JCECE Mathematics Sample Paper-9

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **50** Multiple Choice Questions.
- Each correct answer carries **+1** mark. Incorrect answer: **−0.25** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. The value of $\lim_{x\to 0} \frac{e^{3x}-1}{\sin 2x}$ is

(A) $\frac{2}{3}$

(B) 1

(C) $\frac{3}{2}$

(D) 3

Q2. If A is a nonsingular matrix of order 3 and $|A| = 4$, then $|\operatorname{adj} A|$ is

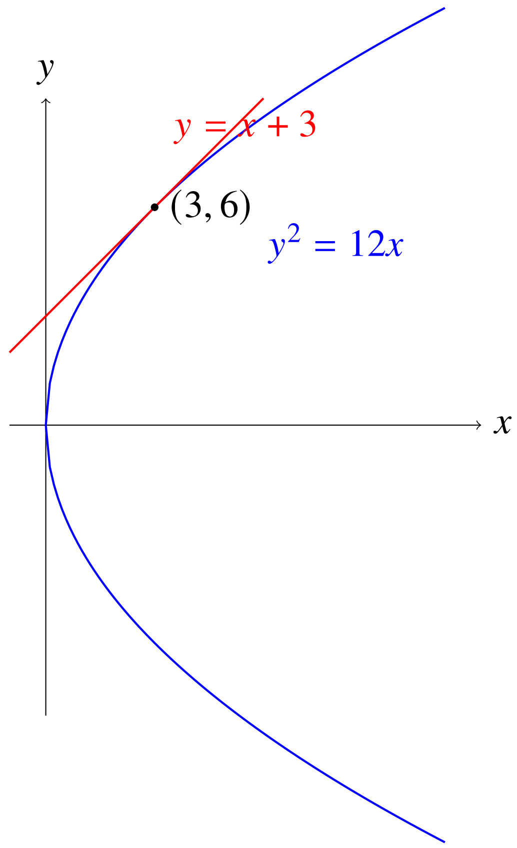
(A) 4

(B) 8

(C) 16

(D) 64

Q3. The tangent to the parabola $y^2 = 12x$ at the point $(3, 6)$ is



(A) $y = x + 3$

(B) $y = 2x$

(C) $2y = x + 9$

(D) $y = x - 3$

Q4. The value of $\int_0^1 \frac{3x^2}{1+x^3}\,dx$ is

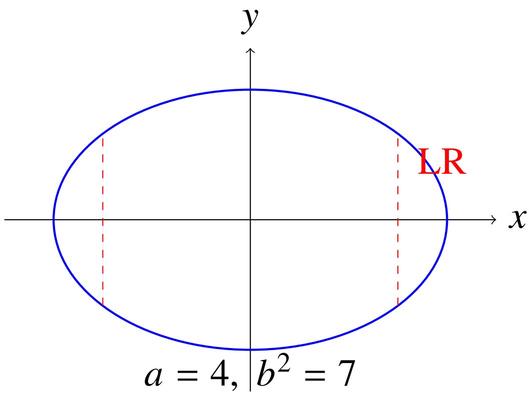
(A) $\ln 2$

(B) $2\ln 2$

(C) $\frac{1}{2}\ln 2$

(D) 1

Q5. For the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$, the length of the latus rectum is



(A) $\frac{7}{2}$

(B) $\frac{14}{3}$

(C) $\frac{8}{7}$

(D) $\frac{16}{7}$

Q6. Two fair dice are thrown. The probability that the sum appearing on the dice is a prime number is

(A) $\frac{1}{3}$

(B) $\frac{5}{12}$

(C) $\frac{7}{18}$

(D) $\frac{1}{2}$

Q7. The number of solutions of $\sin^2 x - \sin x = 0$ in $[0, 2\pi]$ is

(A) 2

(B) 3

(C) 4

(D) 5

Q8. If $f(x) = \begin{cases} \dfrac{1 - \cos kx}{x^2}, & x \neq 0, \\ 8, & x = 0, \end{cases}$ is continuous at $x = 0$ and $k > 0$, then k is

(A) 2

(B) 3

(C) 4

(D) 6

Q9. An ellipse has semi-major axis 13 and semi-minor axis 5. Its eccentricity is

(A) $\frac{5}{13}$

(B) $\frac{12}{13}$

(C) $\frac{13}{12}$

(D) $\frac{8}{13}$

Q10. The particular solution of $\frac{dy}{dx} = \frac{2x}{y}$ passing through $(1, 2)$ is

(A) $y^2 - 2x^2 = 2$

(B) $y^2 + x^2 = 5$

(C) $2y^2 - x^2 = 7$

(D) $y^2 - 2x^2 = 0$

Q11. If the sum of the first n terms of a sequence is $S_n = 2n^2 - n$, then its 8th term is

(A) 25

(B) 27

(C) 29

(D) 31

Q12. The equation of the line passing through $(2, -1)$ and parallel to $2x - 3y + 5 = 0$ is

(A) $2x - 3y - 7 = 0$

(B) $3x - 2y - 8 = 0$

(C) $2x + 3y - 1 = 0$

(D) $3x + 2y - 4 = 0$

Q13. The roots of $x^2 - (k + 2)x + 2k = 0$ are equal when k equals

(A) -2

(B) 0

(C) 2

(D) 4

Q14. If $y = \ln\left(x + \sqrt{1 + x^2}\right)$, then $\frac{dy}{dx}$ equals

(A) $\frac{1}{\sqrt{1+x^2}}$

(B) $\frac{x}{\sqrt{1+x^2}}$

(C) $\sqrt{1 + x^2}$

(D) $\frac{1}{1+x^2}$

Q15. For $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, the $(2, 1)$ entry of A^{-1} is

(A) -1

(B) 1

(C) -3

(D) 2

Q16. The mean of six observations is 12. If a seventh observation 18 is added, the new mean is

(A) $\frac{84}{7}$

(B) $\frac{90}{7}$

(C) 13

(D) 14

Q17. The radius of the circle $x^2 + y^2 - 6x + 8y - 11 = 0$ is

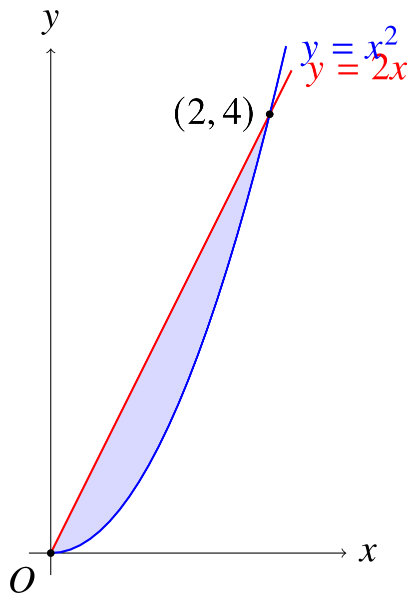
(A) 3

(B) 4

(C) 5

(D) 6

Q18. The area enclosed by the curves $y = 2x$ and $y = x^2$ is



(A) $\frac{2}{3}$

(B) 1

(C) $\frac{4}{3}$

(D) 2

Q19. If $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(A) 0

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

Q20. If $f(x) = \dfrac{x}{x+1}$, then $f(f(1))$ equals

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 1

Q21. For $x > 0$, the minimum value of $x + \frac{9}{x}$ is

(A) 3

(B) 6

(C) 9

(D) 12

Q22. The coefficient of x^2 in $(1 - 2x)^5$ is

(A) 20

(B) 30

(C) 40

(D) 80

Q23. The eccentricity of the hyperbola $\frac{x^2}{25} - \frac{y^2}{144} = 1$ is

(A) $\frac{12}{5}$

(B) $\frac{13}{5}$

(C) $\frac{5}{13}$

(D) $\frac{13}{12}$

Q24. The matrix $\begin{pmatrix} k & 1 \\ 4 & k \end{pmatrix}$ is singular for the positive value of k equal to

(A) 1

(B) 2

(C) 3

(D) 4

Q25. A bag contains 3 red balls and 2 blue balls. Two balls are drawn without replacement. The probability that both are red is

(A) $\frac{3}{10}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{3}{5}$

Q26. The value of $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}\,dx$ is

(A) $\ln 2$

(B) 1

(C) $\frac{1}{2}\ln 2$

(D) 2

Q27. The value of $(1+i)^6$ is

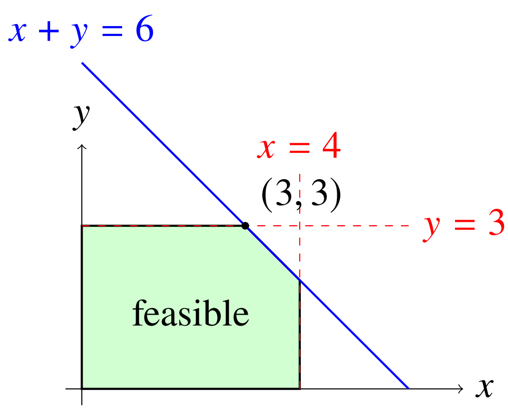
(A) $8i$

(B) $-8i$

(C) 8

(D) -8

Q28. The maximum value of $Z = 2x + 5y$ subject to $x + y \leq 6$, $x \leq 4$, $y \leq 3$, $x \geq 0$, $y \geq 0$ is



(A) 18

(B) 20

(C) 21

(D) 24

Q29. The function $f(x) = |x^2 - 4|$ is not differentiable at

(A) $x = 0$

(B) $x = -2, 2$

(C) $x = 2$ only

(D) $x = -2$ only

Q30. The distance of the origin from the plane $2x - y + 2z = 9$ is

(A) 1

(B) 2

(C) 3

(D) 9

Q31. For the ellipse $\frac{x^2}{49} + \frac{y^2}{24} = 1$, the distance between its foci is

(A) 5

(B) 8

(C) 10

(D) 14

Q32. The solution of $\frac{dy}{dx} - y = e^{2x}$ satisfying $y(0) = 1$ is

(A) $y = e^{2x}$

(B) $y = e^x$

(C) $y = e^{2x} + e^x$

(D) $y = e^{2x} - e^x$

Q33. In an A.P., the second term is 7 and the fifth term is 16. The sum of the first 10 terms is

(A) 165

(B) 170

(C) 175

(D) 180

Q34. The number of stationary points of $f(x) = x^3 - 6x^2 + 9x + 1$ is

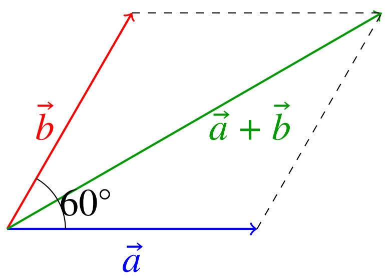
(A) 0

(B) 1

(C) 2

(D) 3

Q35. If $\vec{a}$ and $\vec{b}$ are unit vectors making an angle 60°, then $|\vec{a} + \vec{b}|$ is



(A) 1

(B) $\sqrt{2}$

(C) $\sqrt{3}$

(D) 2

Q36. If $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, then y equals

(A) 1

(B) 2

(C) 3

(D) 4

Q37. The variance of the data 2, 4, 6, 8 is

(A) 4

(B) 5

(C) 6

(D) 8

Q38. The maximum value of $3 + 2\cos x$ is

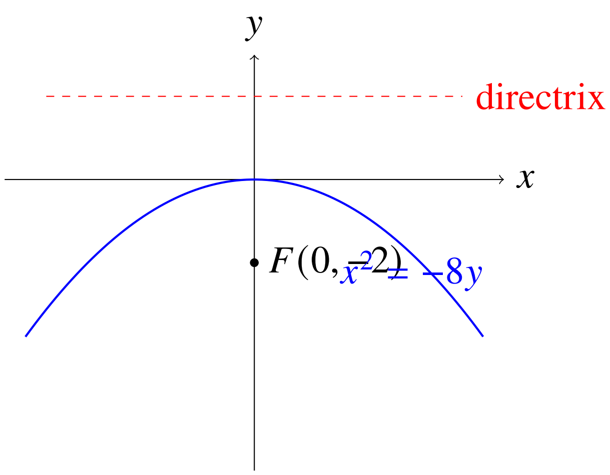
(A) 1

(B) 3

(C) 5

(D) 6

Q39. For the parabola $x^2 = -8y$, the focus is



(A) $(0, 2)$

(B) $(0, -2)$

(C) $(2, 0)$

(D) $(-2, 0)$

Q40. The value of $\int_0^1 \frac{x}{(1+x^2)^2}\,dx$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 1

Q41. The angle between the lines $y = \sqrt{3}x$ and $y = -\dfrac{x}{\sqrt{3}}$ is

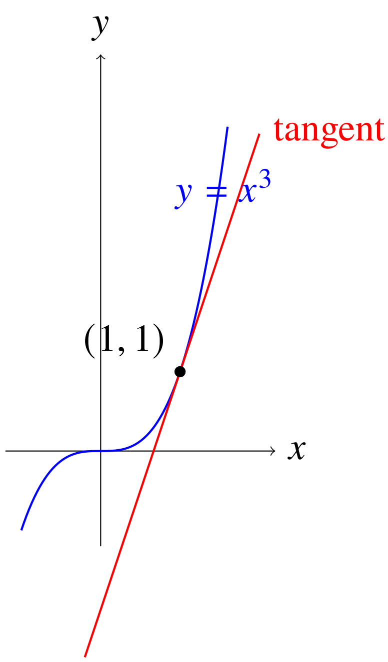
(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q42. The slope of the tangent to the curve $y = x^3$ at $x = 1$ is



(A) 1

(B) 2

(C) 3

(D) 4

Q43. If $\vec{a} = \hat{i} + 2\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$, and $\vec{c} = \hat{j} + \hat{k}$, then $[\vec{a}\ \vec{b}\ \vec{c}]$ equals

(A) 3

(B) 4

(C) 5

(D) 6

Q44. If $z + \bar{z} = 6$ and $z\bar{z} = 13$ with $\text{Im}(z) > 0$, then z is

(A) $3 + i$

(B) $3 + 2i$

(C) $2 + 3i$

(D) $6 + i$

Q45. The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = 0$ are respectively

(A) 2, 2

(B) 2, 3

(C) 3, 2

(D) 1, 3

Q46. The integral $\int \sec^2(3x)\, dx$ is

(A) $\tan 3x + C$

(B) $\frac{1}{3}\tan 3x + C$

(C) $3\tan 3x + C$

(D) $\sec 3x + C$

Q47. If a 2×2 matrix A has eigenvalues 2 and 5, then $|A^{-1}|$ is

(A) 10

(B) $\frac{1}{10}$

(C) 7

(D) $\frac{1}{7}$

Q48. The value of $\lim_{x\to 0} \frac{\tan 5x}{x}$ is

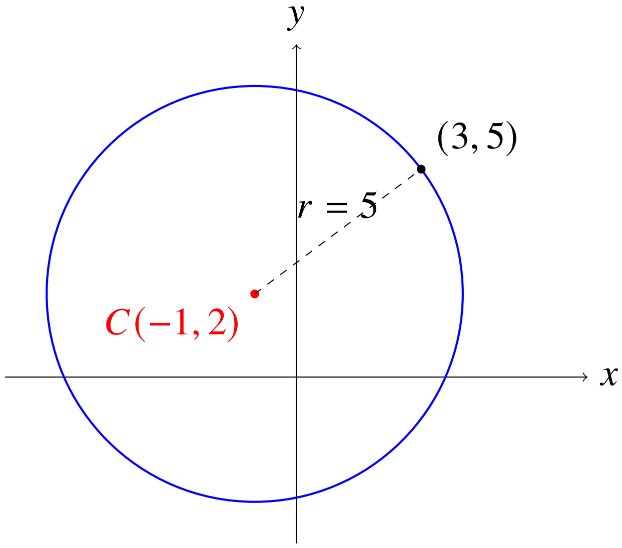
(A) 1

(B) 3

(C) 5

(D) 10

Q49. The circle with centre $(-1, 2)$ and passing through $(3, 5)$ has equation



(A) $(x+1)^2 + (y-2)^2 = 25$

(B) $(x-1)^2 + (y+2)^2 = 25$

(C) $(x+1)^2 + (y-2)^2 = 16$

(D) $(x-3)^2 + (y-5)^2 = 25$

Q50. If $f'(x) = (x-2)(x+1)$, then f is increasing for

(A) $-1 < x < 2$

(B) $x < -1$ or $x > 2$

(C) $x > -1$ only

(D) $x < 2$ only

Detailed Solutions

Q1.

Solution

Concept:
For small values of x, $e^{kx} - 1$ behaves like kx and $\sin mx$ behaves like mx. The limit can also be evaluated by converting both numerator and denominator into standard limits.
Solution:

(a) Write the expression as $\frac{e^{3x}-1}{3x} \cdot \frac{3x}{2x} \cdot \frac{2x}{\sin 2x}$.

(b) As $x \to 0$, $\frac{e^{3x}-1}{3x} \to 1$ by the standard exponential limit.

(c) Also $\frac{2x}{\sin 2x} \to 1$ by the standard sine limit.

(d) The remaining constant factor is $\frac{3x}{2x} = \frac{3}{2}$.

(e) Therefore the required limit is $1 \cdot \frac{3}{2} \cdot 1 = \frac{3}{2}$.

(f) This matches option (C).

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept:
For an $n \times n$ nonsingular matrix, $|\operatorname{adj} A| = |A|^{n-1}$. This follows from $A \operatorname{adj} A = |A| I_n$ and determinant properties.
Solution:

(a) The order of A is 3, so $n = 3$.

(b) Use the formula $|\operatorname{adj} A| = |A|^{n-1}$.

(c) Substitute $|A| = 4$ and $n = 3$.

(d) Thus $|\operatorname{adj} A| = 4^{3-1} = 4^2 = 16$.

(e) The correct option is (C).

Final Answer: $\boxed{16}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept:
For the parabola $y^2 = 4ax$, the tangent at (x_1, y_1) is $yy_1 = 2a(x + x_1)$. This avoids differentiating and is valid when the point lies on the parabola.
Solution:

(a) Compare $y^2 = 12x$ with $y^2 = 4ax$. Hence $4a = 12$ and $a = 3$.

(b) The given point is $(x_1, y_1) = (3, 6)$, and it lies on the curve since $6^2 = 36 = 12 \cdot 3$.

(c) Use the tangent formula $yy_1 = 2a(x + x_1)$.

(d) Substitute $y_1 = 6$, $a = 3$, and $x_1 = 3$: $6y = 6(x + 3)$.

(e) Divide by 6 to get $y = x + 3$.

(f) Hence option (A) is correct.

Final Answer: $\boxed{y = x + 3}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:
An integral of the form $\int \frac{f'(x)}{f(x)} dx$ equals $\ln|f(x)| + C$. Here the derivative of $1 + x^3$ is exactly $3x^2$.
Solution:

(a) Let $u = 1 + x^3$.

(b) Then $du = 3x^2\, dx$.

(c) When $x = 0$, $u = 1$; when $x = 1$, $u = 2$.

(d) The integral becomes $\int_1^2 \frac{du}{u}$.

(e) Evaluating gives $[\ln u]_1^2 = \ln 2 - \ln 1 = \ln 2$.

(f) Therefore option (A) is correct.

Final Answer: $\boxed{\ln 2}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept:
For an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, the length of the latus rectum is $\frac{2b^2}{a}$. The larger denominator identifies the major axis.
Solution:

(a) Here $a^2 = 16$ and $b^2 = 7$ because $16 > 7$.

(b) Thus $a = 4$ and $b^2 = 7$.

(c) Use latus rectum length $= \frac{2b^2}{a}$.

(d) Substitution gives $\frac{2 \cdot 7}{4} = \frac{14}{4} = \frac{7}{2}$.

(e) So the correct answer is option (A).

Final Answer: $\boxed{\frac{7}{2}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:
For two dice, all 36 ordered outcomes are equally likely. A prime sum can be 2, 3, 5, 7, or 11, and each sum has a known number of ordered outcomes.
Solution:

(a) Total outcomes $= 6 \times 6 = 36$.

(b) Prime sums possible are $2, 3, 5, 7, 11$.

(c) Counts are: sum 2: 1, sum 3: 2, sum 5: 4, sum 7: 6, sum 11: 2.

(d) Total favourable outcomes $= 1 + 2 + 4 + 6 + 2 = 15$.

(e) Probability $= \frac{15}{36} = \frac{5}{12}$.

(f) Thus option (B) is correct.

Final Answer: $\boxed{\frac{5}{12}}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:
A trigonometric equation can often be solved by factoring. After finding possible sine values, count all angles in the specified closed interval.
Solution:

(a) Factor the equation: $\sin^2 x - \sin x = \sin x(\sin x - 1) = 0$.

(b) So $\sin x = 0$ or $\sin x = 1$.

(c) In $[0, 2\pi]$, $\sin x = 0$ gives $x = 0, \pi, 2\pi$.

(d) In $[0, 2\pi]$, $\sin x = 1$ gives $x = \frac{\pi}{2}$.

(e) Total number of solutions is $3 + 1 = 4$.

(f) Hence option (C) is correct.

Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept:
Continuity at a point requires the limiting value to equal the defined value. The standard limit $\lim_{u \to 0} \frac{1-\cos u}{u^2} = \frac{1}{2}$ is used for cosine expressions.
Solution:

(a) Continuity at 0 requires $\lim_{x \to 0} \frac{1-\cos kx}{x^2} = 8$.

(b) Write $\frac{1-\cos kx}{x^2} = k^2 \cdot \frac{1-\cos kx}{(kx)^2}$.

(c) As $x \to 0$, $kx \to 0$, so $\frac{1-\cos kx}{(kx)^2} \to \frac{1}{2}$.

(d) Thus the limit is $\frac{k^2}{2}$.

(e) Set $\frac{k^2}{2} = 8$, giving $k^2 = 16$.

(f) Since $k > 0$, $k = 4$; option (C).

Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept:
For an ellipse, $c^2 = a^2 - b^2$ and eccentricity is $e = \frac{c}{a}$, where a is the semi-major axis and b is the semi-minor axis.
Solution:

(a) Given $a = 13$ and $b = 5$.

(b) Compute $c^2 = a^2 - b^2 = 13^2 - 5^2 = 169 - 25 = 144$.

(c) So $c = 12$.

(d) Hence eccentricity $e = \frac{c}{a} = \frac{12}{13}$.

(e) Therefore option (B) is correct.

Final Answer: $\boxed{\frac{12}{13}}$

Answer: (B)
Go Back to Question 9

Q10.

Solution

Concept:
A separable differential equation is solved by bringing all y terms with dy and all x terms with dx. The given point determines the constant of integration.
Solution:

(a) Given $\frac{dy}{dx} = \frac{2x}{y}$.

(b) Rearrange as $y\,dy = 2x\,dx$.

(c) Integrate both sides: $\int y\,dy = \int 2x\,dx$.

(d) This gives $\frac{y^2}{2} = x^2 + C$, or $y^2 - 2x^2 = K$.

(e) Use $(1, 2)$: $K = 2^2 - 2(1)^2 = 4 - 2 = 2$.

(f) Therefore the solution is $y^2 - 2x^2 = 2$.

Final Answer: $\boxed{y^2 - 2x^2 = 2}$

Answer: (A)
Go Back to Question 10

Q11.

Solution

Concept:
The nth term is obtained from the sum formula by $a_n = S_n - S_{n-1}$. This subtracts the sum of the first $n - 1$ terms from the sum of the first n terms.

Solution:

(a) Compute $S_8 = 2(8)^2 - 8 = 128 - 8 = 120$.

(b) Compute $S_7 = 2(7)^2 - 7 = 98 - 7 = 91$.

(c) Therefore $a_8 = S_8 - S_7 = 120 - 91 = 29$.

(d) So the answer is option (C).

Final Answer: $\boxed{29}$

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept:
Parallel lines have proportional coefficients of x and y. A line parallel to $2x - 3y + 5 = 0$ has the form $2x - 3y + c = 0$.

Solution:

(a) Let the required line be $2x - 3y + c = 0$.

(b) It passes through $(2, -1)$.

(c) Substitute the point: $2(2) - 3(-1) + c = 0$.

(d) This gives $4 + 3 + c = 0$, so $c = -7$.

(e) Hence the line is $2x - 3y - 7 = 0$.

(f) The correct option is (A).

Final Answer: $\boxed{2x - 3y - 7 = 0}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept:
A quadratic equation has equal roots when its discriminant is zero. For $ax^2 + bx + c = 0$, the discriminant is $b^2 - 4ac$.
Solution:

(a) Here $a = 1$, $b = -(k + 2)$, and $c = 2k$.

(b) Set discriminant equal to zero: $(k + 2)^2 - 4(1)(2k) = 0$.

(c) This gives $k^2 + 4k + 4 - 8k = 0$.

(d) So $k^2 - 4k + 4 = 0$, or $(k - 2)^2 = 0$.

(e) Hence $k = 2$.

(f) Option (C) is correct.

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept:
The expression $\ln(x + \sqrt{1 + x^2})$ is the inverse hyperbolic sine form $\sinh^{-1} x$. Its derivative is $\frac{1}{\sqrt{1+x^2}}$. We can also verify it directly by differentiation.
Solution:

(a) Let $u = x + \sqrt{1 + x^2}$, so $y = \ln u$.

(b) Then $\frac{dy}{dx} = \frac{u'}{u}$.

(c) Now $u' = 1 + \frac{x}{\sqrt{1+x^2}} = \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$.

(d) Thus $\frac{dy}{dx} = \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}(x+\sqrt{1+x^2})}$.

(e) The common factor $x + \sqrt{1 + x^2}$ cancels, giving $\frac{1}{\sqrt{1+x^2}}$.

(f) Therefore option (A) is correct.

Final Answer: $\boxed{\dfrac{1}{\sqrt{1 + x^2}}}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:

For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the inverse is $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ if the determinant is non-zero.

Solution:

(a) Here $a = 2$, $b = 3$, $c = 1$, and $d = 2$.

(b) The determinant is $2 \cdot 2 - 3 \cdot 1 = 4 - 3 = 1$.

(c) Therefore $A^{-1} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$.

(d) The $(2, 1)$ entry is -1.

(e) Hence option (A) is correct.

Final Answer: $\boxed{-1}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept:

Mean is total sum divided by the number of observations. Adding a new observation changes both the total and the count.

Solution:

(a) Original mean is 12 for 6 observations.

(b) Original total $= 6 \times 12 = 72$.

(c) After adding 18, new total $= 72 + 18 = 90$.

(d) New number of observations $= 7$.

(e) New mean $= \frac{90}{7}$.

(f) Thus option (B) is correct.

Final Answer: $\boxed{\frac{90}{7}}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept:
To find the radius of a circle from its general equation, complete the square in x and y to convert it into standard form.
Solution:

(a) Group the terms: $(x^2 - 6x) + (y^2 + 8y) - 11 = 0$.

(b) Complete squares: $x^2 - 6x = (x - 3)^2 - 9$ and $y^2 + 8y = (y + 4)^2 - 16$.

(c) Substitute: $(x - 3)^2 - 9 + (y + 4)^2 - 16 - 11 = 0$.

(d) Thus $(x - 3)^2 + (y + 4)^2 = 36$.

(e) So $r^2 = 36$ and $r = 6$.

(f) Option (D) is correct.

Final Answer: $\boxed{6}$

Answer: (D)

Go Back to Question 17

Q18.

Solution

Concept:
Area between two curves is the integral of upper curve minus lower curve between their intersection points. The intersection points determine the limits of integration.
Solution:

(a) Find intersections: $2x = x^2$, so $x(x - 2) = 0$.

(b) Thus $x = 0$ and $x = 2$.

(c) On $[0, 2]$, $2x \geq x^2$, so upper curve is $y = 2x$.

(d) Area $= \int_0^2 (2x - x^2)\, dx$.

(e) Evaluate: $\left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$.

(f) Therefore option (C) is correct.

Final Answer: $\boxed{\frac{4}{3}}$

Answer: (C)

Go Back to Question 18

Q19.

Solution

Concept:
The angle between two vectors is found using $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$. If the dot product is zero, the vectors are perpendicular.
Solution:

(a) Compute the dot product: $\vec{a}\cdot\vec{b} = 1(2) + 2(1) + (-2)(2)$.

(b) This equals $2 + 2 - 4 = 0$.

(c) Since neither vector is the zero vector, $\vec{a}\cdot\vec{b} = 0$ implies $\cos\theta = 0$.

(d) Thus $\theta = \frac{\pi}{2}$.

(e) So option (C) is correct.

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept:
For composition, the inner function is evaluated first. Then its output is substituted into the same function again.
Solution:

(a) First compute $f(1) = \frac{1}{1+1} = \frac{1}{2}$.

(b) Now compute $f(f(1)) = f\left(\frac{1}{2}\right)$.

(c) Substitute into $f(x) = \frac{x}{x+1}$.

(d) $f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$.

(e) Therefore option (A) is correct.

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:
For expressions of the form $x + \frac{a}{x}$ with $x > 0$, AM-GM or differentiation can be used. The minimum occurs when the two positive terms are equal.

Solution:

(a) By AM-GM, $x + \frac{9}{x} \geq 2\sqrt{x \cdot \frac{9}{x}}$.

(b) The right side is $2\sqrt{9} = 6$.

(c) Equality occurs when $x = \frac{9}{x}$.

(d) This gives $x^2 = 9$, and since $x > 0$, $x = 3$.

(e) Therefore the minimum value is 6.

(f) Option (B) is correct.

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:
The general term in $(a + b)^n$ is $\binom{n}{r} a^{n-r} b^r$. The coefficient of x^2 comes from the term where $(-2x)$ is squared.

Solution:

(a) Choose $r = 2$ in the binomial expansion.

(b) The required term is $\binom{5}{2}(1)^3(-2x)^2$.

(c) This equals $10 \cdot 4x^2 = 40x^2$.

(d) Therefore the coefficient is 40.

(e) The correct option is (C).

Final Answer: $\boxed{40}$

Answer: (C)

Go Back to Question 22

Q23.

Solution

Concept:
For a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we use $c^2 = a^2 + b^2$ and $e = \frac{c}{a}$.
Solution:

(a) Here $a^2 = 25$ and $b^2 = 144$.

(b) So $a = 5$.

(c) Compute $c^2 = 25 + 144 = 169$, hence $c = 13$.

(d) Eccentricity $e = \frac{c}{a} = \frac{13}{5}$.

(e) Thus option (B) is correct.

Final Answer: $\boxed{\frac{13}{5}}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:
A square matrix is singular if and only if its determinant is zero. We compute the determinant and solve for the positive parameter value.
Solution:

(a) The determinant is $k \cdot k - 1 \cdot 4 = k^2 - 4$.

(b) For singularity, set $k^2 - 4 = 0$.

(c) Thus $(k - 2)(k + 2) = 0$.

(d) So $k = 2$ or $k = -2$.

(e) The positive value is $k = 2$.

(f) Therefore option (B) is correct.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept:
When drawing without replacement, the probability of the second event depends on the first draw. Multiplication rule is applied sequentially.

Solution:

(a) Total balls initially = $3 + 2 = 5$.

(b) Probability first ball is red = $\frac{3}{5}$.

(c) After drawing one red ball, red balls left = 2 and total balls left = 4.

(d) Probability second ball is red given first red = $\frac{2}{4} = \frac{1}{2}$.

(e) Therefore required probability = $\frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10}$.

(f) So option (A) is correct.

Final Answer: $\boxed{\dfrac{3}{10}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:
The derivative of $1 + \sin x$ is $\cos x$. Therefore the substitution $u = 1 + \sin x$ converts the integral into a logarithmic integral.

Solution:

(a) Let $u = 1 + \sin x$.

(b) Then $du = \cos x\, dx$.

(c) When $x = 0$, $u = 1 + 0 = 1$.

(d) When $x = \frac{\pi}{2}$, $u = 1 + 1 = 2$.

(e) The integral becomes $\int_1^2 \frac{du}{u} = \ln 2$.

(f) Thus option (A) is correct.

Final Answer: $\boxed{\ln 2}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept:
A complex number can be simplified using powers. Since $(1+i)^2 = 2i$, higher powers become easier to compute by grouping.
Solution:

(a) First compute $(1+i)^2 = 1 + 2i + i^2 = 2i$.

(b) Then $(1+i)^6 = ((1+i)^2)^3 = (2i)^3$.

(c) Now $(2i)^3 = 8i^3$.

(d) Since $i^2 = -1$, $i^3 = -i$.

(e) Therefore $(1+i)^6 = 8(-i) = -8i$.

(f) The correct option is (B).

Final Answer: $\boxed{-8i}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept:
In linear programming, a linear objective function attains its maximum at a vertex of the feasible region. We list the corner points and evaluate the objective function.
Solution:

(a) The feasible vertices are $(0,0)$, $(4,0)$, $(4,2)$, $(3,3)$, and $(0,3)$.

(b) Evaluate $Z = 2x + 5y$ at these points.

(c) At $(0,0)$, $Z = 0$; at $(4,0)$, $Z = 8$.

(d) At $(4,2)$, $Z = 8 + 10 = 18$.

(e) At $(3,3)$, $Z = 6 + 15 = 21$; at $(0,3)$, $Z = 15$.

(f) The maximum value is 21, so option (C) is correct.

Final Answer: $\boxed{21}$

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept:
The absolute value function $|g(x)|$ may fail to be differentiable at points where $g(x) = 0$ and g changes sign. These points form sharp corners.
Solution:

(a) Here $g(x) = x^2 - 4$.

(b) Set $g(x) = 0$: $x^2 - 4 = 0$.

(c) This gives $(x-2)(x+2) = 0$, so $x = 2$ or $x = -2$.

(d) At both points, $x^2 - 4$ changes sign, so $|x^2 - 4|$ has a corner.

(e) Therefore the function is not differentiable at $x = -2$ and $x = 2$.

(f) Option (B) is correct.

Final Answer: $\boxed{x = -2, 2}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept:
The distance of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$.
Solution:

(a) Rewrite the plane as $2x - y + 2z - 9 = 0$.

(b) For the origin, $(x_1, y_1, z_1) = (0, 0, 0)$.

(c) Numerator $= |2(0) - 0 + 2(0) - 9| = 9$.

(d) Denominator $= \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = 3$.

(e) Distance $= \frac{9}{3} = 3$.

(f) Hence option (C) is correct.

Final Answer: $\boxed{3}$

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept:
For an ellipse with major axis along the x-axis, $c^2 = a^2 - b^2$ and the foci are $(-c, 0)$ and $(c, 0)$. Their distance is $2c$.
Solution:

(a) Here $a^2 = 49$ and $b^2 = 24$.

(b) Compute $c^2 = 49 - 24 = 25$.

(c) Thus $c = 5$.

(d) The foci are $(-5, 0)$ and $(5, 0)$.

(e) The distance between them is $2c = 10$.

(f) Therefore option (C) is correct.

Final Answer: $\boxed{10}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept:
A first-order linear differential equation $\frac{dy}{dx} + Py = Q$ is solved using the integrating factor $e^{\int P\,dx}$. Here $P = -1$.
Solution:

(a) The integrating factor is $e^{\int -1\,dx} = e^{-x}$.

(b) Multiply the equation by e^{-x}: $e^{-x}\frac{dy}{dx} - e^{-x}y = e^x$.

(c) The left side is $\frac{d}{dx}(ye^{-x})$.

(d) So $\frac{d}{dx}(ye^{-x}) = e^x$.

(e) Integrate: $ye^{-x} = e^x + C$, hence $y = e^{2x} + Ce^x$.

(f) Use $y(0) = 1$: $1 = 1 + C$, so $C = 0$. Therefore $y = e^{2x}$.

Final Answer: $\boxed{y = e^{2x}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept:
In an arithmetic progression, $a_n = a + (n-1)d$. Once the first term and common difference are known, the sum is $S_n = \frac{n}{2}[2a + (n-1)d]$.
Solution:

(a) Second term: $a + d = 7$.

(b) Fifth term: $a + 4d = 16$.

(c) Subtracting gives $3d = 9$, so $d = 3$.

(d) Then $a + d = 7$ gives $a = 4$.

(e) Now $S_{10} = \frac{10}{2}[2(4) + 9(3)]$.

(f) So $S_{10} = 5(8 + 27) = 5 \cdot 35 = 175$.

Final Answer: $\boxed{175}$

Answer: (C)

Go Back to Question 33

Q34.

Solution

Concept:
Stationary points occur where the first derivative is zero. We differentiate the function and count the real roots of the derivative.
Solution:

(a) Differentiate: $f'(x) = 3x^2 - 12x + 9$.

(b) Factor: $f'(x) = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$.

(c) Set $f'(x) = 0$.

(d) This gives $x = 1$ or $x = 3$.

(e) There are two stationary points.

(f) Thus option (C) is correct.

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept:
The magnitude of a sum of vectors is computed using the dot product formula: $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$.
Solution:

(a) Since $\vec{a}$ and $\vec{b}$ are unit vectors, $|\vec{a}| = |\vec{b}| = 1$.

(b) The angle between them is $60°$, so $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 60° = \frac{1}{2}$.

(c) Now $|\vec{a}+\vec{b}|^2 = 1^2 + 1^2 + 2\cdot\frac{1}{2}$.

(d) This equals $1 + 1 + 1 = 3$.

(e) Therefore $|\vec{a}+\vec{b}| = \sqrt{3}$.

(f) Option (C) is correct.

Final Answer: $\boxed{\sqrt{3}}$

Answer: (C)

Go Back to Question 35

Q36.

Solution

Concept:
A matrix equation can be converted into simultaneous linear equations by multiplying the matrix with the column vector and equating corresponding entries.
Solution:

(a) Matrix multiplication gives $\begin{pmatrix} 2x+y \\ x+y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

(b) Thus $2x + y = 5$ and $x + y = 3$.

(c) Subtract the second equation from the first: $x = 2$.

(d) Substitute into $x + y = 3$: $2 + y = 3$.

(e) Hence $y = 1$.

(f) Therefore option (A) is correct.

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept:
Variance is the mean of squared deviations from the arithmetic mean. For a small data set, direct computation is clear and reliable.
Solution:

(a) The mean is $\bar{x} = \frac{2+4+6+8}{4} = 5$.

(b) Deviations are $-3, -1, 1, 3$.

(c) Squared deviations are $9, 1, 1, 9$.

(d) Their sum is 20.

(e) Variance $= \frac{20}{4} = 5$.

(f) Thus option (B) is correct.

Final Answer: $\boxed{5}$

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept:
The cosine function satisfies $-1 \leq \cos x \leq 1$. To maximize a linear expression in $\cos x$, take the largest possible value of $\cos x$.
Solution:

(a) Since $\cos x \leq 1$, we have $2\cos x \leq 2$.

(b) Therefore $3 + 2\cos x \leq 3 + 2 = 5$.

(c) This value is attained when $\cos x = 1$, for example at $x = 0$.

(d) Hence the maximum value is 5.

(e) Option (C) is correct.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 38

Q39.

Solution

Concept:
For the standard parabola $x^2 = 4ay$, the focus is $(0, a)$. If a is negative, the parabola opens downward and the focus lies below the origin.
Solution:

(a) Compare $x^2 = -8y$ with $x^2 = 4ay$.

(b) Thus $4a = -8$, giving $a = -2$.

(c) The focus is $(0, a)$.

(d) Therefore the focus is $(0, -2)$.

(e) This corresponds to option (B).

Final Answer: $\boxed{(0, -2)}$

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept:
Use the substitution $u = 1 + x^2$ because its differential contains $2x\,dx$. The integral then becomes a simple power integral in u.
Solution:

(a) Let $u = 1 + x^2$.

(b) Then $du = 2x\,dx$, so $x\,dx = \frac{1}{2}du$.

(c) When $x = 0$, $u = 1$; when $x = 1$, $u = 2$.

(d) The integral becomes $\frac{1}{2}\int_1^2 u^{-2}\,du$.

(e) This is $\frac{1}{2}[-u^{-1}]_1^2 = \frac{1}{2}\left(-\frac{1}{2} + 1\right) = \frac{1}{4}$.

(f) Hence option (A) is correct.

Final Answer: $\boxed{\frac{1}{4}}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept:
Two lines are perpendicular if the product of their slopes is -1. The angle between perpendicular lines is $90°$.
Solution:

(a) The slope of $y = \sqrt{3}x$ is $m_1 = \sqrt{3}$.

(b) The slope of $y = -\frac{x}{\sqrt{3}}$ is $m_2 = -\frac{1}{\sqrt{3}}$.

(c) Their product is $m_1 m_2 = \sqrt{3}\left(-\frac{1}{\sqrt{3}}\right) = -1$.

(d) Therefore the lines are perpendicular.

(e) Hence the angle between them is $90°$.

(f) Option (D) is correct.

Final Answer: $\boxed{90°}$

Answer: (D)
Go Back to Question 41

Q42.

Solution

Concept:
The slope of the tangent to a differentiable curve $y = f(x)$ at a point is given by $\frac{dy}{dx}$ evaluated at that point.
Solution:

(a) Given $y = x^3$.

(b) Differentiate: $\frac{dy}{dx} = 3x^2$.

(c) At $x = 1$, the slope is $3(1)^2 = 3$.

(d) Therefore the slope of the tangent is 3.

(e) This matches option (C).

Final Answer: $\boxed{3}$

Answer: (C)
Go Back to Question 42

Q43.

Solution

Concept:
The scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ is the determinant formed from the components of the three vectors. It gives the signed volume of the parallelepiped.
Solution:

(a) Write the vectors as rows: $\vec{a} = (1, 0, 2)$, $\vec{b} = (2, 1, 0)$, $\vec{c} = (0, 1, 1)$.

(b) Compute $\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$.

(c) Expanding along the first row gives $1(1 \cdot 1 - 0 \cdot 1) + 2(2 \cdot 1 - 1 \cdot 0)$.

(d) This equals $1 + 2(2) = 5$.

(e) Therefore the scalar triple product is 5.

(f) Option (C) is correct.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept:
For $z = a + ib$, $z + \bar{z} = 2a$ and $z\bar{z} = a^2 + b^2$. These relations determine the real and imaginary parts.
Solution:

(a) Let $z = a + ib$ with $b > 0$.

(b) Given $z + \bar{z} = 2a = 6$, so $a = 3$.

(c) Also $z\bar{z} = a^2 + b^2 = 13$.

(d) Substitute $a = 3$: $9 + b^2 = 13$.

(e) Thus $b^2 = 4$, and since $b > 0$, $b = 2$.

(f) Therefore $z = 3 + 2i$.

Final Answer: $\boxed{3 + 2i}$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept:
The order is the highest order derivative appearing in the differential equation. The degree is the power of the highest order derivative when the equation is polynomial in derivatives.
Solution:

(a) The highest derivative present is $\frac{d^2y}{dx^2}$.

(b) Therefore the order is 2.

(c) The equation is polynomial in derivatives.

(d) The highest order derivative is squared.

(e) Therefore the degree is 2.

(f) Hence the answer is 2, 2.

Final Answer: $\boxed{2, 2}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept:
Since $\frac{d}{dx}\tan u = \sec^2 u \cdot \frac{du}{dx}$, an adjustment factor is needed when the angle is $3x$.
Solution:

(a) Let $u = 3x$.

(b) Then $du = 3dx$, so $dx = \frac{1}{3}du$.

(c) The integral becomes $\frac{1}{3}\int \sec^2 u\, du$.

(d) This equals $\frac{1}{3}\tan u + C$.

(e) Substitute back $u = 3x$.

(f) Therefore the integral is $\frac{1}{3}\tan 3x + C$.

Final Answer: $\boxed{\frac{1}{3}\tan 3x + C}$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept:
The determinant of a matrix is the product of its eigenvalues. Also, for a nonsingular matrix, $|A^{-1}| = \frac{1}{|A|}$.
Solution:

(a) The eigenvalues of A are 2 and 5.

(b) Therefore $|A| = 2 \cdot 5 = 10$.

(c) Since $|A| \neq 0$, the inverse exists.

(d) Now $|A^{-1}| = \frac{1}{|A|} = \frac{1}{10}$.

(e) Thus option (B) is correct.

Final Answer: $\boxed{\frac{1}{10}}$

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept:
The standard trigonometric limit $\lim_{u \to 0} \frac{\tan u}{u} = 1$ is used. We multiply and divide by 5 to create this form.
Solution:

(a) Write $\frac{\tan 5x}{x} = 5 \cdot \frac{\tan 5x}{5x}$.

(b) As $x \to 0$, $5x \to 0$.

(c) Therefore $\frac{\tan 5x}{5x} \to 1$.

(d) Hence the limit is $5 \cdot 1 = 5$.

(e) So option (C) is correct.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 48

Q49.

Solution

Concept:
The standard equation of a circle with centre (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. The radius is the distance from the centre to any point on the circle.
Solution:

(a) The centre is $(-1, 2)$, so the equation has the form $(x + 1)^2 + (y - 2)^2 = r^2$.

(b) Find r using the point $(3, 5)$.

(c) $r^2 = (3 - (-1))^2 + (5 - 2)^2 = 4^2 + 3^2 = 16 + 9 = 25$.

(d) Therefore the circle is $(x + 1)^2 + (y - 2)^2 = 25$.

(e) The correct option is (A).

Final Answer: $\boxed{(x + 1)^2 + (y - 2)^2 = 25}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept:
A differentiable function is increasing on intervals where its derivative is positive. Therefore we solve the inequality $(x - 2)(x + 1) > 0$.
Solution:

(a) The critical points are obtained from $(x - 2)(x + 1) = 0$.

(b) So $x = 2$ and $x = -1$.

(c) Check signs of the product on intervals $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$.

(d) For $x < -1$, both factors are negative, so the product is positive.

(e) For $-1 < x < 2$, the product is negative; for $x > 2$, both factors are positive.

(f) Thus f is increasing for $x < -1$ or $x > 2$, which is option (B).

Final Answer: $\boxed{x < -1 \text{ or } x > 2}$

Answer: (B)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	C	3	A	4	A	5	A
6	B	7	C	8	C	9	B	10	A
11	C	12	A	13	C	14	A	15	A
16	B	17	D	18	C	19	C	20	A
21	B	22	C	23	B	24	B	25	A
26	A	27	B	28	C	29	B	30	C
31	C	32	A	33	C	34	C	35	C
36	A	37	B	38	C	39	B	40	A
41	D	42	C	43	C	44	B	45	A
46	B	47	B	48	C	49	A	50	B