2, or 3.
2. $h + t + u = 10$, where $t, u \in \{0, 1, \ldots, 9\}$.
We count the number of possibilities for each value of h.
Case h = 1: $1 + t + u = 10 \implies t + u = 9$. The pairs (t, u) can be (0,9), (1,8), ..., (9,0). This gives 10 numbers.
Case h = 2: $2 + t + u = 10 \implies t + u = 8$. The pairs (t, u) can be (0,8), (1,7), ..., (8,0). This gives 9 numbers.

Case h = 3: $3 + t + u = 10 \implies t + u = 7$. The pairs (t, u) can be (0,7), (1,6), ..., (7,0). This gives 8 numbers.
Total number of possible numbers $= 10 + 9 + 8 = 27$. This is the size of our sample space.

Step 2: Finding the Number of Favorable Outcomes
A number is divisible by 4 if the number formed by its last two digits is divisible by 4. We check the lists from Step 1.
For h = 1 (t+u=9): The numbers are 109, 118, 127, **136**, 145, 154, 163, **172**, 181, 190. The numbers formed by the last two digits divisible by 4 are 36 and 72. This gives 2 favorable outcomes.
For h = 2 (t+u=8): The numbers are **208**, 217, 226, 235, **244**, 253, 262, 271, **280**. The numbers formed by the last two digits divisible by 4 are 08, 44, and 80. This gives 3 favorable outcomes.
For h = 3 (t+u=7): The numbers are 307, **316**, 325, 334, 343, **352**, 361, 370. The numbers formed by the last two digits divisible by 4 are 16 and 52. This gives 2 favorable outcomes.
Total number of favorable outcomes $= 2 + 3 + 2 = 7$.

Step 3: Calculating the Probability
Probability is the ratio of favorable outcomes to the total number of outcomes.

$$P(\text{n is divisible by 4}) = \frac{\text{Total Favorable Outcomes}}{\text{Total Possible Outcomes}} = \frac{7}{27}$$

Step 4: Final Answer
The probability is $\frac{7}{27}$.

Quick Tip

For probability questions involving number properties, first systematically list or count all possible outcomes that fit the given conditions. This is your denominator. Then, apply the specific property (e.g., divisibility rule) to this list to count the favorable outcomes (your numerator). Always be careful with the divisibility rules; for 4, it's the last two digits.

---

5. Suppose a, b and c are three real numbers such that Max(a, b, c) + Min(a, b, c) = 15, and Median(a, b, c) - Mean(a, b, c) = 2. Then the median of a, b and c is

(A) 11
(B) 9.5
(C) 10.5
(D) 10

Correct Answer: (C) 10.5

Solution:

Step 1: Define Variables based on Order
To handle Max, Min, and Median, let's assume an ordering for the three real numbers without loss of generality. Let $x \leq y \leq z$ be the three numbers a, b, and c in non-decreasing order. Then:
Min(a, b, c) = x
Median(a, b, c) = y
Max(a, b, c) = z
Mean(a, b, c) = $\frac{x+y+z}{3}$

Step 2: Formulate Equations from the Given Information
Using our ordered variables, the given conditions become:
1. $z + x = 15$
2. $y - \frac{x+y+z}{3} = 2$
Our goal is to find the median, which is y.

Step 3: Solve the System of Equations for y
Let's simplify the second equation:

$$y - \frac{x + y + z}{3} = 2$$

Multiply the entire equation by 3 to eliminate the fraction:

$$3y - (x + y + z) = 6$$
$$3y - x - y - z = 6$$

Combine the 'y' terms:

$$2y - x - z = 6$$

Rearrange the terms to group x and z:

$$2y - (x + z) = 6$$

Now, we can use the first equation, $x + z = 15$, and substitute it into this equation.

$$2y - (15) = 6$$
$$2y = 6 + 15$$
$$2y = 21$$
$$y = \frac{21}{2} = 10.5$$

Step 4: Final Answer
The median of the three numbers is y, which we found to be 10.5.

Quick Tip

When a problem involves min, max, and median of a set of numbers, it's almost always helpful to first assume an order for the numbers (e.g., $a \leq b \leq c$). This immediately translates the statistical terms into simple algebraic variables, making it much easier to set up and solve the equations.

---

6. Let A(1,3) and B(5,1) be two points. If a line with slope m intersects AB at an angle of 45°, then the possible values of m are

(A) $5, -\frac{1}{5}$
(B) $7, \frac{1}{7}$
(C) $3, \frac{1}{3}$
(D) $-3, \frac{1}{3}$

Correct Answer: (D) $-3, \frac{1}{3}$

Solution:

Step 1: Find the slope of the line segment AB
Let the slope of the line segment AB be m_{AB}. The coordinates are A(1,3) and B(5,1).
Using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$:

$$m_{AB} = \frac{1-3}{5-1} = \frac{-2}{4} = -\frac{1}{2}$$

Step 2: Use the formula for the angle between two lines
The angle θ between two lines with slopes m_1 and m_2 is given by the formula:

$$\tan\theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

In this problem, $\theta = 45°$, so $\tan\theta = \tan 45° = 1$.
Let $m_1 = m_{AB} = -1/2$ and the slope of the other line be $m_2 = m$.
Substituting the values into the formula:

$$1 = \left| \frac{m - (-\frac{1}{2})}{1 + (-\frac{1}{2})m} \right|$$

$$1 = \left| \frac{m + \frac{1}{2}}{1 - \frac{m}{2}} \right|$$

To simplify, multiply the numerator and denominator inside the absolute value by 2:

$$1 = \left| \frac{2(m + \frac{1}{2})}{2(1 - \frac{m}{2})} \right| = \left| \frac{2m+1}{2-m} \right|$$

Step 3: Solve the absolute value equation for m
The equation $|X| = 1$ implies that $X = 1$ or $X = -1$. We have two cases:

Case 1: $\frac{2m+1}{2-m} = 1$

$$2m + 1 = 2 - m$$
$$3m = 1$$
$$m = \frac{1}{3}$$

Case 2: $\frac{2m+1}{2-m} = -1$

$$2m + 1 = -(2 - m)$$
$$2m + 1 = -2 + m$$
$$m = -3$$

Step 4: Final Answer
The two possible values for the slope m are $\frac{1}{3}$ and -3. This corresponds to option (D).

Quick Tip

The formula for the angle between two lines, $\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$, is essential for this type of problem. Remember that the absolute value leads to two separate equations, which will typically give you two possible slopes for the second line.

---

7. If $y = a + b\log_e x$ then which of the following is true?

(A) $x\log_e y$ is proportional to x
(B) e^y is proportional to x^b
(C) $y - a$ is proportional to x^b
(D) $\frac{1}{y-a}$ is proportional to x^b

Correct Answer: (B) e^y is proportional to x^b *(Note: The checkmark in the provided exam paper is on option (C), which is incorrect. The correct answer is (B).)*

Solution:

Step 1: Manipulate the given equation
We are given the equation $y = a + b\log_e x$, which is the same as $y = a + b\ln x$. Our goal is to rearrange this equation to match one of the proportionality statements.

Step 2: Analyze Option (B): e^y is proportional to x^b
The statement "e^y is proportional to x^b" means that $e^y = k \cdot x^b$ for some constant k.
Let's start with the given equation and try to derive an expression for e^y.

$$y = a + b\ln x$$

Exponentiate both sides with base e:

$$e^y = e^{(a+b\ln x)}$$

Using the exponent rule $e^{u+v} = e^u \cdot e^v$:

$$e^y = e^a \cdot e^{b\ln x}$$

Now, use the logarithm power rule $k\ln z = \ln z^k$, which implies $e^{k\ln z} = z^k$.
So, $e^{b\ln x} = x^b$.
Substituting this back into our equation for e^y:

$$e^y = e^a \cdot x^b$$

Since 'a' is a constant, e^a is also a constant. Let's call this constant $k = e^a$.
The equation becomes $e^y = k \cdot x^b$. This perfectly matches the definition of proportionality. Thus, option (B) is true.

Step 3: Analyze Option (C): $y - a$ is proportional to x^b
The statement "$y - a$ is proportional to x^b" means that $y - a = k \cdot x^b$ for some constant k.
From the original equation, we can isolate $y - a$:

$$y - a = b\ln x$$

Comparing the two expressions for $y - a$, we would need $b\ln x = k \cdot x^b$. This equality does not hold for all values of x, so this statement is false.

Step 4: Final Answer
The correct derivation shows that e^y is proportional to x^b. Option (B) is the correct statement. The mark on option (C) in the source document appears to be an error.

Quick Tip

When dealing with logarithmic equations and checking for relationships, try to isolate terms or use inverse functions (like exponentiation for logarithms) to simplify the expression. Test each option by translating the proportionality statement into an equation (P is proportional to Q means $P = kQ$) and see if it can be derived from the given information.

---

8. Consider a triangle with side lengths 4 meters, 6 meters, and 9 meters. A dog runs around the triangle in such a way that the shortest distance of the dog from the triangle is exactly 1 meter. The total distance covered (in meters) by the dog in one round is

(A) $22 - 2\pi$
(B) 22
(C) $19 + 2\pi$

(D) $22 + 2\pi$

Correct Answer: (C) $19 + 2\pi$

Solution:

Step 1: Visualize the Path of the Dog
The dog maintains a constant shortest distance of 1 meter from the triangle. The path of the dog can be broken down into two types of segments:
1. **Straight segments:** When the dog is moving alongside the sides of the triangle, its path is a straight line parallel to each side, at a distance of 1 meter.
2. **Curved segments:** When the dog moves around the vertices of the triangle, its path is a circular arc to maintain the 1-meter distance.

Step 2: Calculate the Total Length of the Straight Segments
The lengths of the straight parts of the dog's path are equal to the lengths of the sides of the triangle.
Total length of straight paths = Sum of the side lengths of the triangle.

$$\text{Length}_{straight} = 4 + 6 + 9 = 19 \text{ meters}$$

Step 3: Calculate the Total Length of the Curved Segments
At each vertex, the dog traces a circular arc of radius $r = 1$ meter. The angle of each arc corresponds to the exterior angle at that vertex.
Let the interior angles of the triangle be A, B, and C. The dog turns through the exterior angles, which are $180^\circ - A$, $180^\circ - B$, and $180^\circ - C$.
The sum of the exterior angles of any convex polygon is always 360° or 2π radians.
So, the three circular arcs at the vertices, when combined, form a complete circle with radius $r = 1$ meter.
The total length of the curved paths is the circumference of this circle.

$$\text{Length}_{curved} = 2\pi r = 2\pi(1) = 2\pi \text{ meters}$$

Step 4: Final Answer
The total distance covered by the dog is the sum of the lengths of the straight and curved segments.

$$\text{Total Distance} = \text{Length}_{straight} + \text{Length}_{curved}$$

$$\text{Total Distance} = 19 + 2\pi \text{ meters}$$

This corresponds to option (C).

Quick Tip

This is a classic problem. For any convex polygon (triangle, square, etc.), the length of an outer parallel curve at a distance 'r' is the perimeter of the polygon plus the circumference of a circle with radius 'r' ($P + 2\pi r$). The specific side lengths and angles of the polygon do not affect the total length of the curved part, which is always $2\pi r$.

---

9. The set of all values of x satisfying the inequality $\log_{(x+1/2)}\left[\log_2\left(\frac{x-1}{x+2}\right)\right] > 0$ is

(A) $(-5, -2)$
(B) $(2, 5)$
(C) Null set
(D) $(5, \infty)$

Correct Answer: (C) Null set

Solution:

Step 1: Analyze the Logarithm Inequality
For an inequality $\log_b(A) > 0$, we have two cases based on the base b.

Case 1: The base is greater than 1.
Base: $x + \frac{1}{2} > 1 \implies x > \frac{1}{2}$.
In this case, for the log to be positive, the argument must be greater than 1.
Argument: $\log_2\left(\frac{x-1}{x+2}\right) > 1$.
Since the base of this inner log is 2 (which is ¿ 1), we can write:

$$\frac{x-1}{x+2} > 2^1 \implies \frac{x-1}{x+2} - 2 > 0$$
$$\frac{x-1-2(x+2)}{x+2} > 0 \implies \frac{-x-5}{x+2} > 0 \implies \frac{x+5}{x+2} < 0$$

This inequality is true for $-5 < x < -2$.
We must find the intersection of this solution with the condition for Case 1: $(-5, -2) \cap (\frac{1}{2}, \infty) = \emptyset$. There are no solutions in this case.

Case 2: The base is between 0 and 1.
Base: $0 < x + \frac{1}{2} < 1 \implies -\frac{1}{2} < x < \frac{1}{2}$.
In this case, for the log to be positive, the argument must be between 0 and 1.
Argument: $0 < \log_2\left(\frac{x-1}{x+2}\right) < 1$.
This gives us a system of two inequalities:
a) $\log_2\left(\frac{x-1}{x+2}\right) > 0 \implies \frac{x-1}{x+2} > 2^0 = 1 \implies \frac{x-1-(x+2)}{x+2} > 0 \implies \frac{-3}{x+2} > 0$.
This is true when $x + 2 < 0$, so $x < -2$.
b) $\log_2\left(\frac{x-1}{x+2}\right) < 1 \implies \frac{x-1}{x+2} < 2^1 = 2 \implies \frac{-x-5}{x+2} < 0 \implies \frac{x+5}{x+2} > 0$.
This is true when $x > -2$ or $x < -5$.
The solution for the argument is the intersection of (a) and (b): $(x < -2) \cap (x > -2 \text{ or } x < -5)$,

which gives $x < -5$.
We must find the intersection of this result with the condition for
Case 2: $(-\infty, -5) \cap (-\frac{1}{2}, \frac{1}{2}) = \emptyset$. There are no solutions in this case either.

Step 3: Consider the Domain of the Logarithms
We must also ensure that all arguments of the logarithms are positive.
1. Base of the outer log: $x + 1/2 > 0$ and $x + 1/2 \neq 1$, so $x > -1/2$ and $x \neq 1/2$.
2. Argument of the inner log: $\frac{x-1}{x+2} > 0$. This is true for $x > 1$ or $x < -2$.
3. Argument of the outer log: $\log_2\left(\frac{x-1}{x+2}\right) > 0$. As solved in 2(a), this implies $x < -2$.
To satisfy all domain conditions simultaneously, we need the intersection of $x > -1/2$, ($x > 1$ or $x < -2$), and $x < -2$. The intersection is clearly empty. For instance, we cannot have $x > -1/2$ and $x < -2$ at the same time.

Step 4: Final Answer
Since there are no values of x that satisfy the inequality in either case, and the domain itself is empty, the solution set is the null set.

Quick Tip

When solving complex logarithmic inequalities, always start by establishing the domain. Check that all bases are positive and not equal to 1, and all arguments are positive. Sometimes, the domain conditions themselves will show there are no possible solutions, saving you the work of solving the full inequality.

---

10. Let P(x) be a quadratic polynomial such that $\begin{vmatrix} P(0) & P(1) \\ P(0) & P(2) \end{vmatrix} = 0$. Let P(0) = 2 and P(1) + P(2) + P(3) = 14. Then P(4) equals

(A) -14
(B) -6
(C) 30
(D) 16

Correct Answer: (B) -6

Solution:

Step 1: Evaluate the determinant and use the given information
The given determinant equation is:

$$P(0) \cdot P(2) - P(0) \cdot P(1) = 0$$

Factor out $P(0)$:

$$P(0)[P(2) - P(1)] = 0$$

This implies that either $P(0) = 0$ or $P(2) - P(1) = 0$.
We are given that $P(0) = 2$, which is not zero. Therefore, the second condition must be true:

$$P(2) - P(1) = 0 \implies P(2) = P(1)$$

Step 2: Define the quadratic polynomial and use the conditions to find its coefficients
Let the quadratic polynomial be $P(x) = ax^2 + bx + c$.
From $P(0) = 2$:

$$P(0) = a(0)^2 + b(0) + c = 2 \implies c = 2$$

So, $P(x) = ax^2 + bx + 2$.
From $P(1) = P(2)$:

$$a(1)^2 + b(1) + 2 = a(2)^2 + b(2) + 2$$
$$a + b + 2 = 4a + 2b + 2$$
$$a + b = 4a + 2b$$
$$3a + b = 0 \implies b = -3a$$

So, $P(x) = ax^2 - 3ax + 2$.

Step 3: Use the final condition to solve for 'a'
We are given $P(1) + P(2) + P(3) = 14$. Since $P(1) = P(2)$, this is $2P(1) + P(3) = 14$.
Let's find $P(1)$ and $P(3)$ in terms of 'a'.

$$P(1) = a(1)^2 - 3a(1) + 2 = a - 3a + 2 = -2a + 2$$
$$P(3) = a(3)^2 - 3a(3) + 2 = 9a - 9a + 2 = 2$$

Now substitute these into the equation $2P(1) + P(3) = 14$:

$$2(-2a + 2) + 2 = 14$$
$$-4a + 4 + 2 = 14$$
$$-4a + 6 = 14$$
$$-4a = 8$$
$$a = -2$$

Step 4: Determine the polynomial and calculate P(4)
Now that we have $a = -2$, we can find b:

$$b = -3a = -3(-2) = 6$$

The polynomial is $P(x) = -2x^2 + 6x + 2$.
Finally, we calculate $P(4)$:

$$P(4) = -2(4)^2 + 6(4) + 2$$
$$P(4) = -2(16) + 24 + 2$$

$$P(4) = -32 + 26 = -6$$

Step 5: Final Answer
The value of P(4) is -6.

Quick Tip

For problems involving finding a polynomial, use the given conditions to create a system of equations for the coefficients. The property $P(x_1) = P(x_2)$ for a quadratic implies that the axis of symmetry is at $x = \frac{x_1+x_2}{2}$. Here, $P(1) = P(2)$, so the axis of symmetry is at $x = 1.5$. For $P(x) = ax^2 + bx + c$, the axis of symmetry is $x = -b/2a$. So, $-b/2a = 1.5 = 3/2$, which gives $-b = 3a$ or $b = -3a$, providing a quick way to find the relationship between a and b.

---

11. If $8x^2 - 2kx + k = 0$ is a quadratic equation in x, such that one of its roots is p times the other, and p, k are positive real numbers, then k equals

(A) $(p + \frac{1}{p})$
(B) $(\sqrt{p} + \frac{1}{\sqrt{p}})^2$
(C) $2(p + \frac{1}{p})$
(D) $2(\sqrt{p} + \frac{1}{\sqrt{p}})^2$

Correct Answer: (D) $2(\sqrt{p} + \frac{1}{\sqrt{p}})^2$

Solution:

Step 1: Use Vieta's formulas for the sum and product of roots
For a quadratic equation $Ax^2 + Bx + C = 0$, the sum of the roots is $-B/A$ and the product of the roots is C/A.
In our equation, $8x^2 - 2kx + k = 0$, we have $A = 8$, $B = -2k$, $C = k$.
Let the roots of the equation be α and β.
Sum of roots: $\alpha + \beta = -(-2k)/8 = 2k/8 = k/4$.
Product of roots: $\alpha\beta = k/8$.

Step 2: Use the given relationship between the roots
We are given that one root is p times the other. Let $\beta = p\alpha$.
Now substitute this relationship into the sum and product equations from Step 1.
Sum: $\alpha + p\alpha = k/4 \implies \alpha(1 + p) = k/4$. (Equation 1)
Product: $\alpha(p\alpha) = k/8 \implies p\alpha^2 = k/8$. (Equation 2)

Step 3: Solve for k in terms of p
We have a system of two equations with two unknowns (α and k). We want to eliminate α to find k.

From Equation 1, we can express α in terms of k:

$$\alpha = \frac{k}{4(1+p)}$$

Now, substitute this expression for α into Equation 2:

$$p\left(\frac{k}{4(1+p)}\right)^2 = \frac{k}{8}$$

$$p\frac{k^2}{16(1+p)^2} = \frac{k}{8}$$

Since k is a positive real number, we can divide both sides by k:

$$p\frac{k}{16(1+p)^2} = \frac{1}{8}$$

Now, solve for k:

$$k = \frac{1}{8} \cdot \frac{16(1+p)^2}{p}$$

$$k = \frac{2(1+p)^2}{p}$$

Step 4: Match the result with the given options

The expression we found for k is $k = \frac{2(1+p)^2}{p}$. Let's expand the options to see which one matches.

Option (D) is $2(\sqrt{p} + \frac{1}{\sqrt{p}})^2$.

Let's expand this:

$$2\left((\sqrt{p})^2 + 2(\sqrt{p})(\frac{1}{\sqrt{p}}) + (\frac{1}{\sqrt{p}})^2\right)$$

$$= 2\left(p + 2 + \frac{1}{p}\right)$$

To combine this into a single fraction:

$$= 2\left(\frac{p^2 + 2p + 1}{p}\right)$$

$$= \frac{2(p+1)^2}{p}$$

This matches our derived expression for k.

Step 5: Final Answer

The value of k is $2(\sqrt{p} + \frac{1}{\sqrt{p}})^2$.

Quick Tip

For quadratic equations where roots have a specific relationship (e.g., one is n times the other, or they differ by a constant), always start with Vieta's formulas. Set up the roots based on the relationship (like α and $p\alpha$), substitute them into the sum and product formulas, and then eliminate α to find the desired condition on the coefficients.

---

12. The sum of the first 5 terms of a geometric progression is the same as the sum of the first 7 terms of the same progression. If the sum of the first 9 terms is 24, then the 4th term of the progression is

(A) 24
(B) -24
(C) -48
(D) 48

Correct Answer: (B) -24

Solution:

Step 1: Analyze the first condition
Let the geometric progression (GP) have the first term 'a' and common ratio 'r'.
The sum of the first n terms is given by $S_n = \frac{a(r^n-1)}{r-1}$ (assuming $r \neq 1$).
We are given that $S_5 = S_7$.

$$\frac{a(r^5-1)}{r-1} = \frac{a(r^7-1)}{r-1}$$

Assuming $a \neq 0$ (otherwise the sum would always be 0), we can cancel a and the denominator:

$$r^5 - 1 = r^7 - 1$$
$$r^7 - r^5 = 0$$
$$r^5(r^2-1) = 0$$

This gives two possibilities: $r^5 = 0$ (so $r = 0$) or $r^2 - 1 = 0$ (so $r = 1$ or $r = -1$).
If $r = 1$, then $S_5 = 5a$ and $S_7 = 7a$. For $S_5 = S_7$, we must have $a = 0$, which is a trivial case.
If $r = 0$, the GP is $a, 0, 0, \ldots$. $S_5 = a$ and $S_7 = a$, so this works. $S_9 = a = 24$. The 4th term is 0, which is not an option.
So, the only non-trivial possibility is $r = -1$.

Step 2: Verify the condition $S_5 = S_7$ for r=-1
If $r = -1$, the GP is $a, -a, a, -a, \ldots$.
$S_5 = a - a + a - a + a = a$.
$S_7 = a - a + a - a + a - a + a = a$.
So $S_5 = S_7$ holds true for $r = -1$.

Step 3: Use the second condition to find the first term 'a'
We are given that the sum of the first 9 terms is 24.

$$S_9 = a - a + a - a + a - a + a - a + a = a$$

So, $S_9 = a = 24$.
The first term of the GP is 24.

Step 4: Find the 4th term of the progression
The n-th term of a GP is given by $T_n = ar^{n-1}$.
We need to find the 4th term, T_4.
We have $a = 24$ and $r = -1$.

$$T_4 = a \cdot r^{4-1} = 24 \cdot (-1)^3 = 24 \cdot (-1) = -24$$

Step 5: Final Answer
The 4th term of the progression is -24.

Quick Tip

The condition $S_m = S_n$ for a GP (where $m \neq n$) implies something significant about the terms between m and n. $S_n - S_m = 0$ means the sum of terms from T_{m+1} to T_n is zero. In this case, $T_6 + T_7 = 0$. This means $ar^5 + ar^6 = 0 \implies ar^5(1 + r) = 0$. Assuming $a \neq 0$ and $r \neq 0$, this forces $r = -1$.

---

13. A and B take part in a rifle shooting match. The probability of A hitting the target is 0.4, while the probability of B hitting the target is 0.6. If A has the first shot, post which both strike alternately, then the probability that A hits the target before B hits it is

(A) $\frac{9}{19}$
(B) $\frac{2}{3}$
(C) $\frac{1}{2}$
(D) $\frac{10}{19}$

Correct Answer: (D) $\frac{10}{19}$

Solution:

Step 1: Define the probabilities of success and failure for A and B
Let $P(A)$ be the probability that A hits the target. $P(A) = 0.4 = \frac{2}{5}$.
Let $P(A')$ be the probability that A misses. $P(A') = 1 - 0.4 = 0.6 = \frac{3}{5}$.
Let $P(B)$ be the probability that B hits the target. $P(B) = 0.6 = \frac{3}{5}$.
Let $P(B')$ be the probability that B misses. $P(B') = 1 - 0.6 = 0.4 = \frac{2}{5}$.

Step 2: Identify the scenarios where A can win
A wins if A hits the target before B does. Since they shoot alternately and A goes first, A can win on the 1st shot, 3rd shot, 5th shot, and so on.
Scenario 1: A wins on the 1st shot.
This happens if A hits. The probability is $P(A)$.
$P(\text{A wins on 1st}) = P(A) = \frac{2}{5}$.
Scenario 2: A wins on the 3rd shot.
This means A misses on the 1st shot, B misses on the 2nd shot, and A hits on the 3rd shot.
$P(\text{A wins on 3rd}) = P(A') \times P(B') \times P(A) = \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{12}{125}$.
Scenario 3: A wins on the 5th shot.
This means A misses (1st), B misses (2nd), A misses (3rd), B misses (4th), and A hits (5th).
$P(\text{A wins on 5th}) = P(A') \times P(B') \times P(A') \times P(B') \times P(A) = (\frac{3}{5} \times \frac{2}{5})^2 \times \frac{2}{5} = (\frac{6}{25})^2 \times \frac{2}{5}$.

Step 3: Sum the probabilities using the formula for an infinite geometric series
The total probability of A winning is the sum of the probabilities of all these disjoint scenarios.
$P(\text{A wins}) = P(A) + P(A')P(B')P(A) + P(A')P(B')P(A')P(B')P(A) + \dots$
This is an infinite geometric series with:
First term $a = P(A) = \frac{2}{5}$.
Common ratio $r = P(A') \times P(B') = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$.
The sum of an infinite geometric series is $S = \frac{a}{1-r}$.

$$P(\text{A wins}) = \frac{\frac{2}{5}}{1 - \frac{6}{25}} = \frac{\frac{2}{5}}{\frac{25-6}{25}} = \frac{\frac{2}{5}}{\frac{19}{25}}$$
$$= \frac{2}{5} \times \frac{25}{19} = \frac{2 \times 5}{19} = \frac{10}{19}$$

Step 4: Final Answer
The probability that A hits the target before B is $\frac{10}{19}$.

Quick Tip

Problems involving alternate turns often result in an infinite geometric series. Identify the event that constitutes a "round" (in this case, A misses and B misses), and its probability will be the common ratio 'r'. The probability of the first winning event is the first term 'a'. Then apply the sum formula $S = a/(1-r)$.

---

14. Two swimmers, Ankit and Bipul, start swimming from the opposite ends of a swimming pool at the same time. Ankit can cover the length of the pool once in 10 minutes. Bipul can cover the length of the pool once in 15 minutes. They swim back and forth for 80 minutes without stopping. The number of times they meet each other is

(A) 5
(B) 7
(C) 6

(D) 8

Correct Answer: (B) 7

Solution:

Step 1: Understanding the Question:
Let the length of the swimming pool be L.
Ankit's speed $(v_A) = \frac{L}{10}$ lengths per minute.
Bipul's speed $(v_B) = \frac{L}{15}$ lengths per minute.
They start from opposite ends, so they are moving toward each other initially.

Step 2: Key Formula or Approach:
When two objects move back and forth between two points starting from opposite ends:
1. The first meeting occurs when the sum of their distances is L.
2. Every subsequent meeting occurs when the additional sum of their distances is $2L$.
Time for the first meeting: $t_1 = \frac{L}{v_A+v_B}$
Time interval between subsequent meetings: $\Delta t = \frac{2L}{v_A+v_B}$

Step 3: Detailed Explanation:
Calculate the relative speed:
$v_{rel} = v_A + v_B = \frac{L}{10} + \frac{L}{15} = \frac{3L+2L}{30} = \frac{5L}{30} = \frac{L}{6}$ lengths/min.

Time for the first meeting (t_1):
$t_1 = \frac{L}{L/6} = 6$ minutes.

Time interval between subsequent meetings (Δt):
$\Delta t = \frac{2L}{L/6} = 12$ minutes.

Now, we list the meeting times M_n:
$M_1 = 6$ min
$M_2 = 6 + 12 = 18$ min
$M_3 = 18 + 12 = 30$ min
$M_4 = 30 + 12 = 42$ min
$M_5 = 42 + 12 = 54$ min
$M_6 = 54 + 12 = 66$ min
$M_7 = 66 + 12 = 78$ min
$M_8 = 78 + 12 = 90$ min (Exceeds the 80-minute limit)

The meetings occur at $t = 6, 18, 30, 42, 54, 66, 78$.
Counting these instances, we get a total of 7 meetings.

Step 4: Final Answer:
The swimmers meet each other 7 times within 80 minutes.

Quick Tip

A quick way to solve this is using the "total lengths" formula.
Total lengths covered by both $= \frac{\text{Total Time}}{T_A} + \frac{\text{Total Time}}{T_B} = \frac{80}{10} + \frac{80}{15} = 8 + 5.33 = 13.33$ lengths.
For swimmers starting at opposite ends, the n-th meeting occurs when the total distance is $(2n-1)L$.
So, $2n - 1 \leq 13.33 \implies 2n \leq 14.33 \implies n \leq 7.16$.
The maximum integer value for n is 7.

---

15. Let $S_1 = \{100, 105, 110, 115, \dots\}$ and $S_2 = \{100, 95, 90, 85, \dots\}$ be two series in arithmetic progression. If a_k and b_k are the k-th terms of S_1 and S_2, respectively, then $\sum_{k=1}^{20} a_k b_k$ equals _____

(A) 138250
(B) 137275
(C) 135375
(D) 137225
Correct Answer: 138250

Solution:

Step 1: Understanding the Question:
We are given two arithmetic progressions (APs). We need to find the sum of the products of their corresponding terms for the first 20 terms.
For S_1: First term $A = 100$, common difference $d_1 = 105 - 100 = 5$.
The k-th term $a_k = A + (k-1)d_1 = 100 + (k-1)5 = 5k + 95$.
For S_2: First term $B = 100$, common difference $d_2 = 95 - 100 = -5$.
The k-th term $b_k = B + (k-1)d_2 = 100 + (k-1)(-5) = 105 - 5k$.

Step 2: Key Formula or Approach:
We need to calculate $\sum_{k=1}^{20} a_k b_k$.
The general term of the summation is $(5k + 95)(105 - 5k)$.
Useful summation formulas:
1. $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
2. $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
3. $\sum_{k=1}^{n} C = nC$

Step 3: Detailed Explanation:
First, expand the product $a_k b_k$:
$a_k b_k = 5(k + 19) \cdot 5(21 - k) = 25(k + 19)(21 - k)$
$a_k b_k = 25(21k - k^2 + 399 - 19k) = 25(-k^2 + 2k + 399)$.
Now, apply the summation from $k = 1$ to 20:

$\sum_{k=1}^{20} a_k b_k = 25\left[-\sum_{k=1}^{20} k^2 + 2\sum_{k=1}^{20} k + \sum_{k=1}^{20} 399\right]$

Calculate the individual sums for $n = 20$:
$\sum_{k=1}^{20} k = \frac{20\times 21}{2} = 210.$
$\sum_{k=1}^{20} k^2 = \frac{20\times 21\times 41}{6} = 2870.$
$\sum_{k=1}^{20} 399 = 399 \times 20 = 7980.$

Substitute these values back into the expression:
Sum $= 25\,[-2870 + 2(210) + 7980]$
Sum $= 25\,[-2870 + 420 + 7980]$
Sum $= 25\,[8400 - 2870]$
Sum $= 25 \times 5530$
Sum $= 138250.$

Step 4: Final Answer:
The value of $\sum_{k=1}^{20} a_k b_k$ is 138250.

Quick Tip

When dealing with the product of terms from two APs, simplify the algebraic expression as much as possible before summing. Notice that $a_k b_k = 25(20+(k-1))(20-(k-1)) = 25[20^2 - (k-1)^2]$ is an alternative way to write the expression if you let $j = k-1$. Using symmetry in series often reduces calculation errors.

---

16. Let A and B be two finite sets such that n(A-B), n(A ∩ B), n(B-A) are in an arithmetic progression. Here n(X) denotes the number of elements in a finite set X. If n(A ∪ B) = 18, then n(A) + n(B) is

(A) 36
(B) 30
(C) 27
(D) 24

Correct Answer: (D) 24

Solution:

Step 1: Define variables and use set theory formulas
Let $n(A - B) = x$, $n(A \cap B) = y$, and $n(B - A) = z$.
We are given that x, y, z are in an arithmetic progression (AP). This means the middle term is the average of the other two, so $2y = x + z$.
We know the formula for the union of two sets:

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$$

Substituting our variables and the given value:

$$18 = x + z + y$$

We are asked to find $n(A) + n(B)$.
We also know:
$n(A) = n(A - B) + n(A \cap B) = x + y$
$n(B) = n(B - A) + n(A \cap B) = z + y$
So, $n(A) + n(B) = (x + y) + (z + y) = x + z + 2y$.

Step 2: Solve the system of equations
We have two equations:
1) $2y = x + z$
2) $18 = x + z + y$
Our goal is to find the value of $x + z + 2y$.
From Equation 2, we can express $x + z$ in terms of y:

$$x + z = 18 - y$$

Now substitute this expression for $x + z$ into Equation 1:

$$2y = (18 - y)$$

$$3y = 18$$

$$y = 6$$

Now that we have y, we can find $x + z$:

$$x + z = 18 - y = 18 - 6 = 12$$

Step 3: Calculate the final answer
We need to find $n(A) + n(B) = x + z + 2y$.
We have the values for $x + z$ and y.

$$n(A) + n(B) = (x + z) + 2y = 12 + 2(6) = 12 + 12 = 24$$

Alternatively, using the principle of inclusion-exclusion: $n(A) + n(B) = n(A \cup B) + n(A \cap B)$.
We found $n(A \cap B) = y = 6$, and we are given $n(A \cup B) = 18$. So, $n(A) + n(B) = 18 + 6 = 24$.
Step 4: Final Answer
The value of n(A) + n(B) is 24.

Quick Tip

For set theory problems, drawing a Venn diagram can be very helpful to visualize the relationships. The quantities $n(A - B)$, $n(B - A)$, and $n(A \cap B)$ correspond to the three disjoint regions of the two-set diagram. The key formula here is $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$.

17. If $\log_{25}[5\log_3(1+\log_3(1+2\log_2 x))] = 1/2$ **then x is**

(A) 4
(B) 16
(C) 8
(D) 2

Correct Answer: (B) 16

Solution:

Step 1: Solve the outermost logarithm
The equation is $\log_{25}[A] = 1/2$, where A is the expression in the brackets.
Using the definition of logarithm, $b^y = x \iff \log_b x = y$, we can rewrite the equation as:

$$A = 25^{1/2}$$
$$A = \sqrt{25} = 5$$

So, the expression inside the outermost logarithm is equal to 5.

$$5\log_3(1+\log_3(1+2\log_2 x)) = 5$$

Step 2: Solve the next logarithm
Divide both sides by 5:

$$\log_3(1+\log_3(1+2\log_2 x)) = 1$$

Let the argument be $B = 1+\log_3(1+2\log_2 x)$. We have $\log_3(B) = 1$.
Rewriting in exponential form:

$$B = 3^1 = 3$$

So, $1+\log_3(1+2\log_2 x) = 3$.

Step 3: Solve the next logarithm
Subtract 1 from both sides:

$$\log_3(1+2\log_2 x) = 2$$

Let the argument be $C = 1+2\log_2 x$. We have $\log_3(C) = 2$.
Rewriting in exponential form:

$$C = 3^2 = 9$$

So, $1+2\log_2 x = 9$.

Step 4: Solve the innermost logarithm for x
Subtract 1 from both sides:

$$2\log_2 x = 8$$

Divide by 2:

$$\log_2 x = 4$$

Rewriting in exponential form:

$$x = 2^4 = 16$$

Step 5: Final Answer
The value of x is 16.

Quick Tip

For nested logarithm equations, work from the outside in. Use the definition of logarithm ($\log_b x = y \implies x = b^y$) at each step to "peel away" one layer of the logarithm, simplifying the equation until you can solve for x.

---

18. Area of a regular octagon inscribed in a circle of radius 1 unit is

(A) $2\sqrt{2}$
(B) $\frac{9}{2\sqrt{2}}$
(C) $\sqrt{10}$
(D) $2+\sqrt{2}$

Correct Answer: (A) $2\sqrt{2}$

Solution:

Step 1: Understand the Geometry
A regular octagon can be divided into 8 congruent isosceles triangles, with their common vertex at the center of the circumscribed circle.
The two equal sides of each triangle are the radii of the circle, so their length is $r = 1$.
The angle at the center of the circle for each triangle is the total angle of a circle divided by the number of sides of the octagon.
Angle $\theta = \frac{360^\circ}{8} = 45^\circ$.

Step 2: Calculate the Area of One Isosceles Triangle
The area of a triangle can be calculated using the formula $Area = \frac{1}{2}ab\sin C$, where a and b are two sides and C is the included angle.
For one of our isosceles triangles, the two sides are the radii (a=1, b=1) and the included angle

is $\theta = 45°$.

$$\text{Area of one triangle} = \frac{1}{2}(1)(1)\sin(45°)$$

We know that $\sin(45°) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

$$\text{Area of one triangle} = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$$

Step 3: Calculate the Total Area of the Octagon
The total area of the octagon is 8 times the area of one of these triangles.

$$\text{Area of Octagon} = 8 \times (\text{Area of one triangle})$$

$$\text{Area of Octagon} = 8 \times \frac{\sqrt{2}}{4} = 2\sqrt{2}$$

Step 4: Final Answer
The area of the regular octagon is $2\sqrt{2}$ square units.

Quick Tip

The area of any regular n-sided polygon inscribed in a circle of radius 'r' can be found using the formula $Area = \frac{1}{2}nr^2\sin(\frac{360°}{n})$. This is a direct application of dividing the polygon into 'n' congruent triangles and using the area formula $\frac{1}{2}ab\sin C$ for each.

---

19. The number of integers greater than 5000 and divisible by 5 that can be formed with the digits 1, 3, 5, 7, 8, 9 where no digit is repeated is

(A) 240
(B) 180
(C) 120
(D) 276

Correct Answer: (D) 276

Solution:

Step 1: Understanding the Question:
We need to find the count of unique integers formed using the digits $\{1, 3, 5, 7, 8, 9\}$ such that the number is greater than 5000 and divisible by 5.
For a number to be divisible by 5, its units digit must be 0 or 5. Since 0 is not in the set, the units digit must be 5.
Since we have 6 digits, we can form numbers with 4, 5, or 6 digits that satisfy the conditions.

Step 2: Key Formula or Approach:
Apply the fundamental principle of counting (Permutations) for each case:
Total = (4-digit numbers > 5000) + (all 5-digit numbers) + (all 6-digit numbers).

Step 3: Detailed Explanation:
Case 1: 4-digit numbers
The units place is fixed as '5' (1 way).
The thousands place must be greater than or equal to 5. Since '5' is already at the units place, the thousands place can only be filled by $\{7, 8, 9\}$. So, 3 ways.
The remaining 2 places can be filled from the remaining 4 digits in ${}^4P_2 = 4 \times 3 = 12$ ways.
Total 4-digit numbers $= 3 \times 4 \times 3 \times 1 = 36$ ways.

Case 2: 5-digit numbers
Any 5-digit number formed using these digits will be greater than 5000.
The units place is fixed as '5' (1 way).
The remaining 4 places can be filled from the remaining 5 digits in ${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$ ways.

Case 3: 6-digit numbers
Any 6-digit number formed using these digits will be greater than 5000.
The units place is fixed as '5' (1 way).
The remaining 5 places can be filled from the remaining 5 digits in $5! = 120$ ways.

Step 4: Final Answer:
Total integers $= 36 + 120 + 120 = 276$.

Quick Tip

In constraints like "greater than 5000", always check for numbers with more digits than the constraint (e.g., 5-digit and 6-digit numbers), as they are automatically greater than the 4-digit limit.
Handle the most restrictive condition (divisibility by 5) first by fixing the units place.

---

20. Let $f(x) = a^2x^2 + 2bx + c$ where, $a \neq 0$, b, c are real numbers and x is a real variable then

(A) f(x) has no minimum and no maximum
(B) f(x) has a maximum and a minimum
(C) f(x) has a minimum and no maximum
(D) f(x) has a maximum and no minimum

Correct Answer: (C) f(x) has a minimum and no maximum

Solution:

Step 1: Identify the type of function
The function $f(x) = a^2x^2 + 2bx + c$ is a quadratic function, which represents a parabola.
The general form of a quadratic function is $f(x) = Ax^2 + Bx + C$.
In this case, the coefficient of the x^2 term is $A = a^2$.

Step 2: Analyze the coefficient of the x^2 term
We are given that 'a' is a real number and $a \neq 0$.
This means that the coefficient $A = a^2$ will always be a positive number. $(a^2 > 0)$.

Step 3: Relate the sign of the leading coefficient to the shape of the parabola
For a quadratic function $f(x) = Ax^2 + Bx + C$:
- If the leading coefficient A is positive $(A > 0)$, the parabola opens upwards. An upward-opening parabola has a minimum value at its vertex and no maximum value (it goes to $+\infty$).
- If the leading coefficient A is negative $(A < 0)$, the parabola opens downwards. A downward-opening parabola has a maximum value at its vertex and no minimum value (it goes to $-\infty$).

Step 4: Conclude based on the given function
In our function, the leading coefficient is $A = a^2$. Since $a \neq 0$, a^2 is strictly positive.
Therefore, the parabola opens upwards.
This means the function $f(x)$ has a minimum value and no maximum value.

Step 5: Final Answer
The function has a minimum and no maximum. This corresponds to option (C).

Quick Tip

The behavior of a quadratic function (parabola) is entirely determined by the sign of its leading coefficient (the coefficient of x^2). If it's positive, the parabola opens up (like a "U") and has a minimum. If it's negative, it opens down (like an "n") and has a maximum. Notice the trick in this question: the coefficient is a^2, which guarantees it's positive.

---

21. Anindita invests a total of 1 lakh rupees distributed across three schemes A, B and C for a period of two years. These schemes offer an interest rate of 10%, 8% and 12% per annum, respectively, each compounded annually. If the initial investment amount in scheme A is 30000 rupees and the total interest earned from all the three schemes during the first year is 10600 rupees, then the total interest earned, in rupees, from all the three schemes for the second year is _____

(A) 10308
(B) 19708
(C) 11748
(D) 22348

Correct Answer: (C) 11748

Solution:

Step 1: Understanding the Question:
We need to find the total compound interest earned in the second year. Total investment is $1,00,000$ rupees.
Scheme A investment $P_A = 30,000$.
Let investments in B and C be P_B and P_C. Thus, $P_B + P_C = 1,00,000 - 30,000 = 70,000$.
Interest rates: $r_A = 10\%$, $r_B = 8\%$, $r_C = 12\%$.

Step 2: Key Formula or Approach:
Interest for Year 1: $I = P \cdot r$
Amount after Year 1: $A = P(1 + r)$
Interest for Year 2: $I_2 = A \cdot r = P(1 + r) \cdot r$

Step 3: Detailed Explanation:
Interest earned in Year 1 from Scheme A: $0.10 \times 30,000 = 3,000$.
Total Year 1 interest is $10,600$. Interest from B and C: $10,600 - 3,000 = 7,600$.
Let $P_B = x$. Then $P_C = 70,000 - x$.
$0.08x + 0.12(70,000 - x) = 7,600$
$0.08x + 8,400 - 0.12x = 7,600$
$-0.04x = -800 \implies x = 20,000$.
So, $P_B = 20,000$ and $P_C = 50,000$.
Now calculate total interest for Year 2:
Amount in A after 1st year: $30,000 + 3,000 = 33,000$. Int $A_2 = 33,000 \times 0.10 = 3,300$.
Amount in B after 1st year: $20,000 \times 1.08 = 21,600$. Int $B_2 = 21,600 \times 0.08 = 1,728$.
Amount in C after 1st year: $50,000 \times 1.12 = 56,000$. Int $C_2 = 56,000 \times 0.12 = 6,720$.
Total interest Year $2 = 3,300 + 1,728 + 6,720 = 11,748$.

Step 4: Final Answer:
The total interest for the second year is $11,748$ rupees.

Quick Tip

Interest in the second year is simply (Rate) $\times$ (Amount at the end of the first year).
Also, Total Interest Year 2 = (Total Interest Year 1) + $\sum$(Interest on Interest).
Interest on Interest $= 3000(0.1) + P_B(0.08)^2 + P_C(0.12)^2$.

---

22. If $a_1, a_2, ..., a_8$ are the roots of the equation $x^8 + x^7 + \cdots + x + 1 = 0$, then the value of $a_1^{2025} + a_2^{2025} + \cdots + a_8^{2025}$ is

(A) 0
(B) 8
(C) 4
(D) 2

Correct Answer: (B) 8

Solution:

Step 1: Understanding the Question:
The equation $x^8 + x^7 + \cdots + x + 1 = 0$ is a geometric series.
It can be written as $\frac{x^9-1}{x-1} = 0$, where $x \neq 1$.
The roots $a_1, \ldots, a_8$ are the 9th roots of unity, excluding 1.

Step 2: Key Formula or Approach:
The n-th roots of unity are given by $\omega_k = e^{i\frac{2\pi k}{n}}$ for $k = 0, 1, \ldots, n-1$.
Here $n = 9$, and k ranges from 1 to 8 for the given roots.
We need to evaluate $\sum_{k=1}^{8} a_k^{2025}$.

Step 3: Detailed Explanation:
For each root a_k, $a_k^9 = 1$.
We check if the power 2025 is a multiple of 9:
$2025 = 9 \times 225$.
Since 2025 is a multiple of 9, then for any root a_k:
$a_k^{2025} = (a_k^9)^{225} = 1^{225} = 1$.
Thus, the sum is:
$\sum_{k=1}^{8} a_k^{2025} = \underbrace{1 + 1 + \cdots + 1}_{8 \text{ times}} = 8$.

Step 4: Final Answer:
The value of the sum is 8.

Quick Tip

If a is an n-th root of unity, then $a^m = 1$ if m is a multiple of n, and $\sum a^m = 0$ (including the root 1) if m is not a multiple of n.

---

23. The area of the triangle, formed by the straight lines $y = 0$, $12x - 5y = 0$, and $3x + 4y = 7$, is _____

(A) $\frac{28}{9}$
(B) $\frac{14}{9}$
(C) $\frac{35}{27}$
(D) $\frac{35}{54}$

Correct Answer: (B) $\frac{14}{9}$

Solution:

Step 1: Understanding the Question:
The triangle is bounded by three lines. We need to find the intersection points (vertices) and then calculate the area.
Line 1: $y = 0$ (the x-axis).
Line 2: $12x - 5y = 0$ (passes through origin).
Line 3: $3x + 4y = 7$.

Step 2: Key Formula or Approach:
Area of triangle with base on x-axis is $\frac{1}{2} \times$ Base Length $\times$ Height.

Step 3: Detailed Explanation:
Vertex A (Intersection of $y = 0$ and $12x - 5y = 0$):
$y = 0 \implies 12x = 0 \implies x = 0$. Vertex A is $(0, 0)$.
Vertex B (Intersection of $y = 0$ and $3x + 4y = 7$):
$y = 0 \implies 3x = 7 \implies x = \frac{7}{3}$. Vertex B is $(\frac{7}{3}, 0)$.
Base length $= \frac{7}{3} - 0 = \frac{7}{3}$.
Vertex C (Intersection of $12x - 5y = 0$ and $3x + 4y = 7$):
From $12x - 5y = 0$, $5y = 12x \implies y = \frac{12x}{5}$.
Substitute into $3x + 4y = 7$:
$3x + 4(\frac{12x}{5}) = 7 \implies 15x + 48x = 35 \implies 63x = 35 \implies x = \frac{35}{63} = \frac{5}{9}$.
Height (y-coordinate of C) $= \frac{12}{5} \times \frac{5}{9} = \frac{12}{9} = \frac{4}{3}$.
Area $= \frac{1}{2} \times \frac{7}{3} \times \frac{4}{3} = \frac{14}{9}$.

Step 4: Final Answer:
The area is $\frac{14}{9}$.

Quick Tip

When one side of the triangle is the x-axis, the y-coordinate of the opposite vertex is the height. This simplifies the area calculation significantly compared to using the coordinate formula.

---

24. Given that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$, the value of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$ is

(A) $\frac{\pi^2}{6} - 1$
(B) $\frac{\pi^2}{12}$
(C) $\frac{\pi^2}{8}$
(D) $\frac{\pi^2}{6}$

Correct Answer: (C) $\frac{\pi^2}{8}$

Solution:

Step 1: Understanding the Question:
We are given the sum of the series of the reciprocal of squares of all natural numbers. We need to find the sum for only the odd natural numbers.

Step 2: Key Formula or Approach:
Let $S = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.
Split S into odd and even terms: $S = S_{\text{odd}} + S_{\text{even}}$.

Step 3: Detailed Explanation:
$S_{\text{even}} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots = \sum_{n=1}^{\infty} \frac{1}{(2n)^2}$
$S_{\text{even}} = \frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{4}S$.
Substitute this into the split equation:
$S = S_{\text{odd}} + \frac{1}{4}S$
$S_{\text{odd}} = S - \frac{1}{4}S = \frac{3}{4}S$
$S_{\text{odd}} = \frac{3}{4} \times \frac{\pi^2}{6} = \frac{\pi^2}{8}$.

Step 4: Final Answer:
The value of the series is $\frac{\pi^2}{8}$.

Quick Tip

For any convergent series where even terms are a constant fraction of the whole series, the odd terms can be found by simple subtraction. Here, even terms are exactly $\frac{1}{4}$ of the total.

---

25. Which of the following numbers is divisible by $3^{10} + 2$?

(A) $3^{30} + 2$
(B) $3^{20} + 4$
(C) $3^{30} + 8$
(D) $3^{20} + 8$

Correct Answer: (C) $3^{30} + 8$

Solution:

Step 1: Understanding the Question:
We are looking for an expression that has $3^{10} + 2$ as a factor. Let $x = 3^{10}$. We want to find an expression divisible by $x + 2$.

Step 2: Key Formula or Approach:
Use the algebraic identity $a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + \cdots + b^{n-1})$ for odd n.
Specifically, $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.

Step 3: Detailed Explanation:
Let $x = 3^{10}$. The divisor is $x + 2$.
Evaluate the options in terms of x:
(A) $3^{30} + 2 = (3^{10})^3 + 2 = x^3 + 2$. Not divisible by $x + 2$.
(B) $3^{20} + 4 = (3^{10})^2 + 4 = x^2 + 4$. Not divisible by $x + 2$.
(C) $3^{30} + 8 = (3^{10})^3 + 2^3 = x^3 + 2^3$.
Using the sum of cubes formula: $x^3 + 2^3 = (x+2)(x^2 - 2x + 4)$.
Since $(x + 2)$ is a factor, $3^{30} + 8$ is divisible by $3^{10} + 2$.

Step 4: Final Answer:
Option (C) is the correct answer.

Quick Tip

For divisibility problems involving powers, substitute a variable for the common base and use polynomial remainder theorem or algebraic identities. If $P(x)$ is divisible by $x + 2$, then $P(-2) = 0$. For $x^3 + 8$, $(-2)^3 + 8 = 0$.

Comprehension:
The table given below provides the details of monthly sales (in lakhs of rupees) and the value of products returned by the customers (as a percentage of sales) of an e-commerce company for three product categories for the year 2024. Net sales (in lakhs of rupees) is defined as the difference between sales (in lakhs of rupees) and the value of products returned (in lakhs of rupees).

	Sales (in lakhs of rupees)			Value of products returned (as a percentage of sales)		
Month	Apparel	Footwear	Electronics	Apparel	Footwear	Electronics
January	262	104	289	13%	7%	2%
February	279	113	387	16%	9%	3%
March	236	121	283	20%	7%	2%
April	258	58	325	16%	8%	1%
May	249	69	359	12%	6%	4%
June	230	111	321	19%	5%	3%
July	244	119	341	17%	9%	4%
August	252	60	336	16%	6%	2%
September	288	118	355	10%	9%	5%
October	222	108	383	15%	8%	2%
November	228	93	282	14%	9%	4%
December	221	86	268	18%	10%	1%

26. Among the following four months, for which month the value of the Footwear returned (in lakhs of rupees) was the highest?

(A) March
(B) July
(C) June
(D) September

Correct Answer: (B) July

Solution:

Step 1: Understanding the Question:
Value returned = Sales × Return %. We calculate this for Footwear in the specified months.

Step 3: Detailed Explanation:
March: $121 \times 7\% = 8.47$ lakhs.
June: $111 \times 5\% = 5.55$ lakhs.
July: $119 \times 9\% = 10.71$ lakhs.
September: $118 \times 9\% = 10.62$ lakhs.
The highest value is in July (10.71).

Step 3: Final Answer:
The value of Footwear returned was highest in July.

Quick Tip

To save time, compare products by mentally rounding. July has higher sales than September and the same return percentage, so July's returns must be higher.

---

27. By what percentage the net sales for June increased as compared to May in the Footwear category?

(A) 7.21 percent
(B) 18.97 percent
(C) 62.58 percent
(D) 60.87 percent

Correct Answer: (C) 62.58 percent

Solution:

Step 1: Understanding the Question:
Net Sales = Sales $\times(1 - \text{Return \%})$. We compare June and May.

Step 2: Detailed Explanation:
May Footwear: Net Sales $= 69 \times (1 - 0.06) = 69 \times 0.94 = 64.86$.
June Footwear: Net Sales $= 111 \times (1 - 0.05) = 111 \times 0.95 = 105.45$.
Percentage increase $= \frac{105.45 - 64.86}{64.86} \times 100$
$= \frac{40.59}{64.86} \times 100 \approx 62.58\%$.

Step 3: Final Answer:
The net sales increased by 62.58%.

Quick Tip

Percentage increase $= \left(\frac{\text{Final}}{\text{Initial}} - 1\right) \times 100$. Estimating: $105/65 \approx 1.6$, which implies a 60% increase. Only options (C) and (D) are close.

---

28. Which month had highest percentage decline in monthly sales as compared to previous month for the Apparel category?

(A) October
(B) June
(C) March
(D) December

Correct Answer: (A) October

Solution:

Step 1: Understanding the Question:
Decline % $= \frac{\text{Prev Month Sales} - \text{Curr Month Sales}}{\text{Prev Month Sales}} \times 100$ for Apparel.

Step 2: Detailed Explanation:
March: from Feb (279 to 236): $\frac{279 - 236}{279} = \frac{43}{279} \approx 15.4\%$.
June: from May (249 to 230): $\frac{249 - 230}{249} = \frac{19}{249} \approx 7.6\%$.
October: from Sep (288 to 222): $\frac{288 - 222}{288} = \frac{66}{288} \approx 22.9\%$.
December: from Nov (228 to 221): $\frac{228 - 221}{228} \approx 3.1\%$.
The highest decline is in October.

Step 3: Final Answer:
October had the highest percentage decline.

Quick Tip

Comparing the fractions: 66/288 is significantly larger than 43/279 or 19/249. Always estimate fractions before doing precise division.

---

29. For which categories the value of the products returned (as a percentage of sales) increased for three consecutive months?

(A) Only Footwear
(B) Both Apparel and Footwear
(C) Only Apparel
(D) Only Electronics

Correct Answer: (B) Both Apparel and Footwear

Solution:

Step 1: Understanding the Question:
We need to find categories where the return percentage increased for at least 3 months in a row ($M_1 < M_2 < M_3$).

Step 2: Detailed Explanation:
Apparel: Jan (13%), Feb (16%), Mar (20%). This is a sequence of increases over 3 consecutive months.
Footwear: Oct (8%), Nov (9%), Dec (10%). This is also a sequence of increases over 3 consecutive months.
Electronics: Jan(2%), Feb(3%), Mar(2%)[Dec]; Apr(1%), May(4%), June(3%)[Dec]. No 3-month increasing streak.
Thus, both Apparel and Footwear categories satisfy the condition.

Step 3: Final Answer:
The correct option is Both Apparel and Footwear.

Quick Tip

Consecutive months means a streak like month 1 $\rightarrow$ month 2 $\rightarrow$ month 3. Here, the values are 13-16-20 and 8-9-10.

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30. Among the following four months, for which month the contribution of the Apparel category in the total monthly sales was the highest?

(A) January
(B) April
(C) August
(D) December

Correct Answer: (B) April

Solution:

Step 1: Understanding the Question:
Contribution % = $\frac{\text{Apparel Sales}}{\text{Total Sales}} \times 100$. Total Sales = Apparel + Footwear + Electronics.

Step 2: Detailed Explanation:
January: $\frac{262}{262+104+289} = \frac{262}{655} = 40.0\%$.
April: $\frac{258}{258+58+325} = \frac{258}{641} \approx 40.25\%$.
August: $\frac{252}{252+60+336} = \frac{252}{648} \approx 38.89\%$.
December: $\frac{221}{221+86+268} = \frac{221}{575} \approx 38.43\%$.
Highest contribution is in April.

Step 3: Final Answer:
The highest contribution was in April.

Quick Tip

April's total denominator is lower than January's while the numerator is nearly the same, which mathematically results in a higher percentage.

Verbal Ability

Comprehension:
Read the following passage and choose the answer that is closest to each of the questions that are based on the passage.

Meta is recalibrating content on its social media platforms as the political tide has turned in Washington, with Mark Zuckerberg announcing last week that his company plans to fire its US fact-checkers. Fact-checking evolved in response to allegations of misinformation and is being watered down in response to accusations of censorship. Social media does not have solutions to either. Community review — introduced by Elon Musk at X and planned by Zuckerberg for Facebook and Instagram — is not a significant improvement over fact-checking. Having Washington lean on foreign governments over content moderation does not benefit free speech. Yet, that is the nature of the social media beast, designed to amplify bias. Information and

misinformation continue to jostle on social media at the mercy of user discretion. Social media now has enough control over all other forms of media to broaden its reach. It is the connective tissue for mass consumption of entertainment, and alternative platforms are reworking their engagement with social media. Technologies are shaping up to drive this advantage further through synthetic content targeted precisely at its intended audience. Meta's algorithm will now play up politics because it is the flavour of the season. The Achilles' Heel of social media is informed choice which could turn against misinformation. Its move away from content moderation is driven by the need to be more inclusive, yet unfiltered content can push users away from social media towards legacy forms that have better moderation systems in place. Lawmakers across the world are unlikely to give social media a free run, even if Donald Trump is working on their case. Protections have already been put in place across jurisdictions over misinformation. These may be difficult to dismantle, even if the Republicans pull US-owned social media companies further to the right. Media consumption is, in essence, evidence-based judgement that mediums must adapt to. Content moderation, not free speech, is the adaptation mechanism. Musk and Zuckerberg are not exempt.

1. The writer's conclusion is that information available on social media is linked to

(A) the policies of the governments in power.
(B) the global legal systems' support of free speech.
(C) the need for deregulation.
(D) the individual's right to free speech.

Correct Answer: (A) the policies of the governments in power.

Solution:

Step 1: Understanding the Question:
The question asks for the writer's conclusion regarding what controls or influences the information found on social media platforms.

Step 2: Detailed Explanation:
The passage begins by noting that "Meta is recalibrating content... as the political tide has turned in Washington".
It highlights that content moderation is being "watered down" and that companies are adapting to political pressures.
The final section of the passage mentions that "mediums must adapt to" evidence-based judgement and that content moderation is an "adaptation mechanism".
The overarching theme is that social media platforms adjust their content policies based on the prevailing political climate and governmental pressures (e.g., "Washington lean on foreign governments", "Lawmakers across the world").
Thus, the information is linked to the policies of the governments in power.

Step 3: Final Answer:
The correct option is (A).

Quick Tip

In reading comprehension, the conclusion is often found by looking at the relationship between the opening and closing statements. Here, the mention of "political tide in Washington" at the start and "lawmakers" at the end strongly links content to government policy.

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2. The writer argues that social media

(A) is in a difficult position because it cannot adapt to new policies.
(B) remains unaffected by global debates amongst lawmakers on misinformation.
(C) has become the preferred way to access entertainment.
(D) flourishes because it can publish any material.

Correct Answer: (C) has become the preferred way to access entertainment.

Solution:

Step 1: Understanding the Question:
The question asks for a specific argument made by the writer regarding the current state or function of social media.

Step 2: Detailed Explanation:
The passage states, "Social media now has enough control over all other forms of media to broaden its reach. It is the connective tissue for mass consumption of entertainment...".
This directly supports the idea that social media has become a primary, preferred medium for accessing entertainment for the masses.
Option (A) is incorrect as the passage says mediums "must adapt".
Option (B) is incorrect as lawmakers are noted to "not give social media a free run".
Option (D) is not explicitly argued; in fact, censorship and moderation are discussed as ongoing constraints.

Step 3: Final Answer:
The correct option is (C).

Quick Tip

Look for explicit keywords like "entertainment" or "mass consumption" in the text to verify factual arguments made by the author.

---

3. Technologies are enabling social media to

(A) accept the current trends as emphasised by algorithms.
(B) readjust its interaction with competitors.
(C) enlarge its sphere of influence and persuasion.
(D) understand that algorithms cannot control its content.

Correct Answer: (C) enlarge its sphere of influence and persuasion.

Solution:

Step 1: Understanding the Question:
The question asks how technology is aiding social media platforms according to the text.

Step 2: Detailed Explanation:
The passage mentions: "Technologies are shaping up to drive this advantage further through synthetic content targeted precisely at its intended audience."
By using targeted, synthetic content, platforms can more effectively influence and persuade specific demographics, thereby enlarging their overall sphere of influence.
This is presented as an "advantage" that technology drives further.

Step 3: Final Answer:
The correct option is (C).

Quick Tip

"Targeted content" is synonymous with "persuasion" and "influence" in the context of digital media.

---

4. The writer implies that

(A) older forms of media will regain users because of their controls.
(B) social media can never be discarded by its users.
(C) social media's innate strength is the user's inability to fact check.
(D) uncensored content will always have more appeal than controlled content.

Correct Answer: (A) older forms of media will regain users because of their controls.

Solution:

Step 1: Understanding the Question:
The question asks for an implication (something not explicitly stated but suggested) by the

author.

Step 2: Detailed Explanation:
The passage states, "...unfiltered content can push users away from social media towards legacy forms that have better moderation systems in place."
"Legacy forms" refers to older, traditional media (newspapers, TV, etc.).
The writer is implying that the lack of moderation on social media might result in a migration of users back to older media types that offer better content control and moderation.

Step 3: Final Answer:
The correct option is (A).

Quick Tip

"Legacy forms" is a standard term for traditional media. The text links user movement to "better moderation systems", which equates to "controls" in the options.

---

5. The inherent downside associated with social media is that it

(A) reinforces existing objectivity among the users.
(B) creates and spreads much innate and acquired prejudice.
(C) does not address the problem of the digital divide.
(D) results in unremitting expansion of freedom of expression.

Correct Answer: (B) creates and spreads much innate and acquired prejudice.

Solution:

Step 1: Understanding the Question:
The question identifies a "downside" or negative aspect of social media mentioned in the text.

Step 2: Detailed Explanation:
The text explicitly states: "...that is the nature of the social media beast, designed to amplify bias."
"Amplify bias" directly correlates to creating and spreading prejudice (innate or acquired).
Option (A) is the opposite of the text.
Option (C) is not mentioned.
Option (D) is not supported, as the text discusses censorship and adaptation mechanisms.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

The word "bias" in a text about media is a strong indicator of "prejudice" in the answer choices.

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6. Social media has succeeded in

(A) finding alternative means for fact-checking.
(B) ignoring technology and artificial content.
(C) becoming independent of other media.
(D) controlling other media that depend on it.

Correct Answer: (D) controlling other media that depend on it.

Solution:

Step 1: Understanding the Question:
The question asks about a success or achievement of social media mentioned in the passage.

Step 2: Detailed Explanation:
The passage states, "Social media now has enough control over all other forms of media to broaden its reach. It is the connective tissue... and alternative platforms are reworking their engagement with social media."
This indicates that other media forms now depend on or are influenced/controlled by social media's reach and infrastructure.

Step 3: Final Answer:
The correct option is (D).

Quick Tip

Words like "connective tissue" and "broaden its reach" over "all other forms" imply a level of systemic control or dominance.

Comprehension:
Read the following passage and choose the answer that is closest to each of the questions that are based on the passage.

According to the French philosopher Jean Baudrillard, commodities available for consumption are not inherently negative things. Baudrillard tried to interpret consumption in modern societies by engaging with the 'cargo myth' prevalent among the indigenous Melanesian people

living in the South Pacific. The Melanesians did not know what aeroplanes were. However, they saw that these winged entities descended from the air for white people and appeared to make them happy. They also noted that aeroplanes never descended for the Melanesian people.

The Melanesian natives noted that the white people had placed objects similar to the aeroplane on the ground. They concluded that these objects were attracting the aeroplanes in the air and bringing them to the ground. Through a magical process, the aeroplanes were bringing plenty to the white people and making them happy. The Melanesian people concluded that they would need to place objects that simulated the aeroplane on the ground and attract them from the air. Baudrillard believes that the cargo myth holds an important analogy for the ways in which consumers engage with objects of consumption.

According to Baudrillard, the modern consumer "sets in place a whole array of sham objects, of characteristic signs of happiness, and then waits for happiness to alight". For instance, modern consumers believe that they will get happiness if they buy the latest available version of a mobile phone or automobile. However, consumption does not usually lead to happiness. While consumers should ideally be blaming their heightened expectations for their lack of happiness, they blame the commodity instead.

They feel that they should have waited for the next version of a mobile phone or automobile before buying the one they did. The version they bought is somehow inferior and therefore cannot make them happy. Baudrillard argues that consumers have replaced 'real' happiness with 'signs' of happiness. This results in the endless deferment of the arrival of total happiness.

In Baudrillard's words, "in everyday practice, the blessings of consumption are not experienced as resulting from work or from a production process; they are experienced as a miracle". Modern consumers view consumption in the same magical way as the Melanesian people viewed the aeroplanes in the cargo myth. Television commercials also present objects of consumption as miracles. As a result, commodities appear to be distanced from the social processes which lead to their production. In effect, objects of consumption are divorced from the reality which produces them.

7. Why are consumers unhappy with commodities that they have just bought?

(A) Because television commercials do not create enough hype about commodities.
(B) Because they have exaggerated expectations of commodities.
(C) Because the Law of Diminishing Marginal Commodities comes into play.
(D) Because they focus on improved functionality of commodities.

Correct Answer: (B) Because they have exaggerated expectations of commodities.

Solution:

Step 1: Understanding the Question:
The question asks for the reason behind consumer dissatisfaction after purchasing goods, ac-

cording to the passage.

Step 2: Detailed Explanation:
The passage explicitly states: "While consumers should ideally be blaming their heightened expectations for their lack of happiness, they blame the commodity instead."
"Heightened expectations" is synonymous with "exaggerated expectations".
The text explains that consumers treat commodities as "signs" of happiness rather than the actual source, leading to constant disappointment as reality fails to match their "magical" expectations.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Direct mapping of words like "heightened expectations" to "exaggerated expectations" helps in identifying the correct choice in factual RC questions.

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8. Which of the following is an argument made by Baudrillard?

(A) Production and consumption are magical processes.
(B) Consumers value signs more than the real.
(C) Melanesian people coped with the inequality of colonialism by creating myths.
(D) Television commercials are at the heart of unhappiness experienced by consumers.

Correct Answer: (B) Consumers value signs more than the real.

Solution:

Step 1: Understanding the Question:
We need to identify a core philosophical argument made by Baudrillard as described in the passage.

Step 2: Detailed Explanation:
The passage states: "Baudrillard argues that consumers have replaced 'real' happiness with 'signs' of happiness."
This replacement indicates that the "sign" (the status or image of the product) is what is being valued over the actual, "real" utility or happiness derived from the object.
Option (A) is incorrect because Baudrillard says consumers *view* them as magical, not that they *are* magical.
Option (D) is a secondary point, but the core argument is the shift from "real" to "signs".

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Baudrillard is famous for his theories on "hyperreality" and "simulacra", which center on the idea that symbols (signs) have replaced reality. The text reinforces this specific concept.

---

9. How can consumption be made more satisfying?

(A) By banning television commercials that promise real happiness.
(B) By understanding the connection between production and consumption.
(C) By recognising that commodities produce miraculous change.
(D) By rejecting colonialism and all other forms of economic oppression.

Correct Answer: (B) By understanding the connection between production and consumption.

Solution:

Step 1: Understanding the Question:
The question asks for a solution to the "unhappiness" problem described by Baudrillard.

Step 2: Detailed Explanation:
The text notes that commodities "appear to be distanced from the social processes which lead to their production" and are "divorced from the reality which produces them."
It follows that the dissatisfaction stems from viewing them as "miracles" rather than products of labor.
Therefore, re-establishing the "connection between production and consumption" would ground the consumer in reality, potentially making the experience more satisfying or less prone to exaggerated "magical" disappointment.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

The text identifies the "divorce from reality/production" as the problem. Logically, the solution involves reversing that specific condition.

---

10. How does Baudrillard engage with the cargo myth?

(A) He uses it to describe the suffering of Indigenous people.
(B) He uses it to show that consumers should consume more serious objects.
(C) He uses it as a metaphor to critique modern consumption.
(D) He uses it to show that consumption is a blessing.

Correct Answer: (C) He uses it as a metaphor to critique modern consumption.

Solution:

Step 1: Understanding the Question:
What is the purpose of the Melanesian "cargo myth" story in Baudrillard's analysis?

Step 2: Detailed Explanation:
The text says Baudrillard "tried to interpret consumption in modern societies by engaging with the 'cargo myth'".
He uses the myth as an "analogy" (a metaphor) to describe how modern consumers treat goods as magical items descending from nowhere to bring happiness, much like the Melanesians viewed aeroplanes.
This is a "critique" because it highlights the irrational, sham-based nature of modern consumer behavior.

Step 3: Final Answer:
The correct option is (C).

Quick Tip

An "analogy" in literature or philosophy is almost always a "metaphor" used to illustrate a larger point or critique.

Comprehension:
Read the following passage and choose the answer that is closest to each of the questions that are based on the passage.

According to the French philosopher Jean Baudrillard, commodities available for consumption are not inherently negative things. Baudrillard tried to interpret consumption in modern societies by engaging with the 'cargo myth' prevalent among the indigenous Melanesian people living in the South Pacific. The Melanesians did not know what aeroplanes were. However, they saw that these winged entities descended from the air for white people and appeared to make them happy. They also noted that aeroplanes never descended for the Melanesian people.

The Melanesian natives noted that the white people had placed objects similar to the aeroplane on the ground. They concluded that these objects were attracting the aeroplanes in the air and bringing them to the ground. Through a magical process, the aeroplanes were bringing plenty to the white people and making them happy. The Melanesian people concluded that they would need to place objects that simulated the aeroplane on the ground and attract them from the air. Baudrillard believes that the cargo myth holds an important analogy for the ways in which consumers engage with objects of consumption.

According to Baudrillard, the modern consumer "sets in place a whole array of sham objects, of characteristic signs of happiness, and then waits for happiness to alight". For instance, modern consumers believe that they will get happiness if they buy the latest available version of a mobile phone or automobile. However, consumption does not usually lead to happiness. While consumers should ideally be blaming their heightened expectations for their lack of happiness, they blame the commodity instead.

They feel that they should have waited for the next version of a mobile phone or automobile before buying the one they did. The version they bought is somehow inferior and therefore cannot make them happy. Baudrillard argues that consumers have replaced 'real' happiness with 'signs' of happiness. This results in the endless deferment of the arrival of total happiness.

In Baudrillard's words, "in everyday practice, the blessings of consumption are not experienced as resulting from work or from a production process; they are experienced as a miracle". Modern consumers view consumption in the same magical way as the Melanesian people viewed the aeroplanes in the cargo myth. Television commercials also present objects of consumption as miracles. As a result, commodities appear to be distanced from the social processes which lead to their production. In effect, objects of consumption are divorced from the reality which produces them.

11. What is Baudrillard's position on total happiness?

(A) It comes with patience and waiting.
(B) It results from ethical consumption.
(C) It prioritises production over consumption.
(D) It is perpetually delayed.

Correct Answer: (D) It is perpetually delayed.

Solution:

Step 1: Understanding the Question:
The question asks for Baudrillard's specific view on the attainment of total happiness in a consumerist society as described in the passage.

Step 2: Detailed Explanation:
The passage explicitly states towards the end of the fourth paragraph: "This results in the endless deferment of the arrival of total happiness."

Baudrillard's argument is that because consumers chase "signs" rather than reality, and are constantly looking forward to the "next" product, they never actually reach a state of total happiness.
The word "deferment" means to put off or delay to a later time. Therefore, total happiness is perpetually delayed.

Step 3: Final Answer:
The correct option is (D).

Quick Tip

Vocabulary check: "Deferment" and "delayed" are synonyms. Often, the answer in RC is a paraphrased version of a direct quote from the text.

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12. What is Baudrillard's position on consumption?

(A) It is a positive process.
(B) It is an egalitarian process.
(C) It is an irrational process.
(D) It is a utilitarian process.

Correct Answer: (C) It is an irrational process.

Solution:

Step 1: Understanding the Question:
The question asks how Baudrillard characterizes the process of consumption in modern society.

Step 2: Detailed Explanation:
The passage describes how modern consumers view consumption in the same "magical way" as the Melanesian cargo myth.
It uses terms like "sham objects," "miracles," and "divorced from reality."
Baudrillard points out that consumers blame the commodity for their lack of happiness instead of their own heightened expectations, and they engage with "signs" rather than reality.
Collectively, these descriptions point toward a process that is not based on logic, utility, or production reality, but on a mythical or magical belief. This makes the process "irrational."

Step 3: Final Answer:
The correct option is (C).

Quick Tip

If an author compares a modern habit to a "mythical" or "magical" belief system of a primitive culture, they are likely critiquing the habit as "irrational."

Comprehension:
Complete the following sentences by choosing the most appropriate word/phrase from the options given below.

13. There are so many instances of one or more deer crossing the road, or just standing in the middle of the road, or else _____; it is like the deer cannot hear the noise of the engines or see the headlights.

(A) jumping under the road
(B) staggering with the road
(C) foraging beneath the road
(D) bounding across the road

Correct Answer: (D) bounding across the road

Solution:

Step 1: Understanding the Question:
The sentence requires a verb phrase that describes the movement of deer on or near a road, fitting the context of "crossing" or "standing."

Step 2: Detailed Explanation:
"Bounding" means leaping or moving with high, springing steps, which is a characteristic way deer move.
"Across the road" fits perfectly with the established context of "crossing the road."
Option (A) "jumping under" is physically impossible for a surface road.
Option (B) "staggering with" doesn't describe deer movement well.
Option (C) "foraging beneath" suggests being underground, which is incorrect.

Step 3: Final Answer:
The correct option is (D).

Quick Tip

"Bounding" is the specific word often used to describe the leaping motion of animals like deer.

14. We hope that the government's new policies will _____ a period of economic growth.

(A) turn up
(B) usher in
(C) set in
(D) set forth

Correct Answer: (B) usher in

Solution:

Step 1: Understanding the Question:
The sentence requires a phrasal verb that means to signal the start of or bring about a new period or era.

Step 2: Detailed Explanation:
To "usher in" means to cause or mark the start of something new, such as a period of change or growth. This fits perfectly with "a period of economic growth."
"Turn up" means to appear or arrive.
"Set in" usually refers to something unpleasant beginning and likely to continue (like bad weather).
"Set forth" means to state or describe something clearly.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

"Usher in" is frequently paired with abstract concepts like "era," "period," or "phase" of growth or change.

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15. When she inherited some jewellery from a distant relative, she had no idea of its worth and decided _____.

(A) to have it appraised
(B) to get an approval
(C) to get it appreciated
(D) to have it apprised

Correct Answer: (A) to have it appraised

Solution:

Step 1: Understanding the Question:
The sentence needs a term that means assessing the financial value of an item like jewellery.

Step 2: Detailed Explanation:
To "appraise" means to assess the value or quality of something, specifically for insurance or sale purposes. This fits the context of "no idea of its worth."
"Approval" means formal permission.
"Appreciated" means increased in value or recognized with gratitude.
"Apprised" means to inform or tell someone about something.

Step 3: Final Answer:
The correct option is (A).

Quick Tip

Confusing words: Appraise (value) vs Apprise (inform). Remember that "Praise" is in "Appraise" – you evaluate if it's worthy of praise (value).

---

16. The labourers who were fired broke into the office building and destroyed some of the machinery. Rather than finding a solution to their problems, they _____.

(A) extended their troubles
(B) exaggerated their hardships
(C) exacerbated the situation
(D) extenuated the circumstance

Correct Answer: (C) exacerbated the situation

Solution:

Step 1: Understanding the Question:
The sentence requires a word describing how an already bad situation was made worse by negative actions.

Step 2: Detailed Explanation:
"Exacerbate" means to make a problem, a bad situation, or a negative feeling worse. This perfectly describes the outcome of fired workers destroying property.
"Exaggerated" means to represent something as better or worse than it actually is.
"Extenuated" means to make a guilt or an offense seem less serious or more forgivable.

Step 3: Final Answer:
The correct option is (C).

Quick Tip

Exacerbate is a high-frequency word in exams used specifically to mean "worsen a crisis."

---

17. Without a doubt, the widespread use of renewable energy is a key solution to climate change. However, it is not a _____, as efforts in conservation are equally crucial.

(A) silver lining
(B) silver bullet
(C) red herring
(D) dead ringer

Correct Answer: (B) silver bullet

Solution:

Step 1: Understanding the Question:
The sentence requires an idiom that means a simple and immediate solution to a complicated problem.

Step 2: Detailed Explanation:
A "silver bullet" is a metaphor for a simple, magical solution that solves a very complex problem instantly. The sentence suggests that while renewable energy is important, it's not a standalone, perfect solution because conservation is also needed.
"Silver lining" means a positive aspect of an otherwise negative situation.
"Red herring" is something that misleads or distracts from a relevant or important question.
"Dead ringer" is someone who looks exactly like someone else.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Idioms are context-dependent. If the text says a solution is good but "not enough," look for "silver bullet."

---

18. Everyone wondered how the travel vlogger could go around the world all through the year and _____.

(A) manage his iterative life style
(B) manage his itinerary life style
(C) manage his itinerary in his life style
(D) manage his itinerant life style

Correct Answer: (D) manage his itinerant life style

Solution:

Step 1: Understanding the Question:
The sentence needs an adjective to describe a lifestyle involving constant travel.

Step 2: Detailed Explanation:
"Itinerant" means traveling from place to place. An "itinerant lifestyle" is one characterized by frequent movement, which fits a "travel vlogger going around the world."
"Iterative" refers to the repetition of a process.
"Itinerary" is a noun meaning a planned route or journey, not an adjective for a lifestyle.

Step 3: Final Answer:
The correct option is (D).

Quick Tip

Identify the part of speech required. Here, an adjective is needed to modify "lifestyle." "Itinerant" serves that purpose.

---

19. Deepak is an unpleasant person, but we all _____ because his sister is a close friend of ours.

(A) put along with him
(B) put him aside
(C) put up with him
(D) put him down

Correct Answer: (C) put up with him

Solution:

Step 1: Understanding the Question:
The sentence needs a phrasal verb meaning to tolerate someone unpleasant.

Step 2: Detailed Explanation:
To "put up with" someone means to tolerate or endure an unpleasant person or situation. This fits perfectly with Deepak being "an unpleasant person."
"Put aside" means to save or ignore something.
"Put down" means to humiliate or insult someone.

Step 3: Final Answer:
The correct option is (C).

Quick Tip

Phrasal verbs are best learned in pairs of situation-response. Unpleasant person → Put up with.

Comprehension:
In each of the following sentences, the incorrect part of the sentence is underlined. Choose an alternative from the four given options so that the sentence is rendered correct.

20. Although the new policy aims to increase efficiency, reducing costs, and enhancing emplo some employees feel that the changes are too abrupt and poorly communicated.

(A) increasing efficiency, reducing of costs, and enhancing of employee satisfaction
(B) increase efficiency, reduce costs, and enhance employee satisfaction
(C) increase the efficiency, reduce the costs and enhancing employee satisfaction
(D) increase efficiency, reducing the costs and enhanced employee satisfaction

Correct Answer: (B) increase efficiency, reduce costs, and enhance employee satisfaction

Solution:

Step 1: Understanding the Question:
This is a grammar question focused on parallel structure in a list following the infinitive marker "to."

Step 2: Detailed Explanation:
The phrase "aims to" requires a series of base-form verbs.
The underlined part mixes a base form ("increase") with gerunds ("reducing," "enhancing").
For correct parallel structure, all items in the list must be in the same grammatical form.
Option (B) provides "increase," "reduce," and "enhance," all of which are base-form verbs that

correctly follow "to."

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Rule of Parallelism: Items in a list joined by "and" or "or" must be in the same grammatical format (e.g., all nouns, all adjectives, or all same-form verbs).

Comprehension:
In each of the following sentences, the incorrect part of the sentence is underlined. Choose an alternative from the four given options so that the sentence is rendered correct.

21. Thank goodness, the damage to the car was neglectful.

(A) was negligible
(B) was neglectable
(C) was negligent
(D) was neglecting

Correct Answer: (A) was negligible

Solution:

Step 1: Understanding the Question:
The sentence aims to describe the extent of damage to a car. The word "neglectful" describes a person who fails to take care of something, not the magnitude of an object or event.

Step 2: Detailed Explanation:
"Negligible" means so small or unimportant as to be not worth considering. This correctly describes car damage that is minor enough to cause relief ("Thank goodness").
"Neglectable" is a rare variant and less precise than negligible in this context.
"Negligent" describes a person who fails to exercise proper care.
"Neglecting" is the present participle of the verb neglect and requires an object.

Step 3: Final Answer:
The correct option is (A).

Quick Tip

Confusing adjectives: Use "negligible" for things that are small in amount/size. Use "negligent" or "neglectful" for people who are careless.

---

22. Among scientists, the discovery of the double helix structure of DNA and the genetic code it incorporates is widely regarded to be one of the most significant scientific dis of the twentieth century.

(A) regarded being one of the most significant scientific discoveries
(B) regarded like one of the most significant scientific discovery
(C) regarded for being one of the most significant scientific discoveries
(D) regarded as one of the most significant scientific discoveries

Correct Answer: (D) regarded as one of the most significant scientific discoveries

Solution:

Step 1: Understanding the Question:
This question tests the idiomatic usage of the verb "regard" and the correct pluralization in the phrase "one of the...".

Step 2: Detailed Explanation:
The verb "regard" is idiomatically followed by "as," not "to be" or "like."
The phrase "one of the" must be followed by a plural noun because it implies selection from a group. Therefore, it should be "discoveries," not "discovery."
Option (D) satisfies both requirements: it uses "regarded as" and the plural "discoveries."

Step 3: Final Answer:
The correct option is (D).

Quick Tip

Grammar Rule: "One of the" + Superlative + Plural Noun. Always check for the 's' at the end of the noun in such constructions.

---

23. When I had to leave town due to office work, I had my brother to give food to my dog twice a day.

(A) had my brother feed my dog
(B) had my brother to feed my dog

(C) had my brother who fed my dog
(D) had my brother giving food to my dog

Correct Answer: (A) had my brother feed my dog

Solution:

Step 1: Understanding the Question:
This is a causative verb construction using "have."

Step 2: Detailed Explanation:
In the active causative form, the structure is: [Subject] + [have] + [person] + [base form of verb].
Example: "I had him wash the car."
The underlined phrase incorrectly uses "to give," which is an infinitive. It should be the base form "feed" or "give."
Option (A) "had my brother feed my dog" follows the correct [have + person + base verb] structure.

Step 3: Final Answer:
The correct option is (A).

Quick Tip

Causative verbs: "Have" someone DO something (base verb), but "Get" someone TO DO something (infinitive).

---

24. If the President knew that his allies would let him down so suddenly, he would have handled them with the greatest care.

(A) If the President could know beforehand that his allies would let him down
(B) Had the President known that his allies would let him down
(C) Had the President knowledge that his allies would let him down
(D) If the President knew that his allies can let him down

Correct Answer: (B) Had the President known that his allies would let him down

Solution:

Step 1: Understanding the Question:
The sentence describes a hypothetical past situation and its hypothetical result (Third Conditional).

Step 3: Detailed Explanation:
The result clause is "he would have handled," which signals the Third Conditional (Past Unreal).
The "if" clause for a Third Conditional must use the Past Perfect tense: "If the President had known...".
Alternatively, this can be expressed through inversion: "Had the President known...".
The original underlined text uses "knew" (Simple Past), which is incorrect for this context.
Option (B) uses the correct inverted Past Perfect form "Had the President known."

Step 4: Final Answer:
The correct option is (B).

Quick Tip

Conditional Tense Consistency: If you see "would have + past participle" in one part, you almost certainly need "had + past participle" (or inversion) in the other.

Comprehension:
Each of the paragraphs given below has a sentence missing which is indicated by a blank. From the choices given below each paragraph, choose the sentence that is most logically appropriate to complete the paragraph.

25. On the first day of January 2025, the Indian Meteorological Department [IMD] announced that 2024 was the hottest year on record. A study by the Council on Energy, Environment and Water shows that nearly eight out of ten Indians live in districts that are at risk of either a flood, a cyclone, or a drought. Nearly twenty three States in India are heatwave-prone. _____. In the summer of 2024, India recorded more than 44,000 cases of heatstroke and over 300 heat-related mortalities, as per the bulletin of the Ministry of Health and Family Welfare. Water reservoirs and the energy demand that keeps India powered are impacted too. During a ten-day long heatwave in Delhi, peak power demand was 16% higher than the previous year.

(A) According to the Council, more than 20% of the population is not affected by climate change
(B) The increasing heat stress remains a major challenge, affecting public health and economic productivity
(C) However, the people of these districts are given sufficient compensation for loss of life and property
(D) The record-breaking heat of the summer of 2024 resulted in an unpredictable and delayed monsoon

Correct Answer: (B) The increasing heat stress remains a major challenge, affecting public health and economic productivity

Solution:

Step 1: Understanding the Question:
We need to find a sentence that bridges the information about environmental risks/heatwaves and the subsequent statistics on health and energy demand.

Step 2: Detailed Explanation:
The preceding sentences establish that India is highly prone to extreme weather and heatwaves. The following sentences provide data on health crises (heatstroke/deaths) and economic/infrastructure strain (power demand).
Option (B) "The increasing heat stress remains a major challenge, affecting public health and economic productivity" perfectly synthesizes the paragraph's theme and introduces the two specific areas (health and productivity/energy) discussed in the rest of the text.
Option (A) is a negative, irrelevant statistic. Option (C) discusses compensation, which is not mentioned again. Option (D) discusses the monsoon, which is not the focus of the subsequent sentences.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Paragraph completion: Look for a "thematic bridge." The sentence must summarize what came before and introduce the specific examples that follow.

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26. As globalization held sway over the world, communities, which used to live in relative isolation, sought access to the wider world, and in the process, they parted with their own language and adopted a new lingua franca. The loss of language, however, does not merely mean the loss of a mode of communication or the loss of a few thousand words. _____. So, when a language dies, a way of thinking dies with it.

(A) A certain school of thought regrets the demise of local languages but in recent times revival movements have emerged across the world, especially in India
(B) Since evolution and change in languages is a part of history, most of the languages spoken today would be scarcely recognizable from what they were a few thousand or maybe even a few hundred years ago
(C) A potentially endangered language can sometimes appear to be thriving, or on the other hand, it can show signs of declining
(D) Languages exist not only for the purposes of practical communication; they convey a lin-

guistic community's entire mindset and its culture

Correct Answer: (D) Languages exist not only for the purposes of practical communication; they convey a linguistic community's entire mindset and its culture

Solution:

Step 1: Understanding the Question:
The missing sentence should explain why the loss of a language is more significant than just losing words, leading to the conclusion that a "way of thinking" dies.

Step 2: Detailed Explanation:
The previous sentence says language loss is not "merely" about words. This requires an explanation of what else a language contains.
The final sentence uses the phrase "a way of thinking dies with it."
Option (D) explains that languages carry "a linguistic community's entire mindset and its culture," which provides the direct logical definition of "a way of thinking."
Options (A), (B), and (C) discuss language history or revival, which do not explain the "way of thinking" connection.

Step 3: Final Answer:
The correct option is (D).

Quick Tip

Keyword connection: "Mindset/Culture" in the missing sentence directly leads to "Way of thinking" in the concluding sentence.

---

27. An island in Japan boasts of numerous dairy farms that own nearly one million cows, and supplies 70% of the milk sold in the country. These dairy farms have now begun to use cow manure to produce hydrogen. The methane from cow manure mingles with steam in a high temperature environment to produce hydrogen, which is used to electrify the local zoo. _____.

(A) This shows how Japan has always used technology to help animals
(B) The Indian government too, should replicate this, and use such technology to produce hydrogen
(C) It is a case study of a certain animal that is useful in providing energy for several other animals
(D) This is an exemplary way of creating a sustainable source of energy using innovative technology

Correct Answer: (D) This is an exemplary way of creating a sustainable source of energy using innovative technology

Solution:

Step 1: Understanding the Question:
The sentence should provide a logical conclusion or summary statement for the case study of manure-to-hydrogen energy in Japan.

Step 2: Detailed Explanation:
The paragraph describes a process: cow manure → methane → hydrogen → electricity.
This is a technical process of energy production.
Option (D) "This is an exemplary way of creating a sustainable source of energy using innovative technology" summarizes the entire narrative as a positive, technological achievement in sustainability.
Option (A) focuses only on helping animals, which is a narrow interpretation. Option (B) introduces "India" abruptly, which doesn't fit the flow. Option (C) is factually odd ("useful for other animals").

Step 3: Final Answer:
The correct option is (D).

Quick Tip

For a concluding sentence, choose the one that provides a high-level summary of the central idea presented in the paragraph.

---

28. A report published in Lancet Diabetes and Endocrinology has called for an overhaul of our understanding of obesity. An over-reliance on using Body Mass Index [BMI] as a metric has the peculiar effect of leading to both underdiagnosis and overdiagnosis of the condition. _____. BMI does not give accurate information about how fat is distributed in an individual's body. It frequently fails to capture the true state of health of an individual. A person's BMI may indicate they are "obese", but their organs and bodily functions may be absolutely normal. Every individual is a unique constellation — not only of genes and other biological variables, but also socio-economic conditions and habits.

(A) BMI reading can help the doctor to accurately prescribe the appropriate dosage to reduce fat
(B) This is because BMI does not provide a reliable picture of health, nor any direct measure of fat
(C) Obesity is the end result of multiple factors and BMI can pinpoint the cause of the problem
(D) Further, much of the information on diabetes, obesity or BMI available on social media is

misleading

Correct Answer: (B) This is because BMI does not provide a reliable picture of health, nor any direct measure of fat

Solution:

Step 1: Understanding the Question:
The missing sentence should explain *why* BMI leads to over/underdiagnosis, supporting the argument for an overhaul.

Step 2: Detailed Explanation:
The text critiques BMI as flawed. The subsequent sentences detail how it fails (distribution of fat, failing to capture true health, normal organs in "obese" people).
Option (B) "This is because BMI does not provide a reliable picture of health, nor any direct measure of fat" explains the mechanism of failure mentioned in the previous sentence and introduces the specific examples (fat distribution, health state) that follow.
Option (A) and (C) are supportive of BMI, which contradicts the text. Option (D) introduces "social media," which is irrelevant to the medical journal report.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Paragraph flow: If the sentence before the blank states a problem, the blank often contains the reason ("This is because...") for that problem.

Comprehension:
Some of the sentences below have words that are missing. Choose the best option from those given below to complete them.

29. Since chronic stress can _____ the immune system, making individuals more susceptible to illness and _____ their overall well-being, healthcare practitioners often recommend mindfulness practices and proper sleep to _____ these negative effects.

(A) compromise; impair; counter
(B) endanger; preserve; decrease
(C) paralyse; improve; diminish
(D) undermine; elevate; impede

Correct Answer: (A) compromise; impair; counter

Solution:

Step 1: Understanding the Question:
We need to find three words that logically fit the medical/health context of the sentence regarding "chronic stress."

Step 2: Detailed Explanation:
Blank 1: Stress "weakens" or "harms" the immune system. "Compromise" is the standard medical term for this.
Blank 2: Making individuals susceptible to illness "harms" well-being. "Impair" fits perfectly.
Blank 3: Mindfulness and sleep are used to "offset" or "oppose" these effects. "Counter" fits best.
Option (B) "preserve" well-being is incorrect contextually. Option (C) "improve" well-being doesn't fit the "and" structure with susceptibility to illness. Option (D) "elevate" well-being is also positive, which doesn't fit the description of negative effects.

Step 3: Final Answer:
The correct option is (A).

Quick Tip

Look for "tone consistency." The first two blanks describe negative actions of stress, so both words must be negative. The last blank describes an action taken *against* the negative effects, so it should mean "stop" or "oppose."

---

30. Astronauts who stayed for an _____ period of time at the International Space Station displayed a remarkable level of _____ endurance and mental _____.

(A) expanded; stern; acuity
(B) extended; physical; resilience
(C) explicit; stoic; integrity
(D) extensive; dysfunctional; agility

Correct Answer: (B) extended; physical; resilience

Solution:

Step 1: Understanding the Question:
We need words that describe a long duration of time and the two types of strength (body and mind) shown by astronauts.

Step 2: Detailed Explanation:
Blank 1: A long stay in space is described as an "extended" or "extensive" period.
Blank 2: Endurance in a physical environment is "physical" endurance.
Blank 3: Mental strength or the ability to recover from hardship is "resilience."
Option (B) "extended; physical; resilience" is the most natural and professional collocation for this context.
Option (A) "stern endurance" and Option (C) "stoic endurance" are less common. Option (D) "dysfunctional endurance" is contradictory.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Collocations: "Mental resilience" and "Physical endurance" are very common phrases in English. Matching these standard pairs often leads quickly to the correct answer.

Comprehension:
Some of the sentences below have words that are missing. Choose the best option from those given below to complete them.

31. Psychologists urge users to remember that social media rarely reflects the full complexity of real life. Influencers often _____ a carefully curated online persona, which can _____ unrealistic standards and occasionally _____ negative self-comparisons amongst their followers.

(A) profess; inspire; release
(B) advocate; perpetuate; stimulate
(C) endorse; foster; provoke
(D) maintain; generate; trigger

Correct Answer: (D) maintain; generate; trigger

Solution:

Step 1: Understanding the Question:
The sentence discusses the artificial nature of social media and its psychological impact on followers. We need verbs that describe the management of a persona and its resultant effects.

Step 2: Detailed Explanation:
Blank 1: Influencers "keep up" or "keep going" with a persona. "Maintain" is the perfect word for a sustained image or identity.

Blank 2: These personas "create" or "produce" standards. "Generate" fits the context of creating something like a standard or expectation.
Blank 3: The standards cause a psychological reaction. "Trigger" is the specific psychological term for causing a sudden reaction or comparison.
Option (B) "perpetuate" fits, but "advocate" doesn't fit a persona as well as "maintain." Option (C) "provoke" fits the last blank, but "endorse" doesn't fit the first.

Step 3: Final Answer:
The correct option is (D).

Quick Tip

In social media contexts, a "curated persona" is almost always "maintained." The word "trigger" is also a common keyword in psychological passages describing the cause-effect relationship between external stimuli and internal feelings.

---

32. Art can be _____ because it encourages individuals to express their emotions through a creative outlet, allowing them to process complex feelings, reduce stress, and _____ self-awareness.

(A) acceptable; disturb
(B) therapeutic; enhance
(C) pleasing; decrease
(D) avoidable; mitigate

Correct Answer: (B) therapeutic; enhance

Solution:

Step 1: Understanding the Question:
The sentence describes the positive mental and emotional benefits of art. We need two positive words that describe art's healing nature and its effect on self-awareness.

Step 2: Detailed Explanation:
Blank 1: Because art helps "reduce stress" and "process feelings," it is "therapeutic" (having a healing effect).
Blank 2: Art helps one understand themselves better, which means it "enhances" or "increases" self-awareness.
Option (A) "disturb" and Option (C) "decrease" are negative and contradict the positive tone of the sentence. Option (D) "avoidable" is irrelevant.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Identify the tone of the sentence. "Encourages," "creative outlet," and "reduce stress" are all positive indicators. Therefore, both blanks must contain words with a positive connotation.

---

33. The notion of personhood is _____ on something more than a particular type of genetic material within human beings: it arises only with the larger-scale structural _____ of that material, which permits capacities like _____, thought and moral agency.

(A) interdependent; division; differentiation
(B) dependent; disorganisation; deconstruction
(C) premised; organisation; consciousness
(D) built; distribution; calibration

Correct Answer: (C) premised; organisation; consciousness

Solution:

Step 1: Understanding the Question:
This is a philosophical sentence defining "personhood." We need words that describe a foundational idea, a complex structure, and a high-level cognitive capacity.

Step 2: Detailed Explanation:
Blank 1: A concept or idea is "premised" (based) on a certain assumption.
Blank 2: Complexity in biological systems arises from the specific "organisation" of components.
Blank 3: "Thought" and "moral agency" are high-level traits. "Consciousness" is the primary capacity that belongs in this list.
Option (A) "division" and Option (B) "disorganisation" make no sense in the context of emerging capacities. Option (D) "calibration" is a mechanical term and doesn't fit the biological/philosophical theme.

Step 3: Final Answer:
The correct option is (C).

Quick Tip

When you see a list like "... ____, thought and moral agency," the missing word must be of the same "category." Consciousness is the only word in the options that fits the cognitive level of "thought" and "moral agency."

---

34. While Curcumin, which is an _____ found in turmeric helps to reduce _____, extremely high doses of it can _____ headache and nausea.

(A) alchemy; injury; cause
(B) ingredient; inflammation; induce
(C) enzyme; abrasion; infuse
(D) alkali; infection; promote

Correct Answer: (B) ingredient; inflammation; induce

Solution:

Step 1: Understanding the Question:
The sentence discusses the health benefits and side effects of Curcumin. We need terms relating to components, medical conditions, and causing symptoms.

Step 2: Detailed Explanation:
Blank 1: Curcumin is a specific "ingredient" or substance in turmeric.
Blank 2: Turmeric is well-known for reducing "inflammation."
Blank 3: High doses of medicine or supplements "induce" (bring about) side effects like headaches.
Option (A) "alchemy" is a pseudo-science, not a component. Option (C) "enzyme" is biologically specific and Curcumin is not one. Option (D) "alkali" is a chemical property that doesn't fit the context of a health supplement ingredient.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

The word "induce" is the standard medical verb used to describe how a substance or condition causes a physical symptom (e.g., "induce sleep," "induce labor," "induce nausea").

Comprehension:
CONVERSATION ANALYSIS: Read the following transcript and choose the answer that is

closest to each of the questions that are based on the transcript.

Lucia Rahilly (Global Editorial Director, The McKinsey Podcast): Today we're talking about the next big arenas of competition, about the industries that will matter most in the global business landscape, which you describe as arenas of competition. What do we mean when we use this term?

Chris Bradley (Director, McKinsey Global Institute): If I go back and look at the top ten companies in 2005, they were in traditional industries such as oil and gas, retail, industrials, and pharmaceuticals. The average company was worth about $250 billion. If I advance the clock forward to 2020, nine in ten of those companies have been replaced, and by companies that are eight times bigger than the old guards.

And this new batch of companies comes from these new arenas or competitive sectors. In fact, they're so different that we have a nickname for them. If you're a fan of Harry Potter, it's wizards versus muggles.

Arena industries are wizard-ish; we found that there's a set of industries that play by very different set of economic rules and get very different results, while the rest, the muggles (even though they run the world, finance the world, and energize the world), play by a more traditional set of economic rules.

Lucia Rahilly: Could we put a finer point on what is novel or different about the lens that you applied to determine what's a wizard and what's a muggle?

Chris Bradley: Wizards are defined by growth and dynamism. We looked at where value is flowing and the places where value is moving. And where is the value flowing? What we see is that this set of wizards, which represent about ten percent of industries, hog 45 percent of the growth in market cap. But there's another dimension or axis too, which is dynamism. That is measured by a new metric we've come up with called the "shuffle rate." How much does the bottom move to the top? It turns out that in this set of wizard-ish industries, or arenas, the shuffle rate is much higher than it is in the traditional industry.

Lucia Rahilly: So, where are we seeing the most profit?

Chris Bradley: The economic profit, which is the profit you make minus the cost for the capital you employ is in the wizard industries. It's where R&D happens; they're two times more R&D intensive. They're big stars, the nebulae, where new business is born.

35. In the context of the conversation, "dynamism" most closely refers to

(A) the never-changing reliance on established and unchanging business practices.
(B) the stability and predictability of traditional industries.
(C) the rapid and frequent changes in leadership and market position within an industry.
(D) the slow, gradual growth and morphing of established companies.

Correct Answer: (C) the rapid and frequent changes in leadership and market position within an industry.

Solution:

Step 1: Understanding the Question:
The question asks for the definition of "dynamism" as used by the speaker (Chris Bradley).

Step 2: Detailed Explanation:
Chris Bradley explicitly links dynamism to a new metric called the "shuffle rate."
He explains the shuffle rate as "How much does the bottom move to the top?".
This indicates movement, turnover, and changing positions within the industry hierarchy.
Option (C) "rapid and frequent changes in leadership and market position" accurately captures this concept of "shuffling."

Step 3: Final Answer:
The correct option is (C).

Quick Tip

Contextual mapping: Look for how the speaker *defines* their own terms. Bradley defines dynamism via "shuffle rate," which means change and movement.

---

36. In the context of the conversation, the term "arenas of competition" refers to

(A) government regulations that control business competition.
(B) broad categories of industries where companies engage in competitive activities.
(C) physical locations where businesses compete.
(D) specific companies that are considered to be powerful competitors.

Correct Answer: (B) broad categories of industries where companies engage in competitive activities.

Solution:

Step 1: Understanding the Question:
The question asks for the meaning of "arenas of competition" in the business context discussed.

Step 2: Detailed Explanation:
Lucia Rahilly introduces the term as "the industries that will matter most in the global business landscape."
Chris Bradley uses it interchangeably with "competitive sectors."
They discuss how top companies in 2005 were in "traditional industries" while new ones come

from "new arenas."
Therefore, an arena is a broad category or sector of the industry.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

In economic and business literature, an "arena" or "sector" refers to a group of related industries (e.g., the tech sector, the energy arena).

---

37. "Muggles" refers to industries that

(A) operate under traditional economic principles.
(B) are primarily focused on technological innovation.
(C) are characterized by rapid and frequent changes.
(D) exhibit high levels of market capitalization growth.

Correct Answer: (A) operate under traditional economic principles.

Solution:

Step 1: Understanding the Question:
What is the specific characteristic of the "muggles" group according to the transcript?

Step 2: Detailed Explanation:
Bradley contrasts Wizards with Muggles. He says: "...while the rest, the muggles... play by a more traditional set of economic rules."
Traditional rules are traditional economic principles.
Options (B), (C), and (D) describe the "Wizard" industries (growth, innovation, and change).

Step 3: Final Answer:
The correct option is (A).

Quick Tip

Contrast Analysis: If Wizards are dynamic and innovative, the Muggles (by the speaker's analogy) must be the opposite—static and traditional.

---

38. Which one of the following does "shuffle rate" not measure?

(A) Volatility of market leadership.
(B) Overall profitability of traditional industries.
(C) Relative change within an industry.
(D) Churn within the arena of competition.

Correct Answer: (B) Overall profitability of traditional industries.

Solution:

Step 1: Understanding the Question:
Identify which item in the list is NOT a component or objective of the "shuffle rate" metric.

Step 2: Detailed Explanation:
The "shuffle rate" is explicitly defined as a measure of "dynamism"—specifically "how much the bottom moves to the top."
This describes volatility (A), relative change (C), and churn (D).
Profitability is a different dimension Bradley discusses *separately* ("The economic profit... is in the wizard industries"). While Wizard industries have higher shuffle rates and higher profits, the shuffle rate itself measures *movement*, not profit.
Furthermore, Bradley notes traditional industries have a *lower* shuffle rate, but he doesn't use the shuffle rate to measure their overall profitability.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Negative Question: When asked what a term "does not" mean, look for the choice that introduces a completely different economic category (Profit vs. Churn/Movement).

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39. Which of the following best and correctly summarizes the main idea of the conversation?

(A) Newer, dynamic industries, termed "wizards," are experiencing significantly greater growth and profit compared to traditional industries.
(B) The global economy is shifting back towards traditional industries, as they offer more stable returns.
(C) Traditional industries are consistently more profitable than newer, "wizard-ish" industries.
(D) The terms "wizard" and "muggle" are used to describe the magical elements of business success.

Correct Answer: (A) Newer, dynamic industries, termed "wizards," are experiencing significantly greater growth and profit compared to traditional industries.

Solution:

Step 1: Understanding the Question:
Synthesize the entire conversation into one core message.

Step 2: Detailed Explanation:
The speaker describes a shift from 2005 (traditional giants) to 2020 (new arena giants).
He explains that "Wizards" (dynamic industries) represent 10% of sectors but 45% of growth and most economic profit.
The entire dialogue serves to highlight the dominance and profitability of these new, dynamic sectors over traditional ones.
Option (A) captures this growth/profit contrast perfectly. Options (B) and (C) contradict the text. Option (D) takes the Harry Potter metaphor too literally.

Step 3: Final Answer:
The correct option is (A).

Quick Tip

Main Idea: The main idea should cover the "What" (Wizard industries) and the "So What" (they are dominating growth and profit).

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40. "Wizard" industries are characterized by

(A) a reliance on traditional economic rules and practices.
(B) a higher concentration of economic profit and research and development.
(C) lower research and development spending.
(D) a slower rate of market capitalization growth.

Correct Answer: (B) a higher concentration of economic profit and research and development.

Solution:

Step 1: Understanding the Question:
Identify the defining traits of the "Wizard" industries as stated in the text.

Step 2: Detailed Explanation:
Bradley says: "The economic profit... is in the wizard industries. It's where R&D happens; they're two times more R&D intensive."
This directly supports Option (B).

Option (A) describes Muggles. Options (C) and (D) are the exact opposite of what the speaker says about Wizards.

Step 3: Final Answer:
The correct option is (B).

Quick Tip

Factual Recall: This information is found in the final paragraph of the transcript. Always scan the end of the text for concluding details about the subject.

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41. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.
1. In drought conditions, water often depletes in the topsoil and remains accessible only in the deeper subsoil layers.
2. A new study gives new insights into how the acid changes root growth angles to enable plants to reach out deeper subsoils in search of water.
3. Plants rely on their root systems, the primary organs for interacting with soil, to actively seek water.
4. Abscisic acid plays an important role in helping plants adapt to these challenging conditions.

Correct Answer: 3142

Solution:

Step 1: Understanding the Question:
The task is to rearrange the four sentences into a logically consistent paragraph regarding plant biology and survival during drought.

Step 2: Detailed Explanation:
Sentence 3 acts as the introductory statement, defining the primary role of root systems in seeking water.
Sentence 1 provides context by describing a specific problem: "drought conditions" where water is only in "deeper subsoil layers."
Sentence 4 introduces a solution/adaptation mechanism: "Abscisic acid" which helps plants in these "challenging conditions" (referring back to the drought in 1).
Sentence 2 concludes the thought by detailing "how the acid" (referring back to 4) works by changing "root growth angles" to reach the "deeper subsoils" (mentioned in 1).
The logical flow is: General role $\rightarrow$ Problem $\rightarrow$ Chemical helper $\rightarrow$ Mechanism of helper.

Step 3: Final Answer:
The correct sequence is 3142.

Quick Tip

Look for "noun-pronoun" or "concept-detail" pairs. Sentence 4 introduces "Abscisic acid," and Sentence 2 refers to it as "the acid." This creates a mandatory 4-2 sequence. Similarly, the "deeper subsoils" mentioned in 1 is targeted in 2, linking them.

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42. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.
1. Among its major urban centres, Harappa and Mohenjo-Daro stand out as prime examples of this architectural prowess, revealing large public structures, residential areas, and sophisticated water management systems indicative of a complex societal structure.
2. Showing remarkable sophistication for its time, this ancient culture developed meticulously planned cities, complete with advanced sanitation systems and intricate grid layouts that underscore their profound understanding of urban design and engineering.
3. Economically, the civilization thrived on a foundation of extensive trade networks, connecting them with distant lands, alongside a robust agricultural system that sustained their large populations and facilitated surplus production.
4. Despite its impressive achievements and longevity, the reasons behind the eventual decline of this remarkable civilization remain largely enigmatic, prompting ongoing research and speculation among historians and archaeologists.
5. The Indus Valley Civilization, flourishing in the Bronze Age, represents one of humanity's earliest urban societies, evidenced by archaeological discoveries dating back thousands of years.

Correct Answer: 52134

Solution:

Step 1: Understanding the Question:
We need to arrange the sentences to form a historical overview of the Indus Valley Civilization.

Step 2: Detailed Explanation:
Sentence 5 is the best opening sentence as it introduces the "Indus Valley Civilization" and sets the historical timeframe.
Sentence 2 follows by describing the general characteristics of "this ancient culture," specifically its planned cities.
Sentence 1 provides specific examples (Harappa and Mohenjo-Daro) of the "architectural prowess"

and cities mentioned in sentence 2.
Sentence 3 adds another dimension of its success: the economic and agricultural foundations.
Sentence 4 serves as the logical conclusion, shifting from "achievements" to the "eventual decline" and current status of research.

Step 3: Final Answer:
The correct sequence is 52134.

Quick Tip

Chronological or general-to-specific ordering is common in history-based para jumbles. Start with the name of the subject (5), describe its features (2, 1, 3), and end with its demise or legacy (4).

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43. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.
1. Seven of the ten worst affected countries (including India) are low- and middle income countries.
2. Between 1993 and 2022, India was the sixth worst-affected country in terms of fatalities and damage sustained from extreme weather events wrought by the climate crisis.
3. High income nations, whose economies are founded in industrial era use of fossil fuels, meanwhile, insist that growing economies, especially India and China, shoulder greater responsibility.
4. This reinforces the developing world's contention that it has had to bear a disproportionate burden of climate afflictions despite having contributed little to the crisis.

Correct Answer: 2143

Solution:

Step 1: Understanding the Question:
The sentences discuss the global debate on climate change responsibility and its impact on developing nations like India.

Step 2: Detailed Explanation:
Sentence 2 initiates the paragraph with a specific data point about India's vulnerability to climate change.
Sentence 1 expands this observation from India to the broader group of "ten worst affected countries," noting they are mostly low/middle income.
Sentence 4 uses "This" to refer to the data in 2 and 1, explaining how it supports the developing

world's "contention" of a disproportionate burden.
Sentence 3 introduces the contrasting viewpoint of "High income nations" with the transition word "meanwhile," providing the other side of the global debate.

Step 3: Final Answer:
The correct sequence is 2143.

Quick Tip

"This" in sentence 4 is a demonstrative pronoun that must follow the fact it describes. The fact is the statistics in 2 and 1. "Meanwhile" in 3 indicates a shift to a concurrent but contrasting argument, making it a strong concluding sentence for this debate.

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44. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.
1. Using the wonders of Artificial Intelligence (AI), they quickly improved upon those skills to become far more dexterous.
2. Inside a robotics laboratory of the Toyota Research Institute, a group of robots is busy cooking. There is nothing special about that; robotic chefs have been around for a while.
3. Despite their extraordinary culinary capabilities, these robots are not destined for a career in catering.
4. But these robots are more proficient than most: flipping pancakes, slicing vegetables, and making pizzas with ease.
5. The difference is that instead of being laboriously programmed to carry out their tasks, the Toyota robots have been taught only a basic set of skills.

Correct Answer: 24513

Solution:

Step 1: Understanding the Question:
The paragraph describes an experiment with cooking robots using AI at the Toyota Research Institute.

Step 2: Detailed Explanation:
Sentence 2 sets the scene and introduces the subject: robots cooking in a laboratory.
Sentence 4 provides a contrast ("But") to the claim in 2 that "robotic chefs have been around," highlighting that *these* robots are uniquely proficient.
Sentence 5 identifies the core innovation ("The difference is"): how they are taught rather than programmed.
Sentence 1 elaborates on the learning process mentioned in 5, explaining that AI allowed them

to improve on those "basic skills" to become "dexterous."
Sentence 3 concludes the narrative with an interesting irony: despite their skill, they aren't actually meant for the catering industry.

Step 3: Final Answer:
The correct sequence is 24513.

Quick Tip

Identify "linked pairs." Sentence 5 ends with "basic set of skills" and Sentence 1 starts by saying they "improved upon those skills." This creates a 5-1 link. Sentence 2 establishes a premise that 4 contradicts, creating a 2-4 link.

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45. The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.
1. When we take time to notice these moments, we discover hidden beauty that sparks our creative thoughts because creativity isn't just about rare, amazing events—it's also about finding the special in the ordinary.
2. Creativity is often seen as the ability to look at the world in a new way—to turn everyday sights, sounds, and experiences into art or ideas.
3. In fact, inspiration can come from small details of daily life: the gentle warmth of morning sunlight on a kitchen counter, the steady sound of traffic outside, or the brief smile of a stranger on a busy street.
4. Many people wrongly think that true creativity only comes from big ideas or exciting adventures.

Correct Answer: 2431

Solution:

Step 1: Understanding the Question:
The sentences discuss the nature of creativity and where inspiration comes from.

Step 2: Detailed Explanation:
Sentence 2 provides a broad, positive definition of creativity, making it the perfect opener.
Sentence 4 presents a common misconception ("Many people wrongly think") about the same subject.
Sentence 3 uses "In fact" to provide evidence against the misconception in 4, giving specific examples of "small details of daily life."
Sentence 1 concludes the thought by explaining the importance of "these moments" (referring to the details in 3) and summarizing the essence of creativity.

Step 3: Final Answer:
The correct sequence is 2431.

Quick Tip

Structure of an argument: Definition (2) → Misconception (4) → Counter-fact (3) → Conclusion/Synthesis (1). "In fact" is a classic transition used to introduce a reality that contradicts a common belief.
