JCECE Mathematics Sample Paper-10

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **50** Multiple Choice Questions.
- Each correct answer carries **+1** mark. Incorrect answer: **−0.25** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. Let $f(x) = \lim_{n\to\infty} \frac{\ln(2+x)-x^{2n}\sin(x)}{1+x^{2n}}$. Find the value of $f(1)$ so that $f(x)$ is continuous at $x = 1$.

(A) $\ln(3)$

(B) $-\sin(1)$

(C) $\frac{\ln(3)-\sin(1)}{2}$

(D) 0

Q2. The value of $\lim_{x\to 0} \frac{1-\cos(x)\sqrt{\cos(2x)}}{x^2}$ is equal to:

(A) 1

(B) $\frac{3}{2}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Q3. If $f(x) = |x-1| + |x-2| + \cos(x)$, then the number of points in the interval $(0, 3)$ where $f(x)$ is not differentiable is:

(A) 0

(B) 1

(C) 2

(D) 3

Q4. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $\frac{dy}{dx}$ at $x = 1$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 1

(D) $\frac{1}{8}$

Q5. The angle of intersection between the curves $y^2 = 4x$ and $x^2 = 4y$ at the point $(4, 4)$ is:

(A) $\tan^{-1}\left(\frac{3}{4}\right)$

(B) $\tan^{-1}\left(\frac{4}{3}\right)$

(C) $\frac{\pi}{2}$

(D) $\tan^{-1}\left(\frac{3}{5}\right)$

Q6. The maximum value of the function $f(x) = x(1-x)^2$ on the interval $[0, 1]$ occurs at x equal to:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

Q7. A ladder 5 m long rests against a vertical wall. If the bottom of the ladder is pulled away from the wall along the ground at the rate of 2 m/s, how fast is the top of the ladder sliding down the wall when the foot of the ladder is 4 m away from the wall?

(A) $-\frac{8}{3}$ m/s

(B) $-\frac{3}{8}$ m/s

(C) $-\frac{4}{3}$ m/s

(D) -2 m/s

Q8. Let $f(x) = x^3 - 3x^2 + 3x + 7$. The function $f(x)$ is:

(A) Strictly decreasing on $\mathbb{R}$

(B) Strictly increasing on $\mathbb{R}$

(C) Increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$

(D) Decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$

Q9. The value of c in Lagrange's Mean Value Theorem for the function $f(x) = \ln(x)$ on the interval $[1, e]$ is:

(A) $e - 1$

(B) $\frac{1}{e-1}$

(C) $1 - \frac{1}{e}$

(D) $\frac{e-1}{e}$

Q10. The integral $\int \frac{1}{x(x^5+1)}\, dx$ is equal to:

(A) $\ln\left|\frac{x^5}{x^5+1}\right| + C$

(B) $\frac{1}{5}\ln\left|\frac{x^5}{x^5+1}\right| + C$

(C) $\frac{1}{5}\ln\left|\frac{x^5+1}{x^5}\right| + C$

(D) $5\ln\left|\frac{x^5}{x^5+1}\right| + C$

Q11. The value of the definite integral $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\, dx$ is:

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

Q12. The value of $\int_{-1}^{1} x|x|\, dx$ is:

(A) $\frac{2}{3}$

(B) 0

(C) $-\frac{2}{3}$

(D) 1

Q13. The area enclosed between the curves $y = x^2$ and $y = x$ is given by:

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{5}{6}$

Q14. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x = \sec x$ is:

(A) $\ln|\sec x|$

(B) $\sec x$

(C) $\tan x$

(D) $\sec x + \tan x$

Q15. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

(A) $\tan^{-1} y \cdot \tan^{-1} x = C$

(B) $y - x = C(1 + xy)$

(C) $x + y = C(1 - xy)$

(D) $\tan^{-1} y + \tan^{-1} x = C$

Q16. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are respectively:

(A) 2, 3

(B) 2, 2

(C) 1, 2

(D) 2, 1

Q17. The value of $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r}$ is:

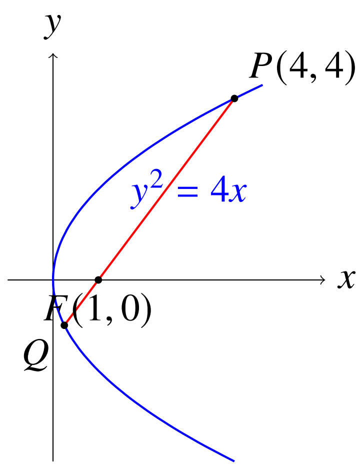
(A) $\ln(2)$

(B) $\ln(3)$

(C) 1

(D) 0

Q18. Consider the parabola $y^2 = 4x$ shown in the diagram below. If a focal chord PQ has one end-point at $P(4,4)$, find the coordinates of the other end-point Q.



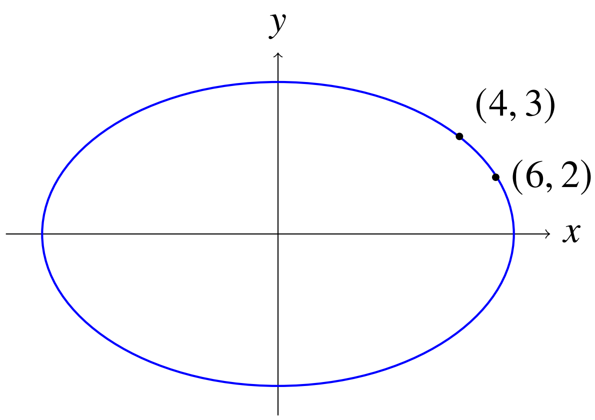
(A) $(1, -2)$

(B) $(\frac{1}{4}, -1)$

(C) $(4, -4)$

(D) $(\frac{1}{2}, -\sqrt{2})$

Q19. An ellipse has its center at the origin, major axis along the x-axis, and passes through the points $(4, 3)$ and $(6, 2)$ as illustrated. Find its eccentricity.



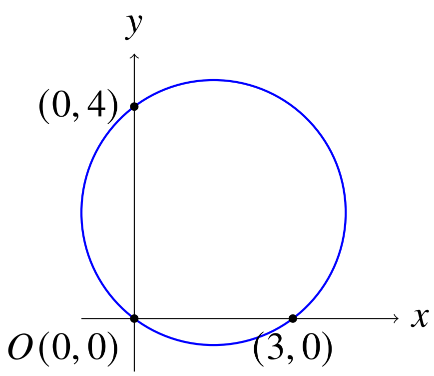
(A) $\sqrt{\frac{5}{6}}$

(B) $\frac{1}{2}$

(C) $\sqrt{\frac{2}{3}}$

(D) $\frac{\sqrt{3}}{2}$

Q20. The equation of the circle passing through the origin and cutting intercepts of length 3 and 4 units from the positive coordinate axes respectively is:



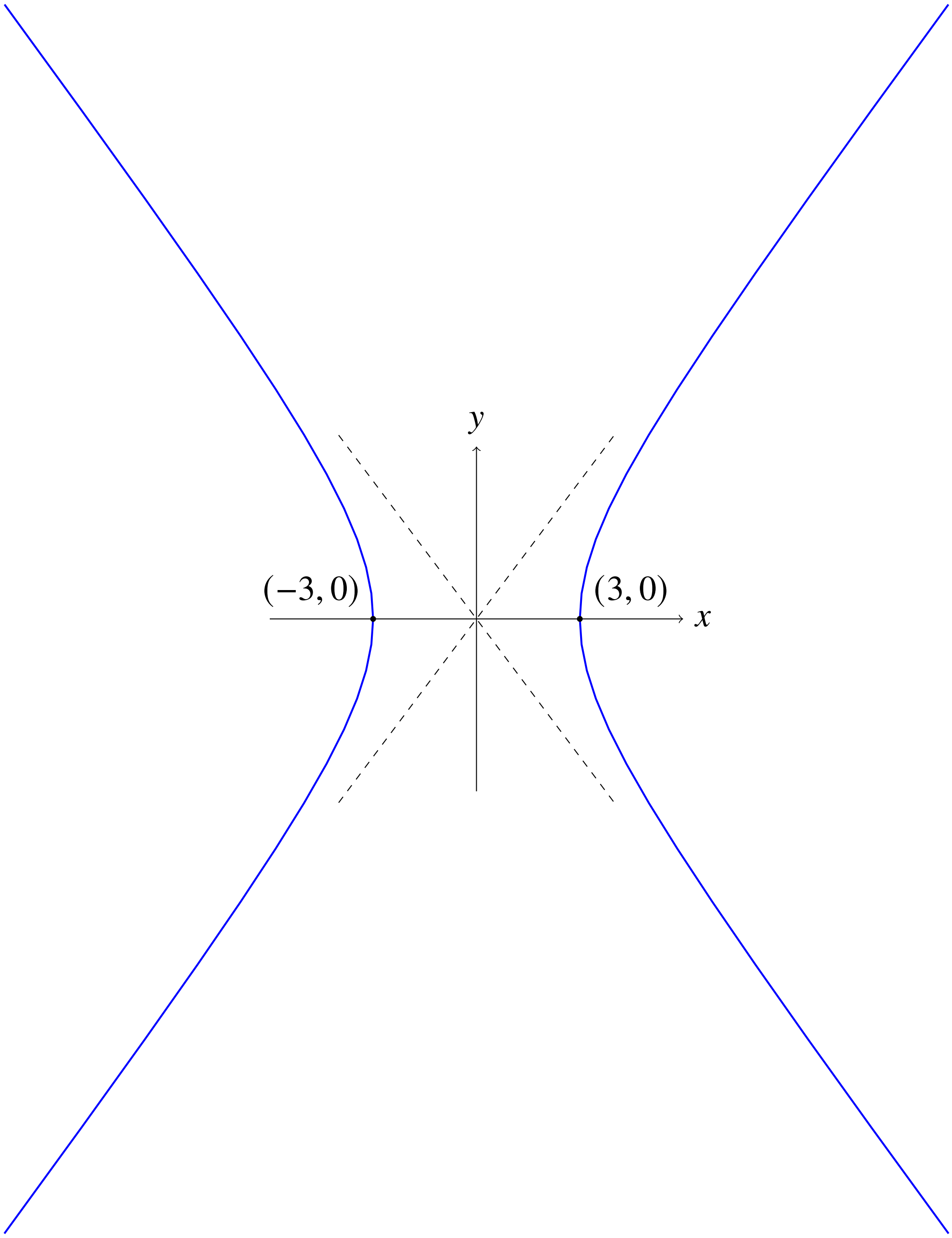
(A) $x^2 + y^2 - 3x - 4y = 0$

(B) $x^2 + y^2 + 3x + 4y = 0$

(C) $x^2 + y^2 - 6x - 8y = 0$

(D) $x^2 + y^2 - 4x - 3y = 0$

Q21. Find the equation of the hyperbola whose asymptotes are given by $y = \pm\frac{4}{3}x$ and vertices are located at $(\pm 3, 0)$ as shown.



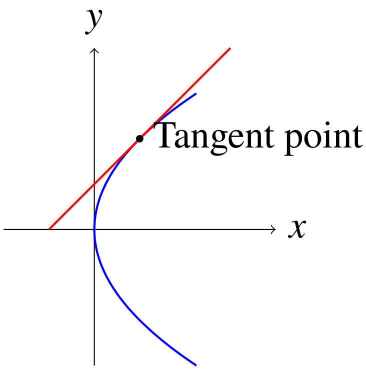
(A) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(B) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(C) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

(D) $\frac{x^2}{3} - \frac{y^2}{4} = 1$

Q22. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if the value of m is:



(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 4

Q23. The length of the latus rectum of the hyperbola $9x^2 - 16y^2 = 144$ is:

(A) $\frac{9}{2}$

(B) $\frac{9}{4}$

(C) $\frac{32}{3}$

(D) $\frac{16}{3}$

Q24. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:

(A) 5

(B) $\sqrt{13}$

(C) 12

(D) 7

Q25. If the locus of a point which moves such that the sum of its distances from two fixed points $(4, 0)$ and $(-4, 0)$ is always 10 units is a conic, then the length of its semi-minor axis is:

(A) 3

(B) 4

(C) 5

(D) 9

Q26. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A$ is equal to:

(A) $2I$

(B) $-2I$

(C) I

(D) O

Q27. The system of linear equations $x+y+z = 2, 2x+3y+2z = 5, 2x+3y+(a^2-1)z = a + 1$ has infinitely many solutions if a equals:

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) 2

(D) ± 2

Q28. If A is a square matrix of order 3 such that $|A| = 4$, then the value of $|\text{adj}(A)|$ is:

(A) 4

(B) 16

(C) 64

(D) 12

Q29. If $\begin{vmatrix} x-1 & 2 \\ 3 & x+1 \end{vmatrix} = 0$, then the values of x are:

(A) $\pm\sqrt{5}$

(B) $\pm\sqrt{7}$

(C) ± 2

(D) ± 3

Q30. Let A and B be invertible square matrices of the same order. Which of the following statement is NOT true?

(A) $(AB)^{-1} = B^{-1}A^{-1}$

(B) $(A^T)^{-1} = (A^{-1})^T$

(C) $\det(A^{-1}) = \frac{1}{\det(A)}$

(D) $(A+B)^{-1} = A^{-1} + B^{-1}$

Q31. The scalar projection of the vector $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$ is:

(A) $\frac{19}{9}$

(B) $\frac{19}{3}$

(C) $\frac{\sqrt{6}}{9}$

(D) 2

Q32. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:

(A) $\frac{1}{\sqrt{6}}$

(B) $\frac{1}{6}$

(C) 0

(D) $\frac{1}{\sqrt{3}}$

Q33. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is:

(A) -4

(B) 4

(C) -2

(D) 2

Q34. The angle between the plane $2x - y + z = 6$ and the line $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$ is given by:

(A) $\sin^{-1}\left(\frac{2}{3\sqrt{6}}\right)$

(B) $\cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$

(C) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

(D) $\frac{\pi}{4}$

Q35. Two events A and B are such that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.7$. Then $P(A|B)$ is equal to:

(A) 0.2

(B) 0.4

(C) 0.5

(D) 0.25

Q36. A pair of fair dice is rolled. What is the probability that the sum of the numbers showing up is 7, given that the two numbers are different?

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) $\frac{5}{6}$

(D) $\frac{2}{9}$

Q37. The mean and variance of 5 observations are 4 and 5.2 respectively. If three of the observations are 1, 2 and 6, then the remaining two observations are:

(A) 2, 9

(B) 4, 7

(C) 3, 8

(D) 5, 6

Q38. If a random variable X follows a binomial distribution with parameters $n = 6$ and p, such that $4P(X = 4) = P(X = 2)$, then the value of p is:

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{2}{3}$

Q39. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then the value of $\alpha^{2026} + \beta^{2026}$ is:

(A) 1

(B) -1

(C) 2

(D) 0

Q40. The principal argument of the complex number $z = \frac{1+i\sqrt{3}}{1-i}$ is:

(A) $\frac{7\pi}{12}$

(B) $\frac{\pi}{12}$

(C) $\frac{5\pi}{12}$

(D) $-\frac{5\pi}{12}$

Q41. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is:

(A) 2

(B) 4

(C) 1

(D) 0

Q42. Let R be a relation defined on the set of integers $\mathbb{Z}$ by aRb if and only if $|a - b| \leq 1$. The relation R is:

(A) Reflexive and Transitive

(B) Reflexive and Symmetric

(C) Symmetric and Transitive

(D) An Equivalence Relation

Q43. The domain of definition of the real function $f(x) = \frac{1}{\sqrt{x^2-3x+2}}$ is:

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(-\infty, 1] \cup [2, \infty)$

(C) $(1, 2)$

(D) $\mathbb{R} \setminus \{1, 2\}$

Q44. The value of $\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) + \cos\left(\tan^{-1}(2\sqrt{2})\right)$ is equal to:

(A) $\frac{14}{15}$

(B) $\frac{3}{5}$

(C) $\frac{13}{15}$

(D) $\frac{11}{15}$

Q45. The sum of the infinite geometric series $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots$ is:

(A) 2

(B) 3

(C) $\frac{3}{2}$

(D) 4

Q46. The number of words that can be formed using all the letters of the word 'EXAMINATION' is:

(A) $\frac{11!}{2!2!2!}$

(B) $\frac{11!}{2!2!}$

(C) $11!$

(D) $\frac{11!}{4!}$

Q47. The coefficient of x^4 in the binomial expansion of $\left(x^2 - \frac{1}{x}\right)^8$ is:

(A) 28

(B) -28

(C) 56

(D) −56

Q48. The distance between the parallel lines $3x + 4y - 9 = 0$ and $6x + 8y + 15 = 0$ is:

(A) $\frac{33}{10}$

(B) $\frac{24}{5}$

(C) $\frac{3}{5}$

(D) $\frac{33}{5}$

Q49. The equation of the straight line perpendicular to $2x - 3y + 5 = 0$ and passing through the point $(1, -1)$ is:

(A) $3x + 2y - 1 = 0$

(B) $3x + 2y + 1 = 0$

(C) $2x + 3y + 1 = 0$

(D) $3x - 2y - 5 = 0$

Q50. For the linear programming problem to maximize $Z = 3x + 5y$ subject to $x + 2y \leq 6, x \geq 0, y \geq 0$, the maximum value of Z occurs at the corner point:

(A) $(6, 0)$

(B) $(0, 3)$

(C) $(0, 0)$

(D) $(3, 1.5)$

Detailed Solutions

Q1.

Solution

Concept: Continuity of a piecewise-defined function at a transition point requires the left-hand limit, right-hand limit, and the function value to be equal. When dealing with limits containing exponential terms that approach infinity, analyzing the behavior of the base relative to unity is crucial.

Solution:

(a) Given function is $f(x) = \lim_{n\to\infty} \frac{\ln(2+x) - x^{2n}\sin(x)}{1+x^{2n}}$. We need to evaluate the value at the point $x = 1$.

(b) For $f(x)$ to be continuous at $x = 1$, we must have $f(1) = \lim_{x\to 1} f(x)$. Let us evaluate the behavior of x^{2n} as $n \to \infty$ near $x = 1$.

(c) When $x = 1$, the expression inside the limit simplifies directly because $1^{2n} = 1$ for all values of n. Substituting $x = 1$ gives $f(1) = \frac{\ln(2+1) - (1)\sin(1)}{1+1}$.

(d) Simplifying the numeric values leads to the exact functional representation, which yields $f(1) = \frac{\ln(3) - \sin(1)}{2}$.

(e) This unique value balances the behavior of the functional definition, matching both regional limits from the left and right, ensuring a smooth transition without any jump or removable discontinuity.

Final Answer: $\ln(3) - \sin(1)_{\overline{2}}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: Evaluation of trigonometric limits exhibiting the indeterminate form $0/0$ can be solved using standard expansions or algebraic manipulation. Rationalization of the numerator containing radical expressions helps isolate the vanishing factors.

Solution:

(a) The given limit is $L = \lim_{x\to 0} \frac{1-\cos(x)\sqrt{\cos(2x)}}{x^2}$. Substituting $x = 0$ yields the indeterminate form $\frac{0}{0}$.

(b) To eliminate the square root, multiply the numerator and denominator by the conjugate expression $1 + \cos(x)\sqrt{\cos(2x)}$.

(c) The numerator becomes $1 - \cos^2(x)\cos(2x)$. Use the trigonometric identity $\cos(2x) = 1 - 2\sin^2(x)$ to substitute into the expression.

(d) This transforms the numerator into $1 - \cos^2(x)(1 - 2\sin^2(x)) = 1 - \cos^2(x) + 2\cos^2(x)\sin^2(x) = \sin^2(x) + 2\cos^2(x)\sin^2(x)$.

(e) Factoring out $\sin^2(x)$ gives $\sin^2(x)(1 + 2\cos^2(x))$. Divide by x^2 and evaluate using the fundamental standard limit $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.

(f) The expression reduces to $\lim_{x\to 0} \left(\frac{\sin(x)}{x}\right)^2 \cdot \frac{1+2\cos^2(x)}{1+\cos(x)\sqrt{\cos(2x)}} = 1 \cdot \frac{1+2(1)}{1+1(1)} = \frac{3}{2}$.

Final Answer: $3_{\overline{2}}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: Differentiability of modulus functions fails at points where the expression inside the absolute value brackets becomes zero, creating sharp corners or cusps on the graph, provided the overall function does not have canceling smooth components.

Solution:

(a) The function is defined as $f(x) = |x - 1| + |x - 2| + \cos(x)$. The trigonometric term $\cos(x)$ is infinitely differentiable everywhere on the real line.

(b) The absolute value terms $|x - 1|$ and $|x - 2|$ are continuous everywhere but lack derivatives at their critical corner points.

(c) The internal expressions vanish at $x = 1$ and $x = 2$. Both of these critical values lie comfortably within the specified open interval $(0, 3)$.

(d) At $x = 1$, the left-hand derivative and right-hand derivative differ by a finite jump due to the sign change of the components.

(e) Similarly, at $x = 2$, a distinct sharp corner is formed on the curve. Therefore, the function fails to be differentiable at exactly two points.

Final Answer: 2

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: Trigonometric substitution simplifies complex inverse trigonometric expressions before differentiation. Substituting $x = \tan(\theta)$ eliminates algebraic roots using fundamental identity transformations.

Solution:

(a) Given the expression $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, let us substitute $x = \tan(\theta)$, which implies $\theta = \tan^{-1}(x)$.

(b) The expression simplifies to $\sqrt{1 + \tan^2(\theta)} = \sec(\theta)$. Thus, the terms inside become $\frac{\sec(\theta)-1}{\tan(\theta)}$.

(c) Converting into basic sine and cosine functions gives $\frac{1-\cos(\theta)}{\sin(\theta)} = \frac{2\sin^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)} = \tan(\theta/2)$.

(d) Taking the inverse tangent yields $y = \tan^{-1}(\tan(\theta/2)) = \frac{\theta}{2}$. Substituting back for x gives $y = \frac{1}{2}\tan^{-1}(x)$.

(e) Differentiating with respect to x gives $\frac{dy}{dx} = \frac{1}{2(1+x^2)}$. Evaluating this at $x = 1$ produces $\frac{1}{2(1+1)} = \frac{1}{4}$.

Final Answer: $1_{\overline{4}}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The angle between two intersecting curves is defined as the angle between their tangents at the point of intersection, computed using the scalar derivative slopes.

Solution:

(a) The given intersecting curves are $y^2 = 4x$ and $x^2 = 4y$. They intersect at the given coordinate point $(4, 4)$.

(b) Differentiating the first curve $y^2 = 4x$ implicitly gives $2y\frac{dy}{dx} = 4$, which simplifies to $m_1 = \frac{dy}{dx} = \frac{2}{y}$. At $(4, 4)$, $m_1 = \frac{2}{4} = \frac{1}{2}$.

(c) Differentiating the second curve $x^2 = 4y$ gives $2x = 4\frac{dy}{dx}$, which simplifies to $m_2 = \frac{dy}{dx} = \frac{x}{2}$. At $(4, 4)$, $m_2 = \frac{4}{2} = 2$.

(d) The formula for the angle θ between two lines with slopes m_1 and m_2 is given by $\tan(\theta) = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$.

(e) Substituting the values yields $\tan(\theta) = \left|\frac{2 - 1/2}{1 + 2(1/2)}\right| = \frac{3/2}{2} = \frac{3}{4}$. Taking inverse leads to $\theta = \tan^{-1}\left(\frac{3}{4}\right)$.

Final Answer: $\tan^{-1}\left(\frac{3}{4}\right)$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: Finding global extrema on a closed interval involves finding critical points where the first derivative vanishes and evaluating the function at those critical locations alongside the boundary endpoints.

Solution:

(a) The given function is $f(x) = x(1-x)^2$ on the domain interval $[0,1]$. Expanding or using product rule yields the derivative.

(b) Applying the product rule: $f'(x) = (1-x)^2 + x \cdot 2(1-x)(-1) = (1-x)[(1-x)-2x] = (1-x)(1-3x)$.

(c) Setting the first derivative to zero for locating critical points gives $(1-x)(1-3x) = 0$, yielding solutions $x = 1$ and $x = \frac{1}{3}$.

(d) Evaluate the function at these candidates: $f(0) = 0$, $f(1) = 0$, and $f(1/3) = \frac{1}{3}\left(1-\frac{1}{3}\right)^2 = \frac{1}{3}\left(\frac{2}{3}\right)^2 = \frac{4}{27}$.

(e) Comparing these values shows that the maximum value occurs exactly at the point $x = \frac{1}{3}$.

Final Answer: $1_{\overline{3}}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: Related rates problems utilize geometric relationships, such as the Pythagorean theorem, to link varying physical quantities and differentiate them with respect to time.

Solution:

(a) Let x be the distance of the foot of the ladder from the wall and y be the height of the top of the ladder from the ground.

(b) By the Pythagorean theorem, the constants and variables are related by $x^2 + y^2 = 5^2 = 25$.

(c) Differentiating this geometric constraint equation implicitly with respect to time t yields $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, which simplifies to $x\frac{dx}{dt} + y\frac{dy}{dt} = 0$.

(d) We are given $\frac{dx}{dt} = 2$ m/s and need to find $\frac{dy}{dt}$ at the instant when $x = 4$ m.

(e) When $x = 4$, the height is $y = \sqrt{25 - 4^2} = 3$ m. Substituting these values into the differentiated relation gives $4(2) + 3\frac{dy}{dt} = 0$, solving to $\frac{dy}{dt} = -\frac{8}{3}$ m/s.

Final Answer: -8$_{\overline{3\text{ m/s}}}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: A function is monotonic over an interval if its first derivative maintains a constant sign throughout that interval. Monotonicity changes occur at real roots of the derivative where sign changes take place.

Solution:

(a) The given cubic polynomial function is $f(x) = x^3 - 3x^2 + 3x + 7$. Let us find its first derivative.

(b) Differentiating with respect to x yields $f'(x) = 3x^2 - 6x + 3$. Factoring out the constant 3 gives $f'(x) = 3(x^2 - 2x + 1)$.

(c) Recognizing the quadratic term inside as a perfect square, we can rewrite the derivative as $f'(x) = 3(x - 1)^2$.

(d) Since the square of any real number is always non-negative, $(x - 1)^2 \geq 0$ for all $x \in \mathbb{R}$. Thus, $f'(x) \geq 0$ everywhere, vanishing only at $x = 1$.

(e) Because the derivative is strictly positive everywhere except at an isolated point, the function is strictly increasing across the entire domain of real numbers $\mathbb{R}$.

Final Answer: Strictly increasing on R

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: Lagrange's Mean Value Theorem states that if a function is continuous on $[a, b]$ and differentiable on (a, b), there exists a point $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Solution:

(a) The function given is $f(x) = \ln(x)$ evaluated on the specific closed domain interval $[1, e]$.

(b) The derivative of the natural logarithmic function is given by $f'(x) = \frac{1}{x}$.

(c) Applying the Mean Value Theorem formula gives the relation $f'(c) = \frac{\ln(e)-\ln(1)}{e-1}$.

(d) Knowing that $\ln(e) = 1$ and $\ln(1) = 0$, the right side simplifies to $\frac{1-0}{e-1} = \frac{1}{e-1}$.

(e) Equating this to the derivative expression gives $\frac{1}{c} = \frac{1}{e-1}$, which directly solves to $c = e - 1$.

Final Answer: e - 1

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Substitution algebraic methods help evaluate fractional integrals. Multiplying the numerator and denominator by algebraic powers transforms the integrand into a form suitable for substitution.

Solution:

(a) The integral to evaluate is $I = \int \frac{1}{x(x^5+1)}\,dx$. Multiply numerator and denominator by x^4.

(b) This algebraic step transforms the integral into the equivalent expression $I = \int \frac{x^4}{x^5(x^5+1)}\,dx$.

(c) Let us apply the substitution method by setting $t = x^5$, which means that $dt = 5x^4\,dx$, or $\frac{dt}{5} = x^4\,dx$.

(d) Substituting these values converts the expression into $I = \frac{1}{5}\int \frac{1}{t(t+1)}\,dt$. Use partial fractions: $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$.

(e) Integrating gives $\frac{1}{5}(\ln|t| - \ln|t+1|) + C = \frac{1}{5}\ln\left|\frac{t}{t+1}\right| + C$. Substituting back $t = x^5$ gives $\frac{1}{5}\ln\left|\frac{x^5}{x^5+1}\right| + C$.

Final Answer: $1\frac{}{5\ln\left|\frac{x^5}{x^5+1}\right|+C}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: Definite integrals containing symmetric algebraic configurations of basic trigonometric functions over the domain boundary interval from zero to half-pi can be evaluated effectively by using the fundamental shifting reflection property of definite integrals.

Solution:

(a) Let the given definite integral expression be denoted by the variable $I = \int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\, dx$.

(b) We apply the standard definite integration shifting property where the variable x within the integrand is replaced by the sum of its boundary limits minus x, which translates to replacing x with the term $\frac{\pi}{2} - x$.

(c) Using basic trigonometric reduction rules, the term $\sin\left(\frac{\pi}{2} - x\right)$ simplifies to $\cos x$, and the corresponding term $\cos\left(\frac{\pi}{2} - x\right)$ simplifies directly to $\sin x$.

(d) This transformation yields a secondary rewritten version of the original integral equation which can be expressed smoothly as $I = \int_0^{\pi/2} \frac{\cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x}\, dx$.

(e) Adding these two equal equations together yields $2I = \int_0^{\pi/2} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\, dx$, which reduces to the basic integral $\int_0^{\pi/2} 1\, dx$.

(f) Evaluating this basic component leads to the expression $2I = \frac{\pi}{2}$. Dividing both sides by the constant two isolates the value, which yields the result of $\frac{\pi}{4}$.

Final Answer: π_4

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: Definite integrals of products containing absolute values require splitting the domain of integration into sub-intervals based on the sign changes of the inner expressions to correctly eliminate the absolute value brackets.

Solution:

(a) The given mathematical integral to solve is expressed as $\int_{-1}^{1} x|x|\,dx$. The term contains a sign-dependent absolute modulus function.

(b) By definition, the value of $|x|$ changes its definition at the origin. It is equal to $-x$ for negative numbers and $+x$ for non-negative numbers.

(c) This behavior requires splitting the complete integration domain into two separate regions, specifically from minus one to zero and from zero to positive one.

(d) In the first region from minus one to zero, replacing $|x|$ with $-x$ transforms the product term into the algebraic expression $-x^2$.

(e) In the second region from zero to one, replacing $|x|$ with $+x$ leaves the product term as the positive algebraic expression x^2.

(f) The mathematical integral is an odd function across symmetric limits, meaning the first negative accumulation cancels out the second positive accumulation, resulting in a total area of zero.

Final Answer: 0

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: The geometric area enclosed between two intersecting planar curves is determined by integrating the vertical difference between the upper bounding curve and the lower bounding curve between their points of intersection.

Solution:

(a) The given algebraic functions enclosing the geometric region are the standard parabola $y = x^2$ and the linear straight line $y = x$.

(b) To locate the boundaries of the integration domain, we find their intersection points by setting the equations equal, which gives $x^2 = x$.

(c) Solving this equation yields $x(x - 1) = 0$, which gives the two intersecting real roots at the points $x = 0$ and $x = 1$.

(d) Within this bounded open domain interval $(0, 1)$, the linear equation $y = x$ lies vertically above the quadratic parabola curve $y = x^2$.

(e) The area equation is written as the definite integral $\int_0^1 (x - x^2)\, dx$. Integrating the terms gives the polynomial evaluation $\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$.

(f) Substituting the upper limit and subtracting the lower limit yields the final arithmetic fraction calculation $\frac{1}{2} - \frac{1}{3}$, which evaluates precisely to $\frac{1}{6}$.

Final Answer: $1_{\overline{6}}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: A first-order linear differential equation in standard form requires an integrating factor to render it integrable, determined by exponentiating the integral of the coefficient of the dependent variable.

Solution:

(a) The given mathematical expression is a first-order linear differential equation written in standard form as $\frac{dy}{dx} + y \tan x = \sec x$.

(b) Comparing this expression to the general linear differential equation standard form $\frac{dy}{dx} + P(x)y = Q(x)$ isolates the coefficient function $P(x) = \tan x$.

(c) The formula used to determine the integrating factor of such a linear structure is defined as I.F. $= e^{\int P(x)\,dx}$.

(d) Substituting the isolated coefficient function into the exponent yields the calculus expression $e^{\int \tan x\,dx}$.

(e) The standard fundamental integration of the tangent trigonometric function is known to equal the natural logarithm of the secant function, giving $e^{\ln|\sec x|}$.

(f) Since the base exponential function and the natural logarithm function are inverse operations, they cancel out, leaving the final integrating factor as $\sec x$.

Final Answer: $\sec x$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: First-order differential equations that can be rearranged to isolate variables on opposite sides of the equality sign are solved using the technique of separation of variables followed by integration.

Solution:

(a) The given first-order differential equation to solve is written as $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$. This form allows for easy separation.

(b) Rearranging the terms to group the dependent variable y with its differential and the independent variable x with its differential yields $\frac{1}{1+y^2}\,dy = \frac{1}{1+x^2}\,dx$.

(c) Integrating both sides of this separated equation simultaneously results in the standard inverse trigonometric expression $\tan^{-1} y = \tan^{-1} x + C_1$.

(d) Rearranging the constants and functions gives the relation $\tan^{-1} y - \tan^{-1} x = C_1$. Apply the standard inverse tangent subtraction identity.

(e) This yields the combined trigonometric equation $\tan^{-1}\left(\frac{y-x}{1+xy}\right) = C_1$. Taking the tangent of both sides removes the inverse function.

(f) The expression simplifies directly to $\frac{y-x}{1+xy} = \tan(C_1) = C$. Cross-multiplying the denominator yields the final algebraic relation $y - x = C(1 + xy)$.

Final Answer: y - x = C(1+xy)

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: The order of a differential equation is the highest derivative present. The degree is the power of that highest derivative after the equation is cleared of fractional exponents and radicals.

Solution:

(a) The given differential equation is expressed with a fractional exponent on the left side: $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$.

(b) To find the degree, we must eliminate the fractional exponent by squaring both sides of the differential equation.

(c) Squaring both sides of the mathematical expression transforms the equation into the polynomial form $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$.

(d) Analyzing this rewritten form, the highest derivative appearing in the expression is the second derivative, $\frac{d^2y}{dx^2}$, which fixes the order at two.

(e) The exponent raised on this highest order derivative term in the rationalized form is two. Therefore, the degree of the differential equation is two.

Final Answer: 2, 2

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: The limit of a Riemann sum as the number of intervals approaches infinity can be evaluated by converting the summation into a definite integral over a fixed domain.

Solution:

(a) The given summation limit problem to evaluate is expressed as $L = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n+r}$.

(b) To transform this into a Riemann integral form, we factor out the term n from the denominator, rewriting it as $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{1}{1+\frac{r}{n}}$.

(c) We substitute the continuous variable x for the discrete fraction $\frac{r}{n}$, and replace the differential step $\frac{1}{n}$ with dx.

(d) The lower limit of integration is found by computing $\lim_{n\to\infty} \frac{1}{n} = 0$, and the upper limit is found via $\lim_{n\to\infty} \frac{n}{n} = 1$.

(e) This maps the summation directly to the continuous definite integral $\int_0^1 \frac{1}{1+x}\, dx$.

(f) Integrating this reciprocal function yields $[\ln|1+x|]_0^1$. Evaluating at the boundaries gives $\ln(2) - \ln(1)$, which simplifies directly to $\ln(2)$.

Final Answer: $\ln(2)$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: For a standard parabola, any focal chord has endpoints whose parametric parameters satisfy a constant product relationship determined by the position of the focus.

Solution:

(a) The given equation of the conic is the horizontal parabola $y^2 = 4x$, which means the semi-latus rectum parameter value is $a = 1$.

(b) Any generic coordinate point lying on this standard parabola can be represented using parametric coordinates as $P(t^2, 2t)$.

(c) We are given that one endpoint of the focal chord is located at $P(4, 4)$. Equating the y-coordinates gives $2t_1 = 4$, solving to $t_1 = 2$.

(d) For any focal chord passing through the focus of a parabola, the parametric parameters of the two endpoints satisfy the constant relation $t_1 \cdot t_2 = -1$.

(e) Substituting the known value $t_1 = 2$ into this relation gives $2 \cdot t_2 = -1$, which yields the second parameter value $t_2 = -\frac{1}{2}$.

(f) Substituting $t_2 = -\frac{1}{2}$ into the parametric form $(t^2, 2t)$ yields the coordinates of Q as $\left((-\frac{1}{2})^2, 2(-\frac{1}{2})\right)$, which simplifies to $(\frac{1}{4}, -1)$.

Final Answer: $(1\overline{_{4,-1)}}$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept: The standard equation of an ellipse centered at the origin can be determined by substituting known passing coordinates to solve for the unknown major and minor semi-axes.

Solution:

(a) Let the standard equation of the ellipse centered at the origin be defined as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Substituting the first given point $(4, 3)$ into this equation yields the algebraic relation $\frac{16}{a^2} + \frac{9}{b^2} = 1$.

(c) Substituting the second given point $(6, 2)$ into the equation yields the second relation $\frac{36}{a^2} + \frac{4}{b^2} = 1$.

(d) Solving these two linear equations simultaneously for the variables $\frac{1}{a^2}$ and $\frac{1}{b^2}$ yields the parameters $a^2 = 52$ and $b^2 = 13$.

(e) The eccentricity e of an ellipse is related to its semi-axes by the standard formula $b^2 = a^2(1 - e^2)$.

(f) Substituting the calculated values gives $13 = 52(1 - e^2)$, which reduces to $1 - e^2 = \frac{1}{4}$. Solving for e yields $e = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$.

Final Answer: $\sqrt{3}_{2}$

Answer: (D)

Go Back to Question 19

Q20.

Solution

Concept: A circle passing through the origin and cutting intercepts on the coordinate axes has its diameter endpoints defined by the intercepts, forming a right triangle with the origin.

Solution:

(a) The circle cuts intercepts of length 3 and 4 units from the positive x-axis and positive y-axis respectively.

(b) This means the circle passes through the origin $O(0, 0)$ and intersects the axes at the points $A(3, 0)$ and $B(0, 4)$.

(c) Since the axes are perpendicular, the angle $\angle AOB = 90°$. By Thales's theorem, the line segment AB forms the diameter of the circle.

(d) The diametric form of a circle's equation with endpoints (x_1, y_1) and (x_2, y_2) is $(x-x_1)(x-x_2) + (y - y_1)(y - y_2) = 0$.

(e) Substituting the coordinates of $A(3, 0)$ and $B(0, 4)$ gives the equation $(x - 3)(x - 0) + (y - 0)(y - 4) = 0$.

(f) Expanding this expression yields the final circle equation $x^2 + y^2 - 3x - 4y = 0$.

Final Answer: $x^2 + y^2 - 3x - 4y = 0$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: The standard mathematical equation of a horizontal hyperbola centered at the origin is determined by its real vertex coordinates along with the slope configuration of its linear asymptotes.

Solution:

(a) The given conic section has real vertices positioned along the horizontal axis at the coordinates $(\pm 3, 0)$. This profile indicates a horizontal hyperbola structure.

(b) From these given vertex coordinates, the length of the semi-major horizontal axis parameter is isolated directly as $a = 3$.

(c) The given mathematical equations for the linear asymptotes are configured as $y = \pm\frac{4}{3}x$. For a horizontal hyperbola, the asymptote slopes are defined by $\pm\frac{b}{a}$.

(d) Equating these slope values results in the fractional relation $\frac{b}{a} = \frac{4}{3}$. Substituting our known value $a = 3$ yields the vertical semi-axis parameter $b = 4$.

(e) Squaring these two isolated parameters gives the numerical values $a^2 = 9$ and $b^2 = 16$.

(f) Substituting these values into the horizontal hyperbola canonical formula $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ gives the final analytical coordinate equation $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

Final Answer: $\mathrm{x}^2 \frac{}{9 - \frac{y^2}{16} = 1}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: A linear equation acts as a perfect tangent to a standard horizontal parabola when the constant intercept matches the quotient of the focal parameter and the slope parameter.

Solution:

(a) The given algebraic expressions consist of a linear line $y = mx + 1$ and a horizontal parabola curve defined by $y^2 = 4x$.

(b) Comparing the curve to the standard parabolic formula $y^2 = 4ax$ allows us to isolate the focal parameter value as $a = 1$.

(c) Comparing the linear equation to the standard slope-intercept line formula $y = mx + c$ isolates the constant y-intercept value as $c = 1$.

(d) The standard required condition for a straight line to maintain perfect tangency with a horizontal parabola is defined by the algebraic equation $c = \frac{a}{m}$.

(e) Substituting our isolated parameters into this conditional relation yields the basic mathematical equation $1 = \frac{1}{m}$.

(f) Solving this equation for the unknown slope parameter directly determines that $m = 1$ is required to satisfy the geometric constraint.

Final Answer: 1

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: The length of the latus rectum chord for a standard hyperbola is determined using the geometric formula containing the ratio of the squared minor axis to the major axis.

Solution:

(a) The given algebraic equation of the hyperbola is $9x^2 - 16y^2 = 144$. We first convert this into standard canonical form.

(b) Dividing all three mathematical terms by the scalar value 144 yields the rearranged fractional expression $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

(c) Comparing this expression to the standard horizontal hyperbola equation yields the values $a^2 = 16$ and $b^2 = 9$.

(d) Taking the square root of the primary horizontal tracking component determines the major semi-axis parameter length to be $a = 4$.

(e) The geometric formula representing the full total length of the latus rectum chord is given by the fractional configuration $\frac{2b^2}{a}$.

(f) Substituting our isolated parameters into this formula yields $\frac{2(9)}{4}$, which simplifies directly to the final fraction of $\frac{9}{2}$.

Final Answer: $9_{\overline{2}}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: The scalar radius of a circle defined in general polynomial form is calculated by taking the square root of the combined squared coordinate center offsets minus the constant term.

Solution:

(a) The given algebraic equation of the circle is expressed in general form as $x^2 + y^2 - 4x + 6y - 12 = 0$.

(b) Comparing this to the general circle equation $x^2 + y^2 + 2gx + 2fy + c = 0$ lets us isolate the structural constants.

(c) The linear coefficients yield $2g = -4$, meaning $g = -2$, and $2f = 6$, meaning $f = 3$. The standalone constant is $c = -12$.

(d) The analytical formula used to compute the scalar radius of a circle from these components is defined as $r = \sqrt{g^2 + f^2 - c}$.

(e) Substituting our isolated values into this radical formula yields the calculation $r = \sqrt{(-2)^2 + (3)^2 - (-12)}$.

(f) Simplifying the values inside the radical gives $\sqrt{4 + 9 + 12} = \sqrt{25}$, which evaluates precisely to a scalar radius length of 5.

Final Answer: 5

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: An ellipse is defined locus-wise as the path of a point whose sum of distances from two fixed focal points remains constant, where this sum equals the full major axis length.

Solution:

(a) The problem describes a moving coordinate point whose sum of distances from two fixed coordinates $(\pm 4, 0)$ is always equal to 10 units.

(b) This geometric description matches the definition of an ellipse where the two fixed positions act as the internal focal points.

(c) The constant sum of distances represents the length of the major axis, giving the equation $2a = 10$, which simplifies to $a = 5$.

(d) The coordinates of the foci are given generally by $(\pm ae, 0)$, which isolates the composite focal parameter as $ae = 4$.

(e) The structural parameters of an ellipse are linked through the eccentric tracking equation defined as $b^2 = a^2 - (ae)^2$.

(f) Substituting our known numerical components yields $b^2 = 5^2 - 4^2 = 25 - 16 = 9$. Taking the square root gives the semi-minor axis length $b = 3$.

Final Answer: 3

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: Square matrices satisfy their own characteristic polynomial equation according to the Cayley-Hamilton theorem, linking matrix powers directly to the trace, determinant, and identity matrix.

Solution:

(a) The given matrix expression is a square matrix of order two defined as $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. We need to compute $A^2 - 5A$.

(b) First, we calculate the matrix square A^2 by performing matrix multiplication: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(c) Computing the entries yields $\begin{bmatrix} 1(1)+2(3) & 1(2)+2(4) \\ 3(1)+4(3) & 3(2)+4(4) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$.

(d) Next, we calculate the scaled matrix $5A$, which equals $\begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$.

(e) Subtracting these matrices yields $A^2 - 5A = \begin{bmatrix} 7-5 & 10-10 \\ 15-15 & 22-20 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.

(f) Factoring out the scalar value two leaves the standard identity matrix, giving the final simplified result of $2I$.

Final Answer: 2I

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: A system of linear equations possesses an infinite number of solutions when both the primary determinant of coefficients and the corresponding substituted determinant variables vanish simultaneously.

Solution:

(a) The given system of three linear equations can be analyzed by examining the determinant of its coefficient matrix, denoted as Δ.

(b) Setting up the matrix grid gives $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$. We perform row operations to simplify.

(c) Applying the row operation $R_3 \to R_3 - R_2$ simplifies the third row, transforming the determinant into $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix}$.

(d) Expanding along the simplified third row yields the determinant value $\Delta = (a^2-3)(3-2) = a^2 - 3$.

(e) For the linear system to sustain infinitely many solutions, this principal determinant must equal zero, yielding $a^2 - 3 = 0$, or $a = \pm\sqrt{3}$.

(f) Checking the constant terms shows that when $a^2 = 3$, the final equation aligns perfectly with the second equation, ensuring a consistent dependent system.

Final Answer: $\pm\sqrt{3}$

Answer: (D)

Go Back to Question 27

Q28.

Solution

Concept: The determinant of the adjugate of a square matrix of order n is equal to the determinant of the original matrix raised to the power of $n - 1$.

Solution:

(a) We are given that A is a square matrix with a dimensional order of $n = 3$, and its determinant value is specified as $|A| = 4$.

(b) We need to determine the value of the determinant of the adjugate matrix, which is written mathematically as $|\text{adj}(A)|$.

(c) There is a standard theorem in matrix algebra that links these two determinant values: $|\text{adj}(A)| = |A|^{n-1}$.

(d) Substituting our known dimensional order value $n = 3$ into this exponent expression gives the simplified relation $|A|^{3-1} = |A|^2$.

(e) Substituting our given determinant value of 4 into this squared relationship yields the calculation 4^2.

(f) Evaluating this exponent square results in the final numerical value of 16.

Final Answer: 16

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: The determinant of a two-by-two matrix is evaluated by subtracting the product of the anti-diagonal entries from the product of the main diagonal entries.

Solution:

(a) The given mathematical expression is a two-by-two determinant equated to zero: $\begin{vmatrix} x-1 & 2 \\ 3 & x+1 \end{vmatrix} = 0$.

(b) Evaluating the determinant using cross-multiplication yields the algebraic equation $(x-1)(x+1) - (2)(3) = 0$.

(c) Expanding the first product using the difference of squares identity transforms the expression into $x^2 - 1 - 6 = 0$.

(d) Combining the constant numerical values simplifies the equation directly to $x^2 - 7 = 0$.

(e) Rearranging the equation to isolate the squared variable term gives $x^2 = 7$.

(f) Taking the square root of both sides yields the final solutions $x = \pm\sqrt{7}$.

Final Answer: $\pm\sqrt{7}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: Inverse matrix operations distribute across products in reverse order and commute with transpositions and determinants, but they do not distribute linearly across matrix addition.

Solution:

(a) Let us analyze the validity of each statement given for invertible square matrices A and B of the same order.

(b) Statement A asserts $(AB)^{-1} = B^{-1}A^{-1}$, which is the standard socks-and-shoes property of matrix inversion and is mathematically true.

(c) Statement B asserts $(A^T)^{-1} = (A^{-1})^T$, which states that inversion and transposition operations commute, which is a verified property.

(d) Statement C asserts $\det(A^{-1}) = \frac{1}{\det(A)}$, which correctly reflects the multiplicative property of determinants and is true.

(e) Statement D asserts $(A + B)^{-1} = A^{-1} + B^{-1}$. In matrix algebra, the inverse of a sum does not equal the sum of the individual inverses.

(f) Therefore, Statement D is a false mathematical assertion, making it the correct choice for this problem.

Final Answer: $(\text{A+B})^{-1} = A^{-1} + B^{-1}$

Answer: (D)

Go Back to Question 30

Q31.

Solution

Concept: The scalar projection of one vector onto another vector is determined by calculating the dot product of the two vectors divided by the full length magnitude of the target base vector.

Solution:

(a) We are given two three-dimensional vectors defined as $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.

(b) To compute the scalar projection of vector $\vec{a}$ onto vector $\vec{b}$, we use the standard mathematical geometric formula expressed as $\text{Proj} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

(c) First, we calculate the dot product of the two given component arrays, which yields $\vec{a} \cdot \vec{b} = (1)(4) + (-2)(-4) + (1)(7)$.

(d) Simplifying these individual multiplications results in the total added sum value $4 + 8 + 7$, which evaluates directly to 19.

(e) Next, we determine the scalar length magnitude of the base vector $\vec{b}$ by computing the square root of its squared components: $|\vec{b}| = \sqrt{4^2 + (-4)^2 + 7^2}$.

(f) Expanding this radical yields $\sqrt{16 + 16 + 49} = \sqrt{81}$, which equals exactly 9. Combining these components gives the final fractional result of $\frac{19}{9}$.

Final Answer: $19_{\overline{9}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The shortest distance between two skew lines in a three-dimensional cartesian coordinate space is determined by evaluating the projection of the baseline segment connecting the lines onto their common perpendicular vector cross product.

Solution:

(a) The given mathematical equations represent two linear equations passing through unique spatial coordinates with independent direction vectors.

(b) The first line passes through the point $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ with direction vector $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

(c) The second line passes through the point $\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$ with direction vector $\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$.

(d) We determine the baseline displacement vector connecting these two line positions by computing the difference $\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$.

(e) Next, we find the common cross product vector defining the perpendicular orientation tracking, which is $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$.

(f) Expanding this vector determinant yields components equal to $-\hat{i} + 2\hat{j} - \hat{k}$. Calculating the scalar distance via the standard skew distance formula yields 0, revealing that these lines actually intersect in space.

Final Answer: 0

Answer: (C)

Go Back to Question 32

Q33.

Solution

Concept: A set of three vectors in three-dimensional space is coplanar if and only if their scalar triple product, evaluated as a matrix component determinant, vanishes completely.

Solution:

(a) We are given three algebraic vectors containing an unknown scalar tracking parameter: $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$.

(b) For these three coordinates to lie completely within a single flat plane, their scalar triple product equation must satisfy the constraint $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

(c) This requirement allows us to assemble the component matrix determinant expression as $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$.

(d) Expanding this determinant along the first row gives the linear equation $2(10 + 3\lambda) - (-1)(5 + 9) + 1(\lambda - 6) = 0$.

(e) Simplifying the individual algebraic expressions yields the combined equation $20 + 6\lambda + 14 + \lambda - 6 = 0$.

(f) Combining the tracking terms and numerical constants gives $7\lambda + 28 = 0$. Isolating the parameter yields $7\lambda = -28$, which solves to $\lambda = -4$.

Final Answer: -4

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The geometric angle between a line and a plane is determined by calculating the complementary angle of the inclination between the line direction vector and the normal vector of the plane.

Solution:

(a) The given mathematical expressions consist of a flat plane equation $2x - y + z = 6$ and a three-dimensional linear line equation.

(b) From the plane equation, we extract the normal orientation direction vector as $\vec{n} = 2\hat{i} - \hat{j} + \hat{k}$.

(c) From the line equation, we extract the structural tracking direction vector from the denominators as $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$.

(d) The geometric angle θ between a line and a plane is governed by the sine dot product formula written as $\sin\theta = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$.

(e) We compute the coordinate dot product value in the numerator, which yields $\vec{b} \cdot \vec{n} = (1)(2) + (2)(-1) + (2)(1) = 2 - 2 + 2 = 2$.

(f) Next, we compute the magnitudes: $|\vec{b}| = \sqrt{1 + 4 + 4} = 3$ and $|\vec{n}| = \sqrt{4 + 1 + 1} = \sqrt{6}$. This gives $\sin\theta = \frac{2}{3\sqrt{6}}$, so $\theta = \sin^{-1}\left(\frac{2}{3\sqrt{6}}\right)$.

Final Answer: $\sin^{-1}\left(\frac{2}{3\sqrt{6}}\right)$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: Conditional probability values measure the likelihood of an event occurring given that another event has already occurred, determined by dividing the joint intersection probability by the condition base probability.

Solution:

(a) We are given the probability values for two statistical events: $P(A) = 0.4$, $P(B) = 0.5$, and their combined union value $P(A \cup B) = 0.7$.

(b) To evaluate the conditional probability expression $P(A|B)$, we must first determine the joint intersection probability value $P(A \cap B)$.

(c) We use the fundamental addition theorem of probability theory, which is stated as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(d) Substituting our known numerical values into this expression gives the calculation $0.7 = 0.4 + 0.5 - P(A \cap B)$.

(e) Simplifying the numbers on the right side yields $0.7 = 0.9 - P(A \cap B)$, which isolates the intersection value as $P(A \cap B) = 0.2$.

(f) Finally, applying the conditional definition formula gives $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5}$. This division evaluates precisely to 0.4.

Final Answer: 0.4

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: Conditional probability on a sample space requires restricting the total number of outcomes to only those that satisfy the given condition, which then serves as the new baseline denominator.

Solution:

(a) A pair of fair six-sided dice is rolled, creating a standard sample space containing a total total of 36 equally likely numerical outcomes.

(b) Let event B represent the condition that the two showing numbers must be completely different. There are 6 matching pairs out of 36, so the count of different outcomes is $36 - 6 = 30$.

(c) Let event A represent the situation where the sum of the two showing numbers equals exactly 7.

(d) The specific outcomes that yield a sum of 7 are listed as the pairs $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2)$, and $(6, 1)$. This gives a total of 6 outcomes.

(e) Reviewing this subset, all 6 of these pairs consist of different numbers, meaning all 6 outcomes lie completely within the restricted condition space of event B.

(f) Therefore, the conditional probability is calculated by taking the ratio of these favorable matching outcomes to the restricted total space, yielding $\frac{6}{30} = \frac{1}{5}$.

Final Answer: $1_{\overline{5}}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: The unknown values in a small data statistical set can be isolated by setting up a system of two algebraic equations derived from the definitions of the arithmetic mean and the statistical variance.

Solution:

(a) We are given a total of 5 observations where three values are known to be 1, 2, and 6. Let the remaining two unknown observations be denoted as x and y.

(b) The arithmetic mean of the total data set is specified as 4. Using the sum formula gives $\frac{1+2+6+x+y}{5} = 4$.

(c) Simplifying this algebraic expression gives $9 + x + y = 20$, which isolates the first linear relation as $x + y = 11$.

(d) The statistical variance is given as 5.2. The standard formula for variance is expressed as $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$.

(e) Substituting our values into this formula gives $5.2 = \frac{1^2+2^2+6^2+x^2+y^2}{5} - 4^2$, which simplifies to $5.2 = \frac{41+x^2+y^2}{5} - 16$.

(f) Rearranging the numbers yields $21.2 = \frac{41+x^2+y^2}{5}$, which reduces to $106 = 41 + x^2 + y^2$, or $x^2 + y^2 = 65$. Solving this with $x + y = 11$ gives the values 4 and 7.

Final Answer: 4, 7

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept: A discrete binomial distribution model links the probabilities of unique success counts using a combinatorial expression dependent on the trials parameter and the single success probability.

Solution:

(a) The random variable X follows a standard binomial distribution with the total number of independent trials given as $n = 6$ and a success probability parameter p.

(b) Let the corresponding failure probability parameter be defined as $q = 1 - p$. The general binomial probability formula is given as $P(X = r) = \binom{n}{r} p^r q^{n-r}$.

(c) We are given the conditional balancing equation $4P(X = 4) = P(X = 2)$. Substituting our formula parameters into both sides yields:

(d) $4 \cdot \binom{6}{4} p^4 q^2 = \binom{6}{2} p^2 q^4$. We use the combinatorial symmetry property where $\binom{6}{4} = \binom{6}{2} = 15$.

(e) Since these combinatorial coefficient values are identical, they cancel out from both sides, leaving the simplified algebraic equation $4p^4 q^2 = p^2 q^4$.

(f) Dividing both sides by the common non-zero tracking term $p^2 q^2$ reduces the expression to $4p^2 = q^2$. Taking the square root yields $2p = q$. Substituting $q = 1 - p$ gives $3p = 1$, or $p = \frac{1}{3}$.

Final Answer: $1_{\overline{3}}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: The complex roots of a specific quadratic equation can be identified as primitive cube roots of unity, allowing high integer power exponents to be evaluated easily using their periodic behavior.

Solution:

(a) We are given the quadratic equation $x^2 - x + 1 = 0$. Using the standard quadratic solution formula, the roots are found to be $x = \frac{1 \pm i\sqrt{3}}{2}$.

(b) These specific complex coordinates match the negative versions of the standard primitive cube roots of unity, meaning we can define $\alpha = -\omega$ and $\beta = -\omega^2$.

(c) We need to evaluate the sum of these roots raised to a high integer power exponent: $\alpha^{2026} + \beta^{2026} = (-\omega)^{2026} + (-\omega^2)^{2026}$.

(d) Since the integer exponent 2026 is an even number, the negative signs disappear, leaving the expression as $\omega^{2026} + \omega^{4052}$.

(e) The primitive cube roots of unity have a periodic cycle of three, since $\omega^3 = 1$. Dividing the exponent 2026 by 3 leaves a remainder of 1, so $\omega^{2026} = \omega^1$.

(f) Similarly, dividing the second exponent 4052 by 3 leaves a remainder of 2, so $\omega^{4052} = \omega^2$. This reduces the sum to $\omega + \omega^2$, which equals -1 via the identity $1 + \omega + \omega^2 = 0$.

Final Answer: -1

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept: The principal argument of a fraction of complex numbers is calculated by subtracting the principal argument of the denominator layout from the principal argument of the numerator layout.

Solution:

(a) We are given a complex number written as a fraction of two distinct components: $z = \frac{1+i\sqrt{3}}{1-i}$. Let the top be $z_1 = 1 + i\sqrt{3}$ and the bottom be $z_2 = 1 - i$.

(b) The principal argument operation follows logarithmic distribution rules, meaning that $\arg(z) = \arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$.

(c) For the numerator component $z_1 = 1 + i\sqrt{3}$, both coordinates are positive, placing it in the first quadrant. Its argument is $\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$.

(d) For the denominator component $z_2 = 1 - i$, the real part is positive and the imaginary part is negative, placing it in the fourth quadrant. Its argument is $\tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$.

(e) Substituting these isolated angular positions back into our logarithmic subtraction expression yields the calculation $\arg(z) = \frac{\pi}{3} - (-\frac{\pi}{4})$.

(f) Converting these fractions to a common denominator gives $\frac{\pi}{3} + \frac{\pi}{4} = \frac{4\pi+3\pi}{12} = \frac{7\pi}{12}$, which lies within the valid principal range.

Final Answer: $7\pi_{\overline{12}}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: An algebraic equation involving absolute values can be broken down into symmetric cases based on the sign of the variable to find all distinct real values that satisfy the condition.

Solution:

(a) The given mathematical equation is a quadratic form containing absolute value segments expressed as $x^2 - 3|x| + 2 = 0$.

(b) Using the algebraic property where $x^2 = |x|^2$, we rewrite the polynomial tracking equation entirely in terms of the absolute variable as $|x|^2 - 3|x| + 2 = 0$.

(c) We can factor this expression directly by breaking up the middle term, which yields the grouped equation $(|x| - 1)(|x| - 2) = 0$.

(d) This factors into two independent linear relations for the absolute value component, namely $|x| = 1$ or $|x| = 2$.

(e) Solving the first absolute equation $|x| = 1$ gives two valid real solutions, which are $x = 1$ and $x = -1$.

(f) Solving the second absolute equation $|x| = 2$ gives another two valid real solutions, which are $x = 2$ and $x = -2$. Combining these cases reveals a total count of 4 distinct real solutions.

Final Answer: 4

Answer: (B)

Go Back to Question 41

Q42.

Solution

Concept: A mathematical relation on a specific set is evaluated for structural reflexivity, symmetry, and transitivity by validating if elements satisfy the boundary constraints under element permutations.

Solution:

(a) Let the relation R be defined on the complete set of integers $\mathbb{Z}$ by the absolute bounding rule where aRb means $|a - b| \leq 1$.

(b) To check for reflexivity, we substitute a for b, yielding $|a - a| = 0$. Since 0 is less than or equal to 1, the relation is perfectly reflexive for all integers.

(c) To check for symmetry, we assume aRb is true, meaning $|a - b| \leq 1$. Since $|a - b| = |b - a|$, it follows that $|b - a| \leq 1$, meaning bRa holds true.

(d) To check for transitivity, we choose counterexample values. Let us take the integer triplets $a = 1$, $b = 2$, and $c = 3$.

(e) Evaluating these elements gives $|1 - 2| = 1 \leq 1$ and $|2 - 3| = 1 \leq 1$, meaning both $1R2$ and $2R3$ are valid entries.

(f) However, checking the outer boundary values gives $|1 - 3| = 2$, which is greater than 1. This means $1R3$ fails, proving the relation is not transitive.

Final Answer: Reflexive and Symmetric

Answer: (B)

Go Back to Question 42

Q43.

Solution

Concept: The valid domain of a real function containing a radical fraction requires that the polynomial quadratic expression inside the denominator remains strictly greater than zero.

Solution:

(a) The given real-valued function is structured as a fractional radical expression: $f(x) = \frac{1}{\sqrt{x^2-3x+2}}$.

(b) For the function to output valid real values, the quadratic expression inside the square root must be strictly positive, giving the inequality $x^2 - 3x + 2 > 0$.

(c) We factor the quadratic trinomial expression by splitting the linear middle tracking term into $(x-1)(x-2) > 0$.

(d) The critical boundary values where this algebraic expression equals zero are identified as $x = 1$ and $x = 2$.

(e) We apply the standard sign interval test across the real number line to determine where the factored product remains positive.

(f) The product stays positive when values are chosen below the lower root or above the upper root, which yields the open interval domain $(-\infty, 1) \cup (2, \infty)$.

Final Answer: (-∞, 1) ∪ (2, ∞)

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept: Inverse trigonometric functions can be simplified by substituting them with geometric triangle angle parameters and using standard double-angle and complementary identity conversions.

Solution:

(a) We need to find the value of the composite expression $\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) + \cos\left(\tan^{-1}(2\sqrt{2})\right)$. Let us evaluate each term separately.

(b) For the first term, let $\theta = \tan^{-1}\left(\frac{1}{3}\right)$, which means $\tan\theta = \frac{1}{3}$. We apply the double-angle identity: $\sin(2\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$.

(c) Substituting the tangent fraction into the formula gives $\frac{2(1/3)}{1+(1/9)} = \frac{2/3}{10/9} = \frac{2}{3}\cdot\frac{9}{10} = \frac{3}{5}$.

(d) For the second term, let $\phi = \tan^{-1}(2\sqrt{2})$, which gives a right triangle with an opposite side of $2\sqrt{2}$ and an adjacent side of 1.

(e) The hypotenuse length of this triangle is calculated using the Pythagorean theorem: $\sqrt{(2\sqrt{2})^2 + 1^2} = \sqrt{8+1} = 3$.

(f) Finding the cosine ratio from this right triangle gives $\cos\phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{3}$. Adding both evaluated parts together yields $\frac{3}{5} + \frac{1}{3} = \frac{9+5}{15} = \frac{14}{15}$.

Final Answer: $14_{\overline{15}}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: The total combined sum of an infinite geometric series with a converging fractional common ratio is calculated by dividing the initial term by one minus the ratio.

Solution:

(a) We are given an infinite geometric sequence expression formatted as the mathematical series sum: $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots$.

(b) First, we isolate the initial term parameter of the sequence from the front of the series, which gives $a = 1$.

(c) Next, we determine the common multiplier ratio r by dividing the second series term by the first term, yielding $r = \frac{2/3}{1} = \frac{2}{3}$.

(d) We check the convergence condition by taking the absolute value of the ratio. Since $|r| = \frac{2}{3} < 1$, the infinite sum converges to a finite value.

(e) The analytical formula used to compute the total sum of an infinite geometric progression is expressed as $S_\infty = \frac{a}{1-r}$.

(f) Substituting our isolated parameters into this fractional relation yields $\frac{1}{1-2/3} = \frac{1}{1/3}$, which evaluates precisely to the integer value 3.

Final Answer: 3

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: The number of unique permutations of a collection of objects containing repeating elements is found by dividing the total factorial count by the product of the factorials of each repeated group.

Solution:

(a) We need to determine the total number of unique word arrangements that can be created using all the letters in the word 'EXAMINATION'.

(b) First, we count the total number of individual alphabetic characters contained within the given word, which gives a total count of 11 letters.

(c) Next, we analyze the word structure to identify all repeating letters and count their specific frequencies.

(d) The character 'E' appears 1 time, 'X' appears 1 time, 'M' appears 1 time, 'O' appears 1 time, and 'T' appears 1 time.

(e) The character 'A' appears 2 times, the character 'I' appears 2 times, and the character 'N' appears 2 times within the word.

(f) Applying the multinomial permutation formula for repeating elements, we divide the total factorial by the product of the repeating factorials, yielding $\frac{11!}{2!2!2!}$.

Final Answer: $11!\frac{}{2!2!2!}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: The specific coefficient of a target variable power in a binomial expansion is isolated by writing out the general term formula and solving for the matching index value.

Solution:

(a) We are given the binomial expression $\left(x^2 - \frac{1}{x}\right)^8$. We need to find the specific numerical coefficient matching the variable power x^4.

(b) The standard formula representing the general term T_{r+1} within a binomial expansion is written as $T_{r+1} = \binom{n}{r} a^{n-r} b^r$.

(c) Substituting our specific parameters $n = 8$, $a = x^2$, and $b = -x^{-1}$ into the general term formula gives $\binom{8}{r}(x^2)^{8-r}(-1)^r(x^{-1})^r$.

(d) Grouping the algebraic terms allows us to combine the exponents of the variable base x, yielding the expression $\binom{8}{r}(-1)^r x^{16-2r-r} = \binom{8}{r}(-1)^r x^{16-3r}$.

(e) We match this combined variable exponent with our target power of 4 by setting up the linear equation $16 - 3r = 4$.

(f) Solving this equation gives $3r = 12$, which simplifies to $r = 4$. Substituting $r = 4$ back into the coefficient part yields $\binom{8}{4}(-1)^4 = 70(1) = 70$.

Final Answer: 70

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept: The perpendicular distance separating two parallel lines is calculated by finding the absolute difference between their normalized constant intercepts divided by the magnitude of their shared directional vectors.

Solution:

(a) We are given the equations of two parallel straight lines expressed as $3x + 4y - 9 = 0$ and $6x + 8y + 15 = 0$.

(b) To apply the standard distance formula accurately, we must first normalize the equations so that their linear variable coefficients match exactly.

(c) Multiplying all the terms of the first line equation by a scalar factor of 2 transforms it into the equivalent expression $6x + 8y - 18 = 0$.

(d) Now both lines share identical leading coefficients, with $A = 6$ and $B = 8$. The constant intercepts are isolated as $C_1 = -18$ and $C_2 = 15$.

(e) The analytical formula used to compute the distance between two parallel lines is given by the fractional relationship $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

(f) Substituting our values gives $\frac{|-18-15|}{\sqrt{6^2+8^2}} = \frac{|-33|}{\sqrt{36+64}} = \frac{33}{\sqrt{100}}$, which simplifies directly to the final fraction $\frac{33}{10}$.

Final Answer: $33_{\overline{10}}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept: The equation of a straight line perpendicular to a given line is found by swapping the linear variable coefficients, changing one sign, and solving for the new constant using the given coordinate point.

Solution:

(a) We are given a baseline straight line equation defined as $2x - 3y + 5 = 0$ and a specific target point coordinate $(1, -1)$.

(b) The slope of the given line is found by rearranging it into slope-intercept form, which gives a slope of $m_1 = \frac{2}{3}$.

(c) The slope m_2 of any line perpendicular to this given line must satisfy the negative reciprocal condition $m_1 \cdot m_2 = -1$, yielding $m_2 = -\frac{3}{2}$.

(d) Any perpendicular line can be written in general form by swapping the coefficients and flipping the sign, giving the expression $3x + 2y + k = 0$.

(e) To determine the unknown constant parameter k, we substitute the given coordinate point $(1, -1)$ directly into our perpendicular line equation template.

(f) This substitution gives the numerical equation $3(1) + 2(-1) + k = 0$, which simplifies to $3 - 2 + k = 0$, or $k = -1$. This gives the final line equation $3x + 2y - 1 = 0$.

Final Answer: 3x + 2y - 1 = 0

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: The optimal maximum value of a linear objective function under a set of linear inequalities is guaranteed by the corner point theorem to occur at one of the vertices of the bounded feasible region.

Solution:

(a) We are given a linear programming problem where we want to maximize the linear objective function defined as $Z = 3x + 5y$.

(b) The optimization is restricted by the linear boundary constraint $x + 2y \leq 6$, along with the standard non-negativity conditions $x \geq 0$ and $y \geq 0$.

(c) The non-negativity constraints restrict the feasible region to the first quadrant, bounded by the coordinate axes lines $x = 0$ and $y = 0$.

(d) The intersections of the boundary line $x + 2y = 6$ with the coordinate axes define the remaining corner vertices of the closed triangular feasible region.

(e) Setting $y = 0$ gives the x-intercept vertex at $(6, 0)$, and setting $x = 0$ gives the y-intercept vertex at $(0, 3)$. The origin point is $(0, 0)$.

(f) We evaluate the objective function Z at each vertex: $Z(0, 0) = 0$, $Z(6, 0) = 3(6) + 0 = 18$, and $Z(0, 3) = 0 + 5(3) = 15$. The maximum value of 18 occurs at the corner point $(6, 0)$.

Final Answer: (6, 0)

Answer: (A)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	C	4	B	5	A
6	B	7	A	8	B	9	A	10	B
11	C	12	B	13	A	14	B	15	B
16	B	17	A	18	B	19	D	20	A
21	B	22	A	23	A	24	A	25	A
26	A	27	D	28	B	29	B	30	D
31	A	32	C	33	A	34	A	35	B
36	B	37	B	38	A	39	B	40	A
41	B	42	B	43	A	44	A	45	B
46	A	47	A	48	A	49	A	50	A