JCECE Mathematics Sample Paper-8

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **50** Multiple Choice Questions.
- Each correct answer carries **+1** mark. Incorrect answer: **−0.25** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. For real constants p and q, if $\lim_{x\to 0} \frac{\sin px - \sin qx}{x} = 6$ and $p + q = 10$, then pq is

(A) 12

(B) 16

(C) 21

(D) 24

Q2. If $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{pmatrix}$, then $|A^2|$ equals

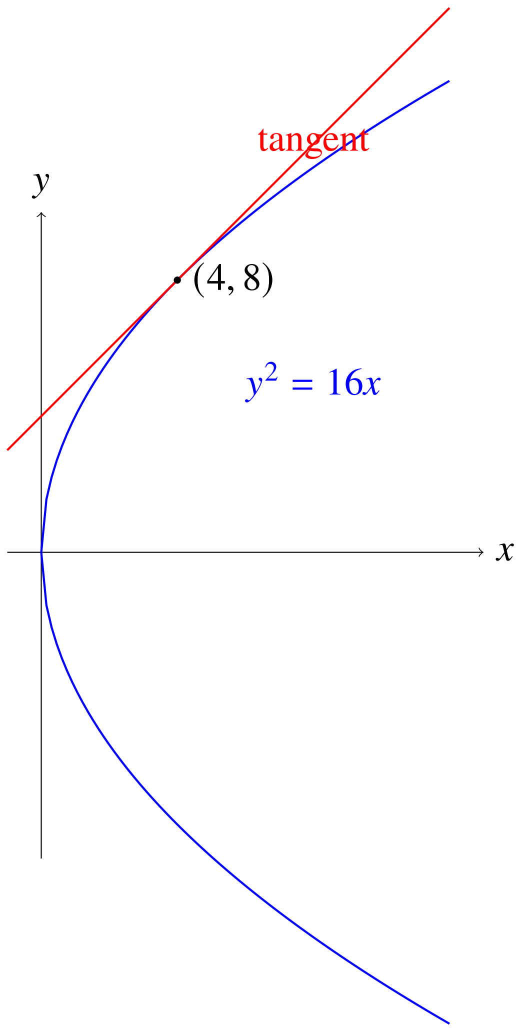
(A) 225

(B) 900

(C) 3600

(D) 30

Q3. The tangent to the parabola $y^2 = 16x$ at the point whose ordinate is 8 is



(A) $y = 2x + 4$

(B) $y = x + 4$

(C) $y = x + 8$

(D) $2y = x + 16$

Q4. The value of $\int_0^1 \frac{2x}{1+x^2}\,dx$ is

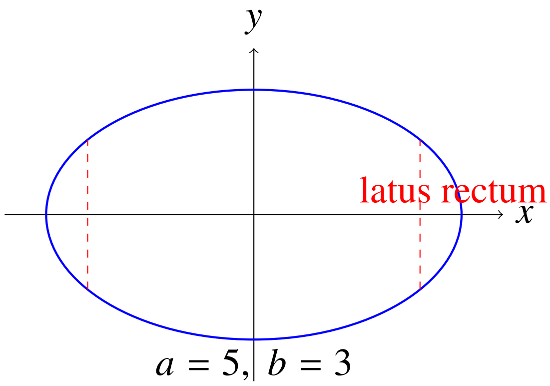
(A) $\ln 2$

(B) 1

(C) $2\ln 2$

(D) $\frac{1}{2}\ln 2$

Q5. For the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, the length of the latus rectum is



(A) $\frac{18}{5}$

(B) $\frac{10}{3}$

(C) $\frac{25}{3}$

(D) 6

Q6. Two fair dice are thrown. The probability that the product of the two numbers is divisible by 6 is

(A) $\frac{1}{3}$

(B) $\frac{7}{18}$

(C) $\frac{5}{12}$

(D) $\frac{1}{2}$

Q7. The number of solutions of $2\cos^2 x - 3\cos x + 1 = 0$ in $[0, 2\pi]$ is

(A) 2

(B) 3

(C) 4

(D) 5

Q8. If $f(x) = \begin{cases} \dfrac{\sqrt{1+kx}-1}{x}, & x \neq 0, \\ 3, & x = 0, \end{cases}$ is continuous at $x = 0$, then k equals

(A) 3

(B) 4

(C) 5

(D) 6

Q9. An ellipse has major axis length 20 and minor axis length 12. Its eccentricity is

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{2}{5}$

(D) $\frac{5}{6}$

Q10. The particular solution of $\frac{dy}{dx} = \frac{x}{y}$ passing through $(2, 3)$ is

(A) $y^2 - x^2 = 5$

(B) $y^2 + x^2 = 13$

(C) $x^2 - y^2 = 5$

(D) $2y^2 - x^2 = 14$

Q11. If the sum of the first n terms of a sequence is $S_n = n^2 + 2n$, then its 10th term is

(A) 19

(B) 20

(C) 21

(D) 22

Q12. The distance of the point $(4, -1)$ from the line $5x - 12y + 9 = 0$ is

(A) $\frac{41}{13}$

(B) $\frac{35}{13}$

(C) $\frac{29}{13}$

(D) $\frac{13}{41}$

Q13. If the roots of $x^2 - 7x + m = 0$ differ by 3, then m is

(A) 8

(B) 10

(C) 12

(D) 14

Q14. If $y = \tan^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ equals

(A) $\frac{1}{x^2+1}$

(B) $\frac{2}{x^2+1}$

(C) $\frac{1}{2(x^2+1)}$

(D) $\frac{x}{x^2+1}$

Q15. For $A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$, the $(1,2)$ entry of A^{-1} is

(A) 2

(B) −2

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

Q16. The mean of five observations is 14. If one observation 10 is replaced by 20, the new mean is

(A) 15

(B) 16

(C) 17

(D) 18

Q17. The length of the chord of the circle $x^2 + y^2 = 25$ cut by the line $y = 3$ is

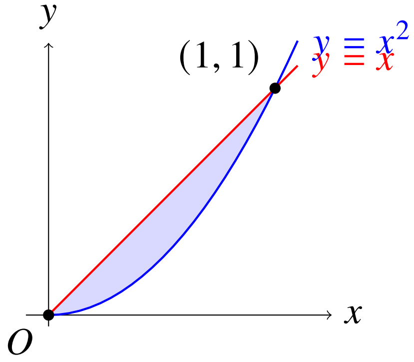
(A) 6

(B) 7

(C) 8

(D) 10

Q18. The area enclosed by $y = x$ and $y = x^2$ is



(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Q19. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$, then the scalar projection of $\vec{a}$ on $\vec{b}$ is

(A) $\frac{4}{3}$

(B) $\frac{5}{3}$

(C) $\frac{6}{3}$

(D) 3

Q20. If $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, then $(g \circ f)(2)$ is

(A) 1

(B) 2

(C) 5

(D) 10

Q21. The maximum value of $x(6 - x)$ for real x is

(A) 6

(B) 8

(C) 9

(D) 12

Q22. The coefficient of x^3 in $(2 + x)^5$ is

(A) 20

(B) 40

(C) 60

(D) 80

Q23. The eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is

(A) $\frac{5}{4}$

(B) $\frac{4}{5}$

(C) $\frac{3}{4}$

(D) $\frac{\sqrt{7}}{4}$

Q24. The matrix $\begin{pmatrix} 1 & k & 2 \\ 0 & 1 & 3 \\ 2 & 1 & 4 \end{pmatrix}$ is singular when k equals

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

Q25. One card is drawn from a standard pack of 52 cards. The probability that it is either a king or a heart is

(A) $\frac{4}{13}$

(B) $\frac{17}{52}$

(C) $\frac{16}{52}$

(D) $\frac{1}{4}$

Q26. If $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$, then I equals

(A) $\frac{\pi}{8}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) 1

Q27. If $z = 3 - 4i$, then $\left|\frac{z}{\bar{z}}\right|$ is

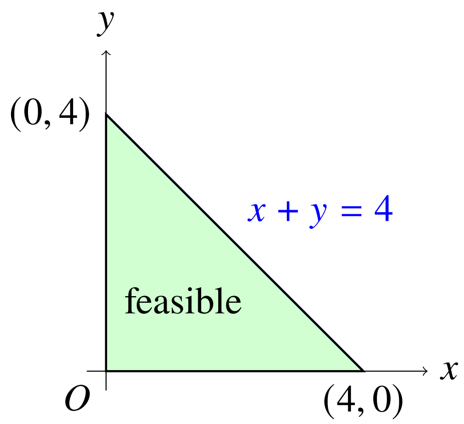
(A) 0

(B) 1

(C) 5

(D) 25

Q28. The maximum value of $Z = 3x + 2y$ subject to $x + y \le 4$, $x \ge 0$, $y \ge 0$ is



(A) 8

(B) 10

(C) 12

(D) 14

Q29. The function $f(x) = |x - 2| + |x + 1|$ is not differentiable at

(A) $x = -1, 2$

(B) $x = 0, 2$

(C) $x = -2, 1$

(D) $x = 1, 2$

Q30. The angle between the line with direction ratios $(1, 2, 2)$ and the plane $x+2y+2z = 5$ is

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q31. For the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$, the distance between its foci is

(A) 4

(B) 6

(C) 8

(D) 10

Q32. The solution of $\frac{dy}{dx} + y = e^x$ satisfying $y(0) = 0$ is

(A) $y = \frac{e^x - e^{-x}}{2}$

(B) $y = e^x - 1$

(C) $y = xe^{-x}$

(D) $y = e^{-x}$

Q33. Three numbers are in A.P. Their sum is 24 and their product is 480. The middle term is

(A) 6

(B) 8

(C) 10

(D) 12

Q34. The number of stationary points of $f(x) = x^4 - 4x^2 + 1$ is

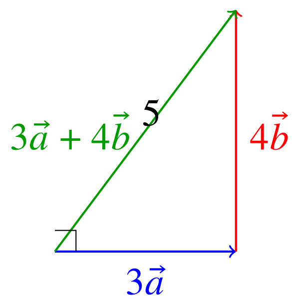
(A) 1

(B) 2

(C) 3

(D) 4

Q35. If $\vec{a}$ and $\vec{b}$ are perpendicular unit vectors, then $|3\vec{a} + 4\vec{b}|$ is



(A) 1

(B) 5

(C) 7

(D) 25

Q36. If $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$ satisfies $AX = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$, then x is

(A) 2

(B) 3

(C) 4

(D) 5

Q37. If every observation of a data set is multiplied by 3, then its variance is multiplied by

(A) 3

(B) 6

(C) 9

(D) 27

Q38. The minimum value of $1 + 2\sin x \cos x$ is

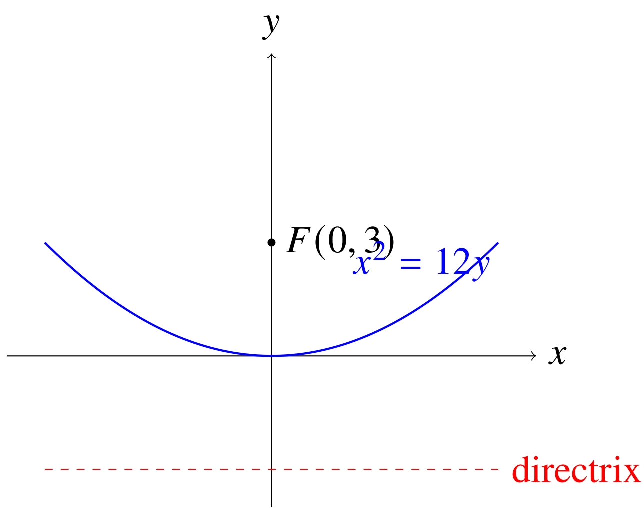
(A) 0

(B) 1

(C) 2

(D) -1

Q39. For the parabola $x^2 = 12y$, the focus is



(A) $(0, 3)$

(B) $(3, 0)$

(C) $(0, -3)$

(D) $(-3, 0)$

Q40. The value of $\int_1^e \frac{\ln x}{x}\,dx$ is

(A) $\frac{1}{2}$

(B) 1

(C) $e - 1$

(D) $\ln 2$

Q41. The equation of the line through $(1, -2)$ with slope -3 is

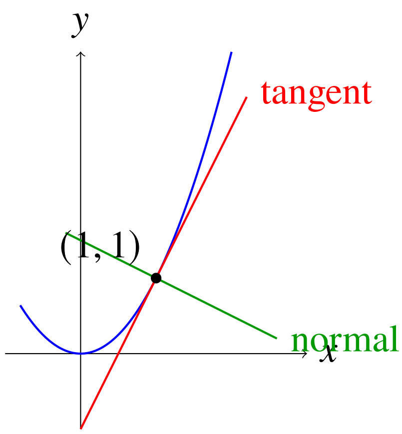
(A) $3x + y - 1 = 0$

(B) $3x + y - 5 = 0$

(C) $x + 3y + 5 = 0$

(D) $3x - y - 5 = 0$

Q42. The slope of the normal to the curve $y = x^2$ at the point $(1, 1)$ is



(A) 2

(B) -2

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Q43. If $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$, and $\vec{c} = \hat{k} + \hat{i}$, then $[\vec{a}\ \vec{b}\ \vec{c}]$ equals

(A) 0

(B) 1

(C) 2

(D) -2

Q44. If $1 + i$ is a root of $x^2 - 2x + c = 0$, then c equals

(A) 1

(B) 2

(C) -2

(D) $2i$

Q45. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 = x$ are respectively

(A) 2, 3

(B) 3, 2

(C) 2, 2

(D) 1, 3

Q46. The integral $\int e^{2x}\,dx$ equals

(A) $2e^{2x} + C$

(B) $\frac{1}{2}e^{2x} + C$

(C) $e^{2x} + C$

(D) $\frac{1}{4}e^{2x} + C$

Q47. If A is a 2×2 matrix with eigenvalues 4 and -1, then $\text{tr}(A^2)$ is

(A) 15

(B) 16

(C) 17

(D) 18

Q48. The value of $\lim_{x \to 0} \dfrac{\ln(1+4x)}{\sin 2x}$ is

(A) 1

(B) 2

(C) 4

(D) $\frac{1}{2}$

Q49. The circle $x^2 + y^2 + 4x - 6y + 9 = 0$ has radius

(A) 1

(B) 2

(C) 3

(D) 4

Q50. If $f'(x) = 3x^2 - 12x + 9$, then f is increasing for

(A) $x < 1$ only

(B) $1 < x < 3$ only

(C) $x < 1$ or $x > 3$

(D) $x > 1$ only

Detailed Solutions

Q1.

Solution

Concept: For small-angle limits, the standard result $\lim_{x\to 0} \frac{\sin kx}{x} = k$ is used. This follows from $\sin u \sim u$ near $u = 0$. The condition converts the limit into a simple equation in the parameters.

Solution:

(a) Using the standard limit, $\lim_{x\to 0} \frac{\sin px - \sin qx}{x} = p - q$.

(b) The given limit is 6, so $p - q = 6$.

(c) We are also given $p + q = 10$.

(d) Adding the two equations gives $2p = 16$, hence $p = 8$.

(e) Substituting in $p + q = 10$, we get $q = 2$.

(f) Therefore $pq = 8 \times 2 = 16$.

Final Answer: $\boxed{16}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: For a triangular matrix, the determinant is the product of its diagonal entries. Also, for any square matrix A, $|A^2| = |A|^2$. These properties avoid unnecessary multiplication of matrices.

Solution:

(a) The matrix is upper triangular, so $|A| = 2 \cdot 3 \cdot 5 = 30$.

(b) We need $|A^2|$.

(c) Using determinant property, $|A^2| = |A|^2$.

(d) Thus $|A^2| = 30^2 = 900$.

(e) This matches option (B).

Final Answer: $\boxed{900}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: For the parabola $y^2 = 4ax$, the point with parameter t is $(at^2, 2at)$ and the tangent is $ty = x + at^2$. This formula is useful because the ordinate directly gives the parameter.

Solution:

(a) Compare $y^2 = 16x$ with $y^2 = 4ax$, so $4a = 16$ and $a = 4$.

(b) At the required point, ordinate $y = 8$.

(c) Since $y = 2at$, we have $8 = 2(4)t$, hence $t = 1$.

(d) The tangent is $ty = x + at^2$.

(e) Substituting $t = 1$ and $a = 4$, we get $y = x + 4$.

(f) Therefore the correct option is (B).

Final Answer: $\boxed{y = x + 4}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: An integral of the form $\int \frac{f'(x)}{f(x)} dx$ is evaluated as $\ln|f(x)| + C$. Here the derivative of $1 + x^2$ is $2x$, exactly the numerator.

Solution:

(a) Let $u = 1 + x^2$.

(b) Then $du = 2x\, dx$.

(c) When $x = 0$, $u = 1$; when $x = 1$, $u = 2$.

(d) Therefore $\displaystyle\int_0^1 \frac{2x}{1 + x^2} dx = \int_1^2 \frac{du}{u}$.

(e) Hence the value is $[\ln u]_1^2 = \ln 2 - \ln 1 = \ln 2$.

Final Answer: $\boxed{\ln 2}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, the length of the latus rectum is $\frac{2b^2}{a}$. The major axis is along the x-axis because $25 > 9$.

Solution:

(a) From $\frac{x^2}{25} + \frac{y^2}{9} = 1$, we have $a^2 = 25$ and $b^2 = 9$.

(b) Thus $a = 5$ and $b = 3$.

(c) Latus rectum length $= \frac{2b^2}{a}$.

(d) Substituting values gives $\frac{2\cdot 9}{5} = \frac{18}{5}$.

(e) The correct option is (A).

Final Answer: $\boxed{\frac{18}{5}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: A product is divisible by 6 if it is divisible by both 2 and 3. We count ordered dice outcomes satisfying this condition and divide by the total 36 outcomes.

Solution:

(a) Total ordered outcomes for two dice are $6 \times 6 = 36$.

(b) Product divisible by 6 means the product has at least one factor 2 and at least one factor 3.

(c) Count the complement: product not divisible by 6 means it fails divisibility by 2 or by 3.

(d) Outcomes with product not divisible by 2: both dice odd, so $3 \times 3 = 9$.

(e) Outcomes with product not divisible by 3: neither die is a multiple of 3, so $4 \times 4 = 16$.

(f) Outcomes counted in both: both dice are odd and not multiples of 3, choices $\{1, 5\}$ for each, so $2 \times 2 = 4$.

(g) Complement count $= 9 + 16 - 4 = 21$.

(h) Favorable count $= 36 - 21 = 15$.

(i) Probability $= \frac{15}{36} = \frac{5}{12}$.

Final Answer: $\boxed{\frac{5}{12}}$

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: A trigonometric equation that is quadratic in $\cos x$ can first be solved algebraically. Then only the roots lying in the range $[-1, 1]$ are used to count angles in the given interval.

Solution:

(a) Factor the equation: $2\cos^2 x - 3\cos x + 1 = (2\cos x - 1)(\cos x - 1) = 0$.

(b) Hence $\cos x = 1$ or $\cos x = \frac{1}{2}$.

(c) In $[0, 2\pi]$, $\cos x = 1$ gives $x = 0, 2\pi$, two solutions.

(d) In $[0, 2\pi]$, $\cos x = \frac{1}{2}$ gives $x = \frac{\pi}{3}, \frac{5\pi}{3}$, two solutions.

(e) Total solutions = 4. Wait, endpoints 0 and 2π are distinct values in the closed interval, so both are counted.

(f) Therefore the correct count is 4.

Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: Continuity at $x = 0$ requires $\lim_{x\to 0} f(x) = f(0)$. For radicals, rationalisation converts the expression into a form where direct substitution is possible.

Solution:

(a) Since $f(0) = 3$, continuity requires $\lim\limits_{x\to 0} \dfrac{\sqrt{1+kx} - 1}{x} = 3$.

(b) Rationalise the numerator: $\dfrac{\sqrt{1+kx} - 1}{x} \cdot \dfrac{\sqrt{1+kx} + 1}{\sqrt{1+kx} + 1}$.

(c) This becomes $\dfrac{1 + kx - 1}{x(\sqrt{1+kx} + 1)} = \dfrac{k}{\sqrt{1+kx} + 1}$.

(d) Taking $x \to 0$, the limit is $\frac{k}{1+1} = \frac{k}{2}$.

(e) Thus $\frac{k}{2} = 3$, so $k = 6$.

Final Answer: $\boxed{6}$

Answer: (D)

Go Back to Question 8

Q9.

Solution

Concept: For an ellipse, a is the semi-major axis and b is the semi-minor axis. The eccentricity is $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:

(a) Major axis length = 20, so $a = 10$.

(b) Minor axis length = 12, so $b = 6$.

(c) Apply $e = \sqrt{1 - \frac{b^2}{a^2}}$.

(d) $e = \sqrt{1 - \frac{36}{100}} = \sqrt{\frac{64}{100}} = \frac{8}{10} = \frac{4}{5}$.

(e) Hence option (B) is correct.

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: A separable differential equation can be solved by collecting y terms with dy and x terms with dx. The constant is then determined from the given point.

Solution:

(a) Given $\frac{dy}{dx} = \frac{x}{y}$.

(b) Rearranging gives $y\,dy = x\,dx$.

(c) Integrate both sides: $\int y\,dy = \int x\,dx$.

(d) Hence $\frac{y^2}{2} = \frac{x^2}{2} + C$, or $y^2 - x^2 = K$.

(e) Use point $(2, 3)$: $K = 3^2 - 2^2 = 9 - 4 = 5$.

(f) Thus the solution is $y^2 - x^2 = 5$.

Final Answer: $\boxed{y^2 - x^2 = 5}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: The nth term of a sequence is obtained from the sum formula by $a_n = S_n - S_{n-1}$ for $n \geq 2$. This removes all earlier terms and leaves only the nth term.

Solution:

(a) $S_n = n^2 + 2n$.

(b) $S_9 = 9^2 + 2(9) = 81 + 18 = 99$.

(c) $S_{10} = 10^2 + 2(10) = 100 + 20 = 120$.

(d) Therefore $a_{10} = S_{10} - S_9 = 120 - 99 = 21$.

Final Answer: $\boxed{21}$

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: The perpendicular distance of (x_1, y_1) from $ax + by + c = 0$ is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$. The absolute value is essential because distance cannot be negative.

Solution:

(a) Here $a = 5$, $b = -12$, $c = 9$, and $(x_1, y_1) = (4, -1)$.

(b) Numerator $= |5(4) - 12(-1) + 9| = |20 + 12 + 9| = 41$.

(c) Denominator $= \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = 13$.

(d) Distance $= \frac{41}{13}$.

Final Answer: $\boxed{\frac{41}{13}}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: For a quadratic $x^2 - Sx + P = 0$, the sum and product of roots are S and P. If the difference of roots is known, use $(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$.

Solution:

(a) Let roots be α and β.

(b) From the equation, $\alpha + \beta = 7$ and $\alpha\beta = m$.

(c) Given $|\alpha - \beta| = 3$.

(d) Use $(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$.

(e) Thus $7^2 - 3^2 = 4m$, so $49 - 9 = 4m$.

(f) Hence $40 = 4m$ and $m = 10$.

Final Answer: $\boxed{10}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: The derivative of $\tan^{-1} u$ is $\frac{u'}{1+u^2}$. When u is a quotient, the quotient rule must be applied before simplification.

Solution:

(a) Let $u = \frac{x-1}{x+1}$.

(b) By quotient rule, $u' = \frac{(x+1)(1)-(x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$.

(c) Now $1 + u^2 = 1 + \left(\frac{x-1}{x+1}\right)^2$.

(d) This is $\frac{(x+1)^2+(x-1)^2}{(x+1)^2} = \frac{2x^2+2}{(x+1)^2} = \frac{2(x^2+1)}{(x+1)^2}$.

(e) Therefore $\frac{dy}{dx} = \frac{\frac{2}{(x+1)^2}}{\frac{2(x^2+1)}{(x+1)^2}} = \frac{1}{x^2+1}$.

Final Answer: $\boxed{\frac{1}{x^2+1}}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the inverse is $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, provided the determinant is non-zero.

Solution:

(a) Here $a = 1, b = 2, c = 3, d = 7$.

(b) Determinant $= 1 \cdot 7 - 2 \cdot 3 = 7 - 6 = 1$.

(c) Thus $A^{-1} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$.

(d) The $(1, 2)$ entry is -2.

Final Answer: $\boxed{-2}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: Mean equals total sum divided by the number of observations. Replacing one observation changes the total sum by the difference between the new and old observation.

Solution:

(a) Original mean $= 14$ and number of observations $= 5$.

(b) Original total $= 5 \times 14 = 70$.

(c) Replacing 10 by 20 increases the total by $20 - 10 = 10$.

(d) New total $= 70 + 10 = 80$.

(e) New mean $= \frac{80}{5} = 16$.

Final Answer: $\boxed{16}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: A chord produced by a horizontal line can be found by substituting the line equation into the circle. The two resulting x-coordinates give the endpoints of the chord.

Solution:

(a) Substitute $y = 3$ in $x^2 + y^2 = 25$.

(b) Then $x^2 + 9 = 25$, so $x^2 = 16$.

(c) Hence $x = \pm 4$.

(d) The endpoints are $(-4, 3)$ and $(4, 3)$.

(e) Chord length $= 4 - (-4) = 8$.

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 17

Q18.

Solution

Concept: Area between two curves is obtained by integrating upper curve minus lower curve between their intersection points. First find the points of intersection to set correct limits.

Solution:

(a) Solve intersections: $x = x^2$, so $x(x - 1) = 0$.

(b) Hence $x = 0$ and $x = 1$.

(c) On $[0, 1]$, $x \geq x^2$, so upper curve is $y = x$ and lower curve is $y = x^2$.

(d) Area $= \int_0^1 (x - x^2)dx$.

(e) $= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

Final Answer: $\boxed{\frac{1}{6}}$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept: The scalar projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$. The denominator is the magnitude of the vector on which projection is taken.

Solution:

(a) Compute dot product: $\vec{a}\cdot\vec{b} = 2(1) + (-1)(2) + 2(2) = 2 - 2 + 4 = 4$.

(b) Magnitude $|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$.

(c) Scalar projection $= \frac{4}{3}$.

(d) Therefore option (A) is correct.

Final Answer: $\boxed{\frac{4}{3}}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: Composition $(g \circ f)(x)$ means $g(f(x))$. We first evaluate the inner function $f(x)$ and then substitute that result into g.

Solution:

(a) First find $f(2) = 2(2) - 3 = 1$.

(b) Now apply g to this value: $g(f(2)) = g(1)$.

(c) Since $g(x) = x^2 + 1$, $g(1) = 1^2 + 1 = 2$.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: A quadratic with negative coefficient of x^2 opens downward and has a maximum at its vertex. Alternatively, complete the square.

Solution:

(a) Let $f(x) = x(6 - x) = 6x - x^2$.

(b) Complete the square: $f(x) = -(x^2 - 6x) = -(x - 3)^2 + 9$.

(c) Since $(x - 3)^2 \geq 0$, the term $-(x - 3)^2 \leq 0$.

(d) Therefore maximum value occurs at $x = 3$ and equals 9.

Final Answer: $\boxed{9}$

Answer: (C)

Go Back to Question 21

Q22.

Solution

Concept: The general term in $(a + b)^n$ is $\binom{n}{r}a^{n-r}b^r$. To get x^3, choose the term where $b = x$ is raised to power 3.

Solution:

(a) In $(2 + x)^5$, take $r = 3$ for the x^3 term.

(b) Coefficient $= \binom{5}{3}2^{5-3}$.

(c) $= 10 \cdot 2^2 = 10 \cdot 4 = 40$.

Final Answer: $\boxed{40}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $c^2 = a^2 + b^2$ and eccentricity is $e = \frac{c}{a}$.

Solution:

(a) Here $a^2 = 16$ and $b^2 = 9$, so $a = 4$.

(b) $c^2 = a^2 + b^2 = 16 + 9 = 25$.

(c) Thus $c = 5$.

(d) Eccentricity $e = \frac{c}{a} = \frac{5}{4}$.

Final Answer: $\boxed{\frac{5}{4}}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: A square matrix is singular exactly when its determinant is zero. We compute the determinant as a function of k and solve the resulting linear equation.

Solution:

(a) Let $D = \begin{vmatrix} 1 & k & 2 \\ 0 & 1 & 3 \\ 2 & 1 & 4 \end{vmatrix}$.

(b) Expand along the first row: $D = 1(1 \cdot 4 - 3 \cdot 1) - k(0 \cdot 4 - 3 \cdot 2) + 2(0 \cdot 1 - 1 \cdot 2)$.

(c) Thus $D = (4 - 3) - k(-6) + 2(-2)$.

(d) $D = 1 + 6k - 4 = 6k - 3$.

(e) For singular matrix, $D = 0$, so $6k - 3 = 0$.

(f) Hence $k = \frac{1}{2}$.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept: When two events overlap, use inclusion-exclusion: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. The king of hearts is counted in both groups and must be subtracted once.

Solution:

(a) Number of kings = 4.

(b) Number of hearts = 13.

(c) The overlap is the king of hearts, so count = 1.

(d) Favorable cards = 4 + 13 − 1 = 16.

(e) Probability = $\frac{16}{52} = \frac{4}{13}$.

Final Answer: $\boxed{\frac{4}{13}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: For definite integrals over $[0, a]$, the transformation $x \mapsto a - x$ is often useful. Here the companion integral with cosine has the same denominator, and adding them simplifies the integrand to 1.

Solution:

(a) Let $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$.

(b) Using $x \mapsto \frac{\pi}{2} - x$, we also have $I = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$.

(c) Add the two forms: $2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$.

(d) Hence $2I = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$.

(e) Therefore $I = \frac{\pi}{4}$.

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: The modulus of a quotient is the quotient of moduli: $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$ for $w \neq 0$. A complex number and its conjugate have the same modulus.

Solution:

(a) Given $z = 3 - 4i$ and $\bar{z} = 3 + 4i$.

(b) $|z| = \sqrt{3^2 + (-4)^2} = 5$.

(c) $|\bar{z}| = \sqrt{3^2 + 4^2} = 5$.

(d) Therefore $\left|\frac{z}{\bar{z}}\right| = \frac{|z|}{|\bar{z}|} = \frac{5}{5} = 1$.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: In a linear programming problem, the maximum or minimum of a linear objective function over a polygonal feasible region occurs at a vertex of the region. Therefore, check the corner points.

Solution:

(a) The constraints are $x \geq 0$, $y \geq 0$, and $x + y \leq 4$.

(b) The feasible region is the triangle with vertices $(0, 0)$, $(4, 0)$, and $(0, 4)$.

(c) Evaluate $Z = 3x + 2y$ at each vertex.

(d) At $(0, 0)$, $Z = 0$.

(e) At $(4, 0)$, $Z = 3(4) + 2(0) = 12$.

(f) At $(0, 4)$, $Z = 0 + 8 = 8$.

(g) The maximum value is 12.

Final Answer: $\boxed{12}$

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept: The absolute value function $|x-a|$ has a sharp corner at $x=a$, where left and right derivatives differ. A sum of absolute value terms can fail to be differentiable at any such corner unless cancellation occurs.

Solution:

(a) The term $|x-2|$ has a corner at $x=2$.

(b) The term $|x+1| = |x-(-1)|$ has a corner at $x=-1$.

(c) At these points, the slope changes abruptly.

(d) There is no cancellation of corners because they occur at different points.

(e) Therefore f is not differentiable at $x=-1$ and $x=2$.

Final Answer: $\boxed{x=-1,2}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: The angle between a line and a plane is the complement of the angle between the line direction vector and the plane normal vector. If the direction vector is parallel to the normal, then the line is perpendicular to the plane.

Solution:

(a) Direction vector of the line is $\vec{d} = (1,2,2)$.

(b) Normal vector of the plane $x+2y+2z=5$ is $\vec{n} = (1,2,2)$.

(c) Since $\vec{d}$ and $\vec{n}$ are the same direction, the line is parallel to the normal of the plane.

(d) Therefore the line is perpendicular to the plane.

(e) The angle between the line and the plane is $\frac{\pi}{2}$.

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (D)

Go Back to Question 30

Q31.

Solution

Concept: For an ellipse centered at origin with major axis along the x-axis, $c^2 = a^2 - b^2$, and the foci are $(\pm c, 0)$. The distance between foci is $2c$.

Solution:

(a) Here $a^2 = 36$ and $b^2 = 20$.

(b) Hence $c^2 = a^2 - b^2 = 36 - 20 = 16$.

(c) Thus $c = 4$.

(d) Foci are $(-4, 0)$ and $(4, 0)$.

(e) Distance between foci $= 2c = 8$.

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept: A first-order linear differential equation $\frac{dy}{dx} + Py = Q$ uses integrating factor $e^{\int P dx}$. Multiplying by the integrating factor makes the left side a derivative.

Solution:

(a) Here $P = 1$, so integrating factor $= e^{\int 1 dx} = e^x$.

(b) Multiply the equation by e^x: $e^x \frac{dy}{dx} + e^x y = e^{2x}$.

(c) The left side is $\frac{d}{dx}(e^x y)$.

(d) Therefore $\frac{d}{dx}(e^x y) = e^{2x}$.

(e) Integrate: $e^x y = \frac{e^{2x}}{2} + C$.

(f) Hence $y = \frac{e^x}{2} + Ce^{-x}$.

(g) Use $y(0) = 0$: $0 = \frac{1}{2} + C$, so $C = -\frac{1}{2}$.

(h) Therefore $y = \frac{e^x - e^{-x}}{2}$.

Final Answer: $\boxed{\dfrac{e^x - e^{-x}}{2}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: For three numbers in arithmetic progression, write them as $a - d$, a, and $a + d$. Their sum is $3a$, so the middle term can often be obtained directly from the sum.

Solution:

(a) Let the numbers be $a - d, a, a + d$.

(b) Their sum is $(a - d) + a + (a + d) = 3a$.

(c) Given sum = 24, so $3a = 24$.

(d) Therefore $a = 8$.

(e) The middle term is 8. The product condition only determines the common difference and is not needed for the middle term.

Final Answer: $\boxed{8}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: Stationary points occur where the first derivative is zero. After differentiating, solve the resulting equation and count real solutions.

Solution:

(a) $f(x) = x^4 - 4x^2 + 1$.

(b) Differentiate: $f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$.

(c) Set $f'(x) = 0$: $4x(x^2 - 2) = 0$.

(d) Hence $x = 0$ or $x^2 = 2$.

(e) Therefore $x = 0, \sqrt{2}, -\sqrt{2}$.

(f) There are three stationary points.

Final Answer: $\boxed{3}$

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: For perpendicular vectors, the square of the magnitude of their linear combination follows the Pythagorean relation because the dot product term is zero.

Solution:

(a) Since $\vec{a}$ and $\vec{b}$ are perpendicular unit vectors, $|\vec{a}| = |\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 0$.

(b) Compute the square: $|3\vec{a} + 4\vec{b}|^2 = (3\vec{a} + 4\vec{b}) \cdot (3\vec{a} + 4\vec{b})$.

(c) This equals $9|\vec{a}|^2 + 24(\vec{a} \cdot \vec{b}) + 16|\vec{b}|^2$.

(d) Substituting values gives $9 + 0 + 16 = 25$.

(e) Therefore $|3\vec{a} + 4\vec{b}| = 5$.

Final Answer: $\boxed{5}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: A matrix equation represents a system of simultaneous linear equations. Equate corresponding entries after multiplication and solve.

Solution:

(a) Multiplying gives $AX = \begin{pmatrix} x + y \\ x - y \end{pmatrix}$.

(b) Therefore $x + y = 6$ and $x - y = 2$.

(c) Add the equations: $2x = 8$.

(d) Hence $x = 4$.

Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 36

Q37.

Solution

Concept: Variance measures squared deviation from the mean. If each observation is multiplied by a constant k, then all deviations are multiplied by k, so variance is multiplied by k^2.

Solution:

(a) Let original observations have variance σ^2.

(b) Multiplying every observation by 3 multiplies each deviation from the mean by 3.

(c) Squared deviations are therefore multiplied by $3^2 = 9$.

(d) Hence the new variance is $9\sigma^2$.

Final Answer: $\boxed{9}$

Answer: (C)

Go Back to Question 37

Q38.

Solution

Concept: Use the identity $2\sin x\cos x = \sin 2x$. Since $\sin 2x$ ranges from -1 to 1, the expression can be bounded directly.

Solution:

(a) $1 + 2\sin x\cos x = 1 + \sin 2x$.

(b) Since $-1 \le \sin 2x \le 1$, we have $0 \le 1 + \sin 2x \le 2$.

(c) Therefore the minimum value is 0.

(d) This occurs when $\sin 2x = -1$.

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: For the standard parabola $x^2 = 4ay$, the vertex is at the origin, the axis is the positive y-axis, and the focus is $(0, a)$.

Solution:

(a) Compare $x^2 = 12y$ with $x^2 = 4ay$.

(b) Therefore $4a = 12$, giving $a = 3$.

(c) For $x^2 = 4ay$, focus is $(0, a)$.

(d) Hence the focus is $(0, 3)$.

Final Answer: $\boxed{(0, 3)}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: The derivative of $\ln x$ is $1/x$. Therefore the substitution $u = \ln x$ changes $\frac{\ln x}{x}dx$ into $u\,du$.

Solution:

(a) Let $u = \ln x$.

(b) Then $du = \frac{1}{x}dx$.

(c) When $x = 1$, $u = 0$; when $x = e$, $u = 1$.

(d) The integral becomes $\int_0^1 u\,du$.

(e) $\int_0^1 u\,du = \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}$.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: The point-slope form of a line is $y - y_1 = m(x - x_1)$. Substitute the given point and slope, then rearrange into general form.

Solution:

(a) Given point $(1, -2)$ and slope $m = -3$.

(b) Point-slope form: $y - (-2) = -3(x - 1)$.

(c) Thus $y + 2 = -3x + 3$.

(d) Rearranging gives $3x + y - 1 = 0$.

(e) Hence option (A) is correct.

Final Answer: $\boxed{3x + y - 1 = 0}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: The derivative gives the slope of the tangent to a curve. The normal is perpendicular to the tangent, so its slope is the negative reciprocal of the tangent slope.

Solution:

(a) For $y = x^2$, derivative is $\frac{dy}{dx} = 2x$.

(b) At $x = 1$, tangent slope $= 2$.

(c) The slope of a line perpendicular to slope 2 is $-\frac{1}{2}$.

(d) Therefore the normal slope is $-\frac{1}{2}$.

Final Answer: $\boxed{-\frac{1}{2}}$

Answer: (D)

Go Back to Question 42

Q43.

Solution

Concept: The scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ equals the determinant formed by the components of the three vectors. It represents the signed volume of the parallelepiped.

Solution:

(a) Components are $\vec{a} = (1, 1, 0)$, $\vec{b} = (0, 1, 1)$, and $\vec{c} = (1, 0, 1)$.

(b) Compute determinant: $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$.

(c) Expand along first row: $= 1(1 \cdot 1 - 1 \cdot 0) - 1(0 \cdot 1 - 1 \cdot 1) + 0$.

(d) $= 1 - (-1) = 2$.

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept: If a number is a root of an equation, substituting it into the polynomial gives zero. Complex arithmetic must be handled carefully, especially $i^2 = -1$.

Solution:

(a) Substitute $x = 1 + i$ into $x^2 - 2x + c = 0$.

(b) Compute $(1 + i)^2 = 1 + 2i + i^2 = 2i$.

(c) Then $x^2 - 2x + c = 2i - 2(1 + i) + c$.

(d) $= 2i - 2 - 2i + c = c - 2$.

(e) For root, $c - 2 = 0$, so $c = 2$.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept: The order of a differential equation is the order of the highest derivative present. The degree is the power of the highest order derivative after the equation is polynomial in derivatives.

Solution:

(a) The highest derivative present is $\frac{d^2y}{dx^2}$.

(b) Therefore the order is 2.

(c) The equation is already polynomial in derivatives.

(d) The highest order derivative $\frac{d^2y}{dx^2}$ is raised to power 3.

(e) Therefore the degree is 3.

Final Answer: $\boxed{2, 3}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: For exponential integrals, $\int e^{ax} dx = \frac{1}{a}e^{ax} + C$ when $a \neq 0$. This is because differentiating e^{ax} gives an extra factor a.

Solution:

(a) Here the exponent is $2x$, so $a = 2$.

(b) Apply the formula: $\int e^{2x} dx = \frac{1}{2}e^{2x} + C$.

(c) Verification: derivative of $\frac{1}{2}e^{2x}$ is $\frac{1}{2} \cdot 2e^{2x} = e^{2x}$.

Final Answer: $\boxed{\frac{1}{2}e^{2x} + C}$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept: If the eigenvalues of A are λ_1 and λ_2, then the eigenvalues of A^2 are λ_1^2 and λ_2^2. The trace equals the sum of eigenvalues.

Solution:

(a) Eigenvalues of A are 4 and -1.

(b) Therefore eigenvalues of A^2 are $4^2 = 16$ and $(-1)^2 = 1$.

(c) Trace of A^2 is the sum of these eigenvalues.

(d) Hence $\text{tr}(A^2) = 16 + 1 = 17$.

Final Answer: $\boxed{17}$

Answer: (C)

Go Back to Question 47

Q48.

Solution

Concept: Use standard limits $\lim_{u\to 0} \frac{\ln(1+u)}{u} = 1$ and $\lim_{v\to 0} \frac{\sin v}{v} = 1$. Convert the expression into these forms by multiplying and dividing by suitable quantities.

Solution:

(a) Write $\dfrac{\ln(1+4x)}{\sin 2x} = \dfrac{\ln(1+4x)}{4x} \cdot \dfrac{4x}{2x} \cdot \dfrac{2x}{\sin 2x}$.

(b) As $x \to 0$, $4x \to 0$ and $2x \to 0$.

(c) The first factor tends to 1.

(d) The middle factor equals 2.

(e) The last factor tends to 1.

(f) Therefore the limit is $1 \cdot 2 \cdot 1 = 2$.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept: Complete the square to convert a circle equation into standard form $(x-h)^2+(y-k)^2 = r^2$. The radius is the square root of the right-hand side.

Solution:

(a) Start with $x^2 + y^2 + 4x - 6y + 9 = 0$.

(b) Group terms: $(x^2 + 4x) + (y^2 - 6y) + 9 = 0$.

(c) Complete squares: $x^2 + 4x = (x + 2)^2 - 4$ and $y^2 - 6y = (y - 3)^2 - 9$.

(d) Substitute: $(x + 2)^2 - 4 + (y - 3)^2 - 9 + 9 = 0$.

(e) Thus $(x + 2)^2 + (y - 3)^2 = 4$.

(f) Radius $= \sqrt{4} = 2$.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 49

Q50.

Solution

Concept: A differentiable function is increasing where its derivative is positive. Therefore solve the inequality $f'(x) > 0$.

Solution:

(a) Given $f'(x) = 3x^2 - 12x + 9$.

(b) Factor: $f'(x) = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3)$.

(c) Since $3 > 0$, the sign depends on $(x - 1)(x - 3)$.

(d) This product is positive when both factors are positive or both are negative.

(e) Therefore $f'(x) > 0$ for $x > 3$ or $x < 1$.

(f) Hence f is increasing on $(-\infty, 1) \cup (3, \infty)$.

Final Answer: $\boxed{x < 1 \text{ or } x > 3}$

Answer: (C)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	A	5	A
6	C	7	C	8	D	9	B	10	A
11	C	12	A	13	B	14	A	15	B
16	B	17	C	18	B	19	A	20	B
21	C	22	B	23	A	24	C	25	A
26	B	27	B	28	C	29	A	30	D
31	C	32	A	33	B	34	C	35	B
36	C	37	C	38	A	39	A	40	A
41	A	42	D	43	C	44	B	45	A
46	B	47	C	48	B	49	B	50	C