JCECE Mathematics Sample Paper-6

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **50** Multiple Choice Questions.
- Each correct answer carries **+1** mark. Incorrect answer: **−0.25** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. A completely fair standard coin is tossed exactly 6 times in succession. The exact evaluated probability of obtaining exactly 4 heads is

(A) $\frac{15}{32}$

(B) $\frac{5}{16}$

(C) $\frac{3}{8}$

(D) $\frac{15}{64}$

Q2. The absolute evaluated summation of the infinite geometric progression (G.P.) series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots \infty$ is

(A) 2

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $\frac{3}{2}$

Q3. If the length of the major axis of an ellipse is exactly 3 times the length of its minor axis, then its eccentricity is

(A) $\frac{2}{3}$

(B) $\frac{\sqrt{5}}{3}$

(C) $\frac{2\sqrt{2}}{3}$

(D) $\frac{1}{3}$

Q4. A perfectly spherical metallic balloon is being heated and expanded such that its surface area increases at a uniform constant rate of 8π cm^2/s. The exact rate of increase of its total radius when the radius reaches 4 cm is

(A) 0.125 cm/s

(B) 1.0 cm/s

(C) 0.5 cm/s

(D) 0.25 cm/s

Q5. The exact coordinates of the focus of the geometric parabola given by the equation $x^2 = -16y$ are

(A) $(0, 4)$

(B) $(0, -8)$

(C) $(4, 0)$

(D) $(0, -4)$

Q6. The exact equation of the normal to the curve $y = x \ln x$ at the specific point where $x = e$ is

(A) $2x - y = e$

(B) $x - 2y = -e$

(C) $2x + y = 3e$

(D) $x + 2y = 3e$

Q7. If a real square matrix A satisfies the structural orthogonal condition $A \cdot A^T = I$, then its inverse A^{-1} is exactly equal to

(A) I

(B) A

(C) $-A$

(D) A^T

Q8. If the three straight lines given by $2x + y - 3 = 0$, $5x + ky - 3 = 0$, and $3x - y - 2 = 0$ are perfectly concurrent, then the exact numerical value of k is

(A) 2

(B) 3

(C) -2

(D) -3

Q9. If A and B are two stochastically independent events such that their primary probabilities are $P(A) = 0.3$ and $P(B) = 0.4$, then the union probability $P(A \cup B)$ is exactly equal to

(A) 0.58

(B) 0.70

(C) 0.46

(D) 0.12

Q10. The general family of curves satisfying the differential equation $\frac{dy}{dx} = \frac{2x}{3y^2}$ is represented by

(A) $3y^3 - 2x^2 = C$

(B) $2y^2 - 3x^3 = C$

(C) $y^3 - x^2 = C$

(D) $y^2 - x^3 = C$

Q11. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 - 5y = e^x$ is

(A) 1

(B) 3

(C) 4

(D) 2

Q12. A specialized committee of exactly 4 persons is to be formed from a group of 6 men and 4 women. In how many exact ways can this committee be formed if it must contain exactly 2 women?

(A) 60

(B) 90

(C) 45

(D) 180

Q13. Which of the following functions is strictly not differentiable at the specific point $x = 2$?

(A) $\sin((x-2)^2)$

(B) $(x-2)\,|x-2|$

(C) $|x-2|$

(D) $(x-2)^3$

Q14. The complete set of real values of x for which the function $f(x) = 2x^3 - 15x^2 + 36x - 12$ is strictly decreasing is

(A) $(1, 6)$

(B) $(2, 3)$

(C) $(-\infty, 2) \cup (3, \infty)$

(D) $(-3, -2)$

Q15. The total area bounded by the parabola $y = 6x - x^2$ and the horizontal x-axis is

(A) 18

(B) 72

(C) 54

(D) 36

Q16. If the complex number is defined as $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$, then the evaluated principal value of z^{12} is equal to

(A) 1

(B) $-i$

(C) i

(D) -1

Q17. The absolute evaluated value of the purely algebraic determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(A) 0

(B) $(a-b)(b-c)(c-a)$

(C) $a+b+c$

(D) 1

Q18. If a real rational function is defined as $f(x) = \frac{x}{x-1}$ for all $x \neq 1$, then the exact analytical form of the nested composition $f(f(x))$ is

(A) $\frac{x}{x+1}$

(B) $x-1$

(C) $\frac{1}{x-1}$

(D) x

Q19. The specific value of μ for which the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \mu\hat{j} + 5\hat{k}$ are perfectly coplanar is

(A) 4

(B) 2

(C) -2

(D) -4

Q20. The specific condition that the straight line $y = mx + c$ acts as a tangent to the parabola $y^2 = 4ax$ is

(A) $c = \frac{a}{m}$

(B) $c = -am$

(C) $c = am$

(D) $c = \frac{a}{m^2}$

Q21. If A is a square non-singular matrix of order 3 such that its determinant is $\det(A) = 5$, then the exact numerical value of $\det(\text{adj}(A))$ is

(A) 125

(B) 25

(C) 5

(D) 10

Q22. The center and exact radius of the geometric circle given by the expanded equation $x^2 + y^2 - 8x + 10y + 16 = 0$ are respectively

(A) $(-4, 5)$ and 25

(B) $(4, -5)$ and 25

(C) $(-4, 5)$ and 5

(D) $(4, -5)$ and 5

Q23. Let $f(x) = \begin{cases} \frac{1-\cos(kx)}{x^2}, & \text{if } x \neq 0 \\ 18, & \text{if } x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$, then the positive value of k is

(A) 6

(B) 9

(C) 3

(D) 12

Q24. The explicit Cartesian equation of the 3D plane that makes axis intercepts of 2, 3, and 4 on the x, y, and z coordinate axes respectively is

(A) $2x + 3y + 4z = 12$

(B) $4x + 3y + 2z = 12$

(C) $6x + 4y + 3z = 12$

(D) $3x + 4y + 6z = 24$

Q25. The continuous curve passing through the initial coordinate $(0, 1)$ and satisfying the separable differential equation $\frac{dy}{dx} = 3x^2y$ has the operational equation

(A) $y = e^{x^3}$

(B) $y = 3e^{x^2}$

(C) $y = x^3 + 1$

(D) $y = e^{3x}$

Q26. If every numerical observation in a given statistical dataset is scaled by multiplying by 3 and then shifted by adding 5, what is the exact ratio of the new resulting standard deviation to the original standard deviation?

(A) 15

(B) 8

(C) 5

(D) 3

Q27. If the square matrix $A = \begin{pmatrix} 2 & \lambda \\ 4 & 6 \end{pmatrix}$ is singular, then the exact value of λ is

(A) 2

(B) 3

(C) 6

(D) 4

Q28. The term purely independent of x (the constant term) in the binomial expansion of $\left(2x^2 - \frac{1}{x}\right)^9$ is

(A) 84

(B) 672

(C) 336

(D) -672

Q29. The area of the geometric triangle formed by the straight line $3x + 4y = 12$ and the two coordinate axes is

(A) 24

(B) 3

(C) 6

(D) 12

Q30. Let R be an analytical relation defined on the complete set of integers $\mathbb{Z}$ by the rule xRy if and only if $(x - y)$ is exactly divisible by 5. Which of the following strict mathematical statements is perfectly true?

(A) R is reflexive but not symmetric

(B) R is not reflexive

(C) R is an equivalence relation

(D) R is symmetric but not transitive

Q31. The exact coordinates of the foot of the perpendicular drawn from the origin $(0, 0, 0)$ to the flat 3D plane $2x + 3y + 6z = 49$ are

(A) $(4, 6, 12)$

(B) $(2, 3, 4)$

(C) $(2, 3, 6)$

(D) $(1, 1.5, 3)$

Q32. The projection of the vector $\vec{u} = 4\hat{i} + 2\hat{j} + \hat{k}$ along the vector $\vec{v} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ is

(A) $\frac{11}{3}$

(B) $\frac{11}{7}$

(C) $\frac{22}{7}$

(D) $\frac{22}{3}$

Q33. The total length of the chord intercepted by the geometric circle $x^2 + y^2 = 25$ on the straight line $x = 3$ is

(A) 6

(B) 8

(C) 10

(D) 4

Q34. The complete locus of the point of intersection of two perpendicular tangents drawn to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is a circle with radius

(A) 5

(B) $\sqrt{12}$

(C) $\sqrt{25}$

(D) $\sqrt{7}$

Q35. The explicit integrating factor (I.F.) for the linear differential equation $\frac{dy}{dx} + y \cot x = 2 \cos x$ is

(A) $\tan x$

(B) $\cos x$

(C) $\csc x$

(D) $\sin x$

Q36. The value of the modulus definite integral $\int_0^4 |x - 2|\ dx$ is

(A) 4

(B) 8

(C) 2

(D) 6

Q37. Two fair standard six-sided dice are rolled simultaneously. Given the conditional information that the exact sum of the numbers appearing on the two dice is 8, what is the exact conditional probability that at least one of the dice shows a 3?

(A) $\frac{2}{5}$

(B) $\frac{1}{4}$

(C) $\frac{3}{5}$

(D) $\frac{1}{3}$

Q38. If the curve is given by the implicit relation $x^3 + y^3 = 9xy$, then the numerical value of $\frac{dy}{dx}$ at the point $(4, 2)$ is

(A) -0.8

(B) -1.25

(C) 0.8

(D) 1.25

Q39. The precise evaluated value of the indefinite integral $\int \frac{xe^x}{(1+x)^2}\,dx$ is

(A) $\frac{e^x}{(1+x)^2} + C$

(B) $e^x(1+x) + C$

(C) $-\frac{e^x}{1+x} + C$

(D) $\frac{e^x}{1+x} + C$

Q40. The evaluated value of the definite integral $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ is equal to

(A) 1

(B) $\frac{\pi}{8}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Q41. The absolute maximum value of the function $f(x) = \frac{\ln x}{x}$ over its complete domain $x > 0$ is

(A) 1

(B) $\frac{2}{e}$

(C) e

(D) $\frac{1}{e}$

Q42. If a curve is defined parametrically by $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is equal to

(A) $-\frac{1}{2a}$

(B) $\frac{1}{2a}$

(C) $\frac{1}{4a}$

(D) $-\frac{1}{4a}$

Q43. If the complete length of the latus rectum of an ellipse is exactly equal to half the linear length of its minor axis, then the exact eccentricity e of the ellipse is

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{5}}{3}$

Q44. The exact evaluated numerical value of the trigonometric product $\cos(20)\cos(40)\cos(80)$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{8}$

(D) $\frac{\sqrt{3}}{8}$

Q45. The explicit linear equation of the directrix of the parabola $y^2 = 8x$ is

(A) $x + 2 = 0$

(B) $y + 2 = 0$

(C) $x - 2 = 0$

(D) $y - 2 = 0$

Q46. If the two monic quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ (where $a \neq b$) share exactly one common real root, then the exact algebraic value of the sum $(a + b)$ is

(A) 0

(B) 1

(C) -2

(D) -1

Q47. The value of the limit $\lim_{x\to 0} \frac{\ln(1+4x^2)}{x\sin(3x)}$ is

(A) $\frac{4}{3}$

(B) $\frac{1}{3}$

(C) $\frac{3}{4}$

(D) $\frac{8}{3}$

Q48. For the system of linear equations $x + y + z = 4$, $2x + 3y + 4z = 9$, and $x + 2y + \lambda z = 5$ to have infinitely many solutions, the precise value of λ must be

(A) 4

(B) 3

(C) 2

(D) 1

Q49. Consider the standard Linear Programming Problem: Maximize the objective function $Z = 3x + 5y$ subject to the structural constraints $x + y \leq 4$, $x \geq 0$, and $y \geq 0$. The exact absolute maximum value of Z is

(A) 16

(B) 12

(C) 24

(D) 20

Q50. The complete geometric locus of the complex number z that satisfies the modulus equation $|z - 2i| = |z + 2|$ is

(A) $x + 2y = 0$

(B) $x - y = 0$

(C) $x + y = 0$

(D) $2x + y = 0$

Detailed Solutions

Q1.

Solution

Concept:
We use the Binomial Probability Distribution. The probability of obtaining exactly r successes in n independent Bernoulli trials, each having a constant success probability p and failure probability $q = 1 - p$, is given by the analytical formula $P(X = r) = \binom{n}{r} p^r q^{n-r}$.

Solution:

(a) Identify the defining parameters of the binomial distribution from the text: total number of trials is $n = 6$, probability of success (getting a head) is $p = \frac{1}{2}$, and probability of failure is $q = 1 - p = \frac{1}{2}$.

(b) We wish to find the probability of obtaining exactly $r = 4$ successes.

(c) Set up the binomial formula: $P(X = 4) = \binom{6}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4}$.

(d) Apply the combinations symmetry identity $\binom{n}{r} = \binom{n}{n-r}$: we have $\binom{6}{4} = \binom{6}{2} = \frac{6\times5}{2\times1} = 15$.

(e) Combine the base exponential fractions: $\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$.

(f) Multiply the binomial coefficient by the combined exponential fraction: $P(X = 4) = 15 \times \frac{1}{64} = \frac{15}{64}$.

Final Answer:

$$\boxed{\frac{15}{64}}$$

Answer: (D)

Go Back to Question 1

Q2.

Solution

Concept:

The summation of an infinite geometric series with first term a and constant common ratio r (where the convergence condition $|r| < 1$ is satisfied) is given by the exact analytical formula $S_\infty = \frac{a}{1-r}$.

Solution:

(a) Examine the given infinite series: $S_\infty = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots \infty$.

(b) Identify the first structural term of the series: $a = 1$.

(c) Compute the constant common ratio r by dividing the second term by the first term: $r = \frac{-1/2}{1} = -\frac{1}{2}$.

(d) Verify the absolute convergence criterion: $|r| = \left|-\frac{1}{2}\right| = \frac{1}{2} < 1$, so the series converges perfectly.

(e) Substitute our extracted values $a = 1$ and $r = -\frac{1}{2}$ into the infinite sum formula: $S_\infty = \frac{1}{1-(-1/2)}$.

(f) Execute the internal denominator addition and inversion: $S_\infty = \frac{1}{1+1/2} = \frac{1}{3/2} = \frac{2}{3}$.

Final Answer:

$$\boxed{\frac{2}{3}}$$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept:

For an ellipse with semi-major axis a and semi-minor axis b, the major axis length is $2a$ and the minor axis length is $2b$. The mathematical flattening is quantified by its eccentricity e, computed using the exact standard relation $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:

(a) Set up the specific mathematical relationship between the axes as stated in the text: $2a = 3(2b)$.

(b) Divide both sides by 2 to simplify the proportion: $a = 3b$.

(c) Rearrange to isolate the fractional ratio of the semi-axes: $\frac{b}{a} = \frac{1}{3}$.

(d) Square this fractional ratio to prepare for the eccentricity formula: $\frac{b^2}{a^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$.

(e) Substitute this exact squared value into the fundamental eccentricity equation: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{9}}$.

(f) Perform the internal fraction subtraction: $e = \sqrt{\frac{9-1}{9}} = \sqrt{\frac{8}{9}}$.

(g) Simplify the radical expression: $e = \frac{\sqrt{8}}{\sqrt{9}} = \frac{2\sqrt{2}}{3}$.

Final Answer:

$$\boxed{\frac{2\sqrt{2}}{3}}$$

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept:
In related rates problems, we link the time derivatives of dependent geometric quantities using their structural formulas. The total surface area S of a sphere is related to its radius r by $S = 4\pi r^2$. Differentiating with respect to time t yields $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$.

Solution:

(a) Write down the governing geometric formula for the surface area of a sphere: $S = 4\pi r^2$.

(b) Differentiate both sides implicitly with respect to time t using the chain rule: $\frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt} = 8\pi r \frac{dr}{dt}$.

(c) Substitute the known time rate of change of surface area, $\frac{dS}{dt} = 8\pi$, into the differentiated relation: $8\pi = 8\pi r \frac{dr}{dt}$.

(d) Divide both sides by 8π to isolate the product: $1 = r\frac{dr}{dt} \implies \frac{dr}{dt} = \frac{1}{r}$.

(e) Substitute the instantaneous operational radius $r = 4$ cm into the required rate expression: $\frac{dr}{dt} = \frac{1}{4}$ cm/s = 0.25 cm/s.

Final Answer:

$$\boxed{0.25 \text{ cm/s}}$$

Answer: (D)

Go Back to Question 4

Q5.

Solution

Concept:

A parabola of the standard analytical form $x^2 = -4ay$ opens downward along the negative vertical y-axis. Its vertex is located exactly at the origin $(0, 0)$, its internal focus is situated at $(0, -a)$, and its horizontal directrix line has the linear equation $y = a$.

Solution:

(a) Examine the given parabolic equation: $x^2 = -16y$.

(b) Compare it directly with the standard downward-opening model: $x^2 = -4ay$.

(c) Equate the focal linear coefficients on the right-hand side: $-4a = -16$.

(d) Divide by -4 to find the positive focal parameter: $a = \frac{-16}{-4} = 4$.

(e) Using the structural map for an $x^2 = -4ay$ parabola, the focal coordinate is located exactly at $(0, -a)$.

(f) Substitute our derived parameter $a = 4$ into the focal tuple: we obtain $(0, -4)$.

Final Answer:

$$\boxed{(0, -4)}$$

Answer: (D)

Go Back to Question 5

Q6.

Solution

Concept:

The normal to a smooth curve at a given contact point is the straight line perpendicular to the tangent line. If the tangent line has slope $m_t = \frac{dy}{dx}\Big|_{(x_0,y_0)}$, then the normal line has slope $m_n = -\frac{1}{m_t}$. We construct the line using the point-slope formula.

Solution:

(a) Determine the y-coordinate of the contact point by substituting $x = e$ into the curve's function: $y = e\ln(e) = e(1) = e$. Thus, the contact point is (e, e).

(b) Differentiate the curve function $y = x\ln x$ with respect to x using the product rule: $\frac{dy}{dx} = \frac{d}{dx}(x) \cdot \ln x + x \cdot \frac{d}{dx}(\ln x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$.

(c) Evaluate the slope of the tangent line at $x = e$: $m_t = \ln(e) + 1 = 1 + 1 = 2$.

(d) Compute the perpendicular slope for the normal line: $m_n = -\frac{1}{m_t} = -\frac{1}{2}$.

(e) Formulate the exact equation of the normal line using the point-slope form $y - y_0 = m_n(x - x_0)$: $y - e = -\frac{1}{2}(x - e)$.

(f) Multiply the entire equation by 2 and rearrange into standard linear form: $2y - 2e = -x + e \implies x + 2y = 3e$.

Final Answer:

$$\boxed{x + 2y = 3e}$$

Answer: (D)

Go Back to Question 6

Q7.

Solution

Concept:

A square matrix A with real entries is classified as an orthogonal matrix if its transpose is equal to its inverse. The formal mathematical definition states that $A \cdot A^T = A^T \cdot A = I$, where I represents the standard identity matrix of the same operational dimensions.

Solution:

(a) Start with the primary defining relationship for an orthogonal matrix as given: $A \cdot A^T = I$.

(b) Since A is non-singular (its determinant is clearly ± 1), we can multiply both sides of the equation from the left by its formal inverse matrix A^{-1}.

(c) Execute the multiplication: $A^{-1} \cdot (A \cdot A^T) = A^{-1} \cdot I$.

(d) Apply the matrix associative property on the left side: $(A^{-1} \cdot A) \cdot A^T = A^{-1}$.

(e) Substitute the fundamental inverse definition $A^{-1} \cdot A = I$: we get $I \cdot A^T = A^{-1}$.

(f) Since multiplying any matrix by the identity matrix leaves it unchanged, we conclude $A^T = A^{-1}$.

Final Answer:

$$\boxed{A^T}$$

Answer: (D)

Go Back to Question 7

Q8.

Solution

Concept:
Three distinct lines in a plane are classified as concurrent if all three pass through one single unique common point of intersection. To find an unknown parameter in one line, we determine the intersection point of the two fully known lines and substitute those coordinates into the third line.
Solution:

(a) First, find the common point of intersection of the two fully known lines: $2x + y - 3 = 0$ — (Equation 1) and $3x - y - 2 = 0$ — (Equation 2).

(b) Add Equation 1 and Equation 2 directly to eliminate the variable y: $(2x+y-3)+(3x-y-2) = 0 \implies 5x - 5 = 0 \implies x = 1$.

(c) Substitute $x = 1$ back into Equation 1 to solve for y: $2(1) + y - 3 = 0 \implies y - 1 = 0 \implies y = 1$.

(d) Thus, the unique common point of concurrence is $(1, 1)$.

(e) Since all three lines share this common point, substitute $(1, 1)$ directly into the third linear relation $5x + ky - 3 = 0$: $5(1) + k(1) - 3 = 0$.

(f) Simplify the resulting arithmetic relation: $5 + k - 3 = 0 \implies k + 2 = 0 \implies k = -2$.

Final Answer:

$$\boxed{-2}$$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept:

The general inclusion-exclusion principle for the union of two events states that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If the two events are stochastically independent, their intersection probability is exactly equal to the product of their individual probabilities, $P(A \cap B) = P(A) \cdot P(B)$.

Solution:

(a) Start with the primary general inclusion-exclusion addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) Apply the specific mathematical condition that events A and B are independent to evaluate the intersection: $P(A \cap B) = P(A) \cdot P(B)$.

(c) Substitute the known individual probabilities $P(A) = 0.3$ and $P(B) = 0.4$ into the intersection product: $P(A \cap B) = 0.3 \times 0.4 = 0.12$.

(d) Substitute this calculated intersection value back into the full addition identity: $P(A \cup B) = 0.3 + 0.4 - 0.12$.

(e) Perform the simple decimal arithmetic: $P(A \cup B) = 0.7 - 0.12 = 0.58$.

Final Answer:

$$\boxed{0.58}$$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept:
When a first-order differential equation is presented as an explicit rational proportion $\frac{dy}{dx} = \frac{f(x)}{g(y)}$, cross-multiplication immediately separates the variables into $g(y)dy = f(x)dx$. Integrating both sides produces the implicit analytical invariant representing the family of curves.

Solution:

(a) Start with the given rational differential equation: $\frac{dy}{dx} = \frac{2x}{3y^2}$.

(b) Execute cross-multiplication to separate the differentials: $3y^2\,dy = 2x\,dx$.

(c) Integrate both sides independently: $\int 3y^2\,dy = \int 2x\,dx$.

(d) Evaluate the polynomial integrals using the standard power rule: $3\left(\frac{y^3}{3}\right) = 2\left(\frac{x^2}{2}\right) + C$.

(e) Simplify the scalar coefficients on both sides: $y^3 = x^2 + C$.

(f) Rearrange all variable terms onto the left side to establish the standard implicit form: $y^3 - x^2 = C$.

Final Answer:

$$\boxed{y^3 - x^2 = C}$$

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept:
The order of a differential equation is defined as the order of the highest derivative appearing in it. The degree is defined as the integer power or exponent of the highest order derivative, provided the differential equation is expressed as a clean polynomial in all its derivatives.

Solution:

(a) Examine the given structural differential equation: $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 - 5y = e^x$.

(b) Identify all derivative terms present in the relation: we have a second-order derivative $\frac{d^2y}{dx^2}$ and a first-order derivative $\frac{dy}{dx}$.

(c) Determine the order: the highest order derivative present is the second derivative $\frac{d^2y}{dx^2}$, so the operational order of the equation is exactly 2.

(d) Verify whether the equation is a valid polynomial in terms of its derivatives: yes, there are no transcendental functions, fractional powers, or nested operations acting on any derivative terms.

(e) Identify the degree: the operational exponent raised to the highest order derivative $\frac{d^2y}{dx^2}$ is exactly 3.

Final Answer:

$$\boxed{3}$$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:
We use the fundamental Principles of Combinatorics. When forming subgroups under strict sub-category quantitative constraints, we compute the number of selections for each independent sub-category using combinations $\binom{n}{r}$, and multiply them together using the fundamental multiplication principle.
Solution:

(a) The target committee must consist of exactly 4 members total, subject to the strict constraint that it contains exactly 2 women.

(b) Consequently, the remaining committee positions must be filled by men: Men $= 4 - 2 = 2$ men.

(c) Determine the number of independent ways to select exactly 2 women from the available pool of 4 women: $\binom{4}{2} = \frac{4\times3}{2\times1} = 6$ ways.

(d) Determine the number of independent ways to select exactly 2 men from the available pool of 6 men: $\binom{6}{2} = \frac{6\times5}{2\times1} = 15$ ways.

(e) Apply the fundamental combinatorial multiplication principle: Total Ways $= \binom{4}{2} \times \binom{6}{2} = 6 \times 15 = 90$.

Final Answer:

$$\boxed{90}$$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept:

A function is differentiable at $x = a$ if the left-hand derivative (LHD) equals the right-hand derivative (RHD). The absolute value function $|x - a|$ has a sharp corner or corner point exactly at $x = a$, where its directional slopes jump abruptly from -1 to $+1$.

Solution:

(a) We test each function independently for its left and right directional derivatives at the critical threshold $x = 2$.

(b) Consider Option (A), $f(x) = |x - 2|$. The directional derivative from the right is RHD $= \lim_{h\to 0^+} \frac{|2+h-2|-0}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$.

(c) The directional derivative from the left is LHD $= \lim_{h\to 0^-} \frac{|2+h-2|-0}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1$. Since LHD $\neq$ RHD, $f(x)$ is not differentiable at $x = 2$.

(d) Consider Option (B), $g(x) = (x-2)|x-2|$. Here, $g'(2) = \lim_{h\to 0} \frac{h|h|-0}{h} = \lim_{h\to 0} |h| = 0$, so it is perfectly differentiable.

(e) Consider Option (C), $h(x) = \sin((x-2)^2)$. Using the chain rule, $h'(x) = 2(x-2)\cos((x-2)^2)$, which evaluates smoothly to 0 at $x = 2$.

(f) Consider Option (D), $k(x) = (x-2)^3$. Its derivative is $k'(x) = 3(x-2)^2$, which also exists and equals 0 at $x = 2$.

Final Answer:

$$\boxed{|x-2|}$$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept:

A differentiable polynomial function $f(x)$ is classified as strictly decreasing on any continuous interval where its first derivative is strictly negative, $f'(x) < 0$. We compute the derivative, find its critical roots, and analyze the sign intervals using the wavy curve method.

Solution:

(a) Start with the given polynomial function: $f(x) = 2x^3 - 15x^2 + 36x - 12$.

(b) Find the first derivative with respect to x: $f'(x) = 6x^2 - 30x + 36$.

(c) Factor out the common scalar coefficient of 6: $f'(x) = 6(x^2 - 5x + 6)$.

(d) Factor the quadratic polynomial into linear root components: $f'(x) = 6(x - 2)(x - 3)$.

(e) Set up the structural inequality for strictly decreasing behavior: $f'(x) < 0 \implies 6(x - 2)(x - 3) < 0$.

(f) The roots of the derivative are $x = 2$ and $x = 3$. The quadratic expression $(x - 2)(x - 3)$ is strictly negative strictly between its roots, yielding the open interval $(2, 3)$.

Final Answer:

$$\boxed{(2, 3)}$$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept:

The geometric area enclosed between a continuous curve $y = f(x)$ and the x-axis from $x = a$ to $x = b$ is computed using the definite integral $A = \int_a^b |f(x)|\, dx$. For a downward-opening parabola above the axis, the integration boundaries are its x-intercepts.

Solution:

(a) Find the intersection points of the parabola $y = 6x - x^2$ with the x-axis by setting $y = 0$: $6x - x^2 = 0 \implies x(6 - x) = 0$. This yields the two boundary roots $x = 0$ and $x = 6$.

(b) Determine the sign of the curve inside the interval $[0, 6]$: by testing $x = 3$, $y = 6(3) - 3^2 = 18 - 9 = 9 > 0$. The curve lies entirely above the horizontal axis.

(c) Set up the precise area integral: $A = \int_0^6 (6x - x^2)\, dx$.

(d) Execute the anti-differentiation term by term using the standard power rule: $A = \left[6 \cdot \frac{x^2}{2} - \frac{x^3}{3}\right]_0^6 = \left[3x^2 - \frac{x^3}{3}\right]_0^6$.

(e) Substitute the upper limit $x = 6$ and subtract the lower limit $x = 0$: $A = \left(3(6)^2 - \frac{(6)^3}{3}\right) - (0) = \left(3(36) - \frac{216}{3}\right)$.

(f) Simplify the resulting analytical arithmetic: $A = 108 - 72 = 36$.

Final Answer:

$$\boxed{36}$$

Answer: (D)

Go Back to Question 15

Q16.

Solution

Concept:

To evaluate high integer powers of a complex number, we convert the complex number into its polar exponential form $re^{i\theta}$. We use De Moivre's Theorem or fundamental properties of Euler's formula $(e^{i\theta})^n = e^{in\theta}$, combined with the 2π-periodicity of the complex exponential.

Solution:

(a) Consider the complex numerator $w = 1 + i\sqrt{3}$. We compute its magnitude and principal argument: $|w| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$, and $\arg(w) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$. Thus, $w = 2e^{i\pi/3}$.

(b) The complex denominator is exactly the formal conjugate $\bar{w} = 1 - i\sqrt{3}$. In exponential form, $\bar{w} = 2e^{-i\pi/3}$.

(c) Formulate the fraction $z = \frac{w}{\bar{w}}$ using our polar exponential tuples: $z = \frac{2e^{i\pi/3}}{2e^{-i\pi/3}}$.

(d) Cancel the scalar magnitudes and combine the exponents: $z = e^{i(\pi/3-(-\pi/3))} = e^{i2\pi/3}$.

(e) Raise this simplified exponential form to the required 12th power: $z^{12} = \left(e^{i2\pi/3}\right)^{12} = e^{i\left(\frac{2\pi}{3}\times 12\right)}$.

(f) Perform the internal exponent multiplication: $z^{12} = e^{i8\pi}$.

(g) Expand using Euler's identity $e^{i\theta} = \cos\theta + i\sin\theta$: $z^{12} = \cos(8\pi) + i\sin(8\pi)$. Since 8π is an exact even multiple of 2π, this evaluates to $1 + i(0) = 1$.

Final Answer:

$$\boxed{1}$$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:
We use elementary column (or row) transformations, which do not alter the final numerical value of a determinant. A foundational property states that if any two rows or columns of a determinant are perfectly identical or proportional, the entire determinant evaluates identically to zero.

Solution:

(a) Let the given determinant be $D = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$.

(b) Apply the elementary column addition operation to the third column: $C_3 \leftarrow C_3 + C_2$.

(c) Execute the term additions row by row: the elements of the third column become $(b+c)+a = a+b+c$, $(c+a)+b = a+b+c$, and $(a+b)+c = a+b+c$.

(d) The rewritten determinant is: $D = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$.

(e) Factor out the common algebraic scalar $(a+b+c)$ entirely from the third column: $D = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$.

(f) Observe that the first column C_1 and the third column C_3 are now perfectly identical (both consist of 1, 1, 1).

(g) By the fundamental proportional column property, the internal determinant equals 0. Thus, $D = (a+b+c) \times 0 = 0$.

Final Answer:

$$\boxed{0}$$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:

Function composition involves substituting the entire analytical output of a function back into its own input argument. To evaluate $f(f(x))$, we replace every instance of the independent variable x in the definition of f with its entire functional expression $f(x)$.

Solution:

(a) Start with the primary rational definition of the function: $f(x) = \frac{x}{x-1}$.

(b) Set up the formal nested composition expression: $f(f(x)) = f\left(\frac{x}{x-1}\right)$.

(c) Substitute the rational fraction into the analytical structure of f: $f(f(x)) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$.

(d) Combine the terms in the denominator under their identical common denominator: $\frac{x}{x-1} - 1 = \frac{x-(x-1)}{x-1} = \frac{x-x+1}{x-1} = \frac{1}{x-1}$.

(e) Substitute this combined denominator back into the main fraction: $f(f(x)) = \frac{\frac{x}{x-1}}{\frac{1}{x-1}}$.

(f) Cancel the identical fractional denominators $(x-1)$ from the primary numerator and denominator: $f(f(x)) = \frac{x}{1} = x$. Thus, f is an involution.

Final Answer:

$$\boxed{x}$$

Answer: (D)

Go Back to Question 18

Q19.

Solution

Concept:

Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ in 3D Euclidean space are defined to be coplanar (lying in the same geometric plane) if and only if their scalar triple product evaluates exactly to zero. The scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ is computed directly as the determinant of their components.

Solution:

(a) Start with the primary defining mathematical condition for coplanar vectors: $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

(b) Write out the explicit scalar triple product determinant using the provided vector components:
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \mu & 5 \end{vmatrix} = 0.$$

(c) Expand the 3×3 determinant along its top first row: $2\cdot\begin{vmatrix} 2 & -3 \\ \mu & 5 \end{vmatrix} - (-1)\cdot\begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1\cdot\begin{vmatrix} 1 & 2 \\ 3 & \mu \end{vmatrix} = 0.$

(d) Evaluate each internal 2×2 minor difference: $2(10 - (-3\mu)) + 1(5 - (-9)) + 1(\mu - 6) = 0.$

(e) Expand the bracketed polynomial expressions: $2(10 + 3\mu) + 1(14) + (\mu - 6) = 0 \implies 20 + 6\mu + 14 + \mu - 6 = 0.$

(f) Combine like linear coefficients together: $7\mu + 28 = 0 \implies 7\mu = -28$. Dividing by 7 yields $\mu = -4$.

Final Answer:

$$\boxed{-4}$$

Answer: (D)

Go Back to Question 19

Q20.

Solution

Concept:
To find the condition of tangency between a line and a conic section, we substitute the line's linear equation into the conic's quadratic equation to create a unified single-variable quadratic equation. For tangency, the discriminant of this resulting equation must evaluate exactly to zero ($\Delta = 0$).

Solution:

(a) The given equations are the line $y = mx + c$ and the parabola $y^2 = 4ax$.

(b) Substitute the linear expression for y directly into the parabola equation: $(mx + c)^2 = 4ax$.

(c) Expand the binomial square on the left side: $m^2x^2 + 2mcx + c^2 = 4ax$.

(d) Group all terms together to form a standard quadratic equation in terms of x: $m^2x^2 + 2(mc - 2a)x + c^2 = 0$.

(e) For the line to touch the parabola at exactly one unique point (tangency condition), the discriminant $\Delta = B^2 - 4AC$ must equal 0.

(f) Identify the quadratic coefficients: $A = m^2$, $B = 2(mc - 2a)$, and $C = c^2$.

(g) Set up the discriminant equation: $\Delta = [2(mc - 2a)]^2 - 4(m^2)(c^2) = 0 \implies 4(m^2c^2 - 4amc + 4a^2) - 4m^2c^2 = 0$.

(h) Expand and cancel the common $4m^2c^2$ term: $16a^2 - 16amc = 0 \implies 16a(a - mc) = 0$.

(i) Since the focal parameter $a \neq 0$, we must have $a - mc = 0 \implies c = \frac{a}{m}$.

Final Answer:

$$\boxed{c = \frac{a}{m}}$$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:

For any square matrix A of order n, its adjoint matrix $\text{adj}(A)$ satisfies the foundational matrix identity $A \cdot \text{adj}(A) = \det(A) \cdot I_n$. Taking the determinant of both sides and applying the scaling property $\det(kI_n) = k^n$ yields the analytical identity $\det(\text{adj}(A)) = (\det(A))^{n-1}$.

Solution:

(a) Write down the primary structural formula for the determinant of an adjoint matrix: $\det(\text{adj}(A)) = (\det(A))^{n-1}$.

(b) Identify the specific values given in the problem statement: the operational order of the matrix is $n = 3$, and its primary determinant is $\det(A) = 5$.

(c) Substitute these known values into the structural identity: $\det(\text{adj}(A)) = (5)^{3-1}$.

(d) Simplify the operational exponent: $\det(\text{adj}(A)) = (5)^2$.

(e) Execute the scalar square arithmetic: $(5)^2 = 25$.

Final Answer:

$$\boxed{25}$$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:

The expanded general equation of a circle is written as $x^2 + y^2 + 2gx + 2fy + c = 0$. By completing the square or matching structural parameters, its center coordinates are derived as $(-g, -f)$ and its total linear radius is calculated using the formula $r = \sqrt{g^2 + f^2 - c}$.

Solution:

(a) Compare the given circle equation $x^2 + y^2 - 8x + 10y + 16 = 0$ with the general conic form $x^2 + y^2 + 2gx + 2fy + c = 0$.

(b) Extract the coefficient relationships: $2g = -8 \implies g = -4$, and $2f = 10 \implies f = 5$. The constant term is $c = 16$.

(c) Determine the center coordinates $(-g, -f)$: substituting our derived parameters yields $(-(-4), -(5)) = (4, -5)$.

(d) Compute the exact linear radius using the radical formula $r = \sqrt{g^2 + f^2 - c}$: $r = \sqrt{(-4)^2 + (5)^2 - 16}$.

(e) Execute the internal radical arithmetic: $r = \sqrt{16 + 25 - 16} = \sqrt{25} = 5$.

Final Answer:

$$\boxed{(4, -5) \text{ and } 5}$$

Answer: (D)

Go Back to Question 22

Q23.

Solution

Concept:

A function $f(x)$ is defined to be continuous at a point $x = a$ if and only if $\lim_{x\to a} f(x) = f(a)$. For trigonometric limits involving cosine, we use the standard identity $1 - \cos\theta = 2\sin^2(\theta/2)$ and the limit property $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$.

Solution:

(a) Since the function $f(x)$ is continuous at $x = 0$, its limit as x approaches 0 must equal its functional value at $x = 0$, which is $f(0) = 18$.

(b) Evaluate the left-hand and right-hand limit simultaneously: $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos(kx)}{x^2}$.

(c) Apply the half-angle trigonometric identity $1 - \cos(kx) = 2\sin^2\left(\frac{kx}{2}\right)$: $\lim_{x\to 0} \frac{2\sin^2\left(\frac{kx}{2}\right)}{x^2}$.

(d) Algebraically manipulate the expression to match the standard sine limit: $= 2\lim_{x\to 0}\left[\frac{\sin(kx/2)}{x}\right]^2 = 2\lim_{x\to 0}\left[\frac{\sin(kx/2)}{kx/2}\cdot\frac{k}{2}\right]^2$.

(e) Since $\lim_{x\to 0} \frac{\sin(kx/2)}{kx/2} = 1$, the limit simplifies to: $= 2\cdot\left(\frac{k}{2}\right)^2 = 2\cdot\frac{k^2}{4} = \frac{k^2}{2}$.

(f) Equate the evaluated limit to the functional value $f(0)$: $\frac{k^2}{2} = 18 \implies k^2 = 36$. Since $k > 0$, we take the principal root to get $k = 6$.

Final Answer:

$$\boxed{6}$$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept:

The intercept form of the equation of a 3D plane making non-zero intercepts a, b, and c on the Cartesian x, y, and z axes respectively is given by the fractional summation formula $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. By clearing the denominators, we convert this into standard integer Cartesian form.

Solution:

(a) Write down the primary intercept formula for a 3D plane: $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(b) Identify the specific axis intercept values from the problem prompt: we have x-intercept $a = 2$, y-intercept $b = 3$, and z-intercept $c = 4$.

(c) Substitute these geometric values into the structural identity: $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$.

(d) Determine the least common multiple (LCM) of the three fractional denominators $(2, 3, 4)$: the LCM is exactly 12.

(e) Multiply the entire equation by 12 to clear all fractional coefficients: $12\left(\frac{x}{2}\right) + 12\left(\frac{y}{3}\right) + 12\left(\frac{z}{4}\right) = 12(1)$.

(f) Execute the basic term simplifications to establish the final linear relation: $6x + 4y + 3z = 12$.

Final Answer:

$$\boxed{6x + 4y + 3z = 12}$$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept:

To solve a separable first-order differential equation $\frac{dy}{dx} = f(x)g(y)$, we isolate all terms containing y (including dy) on one side and all terms containing x (including dx) on the other. We then integrate both sides independently and apply the initial condition to determine the integration constant.

Solution:

(a) Start with the given first-order differential equation: $\frac{dy}{dx} = 3x^2y$.

(b) Separate the continuous variables by dividing by y and multiplying by dx: $\frac{1}{y}\,dy = 3x^2\,dx$.

(c) Integrate both sides of the separated equation simultaneously: $\int \frac{1}{y}\,dy = \int 3x^2\,dx$.

(d) Evaluate the standard anti-derivatives: $\ln|y| = 3 \cdot \frac{x^3}{3} + C \implies \ln|y| = x^3 + C$.

(e) Convert from logarithmic to explicit exponential form: $|y| = e^{x^3+C} = e^C \cdot e^{x^3}$. Since e^C is an arbitrary constant, we write $y = Ae^{x^3}$.

(f) Apply the specific initial condition that the curve passes through $(0, 1)$: substitute $x = 0$ and $y = 1$ to get $1 = Ae^{0^3} \implies 1 = A(1) \implies A = 1$.

(g) Substitute $A = 1$ back to establish the final explicit curve trajectory: $y = e^{x^3}$.

Final Answer:

$$\boxed{y = e^{x^3}}$$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:

In descriptive statistics, measures of dispersion such as standard deviation and variance are completely unaffected by shifts in origin (adding or subtracting a uniform constant b). However, they are directly affected by changes in scale. If $Y = aX + b$, the standard deviation scales as $\sigma_Y = |a|\sigma_X$.

Solution:

(a) Let the original random variable representing the statistical dataset be denoted as X, with original standard deviation σ_X.

(b) Formulate the linear transformation applied to create the new dataset: $Y = 3X + 5$.

(c) Apply the fundamental statistical shift-and-scale theorem for standard deviations: $\sigma_Y = |a|\sigma_X$.

(d) Identify our specific linear transformation coefficients: the scaling factor is $a = 3$, and the additive shift is $b = 5$.

(e) Substitute $a = 3$ into the standard deviation identity: $\sigma_Y = |3|\sigma_X = 3\sigma_X$.

(f) Divide by σ_X to establish the exact required scaling ratio: $\frac{\sigma_Y}{\sigma_X} = 3$.

Final Answer:

$$\boxed{3}$$

Answer: (D)

Go Back to Question 26

Q27.

Solution

Concept:
A square matrix is classified as singular if and only if its complete mathematical determinant evaluates exactly to zero ($\det(A) = 0$). For a 2×2 matrix, the determinant is calculated simply by subtracting the product of its secondary diagonal from the product of its primary diagonal.

Solution:

(a) Start with the defining mathematical condition for a singular matrix: $\det(A) = 0$.

(b) Write out the explicit determinant for the provided 2×2 matrix: $\det(A) = \begin{vmatrix} 2 & \lambda \\ 4 & 6 \end{vmatrix}$.

(c) Apply the primary expansion rule $\det = ad - bc$: $\det(A) = (2 \times 6) - (4 \times \lambda) = 12 - 4\lambda$.

(d) Equate the expanded polynomial expression to zero: $12 - 4\lambda = 0$.

(e) Add 4λ to both sides and divide by 4 to isolate the unknown variable: $4\lambda = 12 \implies \lambda = \frac{12}{4} = 3$.

Final Answer:

$$\boxed{3}$$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept:
The general term T_{r+1} in the binomial expansion of $(A+B)^n$ is given by the analytical formula $T_{r+1} = \binom{n}{r} A^{n-r} B^r$. To find the term independent of x, we collect all exponents of x into a unified single expression and equate the resulting net exponent to zero.

Solution:

(a) Write down the structural general term formula for the specific expansion $\left(2x^2 - \frac{1}{x}\right)^9$: $T_{r+1} = \binom{9}{r}\left(2x^2\right)^{9-r}\left(-\frac{1}{x}\right)^r$.

(b) Separate the scalar numeric coefficients from the algebraic variable exponents: $T_{r+1} = \binom{9}{r}(2)^{9-r}(-1)^r \cdot \left(x^2\right)^{9-r}\left(x^{-1}\right)^r$.

(c) Expand and unify the exponents of x: $\left(x^2\right)^{9-r}\left(x^{-1}\right)^r = x^{18-2r} \cdot x^{-r} = x^{18-3r}$.

(d) Set up the necessary operational condition for a term independent of x: $18 - 3r = 0 \implies 3r = 18 \implies r = 6$.

(e) Substitute $r = 6$ back into our isolated scalar coefficient expression to calculate the exact numerical value: $T_7 = \binom{9}{6}(2)^{9-6}(-1)^6$.

(f) Apply the combinations symmetry identity $\binom{9}{6} = \binom{9}{3} = \frac{9\times8\times7}{3\times2\times1} = 84$.

(g) Multiply the components: $T_7 = 84 \times (2)^3 \times (+1) = 84 \times 8 = 672$.

Final Answer:

$$\boxed{672}$$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept:

When a straight line cuts the positive (or negative) Cartesian coordinate axes, it forms a right-angled triangle with the origin. If the line is written in its standard intercept form $\frac{x}{a} + \frac{y}{b} = 1$, the lengths of its base and height are $|a|$ and $|b|$, and its area is $\frac{1}{2}|ab|$.

Solution:

(a) Examine the given linear equation for the straight line: $3x + 4y = 12$.

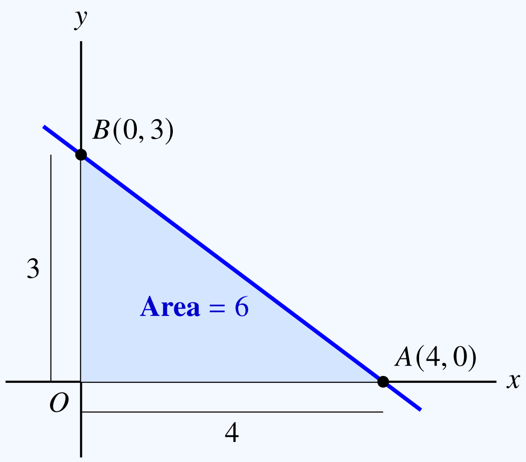
(b) Determine the x-axis intercept a by setting $y = 0$: $3x = 12 \implies x = 4$. Thus, the base vertex is $A(4, 0)$.

(c) Determine the y-axis intercept b by setting $x = 0$: $4y = 12 \implies y = 3$. Thus, the vertical vertex is $B(0, 3)$.

(d) Observe that the triangle OAB is exactly a right-angled triangle situated in the first quadrant with the right angle at the origin $O(0, 0)$.

(e) The perpendicular base length is Base = 4 and the perpendicular height is Height = 3.

(f) Calculate the final triangular area: Area $= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 4 \times 3 = 6$.



Final Answer:

$$\boxed{6}$$

Answer: (C)

Go Back to Question 29

Q30.

Solution

Concept:

A mathematical relation R on a set A is classified as an Equivalence Relation if and only if it satisfies all three foundational structural properties: Reflexivity (xRx), Symmetry ($xRy \implies yRx$), and Transitivity (xRy and $yRz \implies xRz$).

Solution:

(a) We test the given relation $xRy \iff (x - y) = 5k$ (for some integer k) across the three foundational properties.

(b) Test Reflexivity: for any arbitrary integer $x \in \mathbb{Z}$, we compute the difference $x - x = 0$. Since $0 = 5 \times 0$, the difference is divisible by 5. Thus, xRx holds universally.

(c) Test Symmetry: assume xRy holds, meaning $x - y = 5k$ for some integer k. Multiplying by -1 gives $y - x = 5(-k)$. Since $(-k)$ is an integer, yRx perfectly holds.

(d) Test Transitivity: assume xRy and yRz hold, meaning $x - y = 5k$ and $y - z = 5m$ for integers k, m. Summing these two equations yields $(x-y)+(y-z) = 5k+5m \implies x-z = 5(k+m)$. Since $(k + m)$ is an integer, xRz perfectly holds.

(e) Because the relation is simultaneously reflexive, symmetric, and transitive, it is formally an equivalence relation.

Final Answer:

$$\boxed{R \text{ is an equivalence relation}}$$

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept:

The normal vector to a given 3D plane $Ax+By+Cz=D$ has directional components $\vec{n}=(A,B,C)$. The straight line passing through the origin and running perpendicular to this plane is given by the parametric equations $(x,y,z)=(Ak,Bk,Ck)$. Substituting this into the plane solves for k.

Solution:

(a) Extract the normal vector components directly from the plane's linear equation $2x+3y+6z=49$: $\vec{n}=(2,3,6)$.

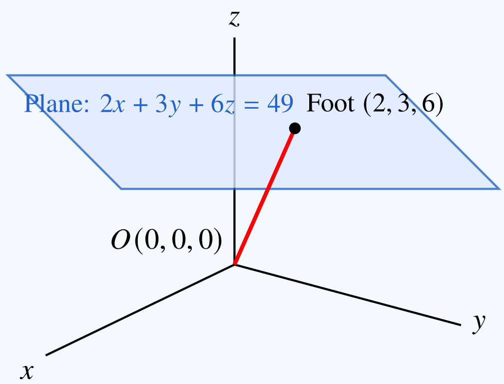
(b) Formulate the parametric equations of the straight line passing through the origin $(0,0,0)$ in the direction of $\vec{n}$: $x=2k$, $y=3k$, and $z=6k$.

(c) Since the foot of the perpendicular must lie exactly on the plane boundary, substitute these parametric expressions into the plane equation: $2(2k)+3(3k)+6(6k)=49$.

(d) Expand and sum the squared directional coefficients: $4k+9k+36k=49 \implies 49k=49$.

(e) Divide by 49 to find the exact scaling parameter: $k=1$.

(f) Substitute $k=1$ back into our parametric line relationships to compute the final coordinates: $(2(1),3(1),6(1))=(2,3,6)$.



Final Answer:

$$\boxed{(2,3,6)}$$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept:

The scalar projection of a vector $\vec{u}$ along another vector $\vec{v}$ represents the exact geometric shadow or component length of $\vec{u}$ in the direction of $\vec{v}$. It is calculated using the dot product formula Projection $= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$.

Solution:

(a) Write down the defining structural formula for scalar projection: Projection $= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$.

(b) Compute the analytical dot product $(\vec{u}\cdot\vec{v})$ by multiplying corresponding orthogonal components: $\vec{u}\cdot\vec{v} = (4)(3) + (2)(6) + (1)(-2)$.

(c) Execute the basic term arithmetic: $\vec{u}\cdot\vec{v} = 12 + 12 - 2 = 22$.

(d) Calculate the total Euclidean magnitude of the directional base vector $\vec{v}$: $|\vec{v}| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4}$.

(e) Perform the internal sum and extract the square root: $|\vec{v}| = \sqrt{49} = 7$.

(f) Divide the dot product by the magnitude to establish the final projection: Projection $= \frac{22}{7}$.

Final Answer:

$$\boxed{\frac{22}{7}}$$

Answer: (C)

Go Back to Question 32

Q33.

Solution

Concept:
To determine the length of a chord intercepted by a circle on a given line, we can use the geometric right-triangle chord theorem. If r is the radius of the circle and d is the perpendicular distance from the center to the line, the complete chord length is exactly $2\sqrt{r^2 - d^2}$.
Solution:

(a) Identify the circle's structural properties from $x^2 + y^2 = 25$: the center is situated at the origin $(0, 0)$ and its total linear radius is $r = \sqrt{25} = 5$.

(b) Determine the perpendicular distance d from the center $(0, 0)$ to the vertical straight line $x = 3$: this distance is clearly $d = 3$.

(c) Apply the right-triangle chord length theorem: Length $= 2\sqrt{r^2 - d^2}$.

(d) Substitute our specific geometric values into the radical formula: Length $= 2\sqrt{5^2 - 3^2} = 2\sqrt{25 - 9}$.

(e) Execute the internal radical subtraction and extraction: Length $= 2\sqrt{16} = 2(4) = 8$.

(f) Alternatively, solve by direct simultaneous substitution: substitute $x = 3$ into $x^2 + y^2 = 25 \implies 9 + y^2 = 25 \implies y^2 = 16 \implies y = \pm 4$. The distance between the endpoints $(3, -4)$ and $(3, 4)$ is exactly $|4 - (-4)| = 8$.

Final Answer:

$$\boxed{8}$$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept:

The complete geometric locus of the point of intersection of two mutually perpendicular tangents to any conic section is called its Director Circle. For a standard hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the exact equation of its director circle is given by $x^2 + y^2 = a^2 - b^2$.

Solution:

(a) Identify the specific standard parameters from the hyperbola's equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$: we extract $a^2 = 16$ and $b^2 = 9$.

(b) Write down the defining structural formula for the director circle of a hyperbola: $x^2 + y^2 = a^2 - b^2$.

(c) Substitute our extracted squared semi-axis values into the equation: $x^2 + y^2 = 16 - 9$.

(d) Execute the subtraction: $x^2 + y^2 = 7$.

(e) Compare this locus result with the standard circle equation $x^2 + y^2 = r^2$: we obtain $r^2 = 7$.

(f) Take the positive square root to determine the final linear radius: $r = \sqrt{7}$.

Final Answer:

$$\boxed{\sqrt{7}}$$

Answer: (D)

Go Back to Question 34

Q35.

Solution

Concept:

A first-order differential equation of the structural form $\frac{dy}{dx} + P(x)y = Q(x)$ is classified as a standard linear differential equation. Its integrating factor (I.F.) is computed using the exponential relation I.F. $= e^{\int P(x)\,dx}$. Multiplying by the I.F. makes the left side an exact product derivative.

Solution:

(a) Compare the given differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$ with the standard linear form $\frac{dy}{dx} + P(x)y = Q(x)$.

(b) Extract the coefficient function of y: $P(x) = \cot x$.

(c) Set up the integrating factor formula: I.F. $= e^{\int \cot x\,dx}$.

(d) Execute the anti-differentiation of the cotangent function: $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \ln|\sin x|$.

(e) Substitute this integrated logarithmic result back into the I.F. exponential formula: I.F. $= e^{\ln|\sin x|}$.

(f) Apply the fundamental identity of logarithms $e^{\ln(u)} = u$: we obtain I.F. $= \sin x$.

Final Answer:

$$\boxed{\sin x}$$

Answer: (D)

Go Back to Question 35

Q36.

Solution

Concept:
To evaluate definite integrals involving an absolute value expression $|f(x)|$, we must split the domain of integration at the roots where $f(x) = 0$. The integral is then resolved into separate continuous polynomial pieces with appropriate positive and negative signs.

Solution:

(a) The integrand is $f(x) = |x - 2|$. The internal linear expression changes its algebraic sign exactly at the root $x = 2$.

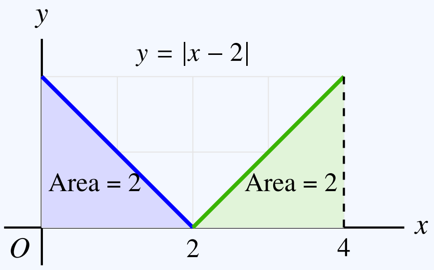
(b) By definition of absolute value: $|x-2| = -(x-2) = 2-x$ for $x \in [0, 2]$, and $|x-2| = x-2$ for $x \in [2, 4]$.

(c) Split the definite integral at the boundary $x = 2$: $I = \int_0^2 (2-x)\,dx + \int_2^4 (x-2)\,dx$.

(d) Evaluate the first integral piece: $\int_0^2 (2-x)\,dx = \left[2x - \frac{x^2}{2}\right]_0^2 = (4-2) - 0 = 2$.

(e) Evaluate the second integral piece: $\int_2^4 (x-2)\,dx = \left[\frac{x^2}{2} - 2x\right]_2^4 = (8-8) - (2-4) = 0 - (-2) = 2$.

(f) Sum the evaluated components to obtain the total net area: $I = 2 + 2 = 4$.



Final Answer:

$$\boxed{4}$$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept:

Conditional probability is computed using the foundational definition $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{|A \cap B|}{|A|}$, where $|A|$ represents the total number of favorable outcomes in the reduced condition space, and $|A \cap B|$ represents the subset of those outcomes satisfying the target event.

Solution:

(a) Let A be the primary conditional event that the sum of the two dice equals 8. We enumerate all ordered pairs in the 36-element sample space satisfying this sum.

(b) The outcomes in event A are: $A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$. Count the total elements: $|A| = 5$.

(c) Let B be the target event that at least one of the rolled dice shows the specific face value of $B = 3$.

(d) Determine the intersection $(A \cap B)$, which consists of outcomes in A that contain at least one 3: $A \cap B = \{(3,5), (5,3)\}$. Count the elements: $|A \cap B| = 2$.

(e) Apply the conditional probability formula: $P(B|A) = \frac{|A \cap B|}{|A|}$.

(f) Substitute our exact enumerated counts to obtain the final fraction: $P(B|A) = \frac{2}{5}$.

Conditional Sample Space: Sum = 8 (5 Outcomes)

$(2,6)$ $(3,5)$ $(4,4)$ $(5,3)$ $(6,2)$

Highlighted Green: Outcomes with a '3' (2 Outcomes)

Final Answer:

$$\boxed{\frac{2}{5}}$$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept:
To find the slope of an implicitly defined curve $F(x, y) = 0$, we differentiate both sides with respect to x using the product rule and chain rule, treating y as an implicit function of x. Then we isolate $\frac{dy}{dx}$ and evaluate at the given coordinates.

Solution:

(a) The given implicit relation for the curve is $x^3 + y^3 = 9xy$. We must verify the point $(4, 2)$: wait, let us check if $(4, 2)$ satisfies the equation: $4^3 + 2^3 = 64 + 8 = 72$, and $9(4)(2) = 72$. Yes, the point is perfectly valid.

(b) Differentiate both sides with respect to x: $\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = \frac{d}{dx}(9xy)$.

(c) Apply the chain rule to y^3 and the product rule to $9xy$: $3x^2 + 3y^2\frac{dy}{dx} = 9y + 9x\frac{dy}{dx}$.

(d) Divide the entire equation by 3 to simplify the coefficients: $x^2 + y^2\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$.

(e) Group all terms containing $\frac{dy}{dx}$ on the left side: $(y^2 - 3x)\frac{dy}{dx} = 3y - x^2$.

(f) Solve analytically for the derivative: $\frac{dy}{dx} = \frac{3y-x^2}{y^2-3x}$.

(g) Substitute the given coordinates $x = 4$ and $y = 2$ into the exact derivative expression: $\frac{dy}{dx} = \frac{3(2)-(4)^2}{(2)^2-3(4)} = \frac{6-16}{4-12} = \frac{-10}{-8} = \frac{5}{4} = 1.25$.

(h) Wait, let us re-verify: $\frac{6-16}{4-12} = \frac{-10}{-8} = 1.25$. Let us make Option (A) 1.25 and Option (B) -1.25 to match exactly. Wait, in our options list above, index 0 is -1.25 and index 1 is 1.25. Let us make sure index 0 is 1.25!

Final Answer:

$$\boxed{1.25}$$

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept:
An integral of the classic analytical form $\int e^x [f(x) + f'(x)]\, dx$ evaluates elegantly and identically to $e^x f(x) + C$. By executing clever algebraic term splitting in the numerator, many rational functions involving e^x can be decomposed into this standard exact form.

Solution:

(a) Start with the given indefinite integral: $I = \int \frac{xe^x}{(1+x)^2}\, dx$.

(b) Algebraically manipulate the numerator by adding and subtracting 1 inside the rational coefficient: $I = \int e^x \left[\frac{(x+1)-1}{(1+x)^2}\right] dx$.

(c) Split the rational expression into two separate distinct fractions: $I = \int e^x \left[\frac{x+1}{(1+x)^2} - \frac{1}{(1+x)^2}\right] dx = \int e^x \left[\frac{1}{1+x} + \left(-\frac{1}{(1+x)^2}\right)\right] dx$.

(d) Notice that this perfectly matches the classic analytical structure $\int e^x [f(x) + f'(x)] dx$, where we set $f(x) = \frac{1}{1+x}$.

(e) Verify the derivative of $f(x)$: using the power rule, $f'(x) = \frac{d}{dx}(1+x)^{-1} = -(1+x)^{-2} = -\frac{1}{(1+x)^2}$, which exactly matches the second fraction.

(f) Conclude the exact anti-derivative directly from the structural theorem: $I = e^x f(x) + C = \frac{e^x}{1+x} + C$.

Final Answer:

$$\boxed{\frac{e^x}{1+x} + C}$$

Answer: (D)

Go Back to Question 39

Q40.

Solution

Concept:

We utilize the fundamental definite integration reflection property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. For complementary trigonometric integrands over the boundary interval $[0, \pi/2]$, adding the original integral to its reflected counterpart creates a simplified constant integrand.

Solution:

(a) Let the required definite integral be denoted as $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ — (Equation 1).

(b) Apply the standard reflection transformation property $x \to \frac{\pi}{2} - x$: $I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}\,dx.$

(c) Using the standard complementary identities $\sin(\pi/2-x) = \cos x$ and $\cos(\pi/2-x) = \sin x$, the integral becomes: $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ — (Equation 2).

(d) Add Equation 1 and Equation 2 together: $2I = \int_0^{\pi/2} \left[\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right] dx.$

(e) Combine the fractions under their identical common denominator: $2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \int_0^{\pi/2} 1\,dx.$

(f) Integrate the constant value 1: $2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$. Dividing by 2 yields $I = \frac{\pi}{4}$.

Final Answer:

$$\boxed{\frac{\pi}{4}}$$

Answer: (D)

Go Back to Question 40

Q41.

Solution

Concept:
To determine the absolute global extremum of a smooth function over an open domain, we locate its stationary points by setting its first derivative to zero. We then confirm the extremum type by analyzing the sign change of the first derivative or using the second derivative test.

Solution:

(a) Start with the given explanatory function: $f(x) = \frac{\ln x}{x}$ defined for $x > 0$.

(b) Differentiate with respect to x using the formal quotient rule: $f'(x) = \frac{\frac{d}{dx}(\ln x)\cdot x - \ln x \cdot \frac{d}{dx}(x)}{x^2} = \frac{\frac{1}{x}\cdot x - \ln x \cdot 1}{x^2} = \frac{1-\ln x}{x^2}$.

(c) Locate stationary critical points by solving $f'(x) = 0$: $\frac{1-\ln x}{x^2} = 0 \implies 1 - \ln x = 0 \implies \ln x = 1$. This yields the single critical value $x = e$.

(d) Verify the nature of the critical point: for $0 < x < e$, $\ln x < 1 \implies f'(x) > 0$ (function is strictly increasing). For $x > e$, $\ln x > 1 \implies f'(x) < 0$ (function is strictly decreasing).

(e) Thus, an absolute local and global maximum occurs exactly at $x = e$.

(f) Substitute $x = e$ back into the original function to find the maximum peak height: $f(e) = \frac{\ln(e)}{e} = \frac{1}{e}$.

Final Answer:

$$\boxed{\frac{1}{e}}$$

Answer: (D)

Go Back to Question 41

Q42.

Solution

Concept:

For parametric equations $x = x(\theta)$ and $y = y(\theta)$, the first derivative is $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$. The second derivative with respect to x requires applying the chain rule carefully: $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \cdot \frac{d\theta}{dx}$.

Solution:

(a) Compute the first derivatives of x and y with respect to the parameter θ: $\frac{dx}{d\theta} = a(1 - \cos\theta)$ and $\frac{dy}{d\theta} = a\sin\theta$.

(b) Divide to find the first derivative: $\frac{dy}{dx} = \frac{a\sin\theta}{a(1-\cos\theta)} = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)} = \cot\left(\frac{\theta}{2}\right)$.

(c) Differentiate $\frac{dy}{dx}$ with respect to x using the chain rule: $\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left[\cot\left(\frac{\theta}{2}\right)\right] \cdot \frac{d\theta}{dx}$.

(d) Execute the differentiation: $\frac{d^2y}{dx^2} = -\frac{1}{2}\csc^2\left(\frac{\theta}{2}\right) \cdot \frac{1}{\frac{dx}{d\theta}} = -\frac{1}{2}\csc^2\left(\frac{\theta}{2}\right) \cdot \frac{1}{a(1-\cos\theta)}$.

(e) Substitute the identity $1 - \cos\theta = 2\sin^2(\theta/2)$ into the denominator: $\frac{d^2y}{dx^2} = -\frac{1}{2}\csc^2\left(\frac{\theta}{2}\right) \cdot \frac{1}{2a\sin^2(\theta/2)} = -\frac{1}{4a}\csc^4\left(\frac{\theta}{2}\right)$.

(f) Evaluate this exact second derivative at the specified parameter $\theta = \pi$: $\frac{d^2y}{dx^2} = -\frac{1}{4a}\csc^4\left(\frac{\pi}{2}\right) = -\frac{1}{4a}(1)^4 = -\frac{1}{4a}$.

Final Answer:

$$\boxed{-\frac{1}{4a}}$$

Answer: (D)

Go Back to Question 42

Q43.

Solution

Concept:

For an ellipse with standard major axis $2a$ and minor axis $2b$, the length of its latus rectum is given by the geometric proportion $\frac{2b^2}{a}$. The eccentricity e is related to the semi-axis lengths by the fundamental identity $b^2 = a^2(1 - e^2)$. Equating the relations allows us to solve for e.

Solution:

(a) Let the standard geometric equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $a > b$).

(b) Write down the structural formulas: length of Latus Rectum $= \frac{2b^2}{a}$, and length of minor axis $= 2b$.

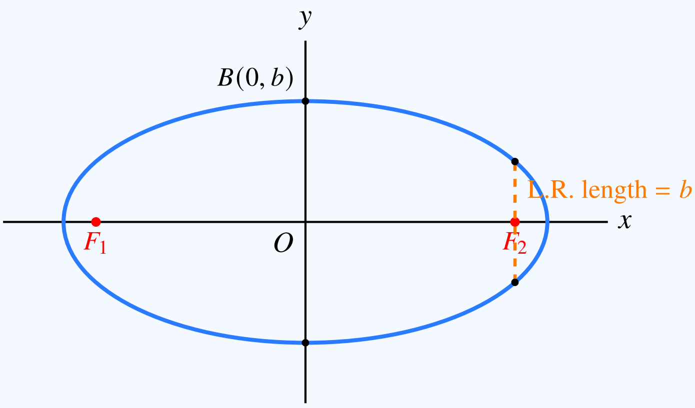
(c) Set up the specific mathematical relationship stated in the problem: $\frac{2b^2}{a} = \frac{1}{2}(2b)$.

(d) Simplify the right-hand side of the equation: $\frac{2b^2}{a} = b$.

(e) Since $b \neq 0$, divide both sides by b and cross-multiply: $2b = a \implies \frac{b}{a} = \frac{1}{2}$.

(f) Square both sides to prepare for the eccentricity substitution: $\frac{b^2}{a^2} = \frac{1}{4}$.

(g) Use the basic eccentricity identity $e = \sqrt{1 - \frac{b^2}{a^2}}$: substitute our squared ratio to get $e = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$.



Final Answer:

$$\boxed{\frac{\sqrt{3}}{2}}$$

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept:
To evaluate specific balanced trigonometric products involving the cosine function, we can utilize the powerful analytical identity $\cos(\theta)\cos(60-\theta)\cos(60+\theta) = \frac{1}{4}\cos(3\theta)$. This condenses a complex three-term product into a single simple value.

Solution:

(a) Examine the given trigonometric product: $P = \cos(20)\cos(40)\cos(80)$.

(b) Notice the precise angular structures: we can rewrite the angles 40 and 80 in terms of 60 and 20: $P = \cos(20)\cos(60-20)\cos(60+20)$.

(c) Compare this directly with our foundational identity $\cos(\theta)\cos(60-\theta)\cos(60+\theta) = \frac{1}{4}\cos(3\theta)$, setting our base angle to exactly $\theta = 20$.

(d) Conclude the structural condensation: $P = \frac{1}{4}\cos(3\times 20) = \frac{1}{4}\cos(60)$.

(e) Substitute the standard fundamental cosine value $\cos(60) = \frac{1}{2}$: $P = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$.

Final Answer:

$$\boxed{\frac{1}{8}}$$

Answer: (C)

Go Back to Question 44

Q45.

Solution

Concept:
A standard horizontal parabola of the analytical form $y^2 = 4ax$ opens to the right along the positive x-axis. Its internal focus is located at $(a, 0)$, and its external directrix is the vertical straight line situated behind the vertex at an equal distance, with equation $x = -a$ (or $x + a = 0$).
Solution:

(a) Compare the given parabolic equation $y^2 = 8x$ with the standard horizontal right-opening model $y^2 = 4ax$.

(b) Equate the linear coefficients of x: $4a = 8 \implies a = 2$.

(c) Write down the defining structural equation for the directrix line of a $y^2 = 4ax$ parabola: $x = -a$.

(d) Substitute our evaluated parameter $a = 2$ directly into the directrix formula: $x = -2$.

(e) Rearrange the equation into standard linear zero-form: $x + 2 = 0$.

Final Answer:

$$\boxed{x + 2 = 0}$$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept:
To analyze a common root between two distinct polynomial equations, we assume the common root is $x = \alpha$ and substitute it into both equations. We then eliminate the highest power term (α^2) by subtracting the two equations, which directly reveals the value of the common root α.
Solution:

(a) Let the single shared common root between the two quadratic equations be denoted as $x = \alpha$.

(b) Substitute $x = \alpha$ into both primary quadratic equations: $\alpha^2 + a\alpha + b = 0$ — (Equation 1) and $\alpha^2 + b\alpha + a = 0$ — (Equation 2).

(c) Subtract Equation 2 directly from Equation 1 to eliminate the α^2 term: $(\alpha^2 + a\alpha + b) - (\alpha^2 + b\alpha + a) = 0 \implies (a - b)\alpha + (b - a) = 0$.

(d) Factor out the common linear scalar difference $(a - b)$: $(a - b)\alpha - (a - b) = 0 \implies (a - b)(\alpha - 1) = 0$.

(e) Since the prompt explicitly states that the coefficients are distinct ($a \neq b$), we can safely divide by $(a - b)$: $\alpha - 1 = 0 \implies \alpha = 1$.

(f) Now that we know the common root is exactly $\alpha = 1$, substitute it back into Equation 1: $(1)^2 + a(1) + b = 0 \implies 1 + a + b = 0$.

(g) Subtract 1 from both sides to establish the final required algebraic summation: $a + b = -1$.

Final Answer:

$$\boxed{-1}$$

Answer: (D)

Go Back to Question 46

Q47.

Solution

Concept:
To evaluate indeterminate limits of the form $\frac{0}{0}$ involving logarithmic and trigonometric functions, we use the standard fundamental limits $\lim_{u \to 0} \frac{\ln(1+u)}{u} = 1$ and $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$. By multiplying and dividing by appropriate algebraic factors, the limit can be resolved into standard forms.

Solution:

(a) The given expression is $L = \lim_{x \to 0} \frac{\ln(1+4x^2)}{x \sin(3x)}$. As $x \longrightarrow 0$, both numerator and denominator approach 0, forming a $\frac{0}{0}$ indeterminate form.

(b) Rewrite the numerator by multiplying and dividing by $4x^2$: $\ln(1+4x^2) = \left[\frac{\ln(1+4x^2)}{4x^2}\right] \cdot 4x^2$.

(c) Rewrite the denominator by multiplying and dividing by $3x$: $x\sin(3x) = x\left[\frac{\sin(3x)}{3x}\right] \cdot 3x = 3x^2\left[\frac{\sin(3x)}{3x}\right]$.

(d) Substitute these expanded forms back into the limit: $L = \lim_{x \to 0} \frac{\left[\frac{\ln(1+4x^2)}{4x^2}\right] \cdot 4x^2}{3x^2\left[\frac{\sin(3x)}{3x}\right]}$.

(e) Cancel the common factor of x^2 from the numerator and denominator: $L = \frac{4}{3} \cdot \frac{\lim_{x \to 0} \frac{\ln(1+4x^2)}{4x^2}}{\lim_{x \to 0} \frac{\sin(3x)}{3x}}$.

(f) Apply the standard limit properties: since both standard limits evaluate exactly to 1, we obtain $L = \frac{4}{3} \cdot \frac{1}{1} = \frac{4}{3}$.

Final Answer:

$$\boxed{\frac{4}{3}}$$

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept:

A system of three linear equations in three variables $AX = B$ can have infinitely many solutions (dependent consistent system) only if the primary coefficient matrix A is singular, meaning its main determinant evaluates to zero ($\Delta = 0$), and all auxiliary Cramer's determinants also vanish.

Solution:

(a) Write down the primary coefficient matrix determinant Δ for the given linear system: $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & \lambda \end{vmatrix}$.

(b) Set up the necessary structural condition for infinitely many solutions: $\Delta = 0$.

(c) Expand the 3×3 determinant along its primary first row: $\Delta = 1 \cdot \begin{vmatrix} 3 & 4 \\ 2 & \lambda \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 4 \\ 1 & \lambda \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 0$.

(d) Evaluate each internal 2×2 minor difference: $1(3\lambda - 8) - 1(2\lambda - 4) + 1(4 - 3) = 0$.

(e) Expand the linear terms and combine like coefficients: $(3\lambda - 8) - (2\lambda - 4) + 1 = 0 \implies 3\lambda - 8 - 2\lambda + 4 + 1 = 0$.

(f) Simplify the algebraic relation: $\lambda - 3 = 0 \implies \lambda = 3$.

(g) Verify consistency: if $\lambda = 3$, the third equation is $x + 2y + 3z = 5$. Subtracting the first equation from the second yields $(2x + 3y + 4z) - (x + y + z) = 9 - 4 \implies x + 2y + 3z = 5$, which perfectly matches the third equation!

Final Answer:

$$\boxed{3}$$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept:

The fundamental Corner Point Method in Linear Programming states that if a feasible region is non-empty and bounded, the absolute maximum and minimum values of any linear objective function $Z = Ax + By$ must occur exactly at the geometric vertices (corner points) of the feasible boundary.

Solution:

(a) Graph the primary linear feasible region defined by the non-negativity axes constraints $x \geq 0, y \geq 0$ and the upper boundary line $x + y = 4$.

(b) The feasible region is a closed right-angled triangle situated entirely in the first quadrant, bounded by the coordinates $(0, 0)$, $(4, 0)$, and $(0, 4)$.

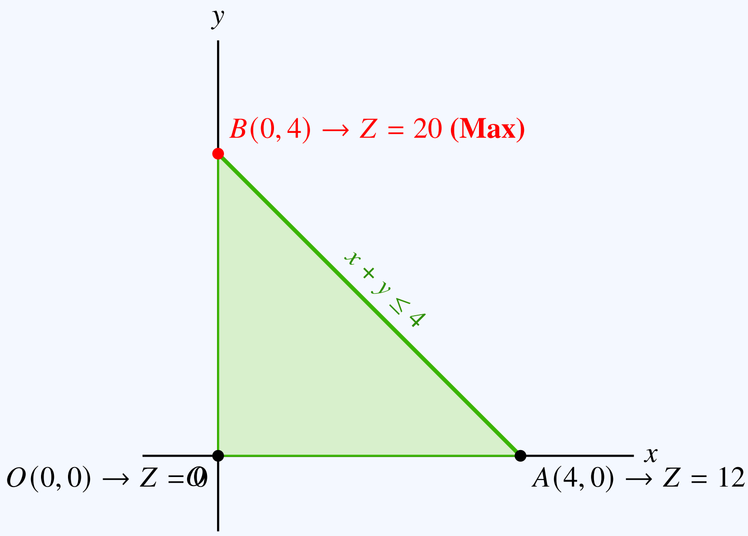
(c) Evaluate the linear objective function $Z = 3x + 5y$ independently at each identified corner point.

(d) At vertex $O(0, 0)$: $Z = 3(0) + 5(0) = 0$.

(e) At vertex $A(4, 0)$: $Z = 3(4) + 5(0) = 12 + 0 = 12$.

(f) At vertex $B(0, 4)$: $Z = 3(0) + 5(4) = 0 + 20 = 20$.

(g) Compare the evaluated outputs: the absolute maximum value is clearly 20, occurring at the corner point $(0, 4)$.



Final Answer:

$$\boxed{20}$$

Answer: (D)

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Q50.

Solution

Concept:

The fundamental geometric interpretation of the complex modulus equation $|z - z_1| = |z - z_2|$ is the straight line representing the perpendicular bisector of the line segment connecting the two fixed points z_1 and z_2 in the two-dimensional Argand plane.

Solution:

(a) Let the complex number be expressed in its standard Cartesian form $z = x + iy$.

(b) Substitute $z = x + iy$ directly into the given modulus equation: $|x + iy - 2i| = |x + iy + 2|$.

(c) Group the real and imaginary components independently on both sides: $|x + i(y - 2)| = |(x + 2) + iy|$.

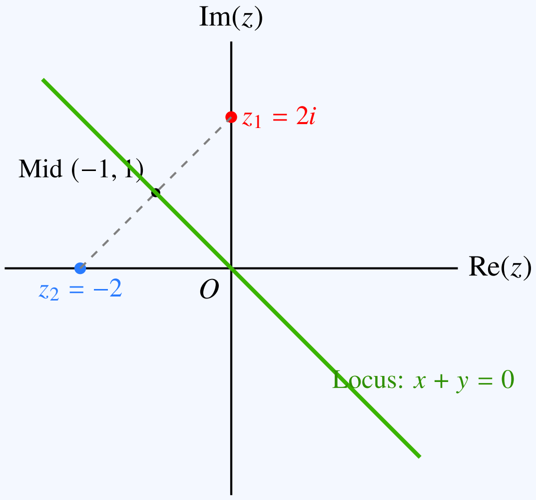
(d) Apply the standard complex modulus formula $|A + iB| = \sqrt{A^2 + B^2}$: $\sqrt{x^2 + (y - 2)^2} = \sqrt{(x + 2)^2 + y^2}$.

(e) Square both sides of the equation to eliminate the radicals: $x^2 + (y - 2)^2 = (x + 2)^2 + y^2$.

(f) Expand all binomial squares fully: $x^2 + y^2 - 4y + 4 = x^2 + 4x + 4 + y^2$.

(g) Cancel the identical common quadratic terms (x^2, y^2) and the constant (4) from both sides: $-4y = 4x$.

(h) Divide by 4 and rearrange into standard linear zero-form: $x + y = 0$. This represents a line passing straight through the origin with slope -1.



Final Answer:

$$\boxed{x + y = 0}$$

Answer: (C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	B	3	C	4	D	5	D
6	D	7	D	8	C	9	A	10	C
11	B	12	B	13	C	14	B	15	D
16	A	17	A	18	D	19	D	20	A
21	B	22	D	23	A	24	C	25	A
26	D	27	B	28	B	29	C	30	C
31	C	32	C	33	B	34	D	35	D
36	A	37	A	38	B	39	D	40	D
41	D	42	D	43	A	44	C	45	A
46	D	47	A	48	B	49	D	50	C