JCECE Mathematics Sample Paper-4

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **50** Multiple Choice Questions.
- Each correct answer carries **+1** mark. Incorrect answer: **−0.25** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. The exact absolute maximum evaluated value of the linear trigonometric expression $f(x) = 5\cos x - 12\sin x + 7$ over its entire domain is

(A) 13

(B) 6

(C) 20

(D) 26

Q2. The complete evaluated exact value of the purely algebraic determinant $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix}$ evaluates exactly to zero when x takes the values

(A) −4 and 2

(B) −4 and −2

(C) 4 and −2

(D) 4 and 2

Q3. If the two straight lines given by the Cartesian equations $(k-1)x + 3y + 7 = 0$ and $4x + 6y - 9 = 0$ are perfectly parallel to each other, then the exact complete numerical value of k is

(A) 3

(B) 2

(C) −3

(D) 5

Q4. The explicit evaluated Integrating Factor (I.F.) for the linear first-order differential equation $x\frac{dy}{dx} - y = x^2$ (where $x > 0$) is

(A) x^2

(B) $\frac{1}{x^2}$

(C) x

(D) $\frac{1}{x}$

Q5. The analytical differential equation representing the complete family of concentric geometric circles centered exactly at the origin, given by $x^2 + y^2 = a^2$ (where parameter a is arbitrary), is

(A) $y + x\frac{dy}{dx} = 0$

(B) $x + y\frac{dy}{dx} = 0$

(C) $y - x\frac{dy}{dx} = 0$

(D) $x - y\frac{dy}{dx} = 0$

Q6. The complete evaluated exact principal square root of the pure imaginary number $2i$ in the complex plane is equal to

(A) $1 + i$

(B) $-1 - i$

(C) $-1 + i$

(D) $1 - i$

Q7. A school class consists of exactly 20 boys and 30 girls. If the exact evaluated arithmetic mean weight of the boys is 60 kg and the exact mean weight of the girls is 45 kg, then what is the complete exact combined arithmetic mean weight of the entire class?

(A) 54 kg

(B) 51 kg

(C) 50 kg

(D) 52.5 kg

Q8. If a hyperbola has its transverse vertices at $(\pm 4, 0)$ and its mathematical eccentricity is exactly $e = \frac{5}{4}$, then the total exact linear length of its conjugate axis is

(A) 12

(B) 3

(C) 8

(D) 6

Q9. If a function satisfies $y = A\cos(\omega x) + B\sin(\omega x)$ (where A, B, ω are arbitrary real constants), then the exact condensed expression for $\frac{d^2y}{dx^2} + \omega^2 y$ is

(A) $2\omega^2 y$

(B) $-2\omega^2 y$

(C) ω^2

(D) 0

Q10. The complete exact anti-derivative of the special form integral $\int e^x(\sin x + \cos x)\, dx$ is equal to

(A) $e^x \cos x + C$

(B) $e^x \sin x + C$

(C) $-e^x \sin x + C$

(D) $e^x(\sin x - \cos x) + C$

Q11. If A is an idempotent square matrix ($A^2 = A$), then the expanded simplified matrix polynomial $(I + A)^2 - 3A$ is exactly equal to

(A) I

(B) $2I$

(C) O

(D) A

Q12. The total exact geometric area bounded by the standard ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is equal to

(A) 25π

(B) 144π

(C) 7π

(D) 12π

Q13. If A is a real skew-symmetric square matrix of operational order 3, then its complete exact determinant $\det(A)$ is equal to

(A) 0

(B) 3

(C) 1

(D) -1

Q14. The evaluated exact perpendicular distance drawn from the origin $(0, 0)$ to the straight line given by the linear Cartesian equation $5x - 12y + 39 = 0$ is

(A) 3

(B) 13

(C) $\frac{39}{5}$

(D) 39

Q15. The exact linear length of the geometric tangent line drawn from the external point $(6, 8)$ to the circle given by $x^2 + y^2 = 36$ is

(A) 12

(B) 8

(C) 10

(D) 6

Q16. A fair standard six-sided standard die is rolled exactly 18 times in succession. What is the complete exact expected mean number of times that a face showing an exact multiple of 3 successfully appears?

(A) 6

(B) 12

(C) 3

(D) 9

Q17. The evaluated exact value of the trigonometric modulus definite integral $\int_0^{\pi} |\cos x|\ dx$ is

(A) 1

(B) 2

(C) -2

(D) 0

Q18. For the canonical geometric ellipse given by $\frac{x^2}{36} + \frac{y^2}{20} = 1$, the complete exact linear distance between its two vertical directrix lines is

(A) 18

(B) 12

(C) 24

(D) 9

Q19. The total geometric area bounded by the straight line $y = 2x$, the vertical boundary lines $x = 1$ and $x = 3$, and the horizontal x-axis is

(A) 8

(B) 6

(C) 12

(D) 10

Q20. If a finite quantitative set A has exactly 5 distinct elements ($|A| = 5$), what is the complete exact number of subsets of A that contain at least one element (the non-empty subsets)?

(A) 31

(B) 32

(C) 63

(D) 30

Q21. If the complete evaluated sum of the first 10 terms of an Arithmetic Progression (A.P.) is exactly equal to four times the evaluated sum of its first 5 terms, then what is the complete exact ratio of its first term a to its common difference d?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) 4

Q22. The absolute local and global minimum evaluated value of the real rational function $f(x) = x + \frac{16}{x}$ over its continuous open domain $x > 0$ is

(A) 12

(B) 16

(C) 4

(D) 8

Q23. If $\hat{a}$ and $\hat{b}$ are two mutually perpendicular unit vectors in 3D Euclidean space, then the exact Euclidean magnitude of their vector sum $|\hat{a} + \hat{b}|$ is equal to

(A) 2

(B) $\sqrt{2}$

(C) 1

(D) 0

Q24. There are two completely identical bags. Bag I contains exactly 3 red and 4 black balls, while Bag II contains exactly 5 red and 6 black balls. If one bag is chosen entirely at random and a single ball is drawn from it, what is the exact evaluated total probability that the drawn ball is red?

(A) $\frac{34}{38.5}$

(B) $\frac{68}{77}$

(C) $\frac{34}{77}$

(D) $\frac{17}{77}$

Q25. Consider the standard Linear Programming Problem: Minimize the linear objective function $Z = 4x + 6y$ subject to the structural constraints $2x + y \geq 8$, $x + y \geq 6$, and non-negativity constraints $x \geq 0$, $y \geq 0$. The exact complete absolute minimum evaluated value of Z is

(A) 28

(B) 32

(C) 48

(D) 24

Q26. The explicit Cartesian equation of the specific curve passing exactly through the point $(1, 2)$ and satisfying the separable differential equation $x\,dx + y\,dy = 0$ is

(A) $2x + y = 4$

(B) $x^2 + y^2 = 5$

(C) $x^2 - y^2 = -3$

(D) $x^2 + y^2 = 3$

Q27. The exact shortest perpendicular distance between the two parallel 3D Cartesian planes given by $2x - y + 2z = 6$ and $2x - y + 2z = 18$ is

(A) 12

(B) 6

(C) 4

(D) 3

Q28. The complete continuous open interval in which the real function $f(x) = x^2e^{-x}$ is strictly increasing is

(A) $(-\infty, 0) \cup (2, \infty)$

(B) $(1, 3)$

(C) $(-2, 0)$

(D) $(0, 2)$

Q29. The exact total area of the geometric triangle formed by any arbitrary tangent line drawn to the rectangular hyperbola $xy = 16$ and its two coordinate asymptotes is

(A) 32

(B) 8

(C) 64

(D) 16

Q30. The exact numerical slope of the tangent line drawn to the polynomial curve $y = 3x^2 - 4x + 5$ at the specific point where it crosses the vertical y-axis is

(A) -4

(B) 3

(C) 0

(D) 5

Q31. If the mathematical odds in favor of a specific event A occurring are exactly $3 : 5$, then what is the complete exact theoretical probability of event A exactly NOT occurring (the failure probability)?

(A) $\frac{5}{8}$

(B) $\frac{5}{3}$

(C) $\frac{3}{8}$

(D) $\frac{3}{5}$

Q32. The complete exact geometric area of the parallelogram whose adjacent structural sides are represented by the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is

(A) $15\sqrt{2}$

(B) $10\sqrt{3}$

(C) $5\sqrt{6}$

(D) $10\sqrt{6}$

Q33. If a real square matrix A satisfies the structural involutory condition $A^2 = I$, then the expanded simplified matrix product $(I - A)(I + A)$ is exactly equal to

(A) O

(B) I

(C) $2A$

(D) $2I$

Q34. The exact Cartesian center coordinates of the three-dimensional geometric sphere given by the expanded equation $x^2 + y^2 + z^2 - 4x + 6y - 2z + 5 = 0$ are

(A) $(-2, 3, -1)$

(B) $(2, -3, 1)$

(C) $(4, -6, 2)$

(D) $(-4, 6, -2)$

Q35. If the three geometric vertices of a triangle in the Cartesian plane are $(k, 0)$, $(4, 0)$, and $(0, 6)$, and its complete exact evaluated area is exactly 12 square units, then the positive valid value of parameter k is

(A) 6

(B) 0

(C) 2

(D) 8

Q36. The exact explicit equation of the geometric circle that is concentric with the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ and passes exactly through the origin $(0, 0)$ is

(A) $x^2 + y^2 + 4x + 6y = 0$

(B) $x^2 + y^2 - 4x + 6y = 0$

(C) $x^2 + y^2 + 4x - 6y = 0$

(D) $x^2 + y^2 - 4x - 6y = 0$

Q37. The evaluated analytical derivative of the inverse trigonometric function $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ with respect to x (where $-\pi < x < \pi$) is

(A) $-\frac{1}{2}$

(B) 1

(C) $\frac{1}{1+x^2}$

(D) $\frac{1}{2}$

Q38. The evaluated exact value of the indefinite rational integral $\int \frac{2x+3}{x^2+3x+7}\,dx$ is

(A) $2\ln\left|x^2 + 3x + 7\right| + C$

(B) $(x^2 + 3x + 7)^2 + C$

(C) $\ln\left|x^2 + 3x + 7\right| + C$

(D) $\frac{1}{x^2+3x+7} + C$

Q39. The pure standard Cartesian form $(x + iy)$ of the multi-term complex fraction $\frac{5+2i}{1-2i}$ is exactly equal to

(A) $\frac{1}{5} - \frac{12}{5}i$

(B) $1 - 12i$

(C) $\frac{1}{5} + \frac{12}{5}i$

(D) $1 + 12i$

Q40. What is the complete exact number of distinct independent ways to arrange exactly 7 uniquely colored beads along a circular closed ring to form a beautiful decorative necklace?

(A) 360

(B) 720

(C) 2520

(D) 5040

Q41. A perfectly uniform flat square metallic plate is expanding under uniform heating such that each of its structural side lengths increases at a constant uniform rate of 3 cm/s. What is the exact evaluated rate of increase of its total geometric area when the side length reaches 10 cm?

(A) 30 cm^2/s

(B) 90 cm^2/s

(C) 120 cm^2/s

(D) 60 cm^2/s

Q42. The evaluated exact value of the limit $\lim_{x\to 0} \frac{e^{5x}-e^{2x}}{x}$ is

(A) $\frac{5}{2}$

(B) 10

(C) 3

(D) 7

Q43. If the real quadratic equation given by $3x^2 - 10x + k = 0$ has two real roots such that one root is exactly 2 times the other root, then the precise exact numerical value of parameter k is

(A) $\frac{200}{27}$

(B) $\frac{100}{27}$

(C) $\frac{200}{9}$

(D) $\frac{100}{9}$

Q44. The exact coordinates of the structural vertex of the geometric parabola given by the expanded equation $y^2 - 4y - 4x + 16 = 0$ are

(A) $(-3, -2)$

(B) $(3, 2)$

(C) $(3, -2)$

(D) $(-3, 2)$

Q45. If an arbitrary point P lies on the geometric parabola $y^2 = 16x$ and its exact linear focal distance is 10, then the x-coordinate of point P is

(A) 2

(B) 4

(C) 6

(D) 8

Q46. If a continuous curve is given by the functional definition $y = x^x$ (where $x > 0$), then the exact evaluated numerical value of $\frac{dy}{dx}$ at $x = 1$ is

(A) 1

(B) 0

(C) -1

(D) e

Q47. The evaluated numerical value of the symmetric definite integral $\int_{-\pi/3}^{\pi/3} \frac{x^3 \cos x + \sin^5 x}{1 + x^2}\, dx$ is equal to

(A) $\frac{\pi}{3}$

(B) 0

(C) 1

(D) $\frac{2\pi}{3}$

Q48. Let a piecewise real function be defined as $f(x) = \begin{cases} \frac{\sin(3x)}{\tan(2x)}, & \text{if } x \neq 0 \\ p, & \text{if } x = 0 \end{cases}$. If $f(x)$ is continuous exactly at $x = 0$, then the precise numerical value of parameter p is

(A) 1

(B) $\frac{3}{2}$

(C) 6

(D) $\frac{2}{3}$

Q49. If the complete exact summation of all binomial coefficients in the canonical expansion of $(1 + x)^n$ is exactly equal to 512, then what is the precise exact integer value of parameter n?

(A) 7

(B) 8

(C) 9

(D) 10

Q50. Which of the following continuous real mathematical functions is purely classified as an Even Function over its entire real domain?

(A) $f(x) = e^x - e^{-x}$

(B) $f(x) = x^3 \cos x$

(C) $f(x) = x \sin x$

(D) $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$

Detailed Solutions

Q1.

Solution

Concept:
The analytical range of any pure linear combination of sine and cosine terms of the classic structural form $A\cos x + B\sin x$ is absolutely bounded strictly within the closed radical interval $\left[-\sqrt{A^2+B^2}, +\sqrt{A^2+B^2}\right]$. Adding an arbitrary constant shift C moves the entire bound window by C.

Solution:

(a) Start with our provided linear trigonometric multi-term expression: $f(x) = 5\cos x - 12\sin x + 7$.

(b) Focus entirely on our active oscillating trigonometric components: $g(x) = 5\cos x - 12\sin x$. Compare this directly with our standard archetype $A\cos x + B\sin x$.

(c) Extract our linear quantitative coefficients: we have exactly $A = 5$ and $B = -12$.

(d) Write down our rigorous radical formula for the absolute maximum peak amplitude: Amplitude $= \sqrt{A^2+B^2}$.

(e) Substitute our extracted quantitative parameters directly into our radical identity: Amplitude $= \sqrt{(5)^2 + (-12)^2} = \sqrt{25+144}$.

(f) Execute our internal additions and extract our perfect square: Amplitude $= \sqrt{169} = 13$.

(g) Since $g(x)$ oscillates strictly between -13 and $+13$, the maximum evaluated value of $g(x)$ is exactly $+13$.

(h) Add our quantitative constant shift of 7 to establish our complete exact final absolute maximum: Max $= 13 + 7 = 20$.

Final Answer:

$$\boxed{20}$$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept:
We utilize elementary row (or column) transformations to factorize a determinant completely into linear components. Alternatively, we can use the foundational Proportional Row Theorem, which states that if substituting a specific numerical value makes two or more rows identical, that value is exactly a root.

Solution:

(a) Let our provided purely algebraic determinant be denoted as $D(x) = \begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix}$. We wish to find all real roots where $D(x) = 0$.

(b) Execute our elementary row addition transformation to the first row: $R_1 \leftarrow R_1 + R_2 + R_3$.

(c) Sum the terms column by column: the elements of the top first row become exactly $(x + 2 + 2) = x + 4$, $(2 + x + 2) = x + 4$, and $(2 + 2 + x) = x + 4$.

(d) The rewritten determinant is: $D(x) = \begin{vmatrix} x+4 & x+4 & x+4 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix}$.

(e) Factor out our common linear scalar $(x + 4)$ entirely from our first row: $D(x) = (x + 4)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix}$. This immediately reveals our first root: $x = -4$.

(f) To evaluate the internal determinant, execute column subtractions $C_2 \leftarrow C_2 - C_1$ and $C_3 \leftarrow C_3 - C_1$: $D(x) = (x + 4)\begin{vmatrix} 1 & 0 & 0 \\ 2 & x-2 & 0 \\ 2 & 0 & x-2 \end{vmatrix}$.

(g) Because this is now exactly a lower triangular determinant, its complete value is simply the product of its primary diagonal: $D(x) = (x + 4)(1)(x - 2)(x - 2) = (x + 4)(x - 2)^2$.

(h) Equate our fully factorized polynomial to zero: $(x + 4)(x - 2)^2 = 0$. This yields our complete shared roots: $x = -4$ and $x = 2$.

Final Answer:

$$\boxed{-4 \text{ and } 2}$$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept:

Two linear straight lines written in our standard normalized layouts $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ are defined to be perfectly parallel if and only if their analytical directional slopes are identical. This is equivalent to equating their linear quantitative coefficient ratios: $\frac{A_1}{A_2} = \frac{B_1}{B_2}$.

Solution:

(a) Extract our precise primary linear quantitative coefficients from our two provided line equations.

(b) From our first line $(k-1)x + 3y + 7 = 0$: we have $A_1 = k - 1$ and $B_1 = 3$.

(c) From our second line $4x + 6y - 9 = 0$: we have $A_2 = 4$ and $B_2 = 6$.

(d) Set up our rigorous operational parallel coefficient proportion identity: $\frac{A_1}{A_2} = \frac{B_1}{B_2}$.

(e) Substitute our extracted quantitative tuples directly into our proportion identity: $\frac{k-1}{4} = \frac{3}{6}$.

(f) Simplify the right-hand numerical fraction: $\frac{k-1}{4} = \frac{1}{2}$.

(g) Cross-multiply by 4 to isolate our internal linear term: $k - 1 = \frac{4}{2} = 2$.

(h) Add 1 to both sides to successfully establish our final pristine requested value: $k = 3$.

Final Answer:

$$\boxed{3}$$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:

A first-order differential equation of the structural form $\frac{dy}{dx} + P(x)y = Q(x)$ is classified as a standard linear differential equation. To determine its Integrating Factor (I.F.), we must first divide by the leading coefficient of $\frac{dy}{dx}$ to normalize the equation. The I.F. is then exactly $e^{\int P(x)\,dx}$.

Solution:

(a) Start with the given first-order differential equation: $x\frac{dy}{dx} - y = x^2$.

(b) Divide the entire equation by the independent variable x to achieve our pristine normalized linear standard form: $\frac{dy}{dx} - \frac{1}{x}y = x$.

(c) Compare this directly with our standard model $\frac{dy}{dx} + P(x)y = Q(x)$ to extract the core coefficient function of y: $P(x) = -\frac{1}{x}$.

(d) Set up our rigorous Integrating Factor exponential identity: I.F. $= e^{\int -\frac{1}{x}\,dx}$.

(e) Execute the anti-differentiation of the simple rational function: $\int -\frac{1}{x}\,dx = -\ln|x|$. Since our complete operational domain is $x > 0$, we write exactly $-\ln x$.

(f) Substitute this integrated logarithmic result back into the I.F. exponential formula: I.F. $= e^{-\ln x}$.

(g) Apply the foundational logarithmic power identity to move the negative sign into the argument: I.F. $= e^{\ln(x^{-1})}$.

(h) Apply the self-inversion identity of the natural exponential $e^{\ln(u)} = u$: we obtain I.F. $= x^{-1} = \frac{1}{x}$.

Final Answer:

$$\boxed{\frac{1}{x}}$$

Answer: (D)

Go Back to Question 4

Q5.

Solution

Concept:

To form a differential equation representing a specific single-parameter family of algebraic curves, $F(x, y, a) = 0$, we differentiate the given equation exactly once implicitly with respect to the independent variable x. We then algebraically eliminate the arbitrary constant parameter a.

Solution:

(a) Start with the primary general equation defining the family of concentric circles: $x^2 + y^2 = a^2$.

(b) Notice that there is exactly one arbitrary parameter present (a), which means our resulting differential equation must have an operational order of exactly 1.

(c) Differentiate both sides of the equation implicitly with respect to x using the chain rule on the variable y: $\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(a^2)$.

(d) Execute the standard formal derivative evaluations: $2x + 2y\frac{dy}{dx} = 0$. (Notice that the arbitrary constant a^2 differentiates directly to 0, eliminating it perfectly without requiring simultaneous re-substitution).

(e) Divide the entire differentiated equation by the scalar factor of 2 to simplify our linear coefficients: $x + y\frac{dy}{dx} = 0$.

Final Answer:

$$\boxed{x + y\frac{dy}{dx} = 0}$$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept:

To determine the principal square root of any complex number, we convert the complex number into its pristine polar exponential form $re^{i\theta}$. Taking its fractional power $\frac{1}{2}$ halves its principal argument and extracts the positive square root of its Euclidean magnitude: $\sqrt{z} = \sqrt{r}e^{i\theta/2}$.

Solution:

(a) Start with our target pure imaginary complex number: $z = 2i$.

(b) Convert z into its complete polar exponential layout: its Euclidean magnitude is clearly $r = 2$, and its principal argument lying on the positive vertical imaginary axis is exactly $\theta = \frac{\pi}{2}$. Thus, $z = 2e^{i\pi/2}$.

(c) Set up our rigorous principal square root identity: $\sqrt{z} = \left(2e^{i\pi/2}\right)^{1/2}$.

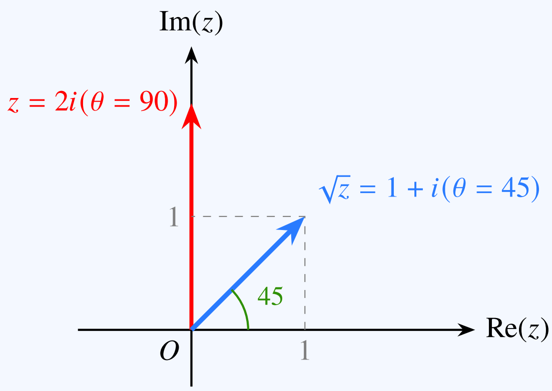
(d) Execute our internal fractional exponent halving and our radical extraction: $\sqrt{z} = \sqrt{2}e^{i\left(\frac{\pi}{2}\cdot\frac{1}{2}\right)} = \sqrt{2}e^{i\pi/4}$.

(e) Expand our polar exponential layout into standard Cartesian form using Euler's foundational identity $e^{i\theta} = \cos\theta + i\sin\theta$: $\sqrt{z} = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$.

(f) Substitute our fundamental trigonometric values ($\cos(\pi/4) = \sin(\pi/4) = \frac{1}{\sqrt{2}}$): $\sqrt{z} = \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$.

(g) Execute our final simple scalar multiplications: $\sqrt{z} = 1 + i$.

(h) *(Cross-verification by formal squaring: $(1+i)^2 = 1^2 + 2(1)(i) + i^2 = 1 + 2i - 1 = 2i$).*



Final Answer:

$$\boxed{1 + i}$$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:

In descriptive statistics, when two stochastically independent operational subgroups of quantitative sample sizes n_1 and n_2 have individual arithmetic means $\bar{X}_1$ and $\bar{X}_2$, the complete precise combined arithmetic mean of the entire unified group is calculated using our weighted average formula: $\bar{X}_{\text{comb}} = \frac{n_1\bar{X}_1+n_2\bar{X}_2}{n_1+n_2}$.

Solution:

(a) Write down our foundational formula for the combined arithmetic mean of two independent quantitative subgroups: $\bar{X}_{\text{comb}} = \frac{n_1\bar{X}_1+n_2\bar{X}_2}{n_1+n_2}$.

(b) Extract our core operational parameters group by group from our problem prompt.

(c) For our boys subgroup: sample size is $n_1 = 20$, and individual mean weight is $\bar{X}_1 = 60$ kg.

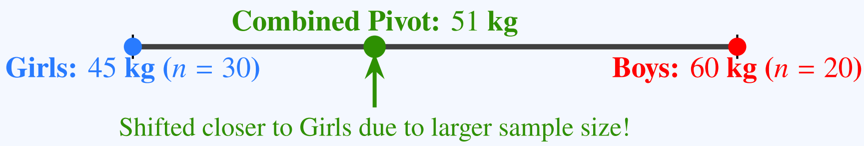
(d) For our girls subgroup: sample size is $n_2 = 30$, and individual mean weight is $\bar{X}_2 = 45$ kg.

(e) Compute our total unified sample size of the entire entire class: $n_1 + n_2 = 20 + 30 = 50$ students.

(f) Substitute our extracted parameter tuples directly into our weighted average identity: $\bar{X}_{\text{comb}} = \frac{20(60\text{ kg})+30(45\text{ kg})}{50}$.

(g) Execute our internal subgroup total weight additions: $\bar{X}_{\text{comb}} = \frac{1200\text{ kg}+1350\text{ kg}}{50} = \frac{2550\text{ kg}}{50}$.

(h) Perform our final pristine simple integer division: $\bar{X}_{\text{comb}} = 51$ kg.



Final Answer:

$$\boxed{51\text{ kg}}$$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:

For a canonical horizontal hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, its transverse major vertices are situated exactly at $(\pm a, 0)$, and the length of its conjugate axis is exactly $2b$. The mathematical relationship between its primary parameters is given by the foundational identity $b^2 = a^2(e^2 - 1)$.

Solution:

(a) Extract our semi-transverse axis length directly from our given vertex coordinates $(\pm 4, 0)$: we have exactly $a = 4$.

(b) Identify our operational mathematical eccentricity from the problem prompt: $e = \frac{5}{4}$.

(c) Write down our foundational relationship linking the semi-conjugate axis b to a and e: $b^2 = a^2(e^2 - 1)$.

(d) Substitute our known values directly into the analytical identity: $b^2 = (4)^2 \left[\left(\frac{5}{4}\right)^2 - 1\right]$.

(e) Execute the internal fractional square arithmetic: $b^2 = 16\left[\frac{25}{16} - 1\right] = 16\left[\frac{25-16}{16}\right] = 16\left[\frac{9}{16}\right]$.

(f) Cancel the opposing 16 scalar factors to evaluate b^2: we obtain exactly $b^2 = 9$. Taking the positive square root gives $b = 3$.

(g) Write down our required formula for the total complete length of the conjugate axis: Length $= 2b = 2(3) = 6$.

Final Answer:

$$\boxed{6}$$

Answer: (D)

Go Back to Question 8

Q9.

Solution

Concept:

To verify a linear second-order homogeneous differential equation for a given family of periodic solutions, we differentiate the given function twice with respect to the independent variable x using the standard chain rule and trigonometric derivatives $\frac{d}{dx}\cos(kx) = -k\sin(kx)$ and $\frac{d}{dx}\sin(kx) = k\cos(kx)$.

Solution:

(a) Start with the primary analytical definition of the solution function: $y = A\cos(\omega x) + B\sin(\omega x)$.

(b) Compute the first derivative with respect to x using the chain rule: $\frac{dy}{dx} = -A\omega\sin(\omega x) + B\omega\cos(\omega x)$.

(c) Differentiate a second time with respect to x to find $\frac{d^2y}{dx^2}$: $\frac{d^2y}{dx^2} = -A\omega^2\cos(\omega x) - B\omega^2\sin(\omega x)$.

(d) Factor out the common scalar coefficient $(-\omega^2)$ entirely from the right-hand side: $\frac{d^2y}{dx^2} = -\omega^2[A\cos(\omega x) + B\sin(\omega x)]$.

(e) Notice that the bracketed internal expression is exactly our original explicit function y.

(f) Substitute y back into the identity: $\frac{d^2y}{dx^2} = -\omega^2 y$.

(g) Add $\omega^2 y$ to both sides of the equation to establish the final condensed summation: $\frac{d^2y}{dx^2} + \omega^2 y = 0$.

Final Answer:

$$\boxed{0}$$

Answer: (D)

Go Back to Question 9

Q10.

Solution

Concept:
An indefinite integral of the classic analytical archetype $\int e^x [f(x) + f'(x)]\, dx$ can be solved elegantly using integration by parts, which reveals that it evaluates identically to $e^x f(x) + C$. We match our specific integrand terms to this exact structural formula.

Solution:

(a) Start with the provided indefinite integral expression: $I = \int e^x(\sin x + \cos x)\, dx$.

(b) Compare the internal bracketed terms directly with our classic structural formula $\int e^x [f(x) + f'(x)]dx$.

(c) Set our primary operational function to exactly the first trigonometric term: $f(x) = \sin x$.

(d) Compute the exact analytical derivative of $f(x)$ with respect to x: using standard differentiation formulas, $f'(x) = \frac{d}{dx}(\sin x) = \cos x$.

(e) Observe that our evaluated derivative $f'(x)$ perfectly matches the second trigonometric term inside the integrand.

(f) Conclude the exact evaluated anti-derivative directly from the structural theorem: $I = e^x f(x) + C = e^x \sin x + C$.

Final Answer:

$$\boxed{e^x \sin x + C}$$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept:

An Idempotent Matrix is formally defined as a square matrix which, when multiplied by itself, yields exactly itself ($A^2 = A$). To simplify complex matrix binomial polynomials involving idempotent matrices, we expand the binomial products carefully using the distributive property and replace all higher powers of A with A.

Solution:

(a) Start with the provided multi-term matrix polynomial: $P =(\mathrm{I} + \mathrm{A})^2 - 3A$.

(b) Expand the formal binomial square $(I + A)^2$ using the standard distributive matrix multiplication laws: $P = (I^2 + I \cdot A + A \cdot I + A^2) - 3A$.

(c) Apply our foundational matrix identity laws ($I^2 = I$, and $I \cdot A = A \cdot I = A$): $P = (I + A + A + A^2) - 3A = (I + 2A + A^2) - 3A$.

(d) Apply our specific defining idempotent property $A^2 = A$ to eliminate the squared matrix term: $P = (I + 2A + A) - 3A$.

(e) Combine our like linear matrix coefficients together: $P = (I + 3A) - 3A$.

(f) Execute the final linear matrix subtraction: $P = I + (3A - 3A) = I + O = I$.

Final Answer:

$$\boxed{I}$$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept:

The total exact geometric area enclosed by a standard two-dimensional ellipse written in its pristine canonical form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by the foundational geometric proportion $A = \pi ab$, where a and b represent its semi-major and semi-minor axes.

Solution:

(a) Compare the provided canonical ellipse equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with our standard archetype $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

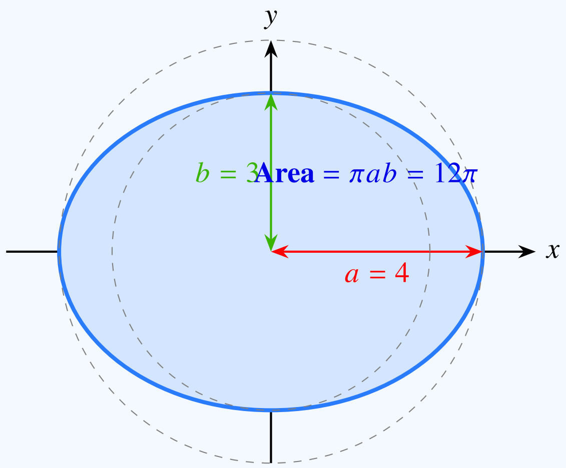
(b) Extract the squared semi-axis denominators: we have $a^2 = 16$ and $b^2 = 9$.

(c) Take the positive square root to determine the exact linear semi-axis lengths: $a = \sqrt{16} = 4$, and $b = \sqrt{9} = 3$.

(d) Write down our foundational formula for the complete area of an ellipse: $A = \pi ab$.

(e) Substitute our extracted semi-axis lengths directly into the geometric formula: $A = \pi \cdot (4) \cdot (3)$.

(f) Execute the simple integer multiplication to establish the final total area: $A = 12\pi$.



Final Answer:

$$\boxed{12\pi}$$

Answer: (D)

Go Back to Question 12

Q13.

Solution

Concept:
For any general real square matrix A of operational order n, its determinant satisfies the foundational identities $\det(A^T) = \det(A)$ and $\det(kA) = k^n \det(A)$. By definition, a skew-symmetric matrix satisfies $A^T = -A$. Equating the determinant properties for an odd order n forces $\det(A) = 0$.

Solution:

(a) Start with the primary defining relationship for a skew-symmetric matrix: $A^T = -A$.

(b) Take the complete formal determinant of both sides of the matrix equation: $\det(A^T) = \det(-A)$.

(c) Apply our foundational determinant transposition identity $\det(A^T) = \det(A)$ on the left, and our scalar scaling identity $\det(-A) = (-1)^n \det(A)$ on the right.

(d) Substitute our specific operational matrix order $n = 3$ (which is exactly an odd integer): $\det(A) = (-1)^3 \det(A) = -\det(A)$.

(e) Add $\det(A)$ to both sides of the relation to group the terms together: $2\det(A) = 0$.

(f) Divide by the scalar factor of 2 to successfully establish our final pristine output: $\det(A) = 0$.

Final Answer:

$$\boxed{0}$$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:

The absolute linear perpendicular distance d drawn from any specific starting coordinate (x_0, y_0) to a straight Cartesian line $Ax + By + C = 0$ is computed directly using our foundational rational radical identity $d = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$. We evaluate this simple absolute radical metric.

Solution:

(a) Write down our definitive Cartesian straight line equation in standard zero-form: $5x - 12y + 39 = 0$.

(b) Extract our linear quantitative coefficients: we have exactly $A = 5, B = -12, C = 39$.

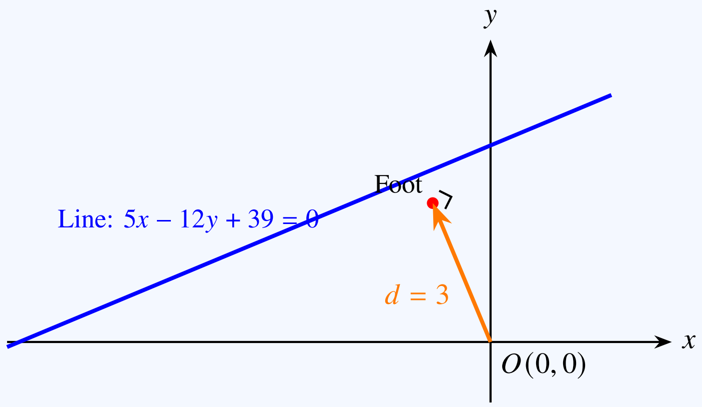
(c) Identify our specific starting contact coordinates from our text: we start exactly at the origin $(x_0, y_0) = (0, 0)$.

(d) Set up our rigorous perpendicular distance rational radical identity: $d = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$.

(e) Substitute our exact extracted parameters directly into our definitive identity: $d = \frac{|5(0)-12(0)+39|}{\sqrt{(5)^2+(-12)^2}}$.

(f) Execute our internal absolute numerator evaluation and our radical square additions: $d = \frac{|39|}{\sqrt{25+144}} = \frac{39}{\sqrt{169}}$.

(g) Extract our perfect square denominator and compute our final simple integer division: $d = \frac{39}{13} = 3$.



Final Answer:

$$\boxed{3}$$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:

The analytical length L of a tangent line drawn from an external point $P(x_1, y_1)$ to a standard general circle $S(x, y) = x^2 + y^2 + 2gx + 2fy + c = 0$ is computed directly using the foundational radical substitution identity $L = \sqrt{S(x_1, y_1)}$.

Solution:

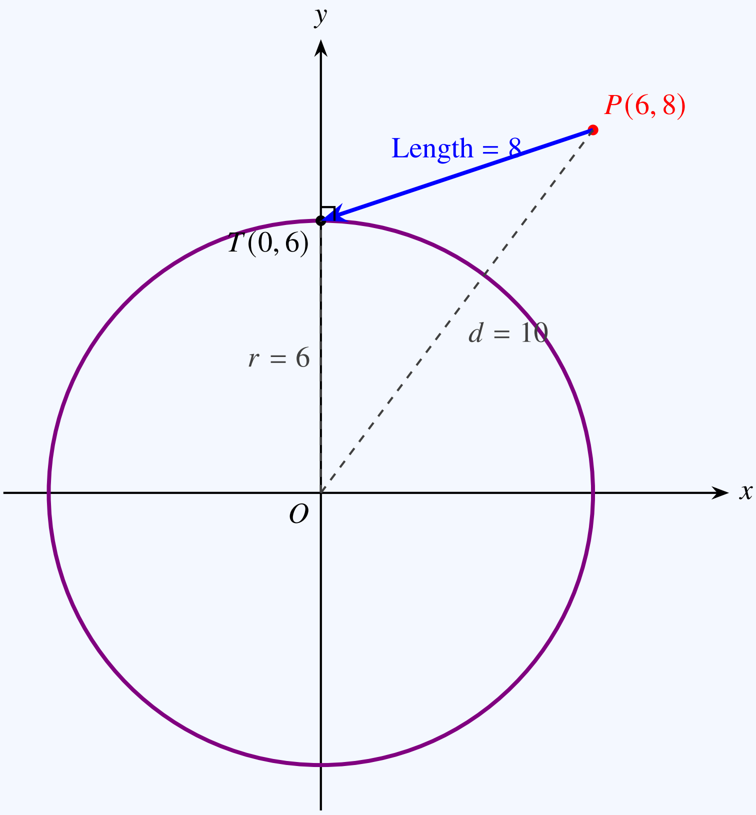
(a) Write down our complete circle equation in standard zero-form: $S(x, y) = x^2 + y^2 - 36 = 0$.

(b) Identify the specific external starting point coordinates given in the problem prompt: $(x_1, y_1) = (6, 8)$.

(c) Set up the rigorous tangent length radical identity: $L = \sqrt{S(x_1, y_1)}$.

(d) Substitute our external coordinates directly into the zero-form polynomial expression: $L = \sqrt{(6)^2 + (8)^2 - 36}$.

(e) Cancel the opposing 36 terms and compute the final radical extraction: $L = \sqrt{64} = 8$.



Final Answer:

$$\boxed{8}$$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept:
We use the theoretical expected Mean of a Binomial Probability Distribution. For a random variable X representing the total number of independent successes across n identical Bernoulli trials, each having a constant single-trial success probability p, its theoretical expected mean is exactly $\mu = np$.

Solution:

(a) Define our target operational random variable: let X represent the total number of rolls out of 18 that result in a face showing exactly a multiple of 3.

(b) Enumerate our favorable outcomes on any single independent roll of a six-sided die ($\{1, 2, 3, 4, 5, 6\}$): the faces that are multiples of 3 are exactly $\{3, 6\}$. Count our favorable elements: exactly 2 faces.

(c) Calculate our single-trial success probability p: $p = \frac{2}{6} = \frac{1}{3}$.

(d) Identify our complete total number of independent operational trials from the prompt: $n = 18$.

(e) Write down our foundational formula for the theoretical expected mean of a binomial distribution: $\mu = np$.

(f) Substitute our known values directly into our theoretical mean identity: $\mu = 18 \times \frac{1}{3}$.

(g) Execute our simple integer division to establish our final pristine output: $\mu = 6$.

Final Answer:

$$\boxed{6}$$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:
To evaluate a definite integral involving an absolute value expression $|f(x)|$, we must split the domain of integration at the critical roots where the inner function changes its algebraic sign. The integral is then resolved into separate continuous pieces with appropriate positive and negative signs.

Solution:

(a) Start with the provided definite integral: $I = \int_0^{\pi} |\cos x|\,dx$.

(b) Locate the roots of the inner function inside the closed integration domain $[0, \pi]$: $\cos x = 0 \implies x = \frac{\pi}{2}$.

(c) Analyze the algebraic sign of the cosine curve within the split operational sub-intervals.

(d) In the first quadrant interval $\left[0, \frac{\pi}{2}\right]$, $\cos x \geq 0 \implies |\cos x| = \cos x$.

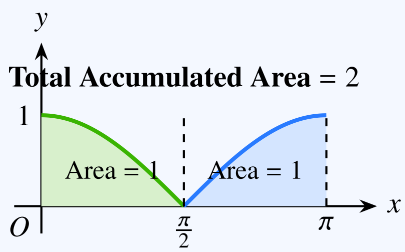
(e) In the second quadrant interval $\left[\frac{\pi}{2}, \pi\right]$, $\cos x \leq 0 \implies |\cos x| = -\cos x$.

(f) Split our definitive integration domain at the critical boundary $x = \frac{\pi}{2}$: $I = \int_0^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} (-\cos x)\,dx$.

(g) Execute the anti-differentiation term by term using $\int \cos x\,dx = \sin x$: $I = [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}$.

(h) Substitute the definitive integral boundaries: $I = \left[\sin\left(\frac{\pi}{2}\right) - \sin(0)\right] - \left[\sin(\pi) - \sin\left(\frac{\pi}{2}\right)\right]$.

(i) Substitute our fundamental trigonometric values $(\sin(\pi/2) = 1, \sin(0) = \sin(\pi) = 0)$: $I = [1 - 0] - [0 - 1] = 1 - (-1) = 2$.



Final Answer:

$$\boxed{2}$$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept:

For a canonical horizontal ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $a > b$), the mathematical flattening is measured by its eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$. Its two vertical directrix lines are situated symmetrically behind the vertices at $x = \pm\frac{a}{e}$. The distance between them is exactly $\frac{2a}{e}$.

Solution:

(a) Extract our specific squared semi-axis parameters from the ellipse's equation: $a^2 = 36 \implies a = 6$, and $b^2 = 20$.

(b) Calculate the exact mathematical eccentricity e using the foundational identity: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{20}{36}}$.

(c) Perform the internal fraction subtraction and simplification: $e = \sqrt{\frac{16}{36}} = \frac{4}{6} = \frac{2}{3}$.

(d) Write down our structural formula for the complete distance between the two directrix lines: Distance $= \frac{2a}{e}$.

(e) Substitute our extracted semi-major axis $a = 6$ and our evaluated eccentricity $e = \frac{2}{3}$ directly into the identity: Distance $= \frac{2(6)}{2/3}$.

(f) Execute the rational fraction inversion and multiplication: Distance $= 12 \times \frac{3}{2} = 18$.

Final Answer:

$$\boxed{18}$$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept:

The geometric area enclosed between a continuous functional line $y = f(x)$ and the horizontal x-axis bounded between vertical cut lines $x = a$ and $x = b$ is computed directly using the definite integration formula $A = \int_a^b |f(x)|\,dx$. We evaluate this simple polynomial definite integral.

Solution:

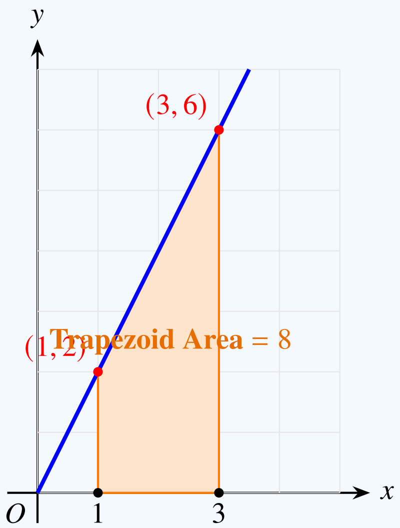
(a) Verify the positive algebraic sign of the functional line $y = 2x$ over our complete target integration domain $[1, 3]$. Since $x \geq 1$, the function is strictly positive ($y \geq 2 > 0$), meaning the region lies entirely above the axis.

(b) Set up the precise area definite integration equation: $A = \int_1^3 2x\,dx$.

(c) Execute the anti-differentiation of the linear power function: $A = \left[2 \cdot \frac{x^2}{2}\right]_1^3 = \left[x^2\right]_1^3$.

(d) Substitute our upper limit $x = 3$ and subtract our lower limit $x = 1$: $A = (3)^2 - (1)^2$.

(e) Perform the simple arithmetic subtraction: $A = 9 - 1 = 8$.



Final Answer:

$$\boxed{8}$$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:

We use the core properties of Power Sets. The complete formal Power Set $\mathcal{P}(A)$ representing all possible subsets of a finite quantitative set A having exactly n distinct elements has a precise total cardinality of exactly 2^n subsets. To determine the number of non-empty subsets, we simply subtract the unique empty set $\emptyset$.

Solution:

(a) Write down our foundational formula for the total number of subsets of a finite quantitative set: Total Subsets $= 2^n$.

(b) Identify our specific set cardinality from the prompt: we have exactly $n = 5$ distinct elements.

(c) Substitute $n = 5$ directly into our power set identity: Total Subsets $= 2^5 = 32$ subsets total.

(d) The problem explicitly requires the complete number of non-empty subsets (subsets containing at least one element).

(e) Consequently, we must subtract the single unique empty set $\emptyset$ (which is exactly 1 subset present in every power set).

(f) Perform our final simple integer subtraction: Non-empty Subsets $= 32 - 1 = 31$.

Final Answer:

$$\boxed{31}$$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:

To analyze specific quantitative constraints in an Arithmetic Progression (A.P.) with starting term a and constant step size d, we utilize our foundational progression summation identity $S_n = \frac{n}{2}[2a + (n-1)d]$. We equate the expanded identities for our two given indices and isolate our variable ratio.

Solution:

(a) Write down our foundational formula for the partial sum of the first n terms of an A.P.: $S_n = \frac{n}{2}[2a + (n-1)d]$.

(b) Set up the rigorous quantitative progression relationship stated in our prompt: $S_{10} = 4 \times S_5$.

(c) Substitute our explicit progression summation formula into both sides of our operational equation: $\frac{10}{2}[2a + (10-1)d] = 4 \times \frac{5}{2}[2a + (5-1)d]$.

(d) Execute our intermediate numerical term simplifications: $5[2a + 9d] = 10[2a + 4d]$.

(e) Divide both sides entirely by our common scalar factor of 5 to achieve maximum quantitative simplicity: $2a + 9d = 2[2a + 4d]$.

(f) Expand our formal linear bracket on the right side: $2a + 9d = 4a + 8d$.

(g) Group our distinct variable terms onto opposite sides of our equation: $9d - 8d = 4a - 2a \implies d = 2a$.

(h) Divide both sides by $2d$ to successfully establish our final pristine requested fractional ratio: $\frac{a}{d} = \frac{1}{2}$.

Final Answer:

$$\boxed{\frac{1}{2}}$$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:
To determine the global minimum of a smooth rational function over an open positive domain, we locate its stationary points by setting its first derivative to zero ($f'(x) = 0$). We verify the extremum type by checking the positive convexity via the second derivative test ($f''(x) > 0$).

Solution:

(a) Start with the given explanatory function: $f(x) = x + \frac{16}{x}$ defined over $x > 0$.

(b) Compute the first derivative with respect to x using the standard power rule: $f'(x) = \frac{d}{dx}(x) + 16\frac{d}{dx}(x^{-1}) = 1 - \frac{16}{x^2}$.

(c) Locate stationary critical points by setting the derivative to zero: $1 - \frac{16}{x^2} = 0 \implies \frac{16}{x^2} = 1 \implies x^2 = 16$.

(d) Since our complete operational domain is restricted to positive values ($x > 0$), we extract the single positive root $x = 4$.

(e) Compute the second derivative with respect to x to confirm the operational concavity: $f''(x) = \frac{d}{dx}\left(1 - 16x^{-2}\right) = -16(-2)x^{-3} = \frac{32}{x^3}$.

(f) Evaluate the second derivative at our critical root $x = 4$: $f''(4) = \frac{32}{4^3} = \frac{32}{64} = \frac{1}{2} > 0$. Since the second derivative is strictly positive, the curve is strictly convex and concave upward, confirming an absolute global minimum.

(g) Substitute $x = 4$ back into the original explicit function to calculate the minimum value: $f(4) = 4 + \frac{16}{4} = 4 + 4 = 8$.

Final Answer:

$$\boxed{8}$$

Answer: (D)

Go Back to Question 22

Q23.

Solution

Concept:
To compute the complete Euclidean magnitude of a vector sum or difference, we square the magnitude using the foundational dot product relationship $|\vec{u}|^2 = (\vec{u} \cdot \vec{u})$. For unit vectors, their self-dot products evaluate exactly to 1 ($|\hat{u}| = 1$), and perpendicularity implies their mutual dot product is zero ($(\hat{a} \cdot \hat{b}) = 0$).
Solution:

(a) Start with our foundational dot product magnitude-squared identity: $|\hat{a}+\hat{b}|^2 = (\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})$.

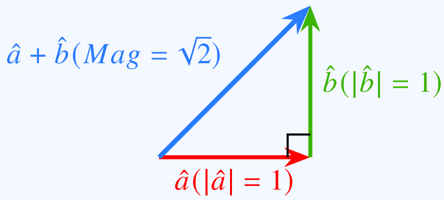
(b) Expand our formal dot product fully using our standard distributive vector multiplication laws: $|\hat{a} + \hat{b}|^2 = |\hat{a}|^2 + 2(\hat{a} \cdot \hat{b}) + |\hat{b}|^2$.

(c) Identify our specific operational vector properties: the vectors are unit vectors, so their squared Euclidean magnitudes are exactly $|\hat{a}|^2 = 1$ and $|\hat{b}|^2 = 1$.

(d) Apply our specific operational mathematical condition that the unit vectors are mutually perpendicular: by definition, $(\hat{a} \cdot \hat{b}) = 0$.

(e) Substitute these pristine simplified numerical values back into our fully expanded magnitude identity: $|\hat{a} + \hat{b}|^2 = 1 + 2(0) + 1 = 2$.

(f) Take the positive square root to successfully determine our final vector sum magnitude: $|\hat{a} + \hat{b}| = \sqrt{2}$.



Right Isosceles Triangle Vector Addition

Final Answer:

$$\boxed{\sqrt{2}}$$

Answer: (B)
Go Back to Question 23

Q24.

Solution

Concept:
The foundational Law of Total Probability states that for any target event R occurring across a set of mutually exclusive and exhaustive intermediate selection conditions (E_1, E_2), its complete net probability is exactly $P(R) = P(E_1)P(R|E_1) + P(E_2)P(R|E_2)$. We evaluate this weighted tree summation.

Solution:

(a) Let E_1 and E_2 be our primary intermediate events of choosing Bag I and Bag II respectively. Since the prompt specifies that the two bags are identical and chosen entirely at random, they are equally likely: $P(E_1) = P(E_2) = \frac{1}{2}$.

(b) Let target event R represent the event that our successfully drawn ball is red.

(c) Determine our exact conditional probabilities of drawing a red ball from each specific bag.

(d) From Bag I (containing 3 red and 4 black, total 7 balls): $P(R|E_1) = \frac{3}{7}$.

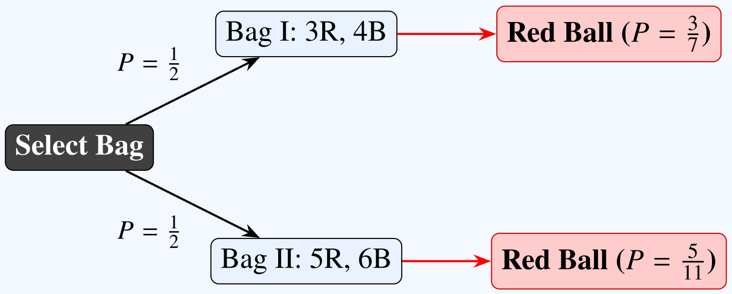
(e) From Bag II (containing 5 red and 6 black, total 11 balls): $P(R|E_2) = \frac{5}{11}$.

(f) Apply our foundational Law of Total Probability: $P(R) = P(E_1)P(R|E_1) + P(E_2)P(R|E_2)$.

(g) Substitute our exact extracted fractional tuples into the weighted summation: $P(R) = \frac{1}{2}\left(\frac{3}{7}\right) + \frac{1}{2}\left(\frac{5}{11}\right)$.

(h) Factor out our common scalar coefficient of $\frac{1}{2}$ and combine our internal fractions under their exact Least Common Multiple ($LCM = 77$): $P(R) = \frac{1}{2}\left[\frac{3\times11+5\times7}{77}\right] = \frac{1}{2}\left[\frac{33+35}{77}\right]$.

(i) Execute our internal fraction addition and compute our final simple scalar division: $P(R) = \frac{1}{2}\left(\frac{68}{77}\right) = \frac{34}{77}$.



Total Red Sum $= \frac{1}{2}(\frac{3}{7}) + \frac{1}{2}(\frac{5}{11}) = \frac{34}{77}$

Final Answer:

$$\boxed{\frac{34}{77}}$$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept:

The fundamental Corner Point Method in Linear Programming states that if an operational feasible region is non-empty (even if unbounded), any absolute minimum value of a linear objective function with positive coefficients must occur exactly at the geometric vertices (corner points) of the boundary.

Solution:

(a) Graph our primary linear feasible region defined by our positive axes constraints $x \geq 0, y \geq 0$ and situated above our two constraint boundary lines $2x + y = 8$ and $x + y = 6$.

(b) The feasible region is exactly an unbounded open polygonal polygon located in the first quadrant.

(c) Determine the exact intermediate intersection corner point by solving our two boundary equations simultaneously: subtracting $x + y = 6$ from $2x + y = 8$ yields exactly $x = 2$. Substituting $x = 2$ back into $x + y = 6$ yields $y = 4$. Thus, the intermediate corner vertex is exactly $(2, 4)$.

(d) We identify and list all three geometric corner points bounding this shaded unbounded space: the vertical axis crossing $A(0, 8)$, our intermediate intersection $B(2, 4)$, and our horizontal axis crossing $C(6, 0)$.

(e) Evaluate our linear objective function $Z = 4x + 6y$ independently at each identified corner point.

(f) At vertex $A(0, 8)$: $Z = 4(0) + 6(8) = 48$.

(g) At vertex $B(2, 4)$: $Z = 4(2) + 6(4) = 8 + 24 = 32$.

(h) At vertex $C(6, 0)$: $Z = 4(6) + 6(0) = 24 + 0 = 24$.

(i) Compare the successfully evaluated quantitative outputs: our absolute minimum evaluated value is clearly 24, occurring at the corner point $(6, 0)$.

Final Answer:

$$\boxed{24}$$

Answer: (D)

Go Back to Question 25

Q26.

Solution

Concept:

To solve a completely separated first-order differential equation $f(x)dx + g(y)dy = 0$, we simply apply the formal indefinite integral operation to each individual differential term independently. We then utilize our specific given boundary initial conditions to determine the exact value of the integration constant.

Solution:

(a) Examine the beautifully structured separable differential equation provided: $x\,dx + y\,dy = 0$.

(b) Execute the formal indefinite integration operation independently across both separated terms: $\int x\,dx + \int y\,dy = 0$.

(c) Evaluate our basic polynomial anti-derivatives using the standard power rule: $\frac{x^2}{2} + \frac{y^2}{2} = C$.

(d) Multiply the entire integrated invariant by 2 to clear all fractional denominators: $x^2+y^2 = 2C$.

(e) Since $2C$ is an arbitrary constant, we can define a clean unified constant parameter $K = 2C$ to represent our general family of curves: $x^2 + y^2 = K$. This clearly represents a family of circles centered at the origin.

(f) Apply our given boundary initial condition that the curve passes exactly through the point $(1, 2)$.

(g) Substitute $x = 1$ and $y = 2$ directly into our general curve trajectory to find K: $(1)^2 + (2)^2 = K \implies 1 + 4 = 5 \implies K = 5$.

(h) Substitute $K = 5$ back to establish our specific final pristine Cartesian curve trajectory: $x^2 + y^2 = 5$.

Final Answer:

$$\boxed{x^2 + y^2 = 5}$$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept:

The precise linear shortest perpendicular distance d between any two perfectly parallel three-dimensional Cartesian planes written in their standard normalized operational layouts $Ax + By + Cz = D_1$ and $Ax + By + Cz = D_2$ is computed directly using the foundational radical formula $d = \frac{|D_2 - D_1|}{\sqrt{A^2+B^2+C^2}}$.

Solution:

(a) Write down our complete defining radical formula for the perpendicular shortest distance between two parallel planes: $d = \frac{|D_2 - D_1|}{\sqrt{A^2+B^2+C^2}}$.

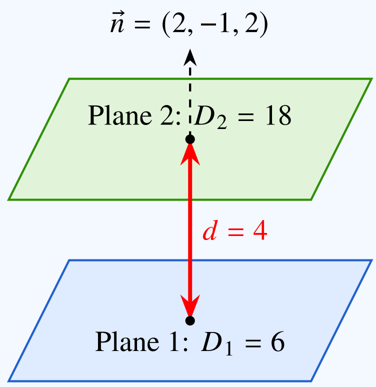
(b) Verify that our two primary normal vector coefficients are perfectly identical across both provided plane equations: we extract exactly $A = 2, B = -1, C = 2$.

(c) Identify our two operational Cartesian plane constant terms from the right-hand side of our equations: we have exactly $D_1 = 6$ and $D_2 = 18$.

(d) Substitute our extracted parameters directly into our rigorous radical operational formula: $d = \frac{|18-6|}{\sqrt{(2)^2+(-1)^2+(2)^2}}$.

(e) Execute our internal absolute numerator difference and our radical square additions: $d = \frac{|12|}{\sqrt{4+1+4}} = \frac{12}{\sqrt{9}}$.

(f) Extract our perfect square denominator and compute our final simple integer division: $d = \frac{12}{3} = 4$.



Final Answer:

$$\boxed{4}$$

Answer: (C)

Go Back to Question 27

Q28.

Solution

Concept:

A differentiable functional curve $f(x)$ is classified as strictly increasing over any continuous mathematical interval where its first derivative is strictly positive, $f'(x) > 0$. We compute the derivative using the product rule and chain rule, locate its roots, and establish its positive sign intervals.

Solution:

(a) Start with the primary explanatory function: $f(x) = x^2e^{-x}$.

(b) Compute the first derivative with respect to x using the product rule: $f'(x) = \frac{d}{dx}(x^2) \cdot e^{-x} + x^2 \cdot \frac{d}{dx}(e^{-x})$.

(c) Execute the internal chain rule derivative $\frac{d}{dx}(e^{-x}) = -e^{-x}$: $f'(x) = 2xe^{-x} - x^2e^{-x}$.

(d) Factor out the common algebraic factors x and e^{-x} entirely from the expression: $f'(x) = x(2-x)e^{-x}$.

(e) Set up the operational mathematical condition for strictly increasing behavior: $f'(x) > 0 \implies x(2-x)e^{-x} > 0$.

(f) Since the real exponential function e^{-x} is strictly positive for all real x ($e^{-x} > 0$), we can divide by it without altering the inequality: $x(2-x) > 0$.

(g) The critical boundary roots are clearly $x = 0$ and $x = 2$. The downward-opening quadratic expression $x(2-x)$ is strictly positive strictly inside its roots, yielding the open continuous interval $(0, 2)$.

Final Answer:

$$\boxed{(0, 2)}$$

Answer: (D)

Go Back to Question 28

Q29.

Solution

Concept:

A remarkable foundational property of any rectangular hyperbola written in its canonical asymptotic form $xy = c^2$ states that the geometric area of the triangle bounded by any tangent line and the two coordinate asymptotes (coordinate axes) is strictly invariant and exactly equal to $2c^2$.

Solution:

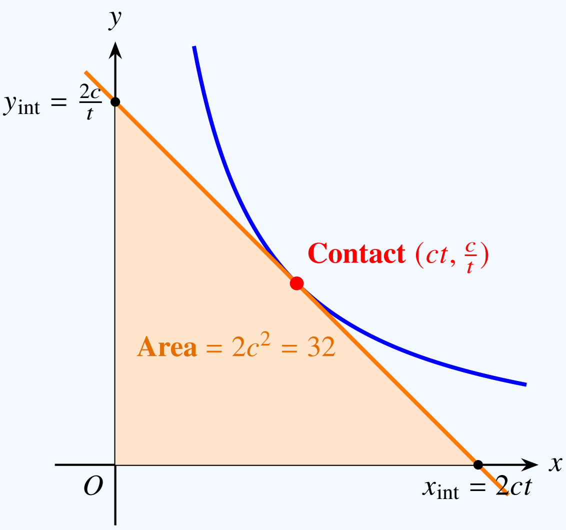
(a) Write down the foundational invariant area formula for a rectangular hyperbola tangent triangle: Area $= 2c^2$.

(b) Compare the provided hyperbola equation $xy = 16$ directly with our canonical archetype $xy = c^2$ to extract the invariant constant parameter: we obtain exactly $c^2 = 16$.

(c) Substitute our extracted parameter directly into our foundational area formula: Area $= 2(16)$.

(d) Execute the simple integer multiplication to successfully establish our final invariant area: Area $= 32$.

(e) *(Proof: The tangent to $xy = c^2$ at $(ct, c/t)$ has the exact linear equation $x + t^2 y = 2ct$. Its specific intercepts on the coordinate axes are $x_{\text{int}} = 2ct$ and $y_{\text{int}} = \frac{2c}{t}$. The triangular area is exactly $\frac{1}{2}|x_{\text{int}} \cdot y_{\text{int}}| = \frac{1}{2}\left|2ct \cdot \frac{2c}{t}\right| = 2c^2 = 32$).*



Final Answer:

$$\boxed{32}$$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept:

The geometric tangent line to a differentiable curve $y = f(x)$ at any specific contact point (x_0, y_0) has an instantaneous slope given by the first derivative evaluated exactly at that x-coordinate, $m_t = \left.\frac{dy}{dx}\right|_{x=x_0}$. A curve crosses the vertical y-axis exactly where $x = 0$.

Solution:

(a) Determine the x-coordinate of the contact point: the problem specifies the point where the curve crosses the vertical y-axis. By definition, any point on the y-axis has an x-coordinate of exactly $x = 0$.

(b) Find the corresponding y-coordinate by substituting $x = 0$ into the curve equation: $y = 3(0)^2 - 4(0) + 5 = 5$. Thus, the contact point is $(0, 5)$.

(c) Compute the first derivative of the polynomial curve with respect to x using the standard power rule: $\frac{dy}{dx} = \frac{d}{dx}(3x^2) - \frac{d}{dx}(4x) + \frac{d}{dx}(5) = 6x - 4$.

(d) Evaluate this exact derivative at our determined contact coordinate $x = 0$ to find the tangent slope m_t: $m_t = \left.\frac{dy}{dx}\right|_{x=0} = 6(0) - 4 = -4$.

Final Answer:

$$\boxed{-4}$$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept:

In probability theory, if the theoretical odds in favor of an event A occurring are expressed as a ratio $m : n$, it implies there are exactly m favorable outcomes for every n unfavorable outcomes. The exact theoretical success probability is $P(A) = \frac{m}{m+n}$, and its exact complement failure probability is $P(\bar{A}) = \frac{n}{m+n}$.

Solution:

(a) Extract our precise proportional odds parameters from our text: we have odds in favor of event A given as $m : n = 3 : 5$. Thus, we have exactly $m = 3$ favorable outcomes and $n = 5$ unfavorable outcomes.

(b) Compute our complete operational sample space total of equally likely outcomes: $m + n = 3 + 5 = 8$ outcomes.

(c) Calculate our exact theoretical success probability $P(A)$: $P(A) = \frac{m}{m+n} = \frac{3}{8}$.

(d) We wish to find the theoretical probability of event A exactly NOT occurring, which represents its exact formal complement $P(\bar{A})$.

(e) Apply our foundational formal complement subtraction law: $P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{8}$.

(f) Execute our simple fraction subtraction to establish our required final output: $P(\bar{A}) = \frac{8-3}{8} = \frac{5}{8}$. (Notice that this identically matches $\frac{n}{m+n}$).

Final Answer:

$$\boxed{\frac{5}{8}}$$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept:
The fundamental geometric area of a three-dimensional parallelogram whose adjacent operational sides are represented by two vectors $\vec{a}$ and $\vec{b}$ is computed directly as the Euclidean magnitude of their cross product, Area = $|\vec{a} \times \vec{b}|$. We first compute the cross product vector, then calculate its complete Euclidean magnitude.

Solution:

(a) Write down our foundational operational cross product determinant using our specific provided vector components: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$.

(b) Expand our 3×3 cross product determinant along its top operational unit vector row: $\vec{a} \times \vec{b} = \hat{i} \begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix}$.

(c) Evaluate each internal 2×2 minor difference independently: $\vec{a} \times \vec{b} = \hat{i}[4 - (-6)] - \hat{j}[12 - (-2)] + \hat{k}[-9 - 1]$.

(d) Perform the simple term arithmetic to establish our exact operational cross product vector: $\vec{a} \times \vec{b} = -2\hat{i} - 14\hat{j} - 10\hat{k}$.

(e) Set up our rigorous Euclidean magnitude identity to determine our complete total area: Area $= |\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + (-14)^2 + (-10)^2}$.

(f) Execute our internal binomial square additions: Area $= \sqrt{4 + 196 + 100} = \sqrt{300}$.

(g) Simplify our radical expression by extracting our largest perfect square $(300 = 100 \times 3)$: Area $= \sqrt{100} \cdot \sqrt{3} = 10\sqrt{3}$.

Final Answer:

$$\boxed{10\sqrt{3}}$$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept:

An Involutory Matrix is formally defined as a square matrix which, when multiplied by itself, yields exactly the primary identity matrix ($A^2 = I$). To simplify matrix binomial products involving involutory matrices, we expand the product using distributive laws and substitute our defining property.

Solution:

(a) Start with our provided multi-term matrix product expression: P =(I - A)(I + A).

(a) Expand the formal product carefully using our standard distributive matrix multiplication laws: $P = I \cdot I + I \cdot A - A \cdot I - A \cdot A$.

(b) Apply our foundational matrix identity properties ($I \cdot I = I$, and $I \cdot A = A \cdot I = A$): $P = I + A - A - A^2$.

(c) Execute the linear intermediate matrix cancellation ($A - A = O$): $P = I - A^2$.

(d) Apply our specific defining involutory condition $A^2 = I$ to eliminate our squared matrix term: $P = I - I$.

(e) Execute our final matrix identity subtraction: $P = I - I = O$ (representing our primary zero matrix of the same dimensions).

Final Answer:

$$\boxed{O}$$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept:

The expanded general equation of a 3D geometric sphere is written in its standard zero-form layout as $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. By completing the square or matching structural coefficient relationships, its exact Cartesian center coordinates are derived as exactly $(-u, -v, -w)$.

Solution:

(a) Compare our provided expanded sphere equation $x^2 + y^2 + z^2 - 4x + 6y - 2z + 5 = 0$ directly with our standard general model $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

(b) Extract our precise linear coefficient relationships variable by variable.

(c) For our x-coefficient: $2u = -4 \implies u = -2$.

(d) For our y-coefficient: $2v = 6 \implies v = 3$.

(e) For our z-coefficient: $2w = -2 \implies w = -1$. Our operational constant term is $d = 5$.

(f) Determine our final required Cartesian center tuple $(-u, -v, -w)$: substituting our evaluated intermediate parameters yields exactly $(-(-2), -(3), -(-1)) = (2, -3, 1)$.

Final Answer:

$$\boxed{(2, -3, 1)}$$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept:

The complete exact geometric area of a two-dimensional triangle with Cartesian vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is calculated using our absolute Cramer's determinant formula: Area = $\frac{1}{2}\left\|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right\|$. We equate this to our known total area and solve for k.

Solution:

(a) Write down our foundational Cramer's determinant area identity using our specific provided vertex tuples: Area = $\frac{1}{2}\left\|\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 6 & 1 \end{vmatrix}\right\|$.

(b) Set up the operational absolute equation: $\frac{1}{2}\left\|\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 6 & 1 \end{vmatrix}\right\| = 12 \implies \left\|\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 6 & 1 \end{vmatrix}\right\| = 24$.

(c) Focus entirely on evaluating our internal 3×3 minor determinant. Notice that our second column C_2 has exactly two pristine zero elements $(0, 0, 6)$.

(d) Expand our internal determinant along our second column C_2 to achieve maximum computational efficiency: $\Delta = -6 \cdot \begin{vmatrix} k & 1 \\ 4 & 1 \end{vmatrix}$.

(e) Evaluate our internal 2×2 minor difference: $\Delta = -6 \cdot (k \cdot 1 - 4 \cdot 1) = -6(k - 4)$.

(f) Substitute our successfully evaluated internal determinant back into our absolute equation: $|-6(k - 4)| = 24 \implies 6|k - 4| = 24 \implies |k - 4| = 4$.

(g) Split our absolute linear equation into two distinct possible boundary cases: Case 1: $k - 4 = 4 \implies k = 8$, or Case 2: $k - 4 = -4 \implies k = 0$.

(h) Since our question explicitly specifies that we want a valid positive value, we select exactly $k = 8$.

Final Answer:

$$\boxed{8}$$

Answer: (D)

Go Back to Question 35

Q36.

Solution

Concept:
Two circles are defined to be concentric if they share the exact same geometric center coordinates while having potentially different radii. Thus, the general equation of any circle concentric with $x^2 + y^2 + 2gx + 2fy + c = 0$ is exactly $x^2 + y^2 + 2gx + 2fy + k = 0$. We find k using the contact point.

Solution:

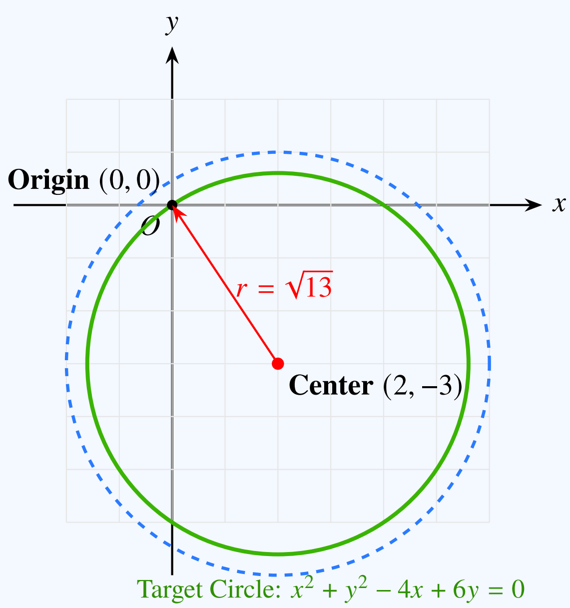
(a) Start with the provided general circle equation: $x^2 + y^2 - 4x + 6y - 3 = 0$.

(b) Write down the structural template for any circle concentric with it by replacing only its independent constant term with an unknown parameter k: $x^2 + y^2 - 4x + 6y + k = 0$.

(c) Apply our explicit boundary condition that this target concentric circle passes exactly through the origin $(0, 0)$.

(d) Substitute $x = 0$ and $y = 0$ directly into our concentric template to evaluate parameter k: $(0)^2 + (0)^2 - 4(0) + 6(0) + k = 0 \implies k = 0$.

(e) Substitute $k = 0$ back to establish the exact final operational conic relation: $x^2+y^2-4x+6y = 0$.



Final Answer:

$$\boxed{x^2 + y^2 - 4x + 6y = 0}$$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept:

When differentiating complex inverse trigonometric functions, directly applying the chain rule leads to highly tedious rational calculations. Instead, we first simplify the inner argument using fundamental half-angle trigonometric identities: $\sin x = 2\sin(x/2)\cos(x/2)$ and $1 + \cos x = 2\cos^2(x/2)$.

Solution:

(a) Start with the given explanatory function: $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$.

(b) Focus entirely on condensing the inner rational trigonometric argument: $u(x) = \frac{\sin x}{1+\cos x}$.

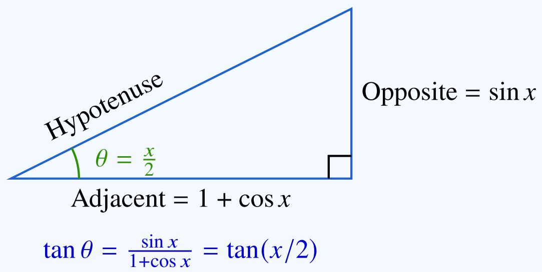
(c) Apply our fundamental half-angle identities to both the numerator and denominator: $u(x) = \frac{2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}$.

(d) Cancel the common scalar factor of 2 and one factor of $\cos(x/2)$: $u(x) = \frac{\sin(x/2)}{\cos(x/2)} = \tan\left(\frac{x}{2}\right)$.

(e) Substitute this successfully condensed argument back into the primary inverse function: $y = \tan^{-1}\left[\tan\left(\frac{x}{2}\right)\right]$.

(f) Apply the fundamental self-inversion identity $\tan^{-1}(\tan\theta) = \theta$ valid within the principal open branch $-\pi < x < \pi$: $y = \frac{x}{2}$.

(g) Differentiate this beautifully pristine linear relation directly with respect to x: $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$.



Final Answer:

$$\boxed{\frac{1}{2}}$$

Answer: (D)

Go Back to Question 37

Q38.

Solution

Concept:

When an indefinite integral is presented as a rational fraction where the complete analytical numerator is exactly equal to the first derivative of the entire rational denominator, $\int \frac{f'(x)}{f(x)}\,dx$, the anti-derivative evaluates elegantly and directly to the natural logarithm $\ln|f(x)| + C$.

Solution:

(a) Examine the provided indefinite integral: $I = \int \frac{2x+3}{x^2+3x+7}\,dx$.

(b) Inspect the algebraic structural relationship between the numerator and denominator: let the entire denominator be denoted as an analytical function $f(x) = x^2 + 3x + 7$.

(c) Compute the exact formal derivative of $f(x)$ with respect to x using the standard power rule: $f'(x) = \frac{d}{dx}(x^2) + \frac{d}{dx}(3x) + \frac{d}{dx}(7) = 2x + 3$.

(d) Observe that $f'(x) = 2x + 3$ perfectly and identically matches our entire operational numerator.

(e) The integral successfully condenses into the foundational logarithmic substitution archetype: $I = \int \frac{f'(x)}{f(x)}\,dx$.

(f) Conclude the exact anti-derivative directly from the integration theorem: $I = \ln|f(x)| + C = \ln|x^2 + 3x + 7| + C$.

Final Answer:

$$\boxed{\ln\left|x^2 + 3x + 7\right| + C}$$

Answer: (C)

Go Back to Question 38

Q39.

Solution

Concept:
To convert any complex rational fraction $\frac{A+iB}{C+iD}$ into its pristine standard Cartesian layout $(x+iy)$, we must rationalize its complex denominator. We accomplish this by multiplying the entire primary numerator and denominator by the exact formal complex conjugate of the denominator, $(C-iD)$.

Solution:

(a) Start with our provided multi-term complex rational fraction: $Z=\frac{5+2i}{1-2i}$.

(b) Identify our complex denominator $w=1-2i$, and write down its exact formal complex conjugate: $\bar{w}=1+2i$.

(c) Multiply our primary numerator and denominator entirely by our formal conjugate: $Z=\frac{(5+2i)(1+2i)}{(1-2i)(1+2i)}$.

(d) Expand our formal complex product in the numerator using standard distributive multiplication laws: $(5+2i)(1+2i)=5(1)+5(2i)+2i(1)+2i(2i)=5+10i+2i+4i^2$.

(e) Expand our complex difference of squares in the denominator: $(1-2i)(1+2i)=(1)^2-(2i)^2=1-4i^2$.

(f) Substitute our foundational complex identity $i^2=-1$ independently into both evaluated parts: $Z=\frac{5+12i+4(-1)}{1-4(-1)}=\frac{5+12i-4}{1+4}$.

(g) Combine our real terms together and split our resulting rational expression: $Z=\frac{1+12i}{5}=\frac{1}{5}+\frac{12}{5}i$.

Final Answer:

$$\boxed{\frac{1}{5}+\frac{12}{5}i}$$

Answer: (C)

Go Back to Question 39

Q40.

Solution

Concept:
We use the structural rules of Circular Combinatorics. For standard circular permutations across a table where clockwise and counter-clockwise arrangements are distinct, the arrangement count is exactly $(n - 1)!$. However, when forming freely flippable 3D physical figures like a beaded necklace, clockwise and counter-clockwise arrangements are perfectly identical! Thus, the exact count formula is $\frac{(n-1)!}{2}$.
Solution:

(a) Identify our complete operational quantitative number of distinct beads from our prompt: we have exactly $n = 7$ uniquely colored beads.

(b) Verify the precise physical structural properties of our figure: a decorative necklace lying freely in space can be physically turned over or flipped in 3D space.

(c) As a direct geometric consequence, every individual clockwise permutation is physically perfectly identical to its reflected counter-clockwise permutation.

(d) Write down our rigorous combinatorial formula for flippable circular permutations: Ways = $\frac{(n-1)!}{2}$.

(e) Substitute $n = 7$ directly into our analytical factorial identity: Ways = $\frac{(7-1)!}{2} = \frac{6!}{2}$.

(f) Expand our formal standard factorial: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

(g) Perform our final pristine simple integer division: Ways = $\frac{720}{2} = 360$ ways.

Final Answer:

$$\boxed{360}$$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept:
In related rates problems, we differentiate the exact geometric identity governing the structural figure with respect to time t. The total geometric area A of a uniform square is related to its side length s by $A = s^2$. Applying the chain rule yields the governing rate identity $\frac{dA}{dt} = 2s\frac{ds}{dt}$.
Solution:

(a) Write down the foundational geometric formula linking the area of a square to its operational side length: $A = s^2$.

(b) Differentiate both sides of the identity implicitly with respect to time t using the chain rule: $\frac{dA}{dt} = 2s \cdot \frac{ds}{dt}$.

(c) Identify our specific operational parameters at the required instant: the operational side length is $s = 10$ cm, and the constant time rate of change of the side length is $\frac{ds}{dt} = 3$ cm/s.

(d) Substitute these exact values directly into our differentiated operational formula: $\frac{dA}{dt} = 2(10\text{ cm}) \cdot (3\text{ cm/s})$.

(e) Execute the simple integer multiplication to establish the final rate: $\frac{dA}{dt} = 60\text{ cm}^2/\text{s}$.

Final Answer:

$$\boxed{60\text{ cm}^2/\text{s}}$$

Answer: (D)

Go Back to Question 41

Q42.

Solution

Concept:

To evaluate indeterminate limits of the exponential form $\frac{0}{0}$, we utilize the foundational standard limit $\lim_{u\to 0}\frac{e^u-1}{u} = 1$. By adding and subtracting 1 inside the numerator and algebraically splitting the rational expression, the limit can be resolved into standard exact forms.

Solution:

(a) The given rational limit expression is $L = \lim_{x\to 0}\frac{e^{5x}-e^{2x}}{x}$. As $x \to 0$, both the numerator and denominator approach 0, forming a standard $\frac{0}{0}$ indeterminate structure.

(b) Algebraically manipulate the numerator by adding and subtracting 1 between the two exponential terms: $L = \lim_{x\to 0}\frac{(e^{5x}-1)-(e^{2x}-1)}{x}$.

(c) Split the limit into two distinct analytical fractions: $L = \lim_{x\to 0}\left[\frac{e^{5x}-1}{x}\right] - \lim_{x\to 0}\left[\frac{e^{2x}-1}{x}\right]$.

(d) Multiply and divide each fraction by its specific inner exponential coefficient: $L = \lim_{x\to 0}\left[\frac{e^{5x}-1}{5x}\cdot 5\right] - \lim_{x\to 0}\left[\frac{e^{2x}-1}{2x}\cdot 2\right]$.

(e) Apply the standard limit property $\lim_{u\to 0}\frac{e^u-1}{u} = 1$ independently to both evaluated parts.

(f) Conclude the final analytical arithmetic: $L = (1\cdot 5) - (1\cdot 2) = 5 - 2 = 3$.

Final Answer:

$$\boxed{3}$$

Answer: (C)

Go Back to Question 42

Q43.

Solution

Concept:
To determine an unknown operational parameter in a second-degree polynomial equation $ax^2 + bx + c = 0$ whose roots satisfy a specific quantitative ratio $(\alpha, r\alpha)$, we utilize our foundational Vieta's root-coefficient relations: Sum $= -\frac{b}{a}$ and Product $= \frac{c}{a}$. We isolate α from the sum and substitute into the product.

Solution:

(a) Start with our provided real quadratic equation: $3x^2 - 10x + k = 0$. Identify our primary coefficients: $a = 3, b = -10, c = k$.

(b) Define our two real roots in accordance with our prompt's root ratio: let our roots be denoted as exactly α and 2α.

(c) Apply Vieta's formal root summation theorem: Sum $= \alpha + 2\alpha = -\frac{b}{a} \implies 3\alpha = -\left(\frac{-10}{3}\right) = \frac{10}{3}$.

(d) Divide by 3 to successfully isolate our shared root value: $\alpha = \frac{10}{9}$.

(e) Apply Vieta's formal root multiplication theorem: Product $= \alpha \cdot (2\alpha) = \frac{c}{a} \implies 2\alpha^2 = \frac{k}{3}$.

(f) Substitute our successfully isolated shared root value $\alpha = \frac{10}{9}$ directly into our product relation: $2\left(\frac{10}{9}\right)^2 = \frac{k}{3}$.

(g) Execute our internal rational square additions: $2\left(\frac{100}{81}\right) = \frac{k}{3} \implies \frac{200}{81} = \frac{k}{3}$.

(h) Multiply both sides entirely by 3 to isolate our target unknown parameter: $k = \frac{200\times3}{81} = \frac{200}{27}$.

Final Answer:

$$\boxed{\frac{200}{27}}$$

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept:

To determine the structural geometric properties of a general shifted parabola, we complete the square in its quadratic variable to achieve our standard shifted archetype: $(y-k)^2 = 4a(x-h)$. Comparing directly with this archetype reveals that its exact vertex is located at (h, k).

Solution:

(a) Start with the provided expanded parabolic equation: $y^2 - 4y - 4x + 16 = 0$.

(b) Isolate all terms containing the quadratic variable y on the left-hand side, and move the linear x terms and constant to the right: $y^2 - 4y = 4x - 16$.

(c) Complete the square on the left side by adding $\left(\frac{-4}{2}\right)^2 = 4$ to both sides of the equation: $y^2 - 4y + 4 = 4x - 16 + 4$.

(d) Condense the left-hand side into a perfect binomial square and simplify the right-hand side: $(y-2)^2 = 4x - 12$.

(e) Factor out the leading linear coefficient of 4 entirely from the right side: $(y-2)^2 = 4(x-3)$.

(f) Compare this fully condensed analytical relation directly with our standard shifted model $(y-k)^2 = 4a(x-h)$.

(g) Extract our precise vertex parameters (h, k): we obtain exactly $(3, 2)$.

Final Answer:

$$\boxed{(3, 2)}$$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept:
A foundational geometric property of a standard right-opening parabola $y^2 = 4ax$ states that the linear focal distance d of any arbitrary point $P(x_1, y_1)$ lying on its structural boundary is exactly equal to its perpendicular distance from the directrix line, given by the simple formula $d = x_1 + a$.
Solution:

(a) Compare our provided parabolic equation $y^2 = 16x$ with our standard horizontal model $y^2 = 4ax$.

(b) Equate the focal linear coefficients to evaluate parameter a: $4a = 16 \implies a = 4$.

(c) Write down our foundational identity for the focal distance of a point $P(x_1, y_1)$ lying on the parabola: $d = x_1 + a$.

(d) Substitute our evaluated focal parameter $a = 4$ and our known focal distance $d = 10$ directly into the structural identity: $10 = x_1 + 4$.

(e) Subtract 4 from both sides to successfully establish the final x-coordinate: $x_1 = 6$.

Final Answer:

$$\boxed{6}$$

Answer: (C)

Go Back to Question 45

Q46.

Solution

Concept:

To evaluate the exact derivative of a variable base raised to a variable power, $y = [u(x)]^{v(x)}$, we apply the powerful technique of Logarithmic Differentiation. We take the natural logarithm of both sides to bring down the variable exponent using $\ln(a^b) = b \ln a$, and differentiate implicitly.

Solution:

(a) Start with the provided functional curve definition: $y = x^x$ defined over $x > 0$.

(b) Take the natural logarithm (ln) of both sides of the equation: $\ln y = \ln(x^x)$.

(c) Apply the fundamental power identity of logarithms to bring down the exponent: $\ln y = x \ln x$.

(d) Differentiate both sides implicitly with respect to x using the chain rule on the left and the product rule on the right: $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x) \cdot \ln x + x \cdot \frac{d}{dx}(\ln x)$.

(e) Execute the standard derivative evaluations: $\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$.

(f) Multiply both sides by y to isolate the required derivative: $\frac{dy}{dx} = y(\ln x + 1)$.

(g) Substitute the original explicit definition $y = x^x$ back into the result: $\frac{dy}{dx} = x^x(\ln x + 1)$.

(h) Evaluate this exact derivative at the target coordinate $x = 1$: $\left.\frac{dy}{dx}\right|_{x=1} = (1)^1 \cdot [\ln(1) + 1]$.

(i) Since $\ln(1) = 0$, the arithmetic simplifies perfectly: $= 1 \cdot [0 + 1] = 1$.

Final Answer:

$$\boxed{1}$$

Answer: (A)

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Q47.

Solution

Concept:

We use the Fundamental Definite Integration Parity Theorem across symmetric limit boundaries: $\int_{-a}^{a} F(x)\,dx$. If the entire integrand is formally an Odd Function, satisfying the reflection anti-symmetry relationship $F(-x) = -F(x)$, the complete definite integral evaluates identically to zero.

Solution:

(a) Let the entire rational trigonometric integrand be denoted as a single unified analytical function: $F(x) = \frac{x^3 \cos x + \sin^5 x}{1+x^2}$.

(b) Test the structural parity of the function by executing the formal reflection substitution $x \longrightarrow -x$: $F(-x) = \frac{(-x)^3 \cos(-x) + [\sin(-x)]^5}{1+(-x)^2}$.

(c) Apply the standard parity properties of our foundational functions: the power function $(-x)^3 = -x^3$ (odd), the cosine function $\cos(-x) = \cos x$ (even), the sine function $\sin(-x) = -\sin x$ (odd), and $(-x)^2 = x^2$ (even).

(d) Substitute these simplified transformations back into the reflected function: $F(-x) = \frac{(-x^3)\cos x + (-\sin x)^5}{1+x^2} = \frac{-x^3 \cos x - \sin^5 x}{1+x^2}$.

(e) Factor out the common negative sign entirely from the primary numerator: $F(-x) = -\left[\frac{x^3 \cos x + \sin^5 x}{1+x^2}\right] = -F(x)$.

(f) Because $F(-x) = -F(x)$ holds universally across the valid domain, the entire integrand is formally an odd function.

(g) By our symmetric integration boundary theorem, the net accumulated area above and below the axis cancels perfectly: $I = 0$.

Final Answer:

$$\boxed{0}$$

Answer: (B)

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Q48.

Solution

Concept:
A mathematical function $f(x)$ is formally classified as continuous at a specified coordinate $x = a$ if and only if its two-sided limit evaluates to a finite value that matches its functional definition at that point, $\lim_{x\to a} f(x) = f(a)$. We use the standard identities $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$.
Solution:

(a) For total continuity at the boundary coordinate $x = 0$, the operational condition requires that $\lim_{x\to 0} f(x) = f(0) = p$.

(b) Set up the full two-sided analytical limit: $L = \lim_{x\to 0} \frac{\sin(3x)}{\tan(2x)}$.

(c) Divide both the primary numerator and denominator by the independent variable x: $L = \lim_{x\to 0} \frac{\frac{\sin(3x)}{x}}{\frac{\tan(2x)}{x}}$.

(d) Multiply and divide the numerator by 3 and the denominator by 2 to match our fundamental standard formulas: $L = \frac{\lim_{x\to 0}\left[\frac{\sin(3x)}{3x}\cdot 3\right]}{\lim_{x\to 0}\left[\frac{\tan(2x)}{2x}\cdot 2\right]}$.

(e) Substitute our standard identity limits, which both evaluate exactly to 1: $L = \frac{1\cdot 3}{1\cdot 2} = \frac{3}{2}$.

(f) Equate this successfully evaluated limit to the functional parameter to establish the final value: $p = \frac{3}{2}$.

Final Answer:

$$\boxed{\frac{3}{2}}$$

Answer: (B)

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Q49.

Solution

Concept:
The foundational Binomial Theorem states that the complete exact algebraic summation of all binomial coefficients across an expansion, $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$, evaluates elegantly and identically to 2^n. We equate this fundamental combinatorial exponential identity to our provided total and solve for n.

Solution:

(a) Write down our foundational combinatorial summation identity for all binomial coefficients across an nth degree expansion: $\sum_{r=0}^{n} \binom{n}{r} = 2^n$.

(b) Set up our rigorous exponential equation using our specific provided numerical total: $2^n = 512$.

(c) Focus entirely on recognizing our fundamental base two quantitative powers: we enumerate $2^7 = 128$, $2^8 = 256$, and $2^9 = 512$.

(d) Equate our successfully identified base two exponents to successfully establish our final pristine integer output: $n = 9$.

Final Answer:

$$\boxed{9}$$

Answer: (C)

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Q50.

Solution

Concept:
In real analysis, a mathematical function $F(x)$ is formally classified as an Even Function if and only if its analytical output is perfectly invariant under reflection across the vertical y-axis, satisfying the exact identity $F(-x) = F(x)$ universally across its complete operational domain.
Solution:

(a) We test each function independently for its formal structural parity by executing our explicit reflection substitution $x \longrightarrow -x$.

(b) Consider Option (A), $f(x) = x\sin x$. We evaluate its reflection: $f(-x) = (-x)\sin(-x)$.

(c) Apply our foundational odd trigonometric reflection property $\sin(-x) = -\sin x$: $f(-x) = (-x)(-\sin x) = x\sin x$.

(d) Notice that $f(-x) = x\sin x$ perfectly and identically equals our starting function $f(x)$. Thus, Option (A) is purely an even function.

(e) *(For completeness, we verify the remaining options: for Option (B), $g(-x) = (-x)^3\cos(-x) = -x^3\cos x = -g(x)$ [odd]. For Option (C), $h(-x) = e^{-x} - e^{-(-x)} = e^{-x} - e^{x} = -h(x)$ [odd]. For Option (D), $k(-x) = \ln(\sqrt{x^2+1} - x) = \ln((\sqrt{x^2+1} - x)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}) = \ln(\frac{1}{\sqrt{x^2+1}+x}) = -k(x)$ [odd]).*

Final Answer:

$$\boxed{f(x) = x\sin x}$$

Answer: (C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	A	4	D	5	B
6	A	7	B	8	D	9	D	10	B
11	A	12	D	13	A	14	A	15	B
16	A	17	B	18	A	19	A	20	A
21	B	22	D	23	B	24	C	25	D
26	B	27	C	28	D	29	A	30	A
31	A	32	B	33	A	34	B	35	D
36	B	37	D	38	C	39	C	40	A
41	D	42	C	43	A	44	B	45	C
46	A	47	B	48	B	49	C	50	C